

Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

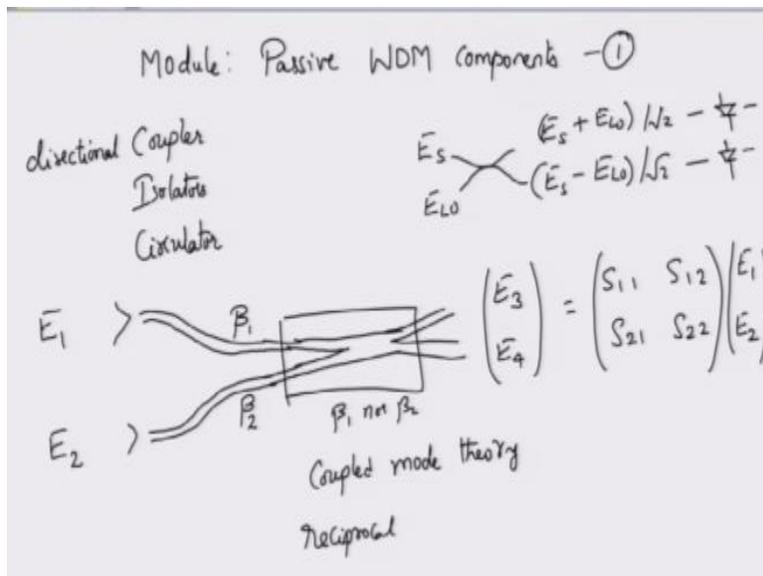
**Course Title
Optical Communications**

**Week – VII
Module – I
Passive WDM components-I**

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Hello and welcome to this online module of online course on optical communications in today's module we will discuss passive WDM components.

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WDM stand for wavelength division multiplexing and it is technology that is used in order to increase the rate of signal transmission over the optical fibers in which multiple channels are multiplexed together each channel caring data at it is own rate there multiplex together so as to

increase the usage of the optical fiber the efficiency of the usage of the optical fiber in order to facilitate wavelength division multiplexing it is necessary to combined different wavelengths to select a particular wavelength it also necessary to combine signals at different ports and then it is also necessary to isolate certain wavelengths.

Isolate signals from reflecting back and damaging certain components so all these operations require us to come up with newer devices optical devices which can accomplish these objectives and in this module we will first talk about two such two or three such components know pleasure isolated and circulators there are additional WDM components that we will take up in the next module we will differ the discussion of active components or the WDM active components such as photo decoders all though decoders are nor technically active and then laser and optical amplifiers for some other time.

Okay so our goal is to understand some of the passive WDM components such as coupler. Which is a basic element we have already looked at couplers when we discussed coherent receivers we also discuss isolators and it is kind of an extinction called as a circulator so let us began by looking at a coupler we have already seen coupler in action when we looked at the coherent detection because for a homodyne deduction of signal we realize that you actually had to have a coupler.

So where in one of the signal was coming in from the transmitted signal and then the other signal was the local oscillator signal and then we obtained two outputs we obtained $E_S + E_{LO} / \sqrt{2}$ and then on the other arm of the coupler we obtained $E_S + E_{LO} / \sqrt{2}$ so we have seen that if you give this outputs to the photo detectors then you will be able to obtain the signal which is basically a interference term between $E_S + E_{LO}$ it is kind of the down conversation process that can be performed for the optical signals.

So couplers in combination with the photo detectors actually perform the operation of the down conversation right so we have seen this device as the front end of a coherent receiver this coupler is a front end of a coherent receiver and coupler for our understanding actually enable us to combine two signals now how exactly does a coupler do that it turns out that what it employees

is a principle suppose you consider a wave guide okay it is certain signal is passing through this wave guide this signal as a propagation constant β okay.

And then you consider one more wave guide and in here let us say another signal is passing through so let us call this one as β_1 and let us call this as β_2 as the two propagation constants it turns out that when you bring these wave guides closer together then what happens is that for the combined wave guide system that is if you actually bring them in such a wave that you fuse the two cores like this and the defuse them again you know defuse in the sense that you simply separate them again in this region in the intermediate region where the wave guides have combined into a single wave guide the propagation constant is neither β_1 nor β_2 .

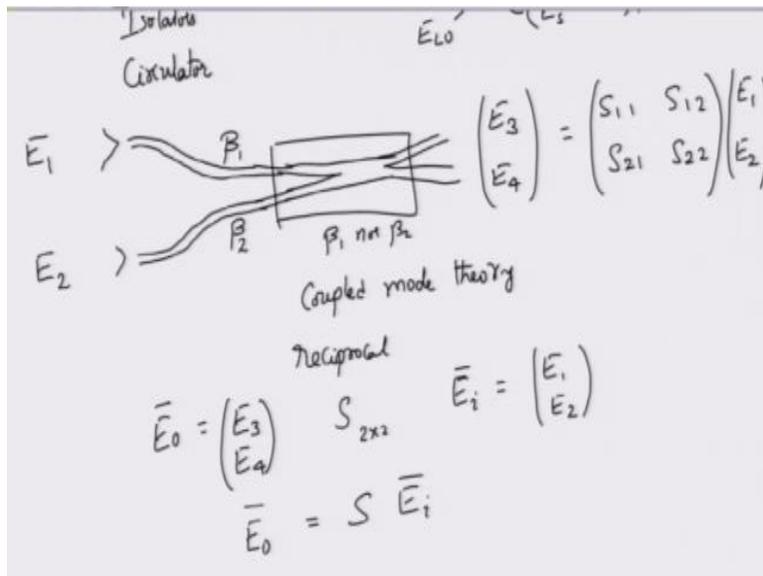
The propagation constant will be a combination of β_1 and β_2 how exactly this combination occurs what is the resultant value for the overall β is determined by what is called as the coupled mode theory coupled mode theory tells us that when you bring two wave guides carrying optical signals together then there is a coupling between the two optical wave guides which changes the overall propagation constant of course it is not just the propagation constant which changes.

What will also changes the amount of the power that goes out of these two ports traditionally are labeled as the input ports so let us call them as the input port carrying electric field E_1 corresponding to an optical signal, this is port 2 carrying field E_2 corresponding to another optical wave, the output ports are labeled as E_3 and E_4 which again represent the electric fields for the output optical waves, okay. This E_3 AND E_4 are actually related by a certain matrix okay, so this matrix is called as a scattering matrix.

And this is the kind of a matrix that you might have been familiar when you have studied microwave theory, okay. So it is kind of the same concept here so for that reason this is sometimes called as a directional coupler, okay. So this is called as a directional coupler because there is a certain direction to its input to output, of course the coupler is also reciprocal, okay. It is reciprocal because you can turn around the input and output.

You can make this coupler work in both ways, so it is like a passive resistor, a resistor is essentially a bilateral or a reciprocal device you can there is no fixed terminal for the resistor to be put in inside a circuit. So coming back to the matrix relationship for the electric field at the output and the electric field at the input this is the relationship I hope this E1 and E2 are visible at the end. Anyway we will write down them slightly better here, so you can see that.

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The output ports are linearly related to the input electric fields, okay. If you define the output electric fields combine them into a certain vector then you have a column vector E_3 and E_4 the matrix is represented by the capital letter S and which will be a 2×2 matrix and then the input can be given as an input vector which is given by E_1 and E_2 , okay. Now you have a matrix relationship which says that output vector is equal to S times input vector.

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$$\begin{aligned}
 \bar{E}_0 &= \begin{pmatrix} E_3 \\ E_4 \end{pmatrix} & S_{2 \times 2} & E_i = \begin{pmatrix} E_1 \\ E_2 \end{pmatrix} \\
 \bar{E}_0 &= S \bar{E}_i & \rightarrow & (\bar{E}_0^T)^* = \bar{E}_i^T S^{T*} \\
 \bar{E}_i^H \bar{E}_i &= \bar{E}_0^H \bar{E}_0 & (A^T)^* & = A^H \\
 &= \bar{E}_i^H \underbrace{S^H S}_{S^H S = I_2} \bar{E}_i \\
 \begin{pmatrix} S_{11}^* & S_{21}^* \\ S_{12}^* & S_{22}^* \end{pmatrix} & \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} & = & \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}
 \end{aligned}$$

Now to actually obtain what is called as a lossless coupler, a lossless coupler would simply mean that the total energy or the total energy from the input ports is preserved at the output ports, nothing is lost in the coupler itself, so the condition is that the total energy as we know is proportional to energy or intensity is proportional to magnitude of the electric field square, so the total electric field in the input port is a some of E1 and E2.

Therefore and since we are assuming E1 and E2 are to be independent of each other, the input energy total input energy is given by taking what is called as the E_i vector multiplying it with E_i vector this hermitian is simply designation for a matrix A you can transpose the matrix and then take the complex conjugate of the elements, so this is short hand notation for hermitian in my notation, okay.

So this would be the total input energy this has to be equal to the total output energy, okay. So this has to be the total equal to the total output energy but I already know what is the relationship between output electric field and the input electric fields, right so output optical electric field and input output electric field are related by the coupler matrix S or the scattering matrix S so taking the transpose of this equation implies that E₀^T will be E_i^T times S transpose, okay.

And then taking the complex conjugate of that would give you complex conjugate on both side this transpose complex conjugate is what I am calling as hermitian, okay. So I will essentially get for this E_0^H to E_0 as $E_i^H S^H S E_i^H$ sorry E_i itself, so clearly if I want to equate the left hand side to the right hand side this term which is S^H or S tagger S must be equal to an identity matrix since this is a 2×2 matrix each of them.

The transpose and the conjugate will also give you a 2×2 matrix multiplying that with another 2×2 matrix will give you an identity which is a 2×2 identity matrix, what is the 2×2 identity matrix? 1, 0, 0 and 1, correct. So this must be equal to the transpose term you can show that this transpose turns out to be $S_{11}^* S_{21}^* S_{12}^*$ and S_{22}^* this is the S^H part multiply this one by S will give you $S_{11} S_{21} S_{12}$ and S_{22} . And individually equating the terms will give you.

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$$\begin{pmatrix} S_{11}^* & S_{21}^* \\ S_{12}^* & S_{22}^* \end{pmatrix} \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{aligned} |S_{11}|^2 + |S_{21}|^2 &= 1 & S_{11}^* S_{12} + S_{21}^* S_{22} &= 0 \\ |S_{12}|^2 + |S_{22}|^2 &= 1 & S_{12}^* S_{11} + S_{22}^* S_{21} &= 0 \end{aligned}$$

Individually if you equate this four terms for reduce the left hand side into a 2×2 matrix and then equate the terms to the right hand side you will get 2 equal, actually you will get four equations so you get $|S_{11}|^2 + |S_{12}|^2 = 1$ $|S_{12}|^2 + |S_{22}|^2 = 1$ there are two additional 0 equations which are $S_{11}^* S_{12} + S_{21}^* S_{22} = 0$ you have the final equation $S_{12}^* S_{11} + S_{22}^* S_{21} = 0$, remember i said that this

coupler is a reciprocal device which means that if I interchange the inputs and the outputs my coupler matrix relation should not be, I mean the power relation should not be variant, right. So I can interchange what I call as input into output so which means that.

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$$\begin{aligned}
 |S_{11}|^2 + |S_{21}|^2 &= 1 & S_{11}^* S_{12} + S_{21}^* S_{22} &= 0 \\
 |S_{12}|^2 + |S_{22}|^2 &= 1 & S_{12}^* S_{11} + S_{22}^* S_{21} &= 0 \\
 S_{11} = S_{22} &= b & \text{Complex numbers} & \\
 S_{12} = S_{21} &= a & \text{Coupling Coeff} & \\
 |b|^2 + |a|^2 &= 1 & b &= \cos \kappa L e^{j\phi_b} \\
 a b^* + a^* b &= 0 & a &= \sin \kappa L e^{j\phi_a}
 \end{aligned}$$

My cross elements must be equal, so S_{11} must be equal to S_{22} , let us call this as equal to b and S_{12} must be equal to S_{21} which I will call this as a , okay. So since these two are equal you can substitute this b and a into above equations and you will see that you get $|S_{11}|^2 + |S_{21}|^2 = |b|^2 + |a|^2 = 1$, please remember that this b and a are complex numbers, okay these are complex numbers. So you can include so this is the one equation the other equation will be the same, because $|S_{12}|^2$ is nothing but $|a|^2$ $|S_{22}|^2$ is nothing but $|b|^2$ what would happen to the right hand side equations, you get $ab^* + a^*b = 0$, okay so you get this other equation.

These two equations determine the constraints on a and b , okay. Once such solution for this equations can be obtained if I consider $b = \cos \kappa L$ where κ is called as the coupling coefficient, okay κ is called as the coupling coefficient and L denotes the length of the coupling region, okay.

This is the length of the coupling region so this would be $b = \cos \kappa L$ so clearly a must be then equal to $\sin \kappa L$, correct so that $|b|^2 + |a|^2$ will be equal to 1, okay.

Now this if you substitute this into this second equation so substituting b and a into the second equation will give you a relationship which can be written as, actually we have to also consider them to be complex number so right, so we will multiply them by certain phase angle $e^{j\phi_b}$ $e^{j\phi_a}$ to b and a , right my magnitude square will remove this $e^{j\phi_b}$ term and $e^{j\phi_a}$ term and therefore they do not really change anything in the first equation, but for the second equation they will actually determine what should be the relationship between ϕ_b and ϕ_a . Remember any complex number can be written in terms of its magnitude and phase, right so magnitude and its phase I can write it.

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The image shows a whiteboard with handwritten mathematical derivations. At the top, there is a simple diagram of a transmission line with ports labeled 'S'. Below it, the following equations are written:

$$\begin{aligned} |S_{11}|^2 + |S_{21}|^2 &= 1 & S_{11}^* S_{12} + S_{21}^* S_{22} &= 0 \\ |S_{21}|^2 + |S_{22}|^2 &= 1 & S_{21}^* S_{11} + S_{22}^* S_{21} &= 0 \end{aligned}$$

Then, the coupling coefficients are defined as:

$$\begin{aligned} S_{11} = S_{22} &= b \\ S_{12} = S_{21} &= a \end{aligned}$$

These are labeled as "Complex numbers" and "Coupling Coeff". The magnitude relationship is given as:

$$|b|^2 + |a|^2 = 1$$

And the phase relationship is derived from the second equation:

$$a b^* + a^* b = 0$$

The coefficients are expressed in terms of magnitude and phase:

$$\begin{aligned} b &= \cos \kappa L e^{j\phi_b} \\ a &= \sin \kappa L e^{j\phi_a} \end{aligned}$$

An exercise is provided to show that the phase difference must be $\pi/2$:

Exercise: $|\phi_a - \phi_b| = \pi/2$ (odd multiples of $\pi/2$)

Hint: $\cos(\phi_a - \phi_b) = 0$

So putting b and a into the second equation will give you an equation in terms of ϕ_b and ϕ_a and I will leave this as a small exercise for you to show that $|\phi_a - \phi_b|$ must be equal to $\pi/2$, okay so you can easily show this one so $|\phi_a - \phi_b|$ must be equal to $\pi/2$ or any odd integer multiples of $\pi/2$, okay. So or it must be odd multiples of $\pi/2$, okay I will give a hint as well where you can get this equation you will essentially find this equation coming from this one, okay. So once you

substitute for b and a into this equation you will end up after simplification with this last equation which is cos of phase difference must be equal to 0 and we know that cos phase difference goes to 0 when that phase difference term will be some odd multiple of $\pi/2$.

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hint: $\cos(\phi_a - \phi_b) = 0$
 $\phi_b = 0$ $\phi_a = \pi/2$
 $S = \begin{pmatrix} \cos kL & j \sin kL \\ j \sin kL & \cos kL \end{pmatrix}$
 $E_2 = 0$
 $E_3 = \cos kL E_1 \rightarrow \cos^2 kL$
 $E_4 = j \sin kL E_1 \rightarrow \sin^2 kL$
 $\cos^2 kL = 0.75$

So we have relationship for ϕ_a and ϕ_b it is common to consider $\phi_b=0$ and therefore $\phi_a=\pi/2$, right this equation still satisfies my phase constraint because the phase difference between these two is equal to $\pi/2$. So putting all these values back you get the coupler matrix as $\cos \kappa L$ $j \sin \kappa L$ you can say j because $e^{j\phi_a}$, $e^{j\phi\pi/2}$ is nothing but j , right so I get $j \sin \kappa L$ and $\cos \kappa L$ so this is your coupler matrix you can show that this coupler matrix satisfies the hermitian property. Okay interestingly now let us suppose that $e_2 = 0$ that is I am trying to use this coupler as a single input to output device, so I will set e_2 is 0 if I do that what will happen to my outputs e_3 will then be equal to $\cos \kappa L$ times e_1 and $e_4 = j \sin \kappa L$ times e_1 .

You can see that whatever the signal that is going in to the port one of the coupler is split in to 2 parts one part goes to e the port 3 and the other part goes to port 4 what would be the power in each of these ports the power is as you know the proportional to magnitude of the electric field

square therefore the power fraction of the power in the first arm that is in the third port is $\cos^2 \kappa L$ power in the second arm is $\sin^2 \kappa L$.

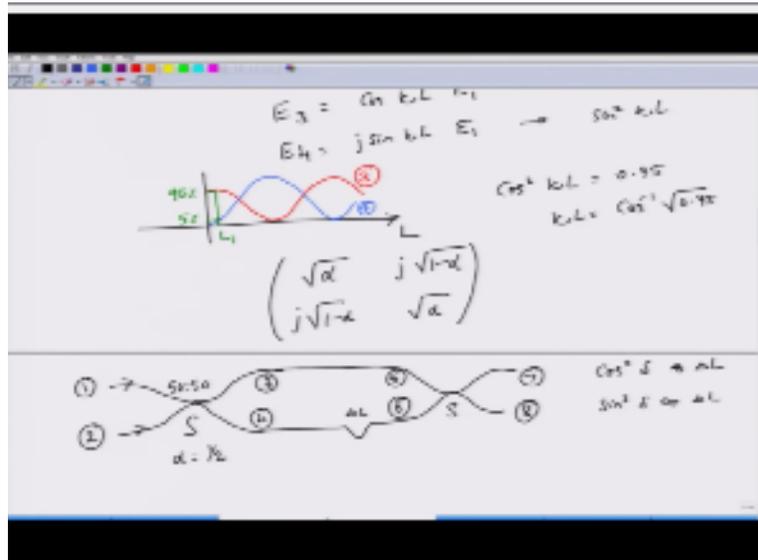
Assuming again loss less coupling the sum these should be equal to 1, but this $\cos^2 \kappa L$ and $\sin^2 \kappa L$ have an interesting behavior right so as I change my coupling length L okay power in the third port initially is maximum the goes to minimum then goes to maximum in a periodic way right whereas power in the forth being proportional to the $\sin^2 \kappa L$ will go in the opposite way. So it goes like this.

So this is your port four output whereas this is one is your port three output there is constant power exchange, you can now determine whatever value of L that you want okay, in order to give a certain fraction of the output power. Suppose I want to send most of the power in to third port then I will chose my length to be equal to sum L1 okay let us also suppose that this power fraction should be about 95% that is to say port 3 gathers 95% of the input power whereas port 2 then must gather only 5% of the input power.

What is the value of L1 required in order to do that you can obtain this value of L one by simply solving this equation $\cos^2 \kappa L = 0.95$ that is the fraction of the power that goes in to port 3 so substitute then you will get $\kappa L = \text{inverse cos of } \sqrt{0.95}$ you can through away the κ result for very obvious reason in this expression. So given coupling co efficient κ you will be able to find out what should be the coupler length L.

Okay so this is how a coupler behaves so for this scattering matrix that we have drawn it actually connects the electric field at the input the electric field at the output you can actually find in literature instead of talking about the amplitude in this way this scattering matrix is written directly in terms of the power coupling ratios as well, for example if the power coupling ratio is α which means input comes in from port one α times input goes in to the port3.

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Of course there is no amplification therefore α must be less than 1 okay in terms of that power ratio power coupling ratio the scattering matrix can be usually written as $\sqrt{\alpha} \ j^{-1} \ \alpha \ \sqrt{1-\alpha} \ j$ and then $\sqrt{\alpha}$ okay clearly this matrix also satisfies the relationship the advantage of writing this one is that you can easily obtain what would be the power ratio right, so α is the power ratio. Okay where in addition to being the front end of the coherent receiver where else can we use couplers wherever you want to couple signals you can use a coupler okay.

Suppose I am constructing what is called as a Mach-Zehnder interferometer okay a Mach-Zehnder interferometer takes input from two arms okay so this is input coming from port one this is input coming from port two and then it splits into two outputs so input three and port four you can connect two waveguides here which will guide light and then you create a small phase I mean the length difference here of path δL and then you combine these two outputs okay.

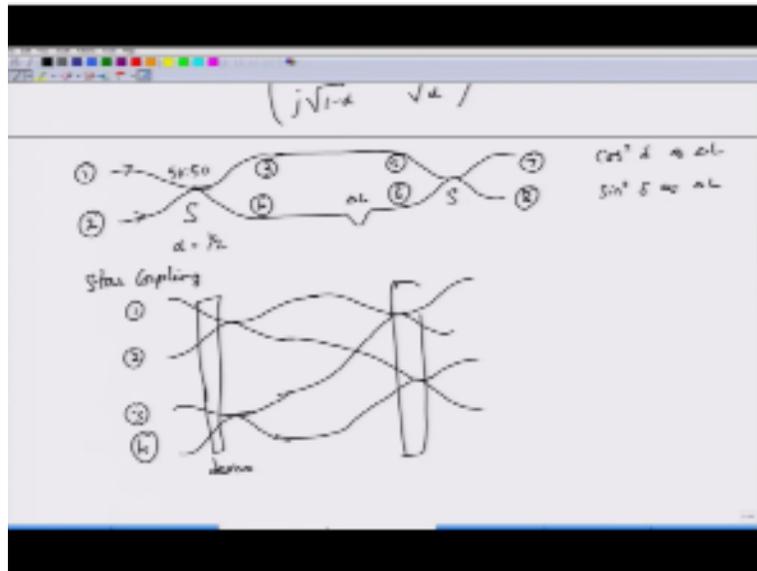
You can combine these two outputs in order at ports say so this is 3 and 4 so let us call this as port 5 and 6 so this port at the output they can call it has 7 and 8 you can then show that output in 7 and 8 okay when there is no input from port 2 will again go as $\cos^2 \delta$ and this will go as $\sin^2(\delta)$, okay where δ is proportional to the path length difference, okay. So you can show this and

in case you are interested you can take this as a simple exercise, that you will have to do is to relate E1 E2 E3 and E4, using this coupler matrix S, you can relate again using another coupler matrix S.

For simplicity assume that this is a 50 to 50 coupler, to say the power coupling ratio $\alpha = 1/2$, 50% of the power course in one arm, 50% of the power in the other arm, okay so this is how you can use couplers in order to construct a mach- zhender interferon meter, infect we will later see that the same mach-zhender interferon meter can be made use as a, or can be implemented as a wavelength filter.

For certain wavelength which the phase difference will then be equal to $2\pi/\lambda * \delta l$, right that wavelength will be constructively interfered at one port and destructively interfered with the other port, so you can actually create wavelength filters by using this mach-zhender interferon meter, the other area where couplers are widely used or use to be widely used is in connecting what is called as static wavelength cross connects. What is this cross connect

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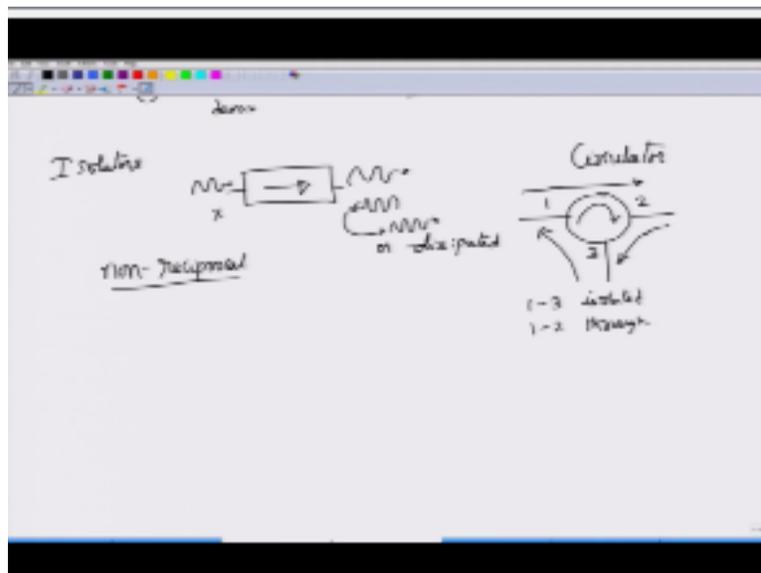


Suppose I have certain wave length of course this is a kind of a simple in that I am considering, suppose I have signals coming in from to arms okay then these where actually used for what is called as star coupling, that is for broad cast purposes if I want to connect many outputs and then broadcast these results on to many outputs then I could use many 3DB couplers, this is one way.

For example I am considering a let me just draw it correctly, so I am considering four inputs here these can be coupled, okay so there is one 3DB coupler here, if I connect these two I will get one more 3DB coupler, I can then connect these two outputs, okay, I can connects these two outputs.

So this was known as a star coupler, okay. You can of course make this in multiple you know in many ways you can implement this infect you can implement this and then put some de multiplexers, okay before you connect them and then put some multiplexers after connecting in order to realize this so called wavelength cross connect, a subject that we are not going to touch for some time now. Okay, this is how we use couplers.

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Let us now quickly jump in to what is called as a isolator? Isolators are very important in at least one major area they are unidirectional devices which means that if I put in a isolator, I have to

specify the direction of the signal propagation, the signal propagates only in the direction that is mentioned in this arrow. For me this would mean signal going from left to right.

So if I sending some signal from left some output you can expect the output where as if is en in input from this side nothing will come out, okay the input basically gets deflected or gets dissipated, okay, so such a behaviors is hall mark of a isolator an extension of a isolator is what is called as a circulator. A circulator is a device which has a certain directional property as well so it is typically we talk about a three port coupler so you have a, 1 to 3 ports for this circulator.

Any signal that goes to port 1, will be carried to port 2 however there will not be any transmission from port 1 to port 3, so we say that 1 to 3 port is isolated, and port 1 to 2 is called as the throughput port or a through port, okay, similarly 2 to 3 is a through port where as 2 to 1 is a isolated port. So you can see that signal can go from 1 to 2, but signal cannot go from 2 to 1, it will be routed instead from 2 to 3.

Of course you send some signal from 3 then this will be routed to 1, okay this is a 3 port coupler you can also find a 4 port coupler in market that will again work in very similar ways, okay how does an isolator work? Isolator turns out to be a non reciprocal device, why is it non reciprocal because, I cannot turn around the leads of an isolator right? So if this is the isolator leads I have to use them in only one direction in this way they are equivalent of a semiconductor diode.

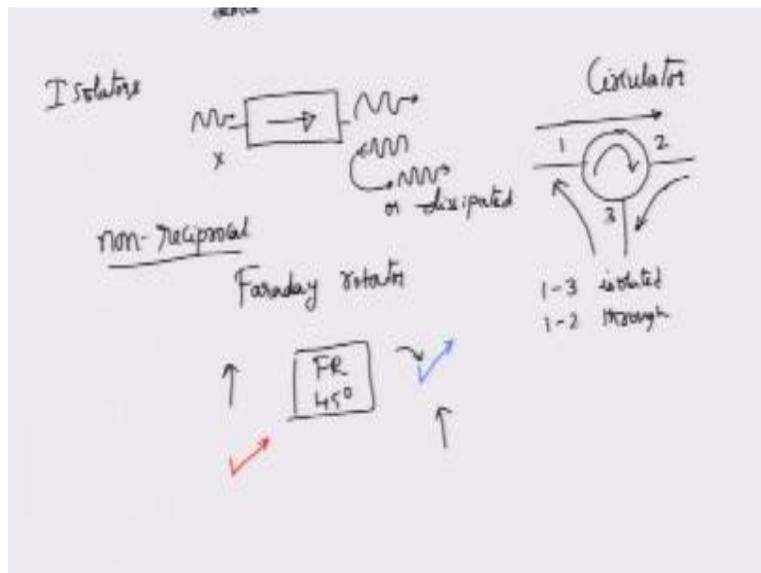
I can't use a diode in, without regard it's terminals, I have to know which one is the anode terminal diode and cathode terminal diode, I have to put them in only one direction. If I put them in other way around that signal will not be past so that is essentially a isolator. So this isolator is built up on single device called as a non reciprocal, it uses inside a non reciprocal device called as a Faraday rotator, Faraday rotator works on the principle of Faraday effect.

If you apply a certain magnetic field, then you can actually change the polarization state of the input field, if the input field, for example of a faraday rotator with the 45 degree rotation means that 35 coming with the vertical polarization, vertical p[polarization means the tip of the electric field vector is vertical in a direction, it's perpendicular to the direction of propagation. So if

coming with such a vertical polarized light, then the faraday rotator will rotate the polarization into the 45 degree angle.

So the output will be 45 degree polarized line, what will happen if I coming with this in this direction, suppose if I start with the input which I vertically polarized, my faraday rotator will again rotate in the clockwise direction only, so it will again rotate this one into 45 degree only,. So if you see that in both ways it will rotate in the same direction, so this is the faraday rotator, this is used inside the isolator.

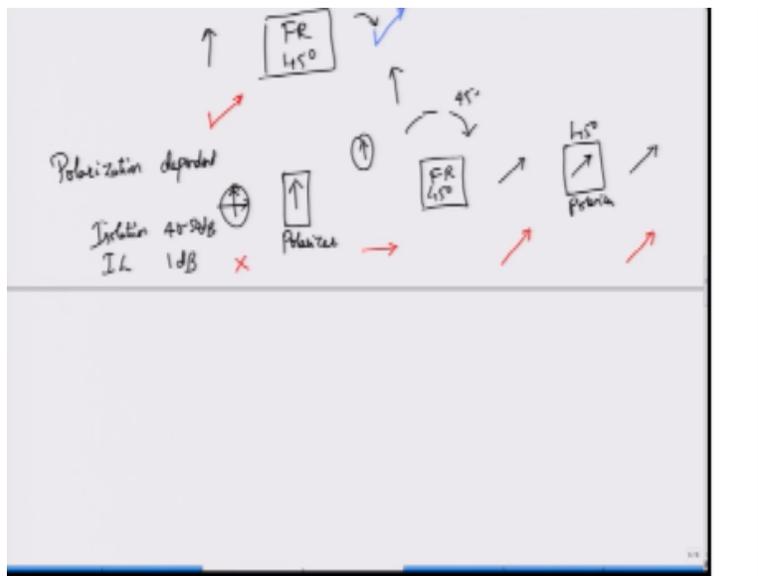
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How does an isolator look like? You assume that initially we are talking about what is called a a polarization pendent isolator, for a polarization dependent isolator we assume that there is a vertical polarizer down, so if I coming with horizontal and vertical polarizer, only the vertical polarizer light will come out, after this vertical polarized light, I put a rotator of 45 degrees so that the polarization become 45degrees. Then what I do is I put a 45 degree polarizer, so this is a 45 degree polarizer which means that only 45 degree polarized light will be coming out of this particular polarizer, so this is a vertical polarizer, this is a faraday rotator which rotates in a clockwise direction from 45 degrees.

Now suppose I want to look at what happens when I go backwards, so I assume that initially I start out with a 45 degree polarized light at the right side? Now when you pass through the 45 degree polarizer, no problem this passes with the polarizer as it is, because these are in the same direction. Next what happens is the faraday rotator will rotate this one in the clockwise direction by 45 degrees, which means that the polarization state becomes horizontal? Now the horizontal polarizer is orthogonal to the vertical polarizer.

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So this polarizer will not transmit anything, so you don't get any transmission, so the isolation basically depends on how good this polarizer is, if this polarizer is good enough in blocking the orthogonal component then the isolation can be pretty high. Typical isolation is about 40 to 50 db and there is some insertion laws which is about 1 to 2 db, is the insertion law when you go from one direction to the direction. We will stop now and consider few more components in the next module. Thank you very much.

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