Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Optical Communications

Week – VI
Module – I
Dispersion in Fibers

by
Prof. Pradeep Kumar K
Dept. of Electrical Engineering
IIT Kanpur

Hello and welcome to this optical communications online course we will discuss today disposition in optical fibers optical fibers have in addition to dispersion other characteristics such as attenuation and nonlinearity we will take up attenuation sometimes later and nonlinearity we will receive it at the end of the course when we talk about dispersion the most visible aspect of dispersion is that if I take a pulse of light and then launch that pulse of light into the fiber.

Then as the pulse propagates through the fiber and arrives at a distance you know arrive at a later distance at the receiver the effect of dispersion is to basically broaden the pulse so you might have started with the pulse whose with is around say 1 nana second but then after propagating through the fiber a certain length and arriving at the receiver then this pulse would have broad into say 1.5 nana seconds.

So this is the visible aspect of dispersion and the reasons for such pulse spreading which is the effect of dispersion is many okay if you consider the multimode fibers multimode fibers as we have seen can support multiple modes they can support they can support the fundamental HE11 mode as well as the next higher order modes righty or the fundamental LP01 mode and the next higher order modes LP11, LP21 and so on so when you launch a pulse the pulse energy goes into all these modes okay.

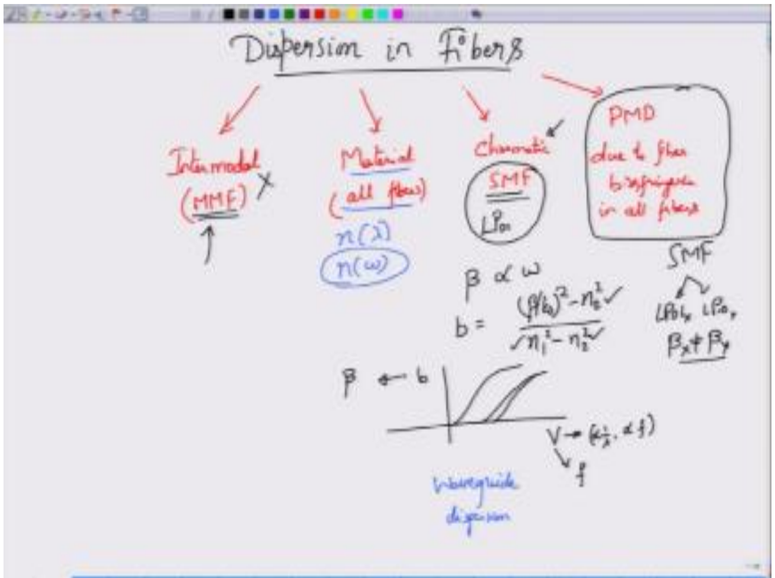
All these modes are excited and they all began to travel towards the receiver what happens is that as these pulse energy in these different mode start to propagate along the fiber right they will propagate so with the different velocity even if you have launched a simple sinusoidal signal this sinusoidal signal will travel at a different velocity through the fiber when it is connected to the different modes that is I different modes propagate at different velocities therefore when these different modes arrive at the receiver right.

Slightly delayed with respect to each other because the overall pulse energy will be magnitude of the sum of electric fields square so you have to cinder the electric field coming in from the different modes and then form a coherent sum take the magnitudes square of that in order to form the intensity or the optical power, power begin proportional to magnitude of E^2 so when you take the square what you see is that this differently delayed components interact with each other or interfere with each other so as to give you an effective increase in the pulse.

So this is the pulse spreading that is cost by having different modes propagate a different velocities and this is the phenomena which is mainly seen in multimode fibers it is in fact the phenomena that is seen in magnetic mode fibers and that is something that limits the amount of band with that one can or the data rate that which one can communicate with the multimode fiber if you remember our geometric optics point of view we talked about a ray of light going are launched inside the core iterating the cladding.

Then you know in a propagating into through the fiber and zigzag way however a different angles corresponding to the angles greater than the critical angle different angles denote different modes so we calculated what happens to the extreme case you had one ray which is propagating directly along the axis and one ray which was propagating at the critical angle and then we found that the one which is propagating here would take larger time compared to the one that is propagating centrally and results in a time delay difference which in turn gives you the spreading of the pulse so this intermodal dispersion is main characterize in a multimode fiber however when you go to single mode fiber.

(Refer Slide Time: 04:04)



This problem inter model dispersion simply disappears the reason being this is a single mode fiber which means that there is only one mode in which the energy can propagate this would be the LP_{01} mode or the LP_{10} mode the XNY modes so they will propagate through the fiber and therefore there is no problem with multiple modes sharing the energy and hence getting some kind of a interference happening at the pulse priding happening.

So in the sense that multi modes have disappear there is only one mode so you do not really expect multi mode dispersion there, however even single mode fibers exhibit a kind of a dispersion which is very important for long distance optical communications, okay this dispersion is called as chromatic dispersion and it arises simply because β is a complicated function of frequency, right.

B is a complicated function of frequency if you remember we talked about this normalized propagation constant B which we define as $(\beta/k_0)^2 - n2^2 / n1^2 - n2^2$ right and then we plotted what happens to this B as a function of the normalized frequency v, right the V number in fact is inversely proportional to λ or it is directly proportional to frequency, frequency or Ω okay, so this v is a parameter you can think of as v changes f changes, okay.

And we see that β is not a linear function but rather for the fundamental mode it goes like this for the next higher order mode it goes like this and then it goes in this particular fashion, so there is actually a non linear relationship between B and V since B is you know for fixed n2 and fixed n1 and fixed n2 B is directly giving you the value of β similarly if you fix all the parameters the numerical aperture and the radius.

This v is giving you how your frequency is changing or as frequency changes v changes so what we have is a non linear relationship between β and F and this in turn causes the dispersion something that we are going to study in the latter half of the module, okay later half of the module, now this dispersion is purely a geometric effect, I can tailor this dispersion by changing the value of β verses F and how can I do that, I can change the values of n1 I can change the values of n2 I can change the core area I can change all these parameters in order to play around and then change the dispersion.

For this reason this is sometimes called as the waveguide dispersion, okay so waveguide dispersion is the geometric dependent dispersion, this dispersion characteristic is because of the circular nature of the optical fiber, if I were to fabricate a rectangular optical fiber then the corresponding β verses v will be different, okay. So this is the pure geometric effect or the fact that you are trying to guide in a particular geometry is called as the waveguide dispersion, okay.

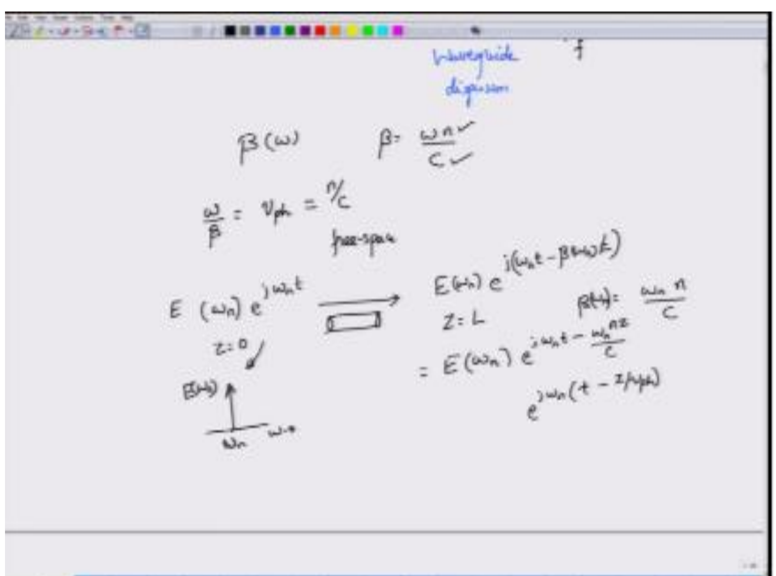
Now in addition to this waveguide dispersion in every fiber that you consider there is an extra dispersion called as material dispersion, material dispersion results because the refractive index becomes a function of λ or equivalently the refractive index becomes a function of Ω frequency, okay. Because you know we fill up with an optical fiber will silicon material and then we also fill that along with silica we fill some dopants.

In order to change the refractive index of the core as well as the cladding and these different particles which are there they would response differently to the electric fields that are applied geo_2 a germanium oxide component would respond differently corresponding to a silica component and when you fill up a certain material with this there will be difference in the material response so as to the electric fields giving rise to a refractive index dependence on the frequency itself and this frequency dependent refractive index.

Turns out to be the reason for material dispersion okay so this is simply saying that refractive index is changing and this change is a function of frequency okay finally there is important dispersion source which is called as the polarization mode dispersion this occurs mainly in the single mode fiber or at least it is very, very prominent in the standard single mode fiber where the two fundamental modes that is LP01x and LP10y that is x polarized and y polarized waves the linearly polarized waves are propagating with their own propagation constants β_x β_y which are not equal.

Even though if you launch the wave at the same frequency for both the x and y polarized modes their propagation constants they will be different and this difference in the propagation co0nstant eventually gives rise to dispersion okay so these are the major sources of dispersion in an optical fiber and we will consider material dispersion in the later class not really important at this point we will consider wave guide dispersion and then model both these dispersions by considering β has a function of.

(Refer Slide Time: 09:27)



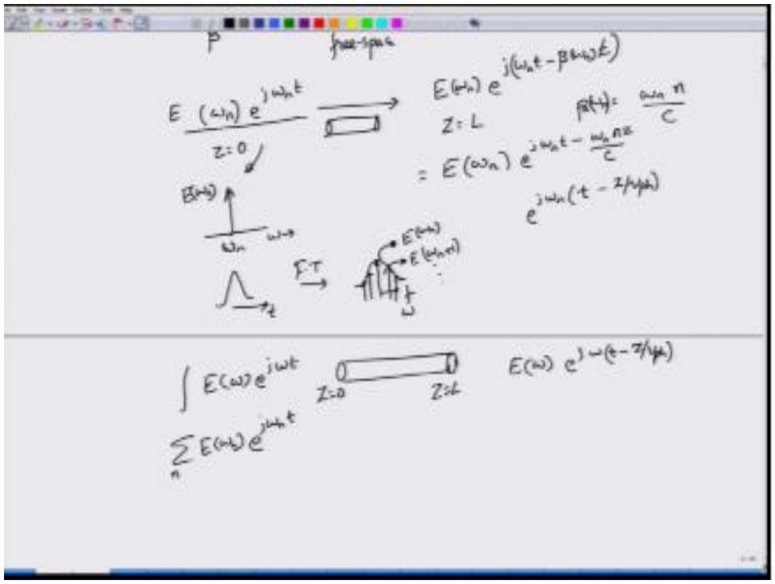
Frequency okay because I know that β is given by ωn/c so for any frequency β at least in the free space is given by ωn/c okay so in this particular case if you now look at the ratio of ω/β from your under graduate electromagnetic classes you know that this ratio of ω/β is called as the phase velocity right so for the free space case where n is independent of frequency c being the speed of flight is independent of frequency and ω being the frequency itself for the free space condition this phase velocity will be equal to c okay.

Assuming that you are sorry the phase velocity will be equal to n/c okay assuming that n is independent of c then all frequency components will have the same phase velocity suppose I consider frequency component ωn I have an amplitude of this frequency component as say e of ωn let me write down the time dependent term as well so this is the pulse which I am launching at z = 0 okay when you launch this was so this is z = 0 and after propagating a length of z okay this is the length of propagations z = L the frequency component arrives here with the phase shift $\omega_n t - \beta$ of ω_n x L okay.

However what is β of ωn β of ωn is the function of β for a given value of ωn and this is simply given by ω_n x n/c this ωn is the nth frequency component that I am considering some frequency component so if you rewrite the electric field that you are seeing at the output it turns out to be e sorry e of ωn this is the amplitude of the electric field at the frequency ωn you have $e^{j\omega nt}$ – ω_n n x z/c right so this is what you get you can remove ej ω_n outside so I can remove this ωn from common factor in this numerator.

So what I get is t – z/vph where vph is the phase velocity now you might object this analysis and say well I am not going to send only one electric field ωn so if you look at it is frequency spectrum right so in terms of the frequency spectrum you will see that this particular sinusoidal signal will have an amplitude of e of ωn and it is present only at ω=ωn this is single frequency component, but we know that we are not going to sent single frequency components through the fiber.

(Refer Slide Time: 12:12)

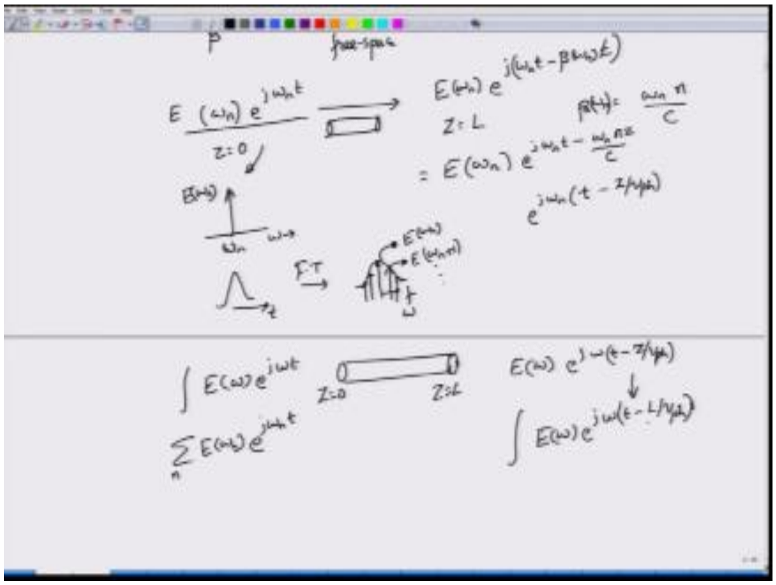


We want to send a pulse through the fiber and a pulse will have a certain frequencies spectrum so if this is the time behavior then taking the Fourier transform will give you the frequency domain picture of this one f or ω equivalently, so there is actually a spread of frequencies around it, right. So that way incident of talking about a single frequency component what I can do is I can talk about E(ω), right I can sum up all this different frequency components summing up a group of frequencies in the limit is equivalent of integrating the whole thing.

So integral of E(ω) $e^{j\omega t}$ clearly this is an integration that I am doing is actually meaning that I take many, many components whose amplitudes can be some E(ωn), right and then I am multiplying this by $e^{j\omega nt}$ so this is essentially many, many frequency component, so you go to the frequency domain picture these are all the different frequency components each having an amplitude so this nth frequency component has an amplitude of E(ωn) this fellow has an amplitude of E(ωn)+1 and so on.

So if you sum all this you get this equation which is what your launching in the fiber, okay. So this expression captures the pulse that is being launched into the fiber at Z=0, at Z=L each frequency component would arrive at a certain phase velocity or a certain phase delay, but luckily for us the phase delay is independent of frequency, so all these frequency components are arriving at the same phase delay, right. So this is very critical, please remember this step that we have just executed.

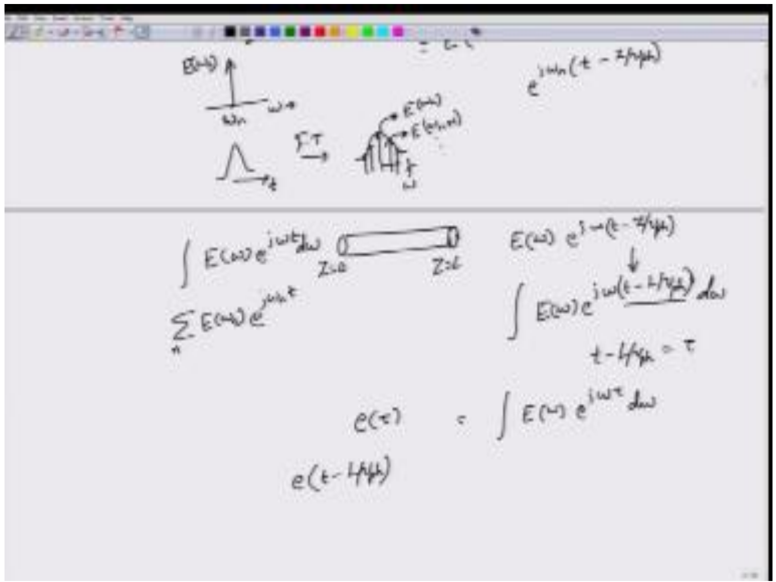
(Refer Slide Time: 13:55)



What we are saying is that, you have a particular frequency component this frequency component propagates through the fiber, okay and when it propagates a length L through the length and arrives at the fiber what it happens, what the time it takes here to arrive would be some L by phase velocity and because this phase velocity is independent for all the frequency components all the frequency components of the input signal arrive at the same time, okay. All frequency components arrive at the same time.

So at Z=L this I should of course write this one as $e^{j\omega t-L/vph}$ and this L/vph is the total time taken by the frequency component ω in arriving at the receiver. So you can multiply this one by E(ω) this integral stills remains the same, integration if you see integration think of this as summation, okay so this is what you get.

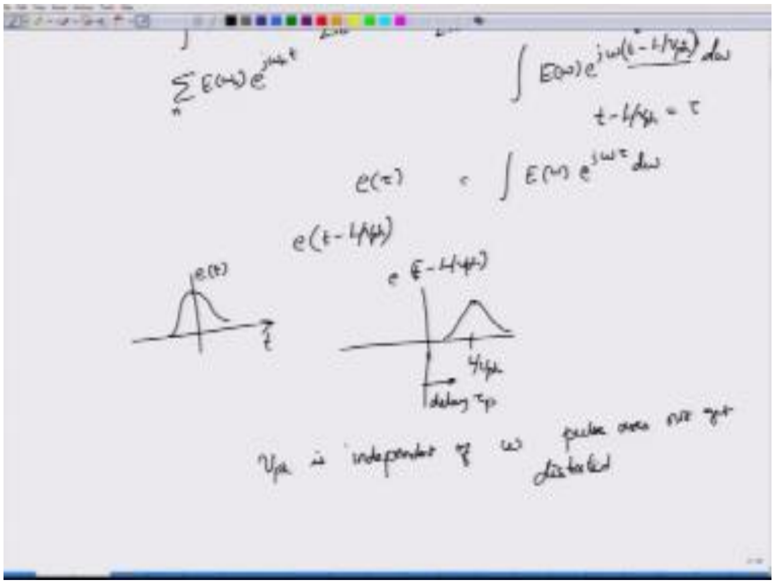
(Refer Slide Time: 14:53)



Now if you want you can take the Fourier inverse transform out of this, so sorry we have to also multiply this one by dω I am representing this in the Fourier transform domain, right. But you look at this, this $e^{-j\omega L/vph}$ right, so this is just like a phase for me, right and if you look at this, this particular thing is simply a constant, right. I can redefine so what I want to say here is that I can take this t-L/vph and call this as τ and I get this E(ω) $e^{j\omega\tau}$dω, right.

And this τ being independent of the frequency, right is just a constant as far as this integration is concern, because this integration is going over ω, so as far as this integral is concerned this τ is a constant and this is a dummy variable, so you can see that the Fourier transform of this one will give you the pulse in the time domain with e(τ), but e(τ) is nothing but e(t-L/vph), right. So go back to your signals and systems.

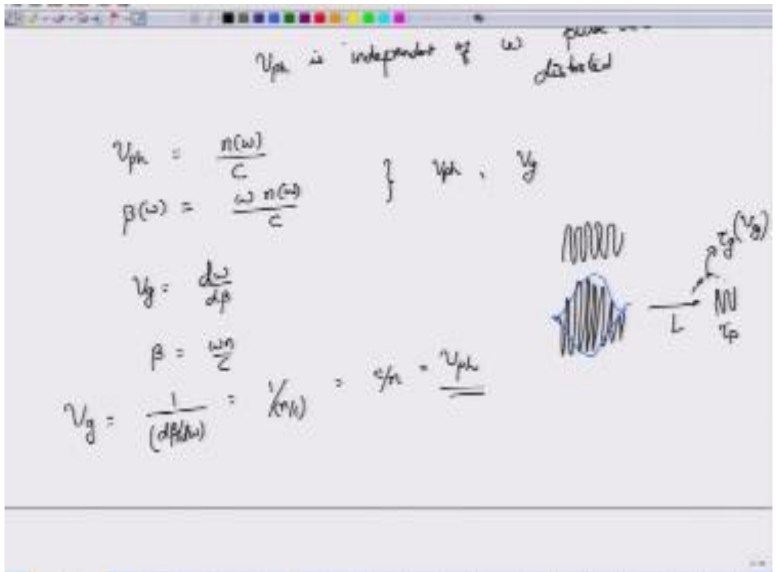
(Refer Slide Time: 16:03)



Theory and then see that if this is your e(t)so this is t and this is you time doming function e(t) then the function e(t) – l/vph is simply the same pulse completely unchanged except that this is delayed by a factor of l/vph so this is the amount of delay τ that you are looking at, this delay incidentally is called as the phase delay τp okay. The important point to note here is that as long as the phase velocity is independent of the frequency right.

The pulse does not get distorted it does not spread it simply arrives after a certain time which is the okay I mean I have sent a pulse now after one micro second or two micro second the pulse would arrive at the receiver it is not a big deal for you as long as the pulse has not sp[read out the pulse has not distorted it will simply arrive after a delay. And this miracle happens only when the phase velocity is independent of frequency.

(Refer Slide Time: 17:14)



However when you considered met4erial dispersion this phase velocity will not be independent of frequency rather what happens is that because your n will be a function of Ω now n is the function of frequency or n is the function of wave length because of this one you will see that phase velocity is now given by $n(\Omega)/C$ right assuming that c is the speed of light is a constant this is what you get.

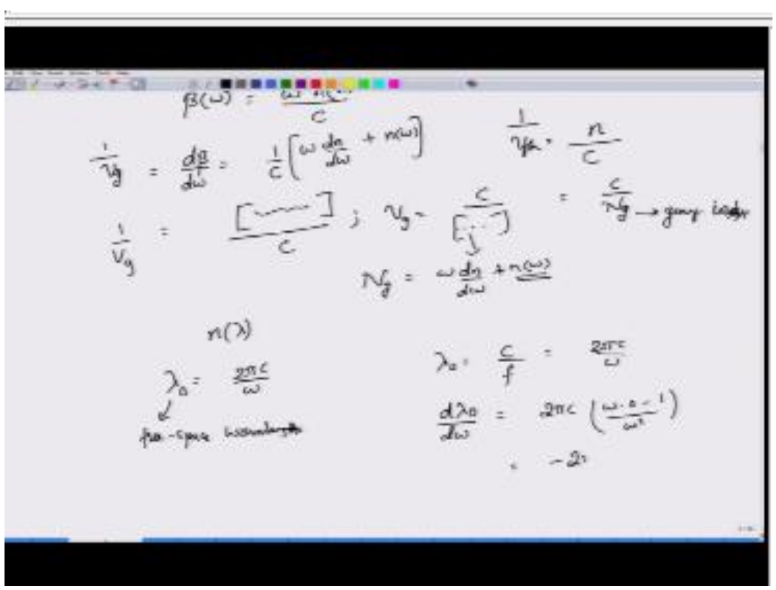
And your $\beta(\Omega)$ the propagation constant for any given frequency Ω is given by $\Omega \times n(\Omega)/C$ clearly this propagation constant will be different because for different values of Ω the value of n will be different okay. So because of this reason what you now see is that energy does not really propagate at the phase velocity but rather energy propagates at what is called as the group velocity.

So whenever you talk of a pulse or envelop especially the one there is the modulated by a carrier frequency when it propagates the carrier will arrive so this is your carrier when you modulate a carrier you might be generating a modulated pulse. Okay so you might be generating a pulse like this, this is the envelop of the pulse which is propagating so I should probably write it like this, so when this pulse is launched in to the fiber it propagates through the fiber of a certain length.

Then the carrier would arrive at a delay of τp whereas the group sorry the envelop would arrive at a delay of τd which is called as the group delay. And this group delay is the function of group velocity vg okay and vg is defined as $d\Omega/d\beta$ what is the group velocity for the case of free space propagation or the propagation that we saw in the previous you know 10 minutes. In previously what we saw was β was given by $\Omega n/c$ and n was independent of frequency correct and if you evaluate this $d\Omega/d\beta$.

Which is nothing but $1/d\beta/d\Omega$ correct just inter changing this equation or rewriting that equation but β is $\Omega n/c$ so $d\beta/d\Omega$ will be equal to n/c right so 1/n/c is nothing but c/ n which is simply equal to the phase velocity of the medium. So in the case when $\beta = \Omega n/c$ and n is independent of the frequency c is just the speed of light then group velocity vg will be equal to phase velocity and this is the reason why the envelop also arrives at the same time so group delay and time delay are equal.

(Refer Slide Time: 20:03)



However in material dispersion case β is a function of frequency, right? It is given by $\omega * n(\omega)/c$, this $n(\omega)$ depends on the material constant, and this is not independent of frequency, so if you now try to take this d β/ d ω, what you get here is 1/c is a constant I can pull this out, you get ω dn/d ω+n(ω),right? So you see here that this d β/d ω is actually given by 1/ group velocity correct?

So group velocity is d ω/ d β, but 1/group velocity is this one, so you can see here that this can be return as [some term/c] for the phase velocity we had some refractive index "n" or rather 1/phase velocity we had n/c here, similarly here you have some number divided by c, for 1/Vg or equivalently for Vg=c/ [...] this term is nothing but ω dn/d ω+n(ω), okay,.

This term is what is normally called as the group index Ng. So you get Vg c/Ng, where Ng is the group index, okay. The group index is the function of frequency because N is the function of frequency, you can actually talk in terms of the wavelength here, you know you can convert this relationship into wavelength relationship, assuming that n is a function of λ that is represent "n" as a function of λ rather than representing this one as the function of frequency.

Then you can show that you take λ = 2 π c/ ω is that correct? Because ω =2 π(f) c/f = λ , so you get λ here, which is the free space wavelength may be we just write this one as λ 0, so λ 0 is the free space wavelength, okay, given by this expression.

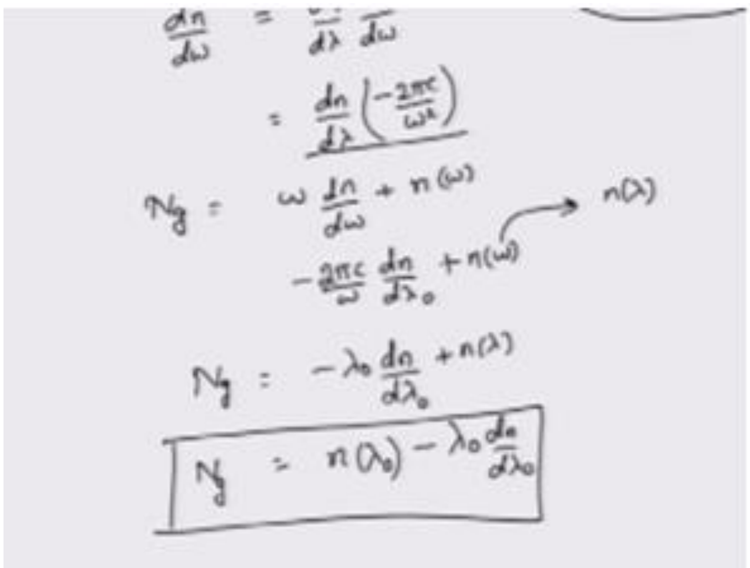
Now if you work to differentiate this one, i know that λ is given by c/f or 2 π c/ ω , any way this is the same this one, so if i now differentiate this λ with respect to ω, okay i can do this what do get, i get here as 2 π c is a constant, what is d / d ω (1/ ω) = ω (0-1)/ $ω^2$ you get here as 2 π c/ $ω^2$ okay.

But what you want is dn/d ω, but dn/d ω can be written as dn/d λ d λ/ d ω, so this d λ, d λ will cancelled with each other, therefore I can write this one as dn/ d λ , I already know what is d λ /d ω, d λ/ d ω is nothing but -2 π c/ $ω^2$ but in the group index or the group index here, Ng is given by ω*dn/ d ω +n(ω) this ω (dn/d ω) can be obtained, because dn/d ω is this fellow which we have just calculated.

Multiplying this one by ω will cancel out an ω here, so what you get is ? $-2\pi c/\omega \, dn/d\lambda + n(\omega)$ rather than $n(\omega)$ we can call this as $n(\lambda)$ because that is what we have said that right? So n can be considered as a function of ω or it can be considered as a function of λ and $2\pi c/\omega$ is nothing but λ_0 therefore this is $-\lambda_0 \, dn/d\lambda_0$, sorry this has to be λ_0 , as we have considered this as the free space wavelength plus $n(\lambda)$. This is the group index Ng, equivalently I can rearrange terms and then write this as $n(\lambda_0) - \lambda_0 \, dn/d\lambda_0$.

So as long as the refractive index changing with the respect to wavelength, you have a group velocity which is dependent on λ and this change in velocity, you know which is dependent on the particular λ that is propagating through thee fiber is called as medical dispersion and causes material dispersion and causes pulse spreading.

(Refer Slide Time: 24:43)

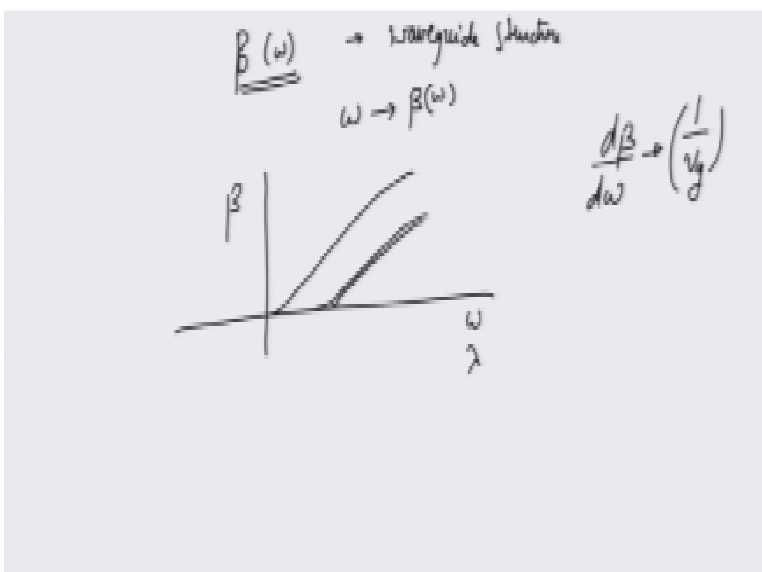


The other dispersion that happens is the chromatic dispersion, so her you have β as a function of frequency ω and this is purely because you have a waveguide structure and this equation for $\beta(\omega)$ is not very easy to find out, because what it require to do is find first given sequence ω, what is the corresponding propagation constant $\beta(\omega)$, however remember how do I calculate $\beta(\omega)$ for a single mode fiber, I have to go back and write the big complicated equation.

In the last two modules we have seen in discussing the mode of the optical fiber, how do I obtain the propagation constant β, I have to first express w in terms of u, find out what I u, from u I have to find out β , so that is the long procedure which I have to follow, this big equation that we wrote down some Jnu+Knu into $k1^2$, you know that equation that we wrote down or even it is simplified version of weekly guided approximation, calculating value of β as a function of ω is a tricky and numerically oriented problem, luckily for us people have solved it earlier.

And the results are these equation which we wrote down slightly earlier in the module today, so this gives you β as a function of ω, if you would like you can also make β as a function of λ, so instead of talking about β as a function of ω you can talk about function of λ and you will be able to obtain through numerical methods how this β actually varies with respect to ω and because β is not a straight line, the derivative of β varies with respect to ω, which will give you 1/group velocity will not be a constant term, I meant to say that.

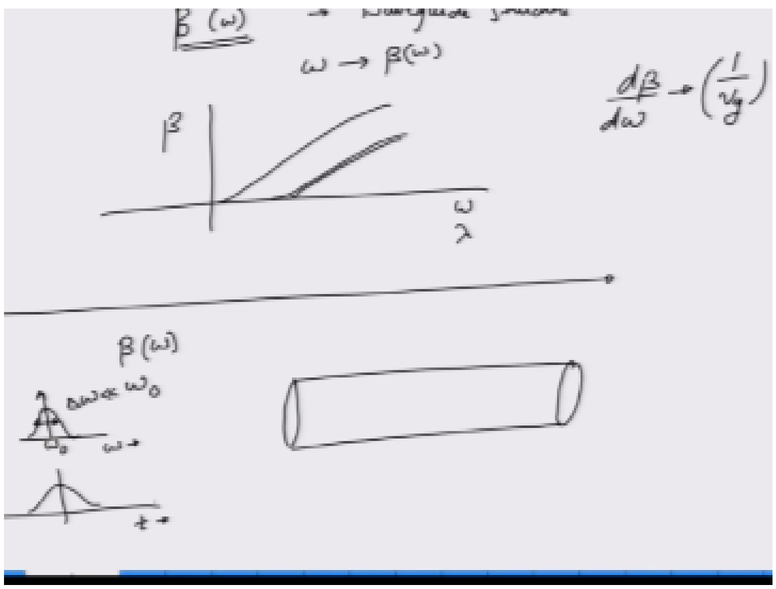
(Refer Slide Time: 26:43)



Because β is non liner function of ω, now regardless of what kind of dispersion that we are considering, what seems to be clear is that I can consider β as a function of frequency ω or equivalently terms of λ and if I consider a modulated spectrum, so I consider a carrier at say ω_0 and there is a modulated spectrum around that with width called this as $\Delta\omega$ is much smaller than responding to the center frequency or the carrier frequency ω_0. So this is the frequency domain, if you look at the time domain, you will actually see a time dependent or the time waveform here.

The objective for us would be try and find what happens to this pulse when you have non 0 dispersion in the fiber and that is what we are going to do in the next module. Thank you very much.

(Refer Slide Time: 27:37)



<u>Acknowledgement</u>

Ministry of Human Resource & Development

Prof. Satyaki Roy

Co-ordinator, NPTEL IIT Kanpur

NPTEL Team

Sanjay Pal

Ashish Singh

Badal Pradhan

Tapobrata Das

Ram Chandra

Dilip Tripathi

Manoj Shrivastava

Padam Shukla

Sanjay Mishra

Shubham Rawat

Shikha Gupta

K. K. Mishra

Aradhana Singh

Sweta

Ashutosh Gairola

Dilip Katiyar

Sharwan

Hari Ram

Bhadra Rao

Puneet Kumar Bajpai

Lalty Dutta

Ajay Kanaujia

Shivendra Kumar Tiwari

an IIT Kanpur Production