Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Optical Communications

Week – VI
Module – II
Dispersion in Fibers(Contd.)

by
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Hello and welcome in this module we will continue the discussion on dispersion in the optical fibers let us look at a specific case of how a Gaussian pulse would broaden as it propagates through the fiber let us assume that.

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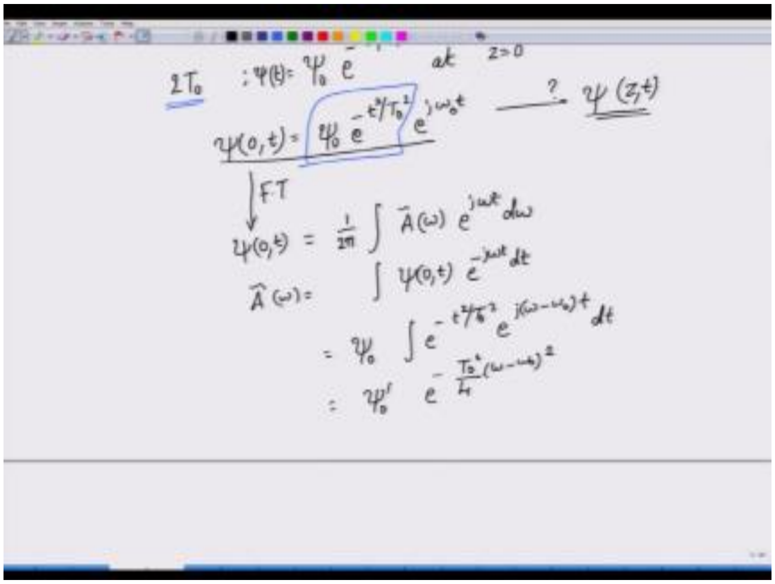

We have Gaussian pulse at the input of the fiber this Gaussian pulse can be represented by giving it is width which we will take it to be some two times To and an expression for this Gaussian pulse will be given as some amplitude we do not really want to bother about the amplitude so let us call this as some $\psi 0\ e^{-t2/T02}$ okay so this is a Gaussian pulse which would be launched in the fiber at z = 0 so at z = 0 you are launching this pulse let me tell you what we mean by this pulse that is being launched actually what I mean to say is that this pulse would not be directly launched.

Okay it would actually be multiplied I mean it could be modulated by a carrier and the modulated pulse can be written as $e^{-t2/T02}$ times $e^{j\omega_0 t}$ this ω_0 is the carrier and this factor I mean this expression that we have written is how it the amplitude of the pulse changes with respect to time but in addition to this one there is also a transfers distribution of the pulse itself so that is f(x,y) or equivalently f(r, Ø) which tells us how the mode is distributed in the transfers plane right so we solved an equation for how the mode would be distributing the transfers plane.

The fundamental mode in terms of r and Ø depends like this so $J_0\ (ur)^{j\eta Ø}$ or rather for the fundamental case this will be independent of frequency but I mean independent of the Azimuthal number but this that transfers field distribution so we will assume that this transfers field distribution does not change so you have in terms of the transfers plane a Gaussian type of a pulse okay or the J_0 type of a pulse okay in time this would be varying as another Gaussian pulse.

So there are two different aspects going on one is in transfers plane what is the distribution of the mode energy that is determined by the solution for the transfer field components which is essentially what we did in the mode theory and how a particular mode that fundamental mode or any other mode is propagating in with respect to z and time is given by this modulated term and how this modulated term will evolve through the fiber as it propagates okay so please keep in mind these two things.

We assume that propagation does not change the transfers destruction the moral distribution of the energy remains as it is as it propagates through the fiber so therefore we do not really worry about what is happening to the transfers plane because we assume that nothing is changing with respect to the transfers field distribution the mode pattern remains the same only in time they are changing as Gaussian pulse there is a certain amplitude ψ_0

And then this pulse as certain with which is 2 times T_0 it is also of course modulated by the carrier e $^{j\omega_0 t}$ let us describe this one as the pulse at z equal to 0 and time t are objective would of course be to obtain ψ (z,t) okay after we have propagated this certain arbitrary distance of z we want to obtain what is the expression for ψ(z,t) the easiest way to do this which also give you a numerical way of solving this problem is to go to the frequency domain.

We can go to the frequency domain as long just is amplitude sixe zero is very small okay in the sense that you do not excite any non linear terms in the fiber which is quite good approximation for most communication systems as long as you are looking at a short distance communication for long distance communication unfortunately non linearity is will be there and this description is not sufficient then so for the linear propagation case we can go to the Fourier transform of this one and the write the Fourier transform of this.

As you know or in fact one can actually write down the Ψ of ot as ∫ of A of ω or A bar of ω to denote the frequency component are the Fourier transform component times e $^{j\omega t}$ d ω so I am writing this Ψ of ot which is a time dependent function in term of it is Fourier transform well there is also 1/2Лwhich we should not forget so we can write this Ψ of ot as A of ω e $^{j\omega t}$ of course what is A of ω, A of ω is nothing but Fourier transform of this fellow right it is Fourier transform Ψ of ot e – $^{j\omega t}$ dt and if you look at what is the situation for this particular time dependent variable.

You can substitute this here the amplitude size 0 being a constant simply move out of the integral and here you have e – t2/t02 e $^{j\omega}$ - – ω0 t dt so clearly this is a modulated waves from therefore first find the Fourier transform of this Gaussian pulse and then shift that spectrum by ω0 in order to get the modulated spectrum so you know in have to get the Fourier transform of this if you do that you know going back to the tables of e – to^2/ to^2 the Fourier transform of that one what you end up with is some other constants which let us call it as Ψ0 prime okay and you have e – to^2/4 ω - $\omega0^2$.

Okay so the Fourier transform also a Gaussian function except that this is now centered at ω0 which is the carrier frequency and it has a width which is 1/to2 so the larger the pulse in the time domain the shorter will be it is spread in the frequency domain and we are assuming that this spread in the frequency domain is much smaller compared to the carrier frequency so that is an assumption that we are making okay so this is what you are.

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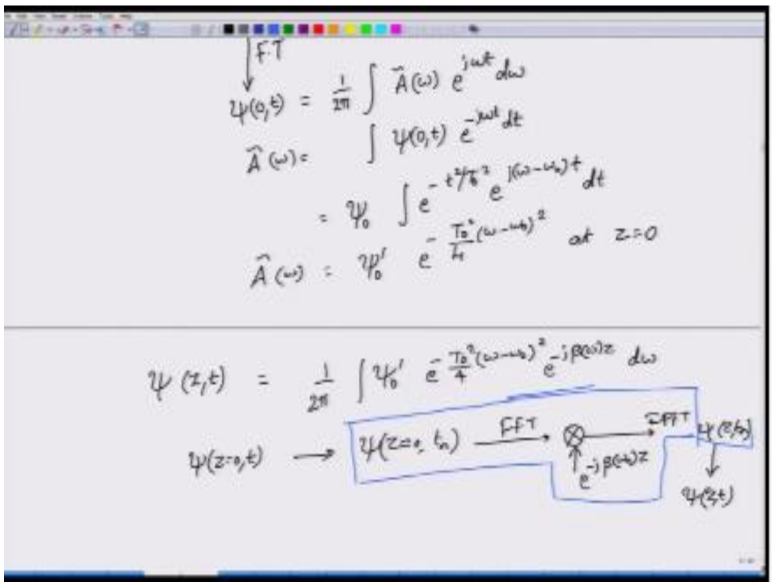

A of ω would look at z = 0 remember we are still at z = 0 we have not yet began to propagate the pulse through the fiber we are still at the fiber input now let us try to propagate this one and here you have to remember from the previous module that I talked about you have to consider Fourier transform as a sum of many, many frequency components each frequency component having a different amplitude and then visualize the propagation as each of these frequency components propagating along the fiber when they propagate along the fiber what will change they will pick up a phase factor of e – jβz correct.

They will pick up a phase factor of e – jβz and that β will be different for different frequency so you have a pulse min time domain you got to the frequency domain you have a pulse in a frequency domain right and this frequency domain picture can be thought of as multiple frequencies in fact it would ban an infinite number of frequencies between the range but never the less we can think of them as multiple frequencies each having different amplitude and then has each of these propagate through the fiber.

It will pick up a phase factor which is dependent on what frequency is propagating so at the output you simply some everything back and go back to the time domain by taking the inverse Fourier transform so that is all you are going to do you start with Fourier transform at here and then you go to the Fourier transform at the or inverse Fourier transform at the output end okay so let us do that one we have already done the Fourier transform part at the input side and we have seen how the spectrum would look like now I go back to the expression of Ψ at any z where I want right so this Ψ at any z after propagating a distance z should be given by the same expression except that I have this size 0 prime which is this constant e – to2 /4 ω - ω 02 times $e^{-j\beta}$ of ω x z this is the phase factor that is picked up.

By the frequency component ω as it propagates through the fiber there is also d of ω okay so I have considered these terms of different frequencies they all have picked up a certain phase factor which is e - $^{j\beta}$ of ω x z and then I am simply summing everything back essentially taking the inverse Fourier transform, so this is all the equation that is there and if you want to solve so you can solve this using a simple numerical method, you start with the pulse at z=0 with respect to time you can discretized this, okay so you can discretized this you know some n number of points by using some discretization technique.

And once you have done that one you can take the Fourier transform after you have taken the Fourier transform to each term you multiply this factor $e^{-j\beta(\omega n)z}$ and then you take the inverse Fourier transform and what you get here would be ψz and tn so at a different distance what you get is your function which you are looking for it would be discrete you can visualize the continuous expression for the same.

So the core of the algorithm here is this, you discretized that is your represent whatever that you want to transmit on a computer by a discrete function take the Fourier transform and do not forget to multiply each term by the appropriate phase factor and then take the inverse Fourier transform that is all that is there to it.

In the assignment we will give you a code which will enable you to perform this assignment and see for yourself as you propagate the pulse through the fiber how that propagation would affect the pulse, you know how the pulse would spread out. So we will come back, we will give you the mat lab implementation for that one you can take a look at that mat lap implementation later. However, for analytical purposes let us try to see whether we can take the inverse Fourier transform this of this expression.

How do I do that, well first of all I need to know how β is changing with respect to z, right. So how is β changing with respect to, sorry not z how is β changing with respect to ω I really do not know how β is changing with respect to ω, because it is quite a complicated expression that I do not even have a proper analytical value for that, analytical expression for that.

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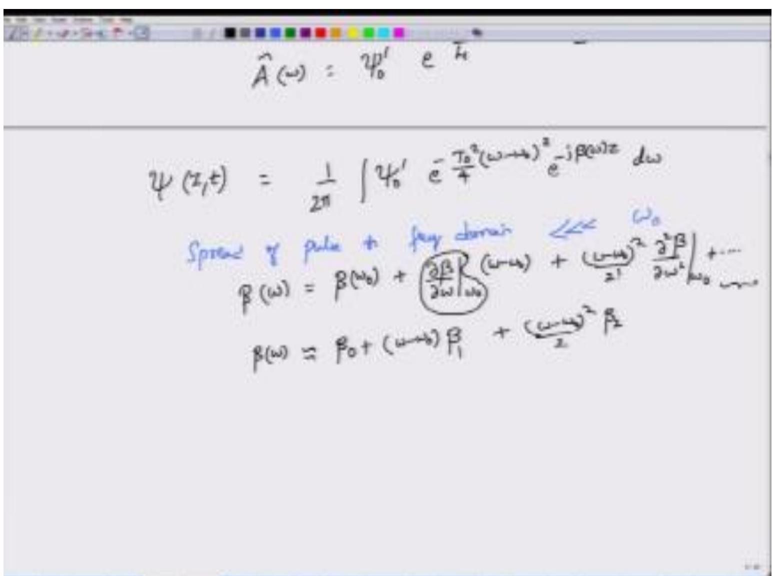

What I know is that the spread of the spectrum that is the spread of the pulse in frequency domain, the frequency domain band width as if you would say is much, much smaller than the carrier frequency $\omega0$, because this is smaller I can approximate this β around a frequency carrier frequency I can write this $\beta(\omega)$ as a tyler series expansion $\beta(\omega0)$ and then you have $+\delta\beta/\delta\omega$

evaluated at ω0 multiplying it by ω-ω0 plus you have ω-$\omega 0^2$/2! 2! Is 2 of course and then have $\delta^2\beta/\delta\omega^2$ evaluated at ω0+ so on.

These terms are going to be small so we can neglect them and this term δβ/δω at ω0 in the literature is denoted as β1, so you have ω-ω0.β1+ω-$\omega 0^2$/2.β2, β2 denotes the second order derivative of β with respect to ω which is evaluated at the carrier frequency ω0 and this β(ω0) can be written asβ0 itself, so this is the approximate expression for β(ω) I have neglected all the third order and higher order terms. In case this neglection is not good you can include those term also, okay.

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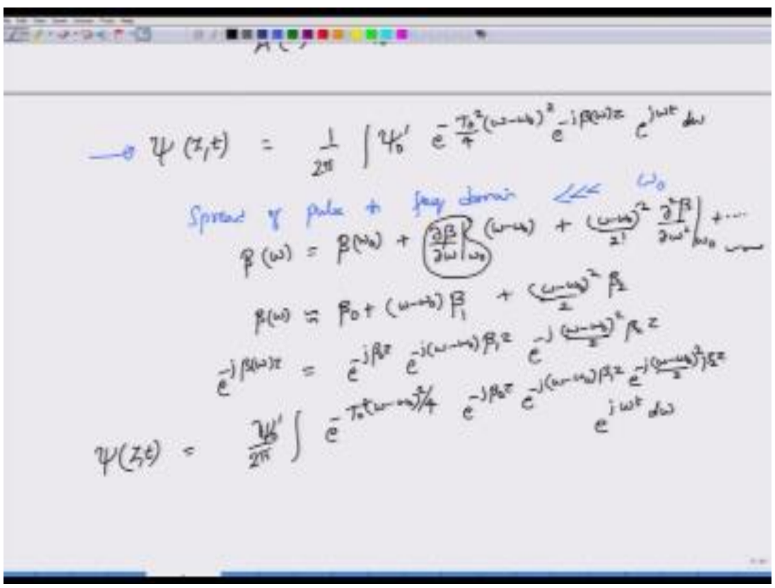

So now we have expanded β(ω) which then makes this $e^{-j\beta\omega z}$ phase factor as $e^{-j\beta 0z}$ $e^{-j(\omega-\omega 0)\beta 1z}$ and there is also this $e^{-j(\omega-\omega)2}$/2 β2z, okay so let us put this one in to this expression into this expression here and then evaluate the inverse forego transform so I get ψ (z t) the pulse envelop at the input is given by 1/2π some constant $\psi_0{}'$ so I am taking that one out of the integral and what I am left here is $e^{-T_0^2}$ **Ω -Ω** o^2/4 $e^{-j\beta 0z}$ $e^{-j\Omega-\Omega 0}$ x β_1z $e^{(j\Omega-\Omega 0)2}$/2β_2z now I remember I have forgotten to include $e^{j\Omega t}$ here in the inc\verse forego transform correct I have $e^{j\Omega t}$ x dΩ that factor I forgot top include. Now let us include that so I have $e^{j\Omega t}$ dΩ.

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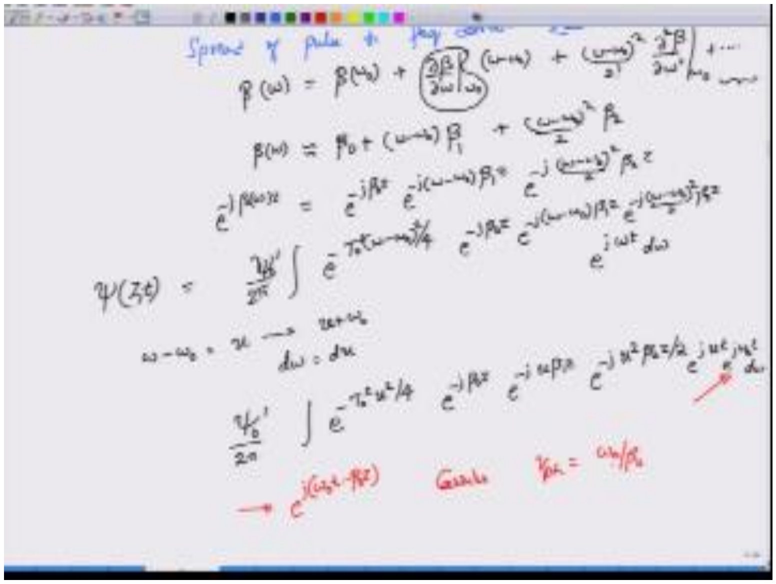

What we will do is we will substitute Ω- Ω_o by some dummy variable u okay and therefore this gives you d Ω= du this integration was anyway going from – infinity and + infinity the integration will remain the same so you get ψ_0' /2π integral e^{-t02} u^2 /4 e^{-j} β_0z e^{-ju} β 1 x z right for Ω- Ω_o I am substituting u and then have e^{-j} u^2 β_2 z/2 e^j but what is Ω is nothing but u+ Ω_o, so I have e^j u_t e^j Ω_0 T d Ω.

Now look at this term e^j Ω_0 T term does it depend on frequency no it does not depend on frequency so you can actually remove this e^j Ω_0 T term outside the integration you can also remove this $e^{-j}\Omega_0$ term be^{-j} β_0 x z term because that is also independent of frequency β_0 evaluated at Ω_0 is actually a constant therefore that term can be removed e^j Ω_0 T tern can also be removed in fact if you remove them you end up with this equation which is e^j Ω_0 T –β_0 x z.

This would actually tell you how the carrier would have propagated if it was not modulated okay so it would have simply told you how that the carrier would have propagated if it is not modulated and this is in fact defining the phase velocity. This is what in meant when I the earlier module I said carrier travels at phase velocity Ω_0/ β the carrier is propagating at this phase velocity. So I have removed this one.

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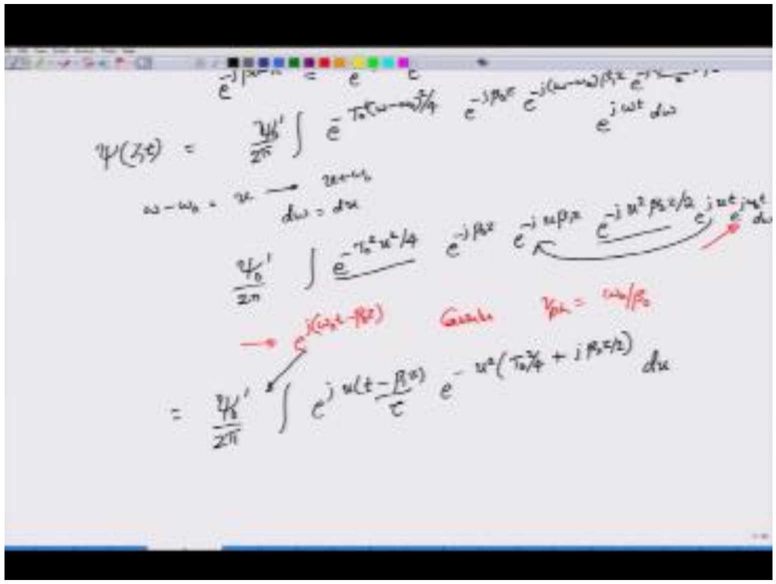

So in this expression I have removed that what is now remaining can be grouped again together okay so there are terms which are u^2 dependent so I can group this term with this one I can write this $e^j u(t)$ $e^j u\beta_1 (z)$ so I can write this one and says $\psi_0' /2\pi$ I am not writing this $e^j \Omega_0 T - \beta_0 x z$ term but you have to assume that the term there so you have e^j u is a common factor so I can write this as $t-\beta_1 x z$ okay interesting I have $t-\beta_1 x z$ and then I have e- first there is u^2 then you have $t_0^2 / 4 + j\beta_2 z/2$ it that okay so this is this one integrated of course with respect to du so this is what I have. Now I can write this t- β 1 , this is the only place where "t" term is there, so I can write this t- β 1 z as some τ, finally I know that the function would actually be inter with respective to τ right? so instead of time it would be τ, and that τ .

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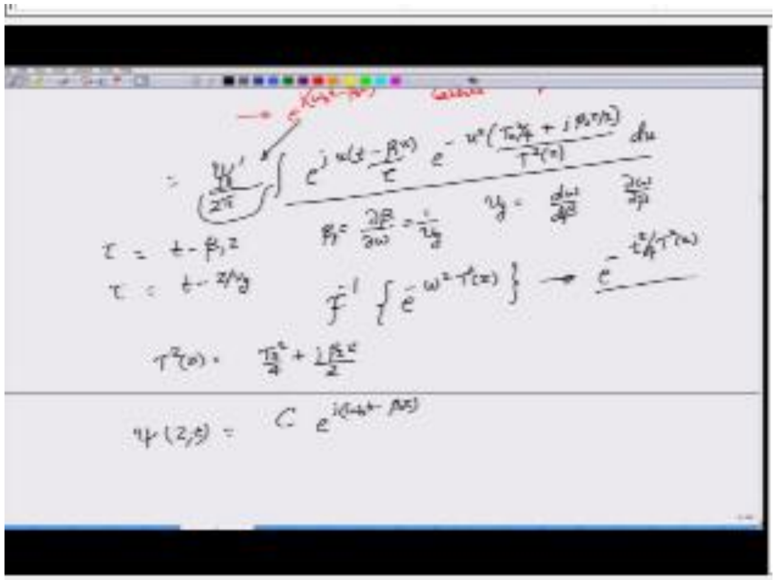

Which is given by t- β 1 z, you will appreciate this one better if you remember what β 1 is, β 1 is actually ∂ β / ∂ ω. But remember we had defined a group velocity as U_g = d ω/ d β or equivalently ∂ ω/ ∂ β, so ∂ β/ ∂ ω must be 1/Vg , so it is like a group ∂ay, β 1 is like a group ∂ay happening, so divided by l will give you the group ∂, okay.

So β 1 is inversely promotional to Vg, so you can write this one as t-z/Vg, so what you can see that eventually the Gaussian pulse would be arriving at a velocity which is dependent on group velocity, not on the phase velocity, the carrier would arrive at the phase velocity where as the pulse would arrive at the group velocity.

What is left? Here this sees one big term which you can think of as some d^2 okay, where d is a constant because inside if you look at it T02 is a constant β 2 is a constant, because that's what we have assumed you have evaluated β 2 at that particular point, it is z is a constant, because you are looking at a particular value of z, right? So this is the constant which you can write this as d^2, and really what is happening to this equation? Including this 1/ 2 π is that you are simply looking at the inverse Fourier transform of this e^{-u2d2} term.

Okay, if you are not comfortable having “u” around, since “u” is a dummy variable, you can simply write this as $e^{-\omega 2\ d2}$ term and I know that this is a Gaussian pulse shape and inverse Fourier transform should be a time dependent term, which would be like e^{-t2}/t^2, give or take some constants are there.

You know there is if this was divided by 4 then this would have been okay, because you have e^{-t2}/d^2 since it is not then you can multiplied and divided by 4, and then this would be $e^{-t2}/4\ d^{2,}$ so the point to note here is that what we have in the bracket here are rather in the expression for the integral is actually nothing but simple Gaussian inverse Fourier transform of a Gaussian function okay.

Rather than calling this as some constant d^2 let us be little more clever and call this as t^2 but because this would depend on some set I am going to call this a $t^2(z)$, okay I get $e^{-t2(z)}$ but at a given set this t^2 is a constant , so please don’t forget that, and this inverse Fourier transform will be $e^{-t2}/t^2(z)$, there is a factor of 4 or something, okay you don’t really have to worry about that factor 4.

Okay so this is what we have interesting what is t^2 (z)? t^2 (z) is given by $T0^2$ /4+J β 2z/2, how is t^2z a complex number? Well it is complex because of its j β 2 terms but what is the implication of this? Let us look at what is the Fourier transform so the final result φ (zt) is given by all those constant of c , because there is a inverse Fourier transform there would be one more constant.

So let us called all those constants as some constant c , okay, and you have the carrier term arriving here which its own phase velocity $e^{j\ \omega\ 0t}$-β 0z and then you have the Gaussian function but remember now, this was the Gaussian Fourier transform but it is not this T that I am looking at, it is actually that this T-Vg that I have to look at, because this is the fourier transform of the Gaussian but Te is itself is given by t-ß1z, so here it is that , so you get $e^{-(t-z/Vg)2/4T2z}$, this is the Gaussian pulse that I have obtained, forget about this factor of 4 here, the point is the pulse width has now become the function of z.

In fact z is this number you can write this one in terms of its, magnitude call this as Tm^2 (z) and also write this angle. So this would be the angle θtm of z, again this is a function of z, because you are evaluating a different values of z, but if you fix the value of z, this is all going to be constant, so I can write down in terms of magnitude and phase angle, so I can write this fellow as $e^{-(t-z/Vg)2/4Tm2z)}$ and then you have angle here which is $e^{j\ \theta tm(z)}$.This $e^{j\ \theta tm(z)}$ can go onto the top and essentially become a constant phase for you.

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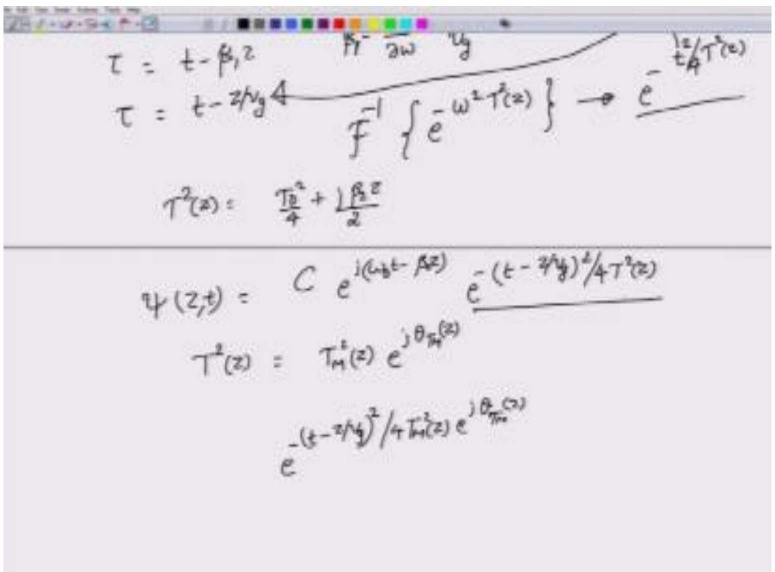

So there is a phase factor associated with this one, so you can evaluate this one or you can write down $1/T^2$ as some magnitude and a phase angle. In any case whatever you do, you will see that there is a magnitude and a phase angle that is present and this phase is the function of z, this give raise what is called as the chirping in optical fibers. Chirping is the phenomenon in which the instantaneous frequency keeps on changing and that chirping is coming because of this θtm (z).
A more careful analysis of you know by going to the tables of a Gaussian function will actually show that, this expression which we have written down, for4 T^2(z), can be written as c/T(z)/T_0 $e^{-(t-z/Vg)2/T2(z)}$ is a phase factor, $e^{j\ \phi(zt)-\beta 0(z)}$, so this would be the expression for ψ(z,t), taking all these constants T_0 is of course the original pulse width or there is a square root here, so T_0 is the original pulse width and T(z) is the pulse width at any point z.

I have just separated them out in terms of it phase and magnitude angle, so this $\phi(zt)$ is basically $\omega^0 t$ coming from the carrier term plus there is also $\varkappa$ $(t\text{-}z/Vg)^2$ and because of this $T^2(z)$ term , there is ½ $\tan^{-1}2\alpha z/T_0^2$, where α itself is a different parameter, α is basically ß$_2$, so you can actually see that if you are little more careful in Fourier transform this is what you're going to get and what is interesting about this is that at z=0, you might have certain pulse, you know a Gaussian pulse, but at a different value of set, you know the pulse would have nicely spread out.

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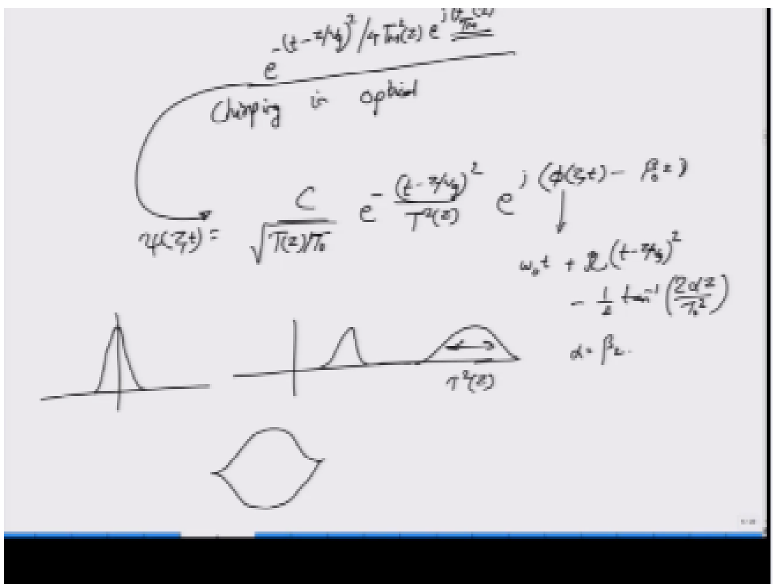

And this width is given by the T^2 (z) or rather it is function of $T^2(z)$, we will also see that, if you look at it in terms of carrier frequency, you will see that, first let me draw the pulse on envelope, now let me fill up the modulated waveform, so the frequency of the carrier is actually increasing, so this phenomenon is called as chirping. You can look at the assignment for some more details, for careful details about the mathematics involved here. I have just given you the intuitive picture, so let me summarize whatever we have done here.

We said that dispersion is a very important topic in optical fiber and that is clearly so, because it causes pulse to spread out and once pulses start to spread then the rate at which you send pulse4, that is you send in one pulses you have to wait until the pulse has to spread properly, because if you send in a pulse before the pulse spreading has been taken into account, then the pulse as it propagate would spread and then it would actually invade into other pulses territory.

So it essentially starts to talk to the other pulses giving rise to what is called the inter symbol interference, so pulse spreads and gives rise to inter symbol interference. Here are many different type of pulse4 spreading phenomenon which is the dispersion phenomenon, material dispersion arises because of the material, waveguide dispersion arise because of waveguide dispersion chromatic dispersion arises because3 of the geometry effect in single mode fibers.

Polarization mode dispersion we will talk about it in later stage and then we also have another dispersion which is multimode fiber dispersion, that is intermodal dispersion, which anyway does not really figure into our course at this point, when calculating how a Gaussian pulse would spread, the reason we took Gaussian is because kind of simple to analyze analytically , you could have taken any other pulse in a numerical method we will supply you can actually see that for yourself, nicely Gaussian pulse spreading out or any other kind of pulse that is spreading out.

The idea is to start at whatever the pulse is given to you at z =0 and then go to the frequency domain picture, when you go to the frequency domain, you will get the spectrum and then you will propagate each component of that spectrum along the fiber. As this component propagates this will pick up $e^{-jß\ (\omega z)}$ phase factor and that has to be taken into account, for the case of Gaussian analyze, you know Gaussian pulses, there is a nice inverts for a transform for a transform that you can do and what you essentially see is that, the pulse not only spreads out, actually it is amplitude also decaying, as you can see when we drew the pictures.

The pulse might not have started at the amplitude but as its propagates its amplitude decays also its frequency changes, there is a chirping that is going on, and this chirping depends on sin(ß2), this ß2 is positive. Corresponding to what is called as normal dispersion, then there is a spreading of pulse. However when ß2 is negative , the pulse initially contracts and then of course begins to spread again, so this kind of a behavior of pulse initially contracting and spreading , there is a region in which the pulse is actually getting short and it's called a anomalous dispersion and that happens when sin(ß2) is negative.

So all this, problems do occur in addition to pulse spreading is also called as chirping and one has to take care of these factors when deciding an optical communication system. So we will stop her and we will take up the question of additional things, additional factors that affect the design of the single mode optical fiber such as attenuation and non linear effects in the , later modules. Thank you very much.

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