**Indian Institute of Technology Kanpur**

**National Programme on Technology Enhanced Learning (NPTEL)**

**Course Title**
**Optical Communications**

**Week – V**
**Module – IV**
# Modes in Optical fiber-II

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Hello and welcome to the online course on optical communications in the previous module we were discussing the modes of an optical fiber we started with 4 step process we looked at the Maxwell's equation expressed the transfer's components $E_R$ $E_Ø$ $H_R$ and $H_Ø$ in terms of the longitudinal components $E_z$ and $H_z$ then we solved the M holes are the wave equation for $E_z$ and $H_z$ and found solutions corresponding to the core region as well as solutions corresponding to the cladding region in core.

We have seen that the solutions are of the form of Bessel functions of the order η and first kind where as in the cladding we have Bessel functions of the second kind and order η right so this is what we have done at the previous module where we had stopped at that point where we had to apply the boundary condition in order to evaluate the arbitrary constants so once you evaluate the arbitrary constants then you can go back and find out $E_r$ $E_Ø$ $H_R$ and $H_Ø$ because you have already obtained Ez and Hz so we will continue the discussion.

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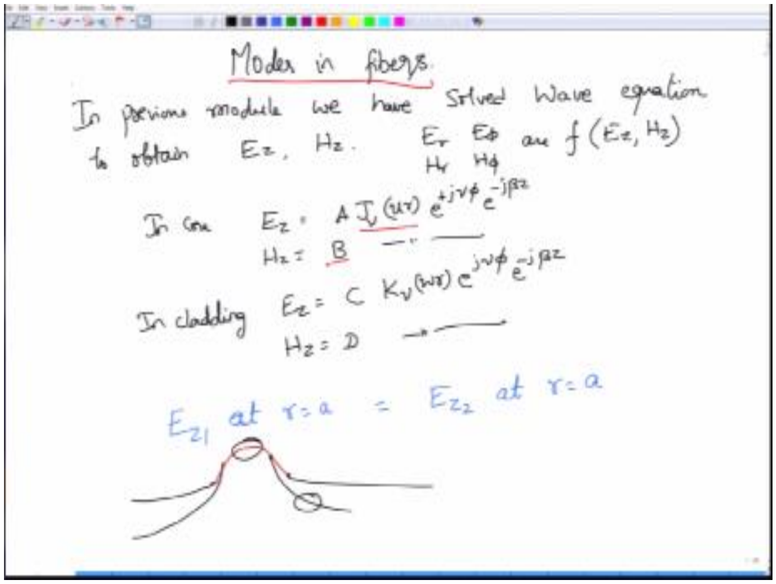


Of modes in an optical fiber I mean and this is just a simple summary of what we had done which I just described to you we know that Er and $E_Ø$ $H_R$ and $H_Ø$ are functions of $E_z$ and $H_Z$ and in core we see that EZ as a function of R will be Bessel function you know of the order η is an integer in this case u and w are positive constants which are related to the k vector and the propagation constant vector β as we have disused in the previous module η again is the order and with respect to Ø it goes as $E^{j \eta Ø}$ right it simply tells you what is the angular distribution of the modes is so if η is equal to 0 then the fields are independent of Ø if η = 1 then you know the functional dependence on Ø is like cos Ø and sin Ø similarly you will have cos2Ø and sin 2Ø indicating the kind of angular rotations or angular wave numbers.

In with respect to z you have $E^{-j \beta z}$ this propagating solution that we are considering inside the fiber therefore in the fiber you want to oscillatory modes with respect to r and angular distribution that is determined by $E^{j \eta Ø}$ as well as propagating terms in the z direction which is $E^{-j\beta z}$ there is a constant a which is multiplying this electric field similarly the equation for Hz will also be exactly in the same form as the equation for $E_z$ and the solution would be then of the same form except that it may have a different arbitrary constant β right.

Oh sorry arbitrary constant B the reasons why B is not the same as a should be obvious because if you take $\delta$ x E then that $\delta$ x E must give you H and the in the process of taking $\delta$ x E there will be additional constant which would come into this equation because of that you have a different constant B which is taken here in the cladding region of course $E_z$ = some constant C times the Bessel function of the second kind right these are the exponentially decaying Bessel function as we have discussed in the previous module and boundary conations.

What it does is to stitch the oscillatory behavior in the core t the d decaying solutions in the clad so if the core mode you know starts for example if the core mode is Bessel function $J_0$ then it is maximum at r= 0 and then as r starts to increase the Bessel function keeps on reducing so let us say at some constant r = a you stop this process okay so you have Bessel function which is nicely decaying like this at some point you stop this one okay this of course would correspond to the actual core radios but you can consider this to be any arbitrary point.

Now at this point the field will have a certain amplitude $E_z$ because it is starting from peak value at r= 0 and it would have drop to a certain value because this is the $J_0$ function which we are considering the Bessel of the Bessel function of the $0^{th}$ order so it drop and it would reach a particular point okay so let us say some value here which is Ez at r= a now Maxwell's equations tells me that the field must be continuous at this point because the tangential electric field across 2 dielectrics must be continuous across that particular boundary therefore at this point if $E_z$ as doped and it has become $E_{z1}$

Then the new field must take off from that point and decay as the radial distance increases so you have to just adjust the amplitudes of the decaying constant as well as the amplitude of the $J_0$ or $J_1$, $J_2$ depending on what mode solution that you are looking for okay so that is essential what the boundary condition does to you this is of course pictorially but what you actually are going to do is step up a set of equations which tell you that $E_{z2}$ which is the electric field in the core region at r = a which is the core and the cladding interface must be equal to Ez2 at r = a so inside the core assuming that we are looking at $J_0$ type of a solution.

This is how you would start you know it would be at certain peak value and then it would start to drop like this and it r =a it would have reached this point next what this condition implies is that the delaying part of the solution must began at the same point and then it has to decay out like this okay and of course the modes are symmetric on both sides so which means that the modes are you know going along for r less than a as such or that r in the other way round as well and again you have to stitch the solution here and the let the mode go to no let the mode basically get decayed suppose you reduce the core radius.

Then what happens well them the stitching as to start at this point right because the field would stop at here for $E_{Z\,1}$ so $E_{Z2}$ must began at the same point and then go in this decaying fashion so this is what the boundary conation is basically doing it is just basically evaluating what is the amplitude of this one and what is amplitude of here and how to adjust these amplitudes such that I am able to evaluable I am scarifies the boundary condition as well as evaluate the arbitrary constants.

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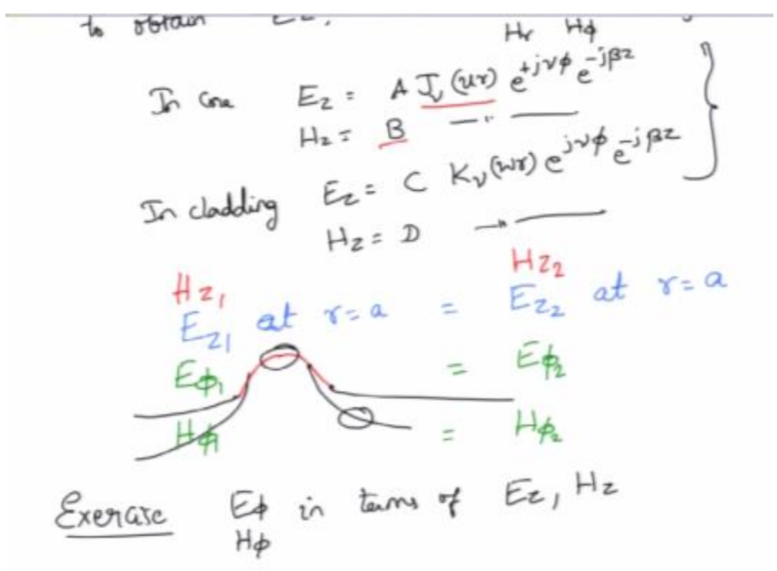


Simpler to the continuity of the tangential electric field components there is also continuity of the tangential magnetic field components so your $H_{Z1}$ must also be continuous at the same boundary okay for the r= a interface $H_z$ is also a tangential component of eth magnetic field so that must be continuous you also get two additional constrains one is that $E_{Ø1}$ which is the angular component of the electric field at r= a but must also be equal to $E_{Ø2}$ which is the angular component of the electric field at r= a so you can imagine you know you take this cylindrical cross section like this and you look at the cross section here r= a you see that $E_z$ will be something that is coming out like this correct.

So to this boundary this $E_Z$ is actually tangential component right to the same boundary this fellow you know the one which is going like this which would be the angular where component $E_Ø$ is would also be tangential however the radial component would be coming out of the surface like this, right. So if you imagine this as cut circular cross section the radial component would be coming out like this, so for this curve at r = a which separates the core and the cladding.

This radial component is a normal component and that is not the constrain that I want to apply, so for r = a in the cylindrical coordinate system $E_z$ and $E_Ø$ are the tangential components. For the electric field $H_z$ and $H_Ø$ are the tangential components for magnetic field so the last one is that $H_{Ø1} = H_{Ø2}$ so you can take a look at this small exercise which will allow you to express $E_Ø$ and well as $H_Ø$ in terms of $E_z$ the longitudinal component $E_z$ and $H_z$ so if you solve this exercise in the previous module which I had recommended you to do so you would then be able to use the values of $E_z$, $H_z$ given in this two equations for the core and the cladding.

And obtain the expression for $E_Ø$ and $H_Ø$ so I am assuming that you will take this exercise seriously and do this one, okay.

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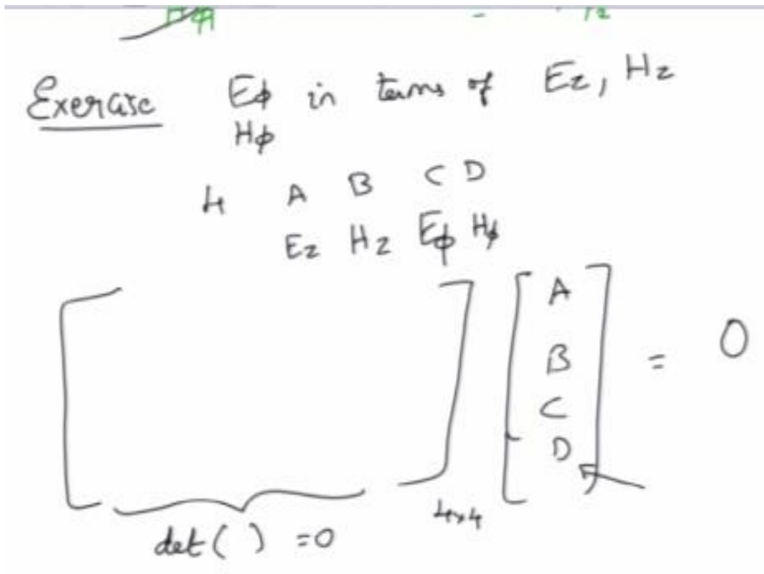


And once you have that you will actually develop 4 equations okay you have 4 unknowns which are A B C and D and there would be 4 equations coming in from $E_z$ A and B are the amplitudes of the electric and magnetic field Z components, C and D are the electric and magnetic field components $E_{\varnothing}$ and $H_{\varnothing}$, so you get 4 equations 4 unknowns the easiest way to solve this one is to write down a matrix right.

So you have the 4 unknowns written out as A B C and D replaced in the matrix here and then there will be a 4 x 4 matrix over her which then should be equal to 0 that is the solution of the linear equation, I mean linear simultaneous equations, now if you want all non zero values of A B C D right, in general you want non zero values of A B C D the trivial solution would be that when you set A B C and D all equal to 0.

In that case there are no modes there are no problem I mean you might be tempted to do that but you would not learn anything from optical fiber if you do really that one, sop rather than that what you have to do is that to make A BC D in general non zero this matrix whatever that matrix that I have obtained by applying the boundary condition this determinant of this matrix must be equal to 0.

So if I do that and this again I can warn you that it is not very nice thing to do it because it takes lot of time it is not difficult but it is just that it is very tedious, okay. So it will take a lot of time but if you really put open and paper together and the spend some time you will be able to find that determinant and the solution of that equation determinant of the matrix equal to 0 which I am not going to write the matrix here. You can refer to the text book for the entries of the matrix, okay.

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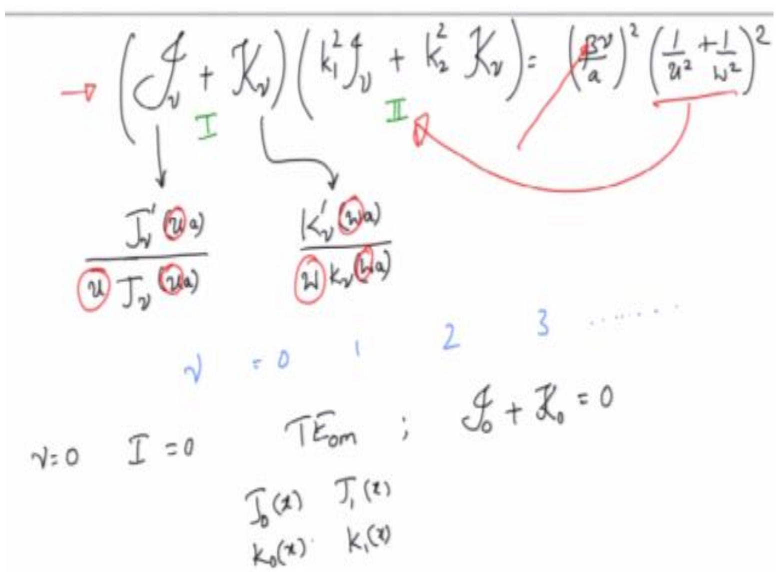

The solutions turn out to be $J_\nu$ $K_\nu$ sorry these are not v these are ν times $k_1^2$ J ν + $k_2^{\ 2}$ J ν = (β ν/ a)$^2$ 1/u$^2$ + I/ w$^2$ the power 2, okay. So you see that this is the solution where of course J ν is actually given by another complicated expression J ν' ( ua) / J ν (ua) there is also u that is getting multiplied k ν is given by kν' which is a derivative of the Bessel function with respect to the argument.

So kν'(wa) / wk ν(wa) and similarly here Jν is this one and Kν is this one so you have substituted all this and then try to solve this equation a simple warning this is going to be very difficult you will not be able to solve this analytically you will have to resort the some numerical technique to solve this problem, okay. To obtain the roots of this equation which will tell you what the value for β is.

Remember the goal here is to actually get β, okay and you cannot simply say Oh! $\beta \nu / a^2$ is equal to this quantity I can take it into the denominator and then take the square root of the whole thing it does not work that way because this fellow u contains β implicitly w contains β implicitly there is a u which contains β this one also contains β this is also β this is also implicitly containing β correct?

Because what is $U^2$? $U^2$ is nothing but $k_1^2 - \beta^2$ so there is β there also, so these equations are not the simple equations that you would be able to solve it these are quite complicated once so the suggestion is that you do not really have to do it analytically it is quite impossible whereas you can use some numerical method to solve for the equations and in the assignment problem we will walk you through the solution of this equation using the numerical technique which you can program using the language such as Mat lab, okay.

So for now just take that this is the equation that you have to solve in order to obtain β, okay. Now there is one parameter which is sitting here. The parameter that is sitting here which is not specified is this order ν am I free to choose this order ν turns out that yes I can choose the order ν and ν starts from 0,1,2 and so on, okay If you choose ν = 0 what happens to the right hand side of this expression, the root equation that we are writing right hand side will simply disappear because it is getting multiplied by $(\beta \nu/a)^2$ and ν is 0 so this entire right hand side goes away.

Then you have to solutions right two possible solutions one is that, this term which I am denoting as 1B = 0 or this second term 2D = 0 it turns out that if 1 = 0 that is Jν + kν = 0 then this corresponds to solutions of the form $TE_{0M}$ okay and this of course will happen only when ν = 0 therefore the solution actually has to be $J_0+K_0 = 0$ and you can then use some recurrence relationship which relates $J_0(x)$, $J_2(x)$, okay.

Similarly $K_0(x)$ and $K_1(x)$ the Bessel functions of the first and second kind you can use some recurrence relationships for the Bessel function.

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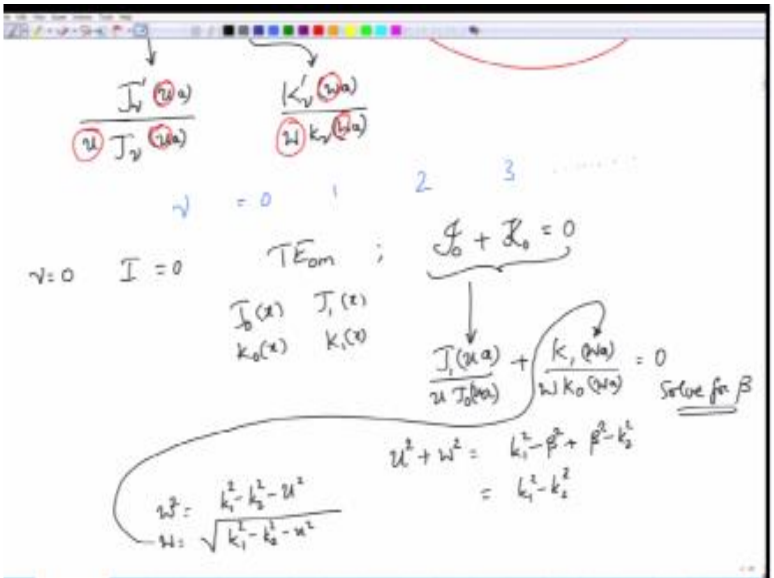


In order to solve this equation or rather write down at this equation in a slightly easier form to solve that would turn out to be $J_1$(u, a) / u$J_0$ (u, a) + the $k_0$ term is $k_1$(wa)/ w$k_0$(wa) so this entire thing must be equal to 0, normally what happens is that if you already have a fiber A the radius gets determined already, right so and $k_1^2$ because in ν $k_1$ $^2$ also gets determined because it is the property of the material.

The unknown factor is still β but β you will be able to obtain by solving this equation right, so you can solve this equation to obtain solve for β given all the other parameters, now you might wonder a little bit here well I have u and w also what do I do about that? Right, you can actually relate $u^2$ and $w^2$ it turns out that if you take $u_2$ + $w_2$ because $u_2$ is nothing but $K_1^2 – β^2$ and $β^2$ – $K_2^2$ the sum of $U^2$ and $w^2$ is nothing but $K_1^2 – K_2^2$ now you can solve this equation and write $W^2$ = $K_1^2 – K_2^2 – U^2$ and then say W= √ of this entire thing so you have $K_1^2$ $K_2^2 – U^2$

So you can put this W into this expression there by eliminating the requirement of having two variables U and W okay so you can put all these things into 1 so this equation that you are looking for which is the equation that tells you the value of β will now

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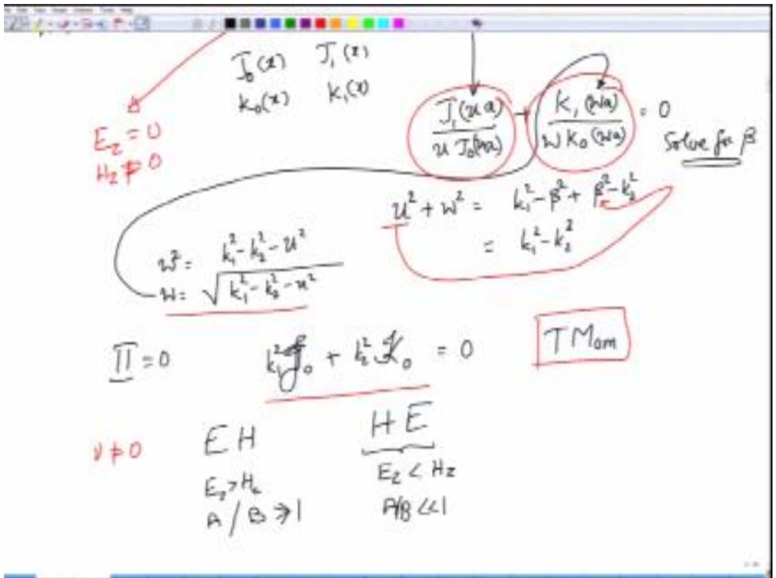


Have only U term sitting inside and when you solve this equation you are going to get a solution for U but U and β are related so one you find U from this you find β okay, so like find U and then find β so this is what you would have to do and when you do that you end up with what is called as at $TE_{0m}$ modes okay now suppose this is not the term that is 0 suppose the second term si equal to 0 which implies that you know go to that equation $K_1^2$ J0 + $K_2^2$ K0 must be equal to 0 again you can sue some reoccurrence relationship so you have $K_1^2$ $J_0$ this is the script $J_0$ = $K_2^2$ $K_0$ = 0 so these are not the basal function these are those expressions for script $J_0$ and script $K_0$ that we have written okay.

So when you take this term equal to 0 the solutions that turn out to be are called $TM_{0m}$ I will just explain to you what the meaning behind this T is 0- I mean $TM_{0m}$ when you take term 1 = 0 I said that the modes are $TE_{0m}$ it means that the electric field component $E_z$ actually vanishes Hz will be non zero and depending on the other this when all the other components are present okay now look at what is happening Ez is 0 which means there is no component along the fiber access all the electric field that is there will be in the form of Er and Eϕ and they are in the plane that is perpendicular to the z axis.

So this is the z axis this is the plane that is perpendicular to it because Ez is 0 the electric field lies in the plane that is perpendicular to the access of propagation so this perpendicular to the access of propagation means that the electric field is transfers to the direction of propagation and because of that we call this as $TE_{0m}$ mode 0 stands for the way in which it would be behaving in terms of radial direction that is radically how the mode would spread out or how would it would it would behave and M stands for how it would change.

Aseamothele you know along angularly how would it change if m is also 0 there it would change well it has no dependence on ϕ and unluckily or luckily for us there is no such mode as $TE_{00}$ mode so M has to be equal to 1 at least in order to get the first mode okay so you get $TE_0$ one mode similarly you get TM01 mode now the meaning of Tm should be very clear to you when I consider the same z directed propagation Hz is 0.

So the magnetic field will lie entirely in the plane that is perpendicular to the z axis Hr and Hϕ okay so this Hr and this is Hϕ so this is called as a transfers magnetic mode similar to $TE_{01}$ the lowest order mode that can propagate is $TM_{01}$ so what do we mean by lowest order values the lowest order propagation means that for a given value of η and m there exists a certain non zero field patter.

If you take TE00 then there is no pattern for that and therefore such a way will not exist at all okay, then what is this pattern, pattern is basically that dependence on r and dependence on ϕ it is the plane that is perpendicular to the access and in that plane if you for a example take $\eta = 0$ then the fields would decay like this they would have a maximum value at the center but then they would decay in this fashion right. In a like a Basel function however if you consider $\eta = 1$ then the solutions will be of the form $J_1$ in that case.

J1 you know that r = 0 starts of it 0 itself and then it goes up like that so at the center of the fiber the field will be equal to 0 so these are the different types of modes that you are going to get we have sin $TE_{0m}$ $TM_{0m}$ the correspond equation I will leave this as a simple exercise for you to write down because I have already written down what is J0 so this is basically $J_0$ and this $K_0$

okay so from $J_0$ and $K_0$ you can substitute that $K_1^2$ you know $K_0^2$ and $1^2$ $K_2^2$ is $K_0^2$ and $2^2$ substitute and then get this equation okay.

Again you can make this substitution for W you can express W in terms of U find U then find β okay that is how you would find the value of β okay so this was $TM_{om}$ in general for η is not equal to 0 you unfortunately end up with two types of waves called as EH waves and HE waves EH waves means that E is greater than H that is $E_z$ amplitude is greater than Hz amplitude Ez amplitude remember was A in the core $H_z$ amplitude was B in the core so A/B if this is much larger than one then we call this as the hybrid EH mode okay or we sometime call as electric hybrid mode.

Similarly HE mode will occur when $E_z$ is less than Hz mode okay so you get A/B much less than one so this called as hybrid electric mode electric hybrid mode and hybrid electric mode okay so if the confusion wise a little bit because you know it is the hybrid mode notations are little confusing this are the old terminology but unfortunately they are struck with us okay if you look at what happens to the situation let us consider this situation suppose I have.

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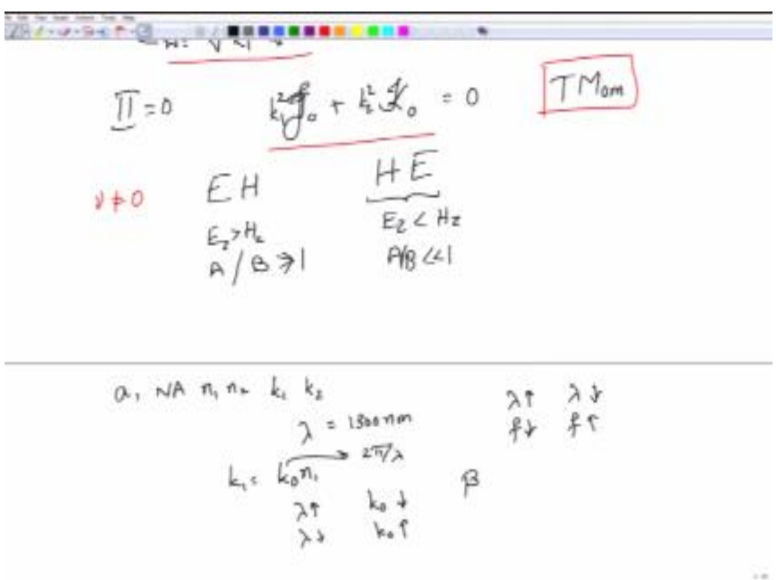

Radius a fixed I have the numerical aperture of the area I mean of the fiber fixed which means that I have six step n1 I have fixed up $n_2$ in turn I have fixed up K1 I have fixed up $K_2$ okay suppose I consider 1λ let say this λ is 1300 nanometer okay and I consider what happens as λ starts to increase or λ to decease λ decreasing means frequency is increasing means frequency is decreasing you can go both ways okay in any way you take whether it is increasing or decreasing as you change λ something else changes right λ is related to K0 which is the free space wave length.

And $K_1$ is given by $K_0$ times $n_1$ I have fixed n1 as I change λ suppose λ increases then $K_0$ decreases because $K_0$ is nothing but 2Л/ λ right so λ is in the free space so this 2Л/λ so as λ increases wave length increases $K_0$ decreases or λ decreases K0 increases what happens to corresponding value of β, β will also change okay so there exist actually a certain value.

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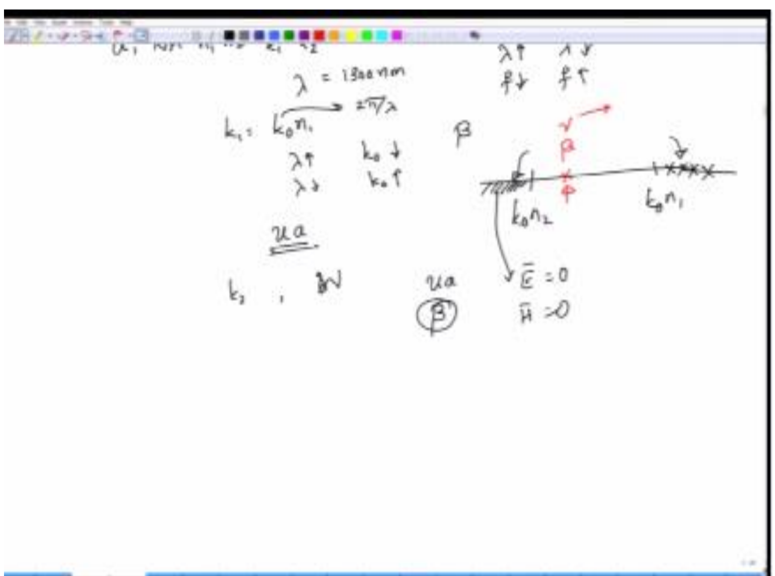

Which is required when you solve that equation there exist a certain value of β which β of course lies in between $k_0n_2$ and $k_0n_1$ it might happen in such a way that the product that you are looking at j0(ua), right so this product has to increase because u is now changing, u is changing because $k_1$ changing β is changing, of course because λ is changing $k_2$ is also changing as well as $k_2$ is also changing as well as w is also changing, so essentially u and w both are changing with all the other fiber parameters fixed except that λ is varying.

It might so happen that the product ua or the equivalently the value of β that you find out from solving the equations might turn up in this region, okay. If it falls below $k_0n_2$ after you have found β then this particular mode does not propagate, which means that the electric field will be 0 here, magnetic field will be equal to 0 here of course it is not exactly 0 because I am looking at guided modes there will be some even sent modes and there would be some radiation modes in the cladding but for the core region there would not be any excitation, okay.

Similarly, it might so happen that as you change λ, right so the value of β that you obtained from solving this equations might increase in such a way that, you may end up in this region in that region again you will see that there would not be any mode which is propagating inside the core. Now suppose by some luck or by some design you are, in this particular position for a given λ, okay which means there is a corresponding β that is associated for that one, okay and there is a corresponding η the order η that also you have found out. For η equal to something you have found out this particular condition. Now remember, for every η when you solve that equation you know you can go back t0 and equation.

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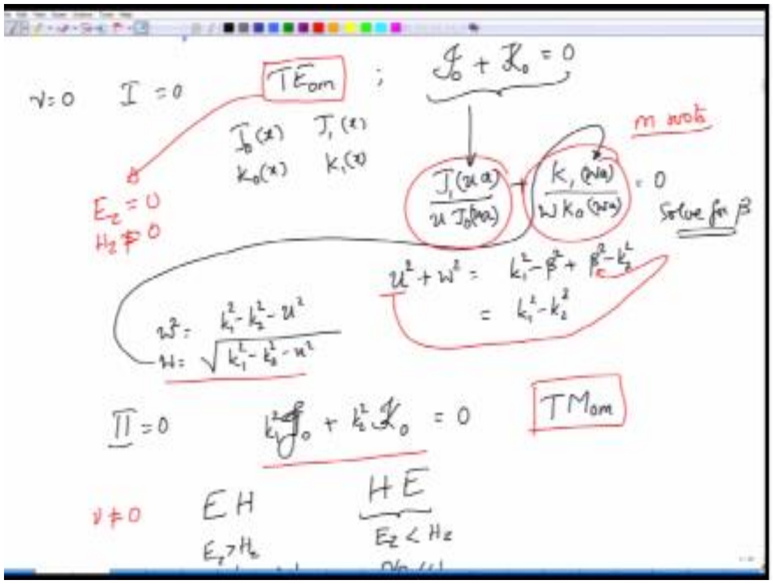


Here for every η that you consider in this case of course η is given η=0 but the solution of this equation has m roots, correct the solution has m roots and correspondent to this m roots of the equation the value of propagation constant will also be different.

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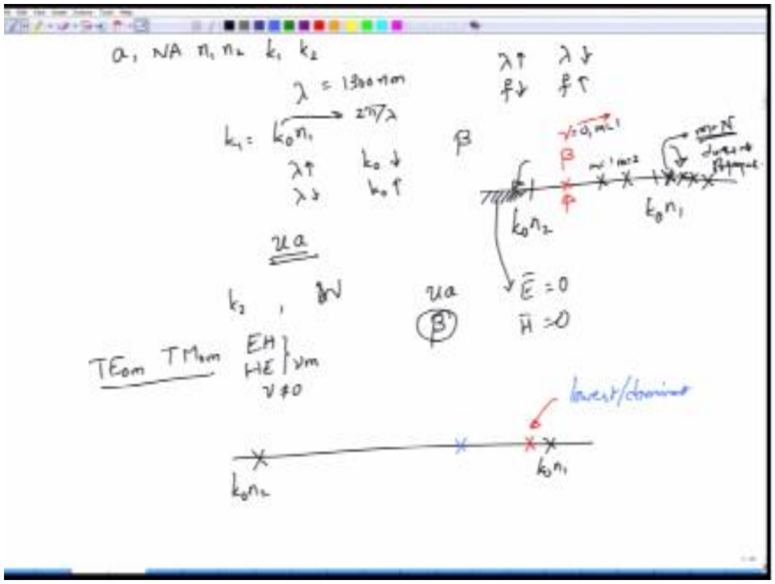

Suppose this is the case with $\eta=0$ and m=1 then corresponding to $\eta=0$, m=2 the value of $\beta$ might go to this one, okay value of $\beta$ might be here. The second solution which corresponds so this is m=1 this is m=2 or rather the third solution that you are going to look at might end up here, then it might also happen that one of these solutions which we have written outside might actually be the solution m equal to some number n, but this mode does not propagate, okay so that does not propagate.

Now let us go back and then look at the modes that we have, we have TE0m, TM0m, EH and HE both are $\eta$ times m, $\eta$ can of course be equal to 1 here and $\eta$ is not equal to 0, because the $\eta=0$ you get these two modes. Now let us ask this question, what is the lowest order mode that is propagating? By lowest order mode what we means is that you start with $k_0n_2$ and $k_0n_1$ as the range of the values of $\beta$ for which the mode can propagate, the lowest order 1 is the one which actually starts of by just being as soon as you start increasing certain values the lowest order mode will be the 1 which has this largest value of $k_0n_1$, okay.

So the lowest order mode starts at this point, whichever mode that satisfies you know they just has value of $\beta$ slightly less than $k_0n_1$ when it begins to propagate, okay. The next mode that will propagate with a different values of η and m combination will be called as the first higher order mode, okay so this is the lowest or the dominant mode which means this mode exist right as soon as the mode conditions exists, okay.

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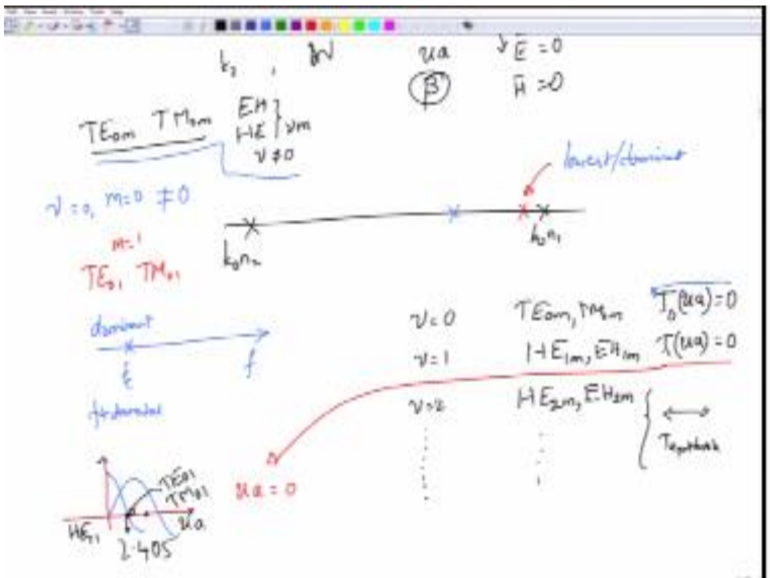


If you go to the rectangular wave guide situation and you might remember something like this, you know your frequency f is increasing, right and at certain particular point you cross what is called as the cutoff frequency, so the mode which has that lowest cutoff frequency is called as the dominant mode or the fundamental mode, okay. For the optical fiber I am drawing it in this wave because I have writing this in terms of λ rather than frequency, so you get this as the lowest order mode or the dominant mode and then you keep doing this one.

Now when you look at the type of modes over here, you might think that the lowest order mode should be the one where m=0 but unfortunately when you calculate the amplitudes for this case with η=0 and m=0 the field would not exists, okay so there would be any field that would exist at this particular mode and similarly TM00 mode also does not exist, okay. So might think oh, well maybe m is not 0 but m=1 can exist that is can TE01 mode and TM01 mode be the fundamental mode turns out that, no.

Why, is because if you solve the equations for the, to solve that big equation and find out the cutoff frequencies or rather the cutoff conditions for each mode you will see that with $\eta=0$, okay TE0m mode and TM0m mode that you are going to get will cutoff when j0ua=0, okay. So when joua=0 and when $\eta=1$ the cutoff frequency for HE1m and EH1m modes will begin with, will be obtained when you solve this equation which is j1ua=0 when $\eta=2$ you get HE2m and EH2m and of course with $\eta$ greater than 2 you will get similar equations over here.

The root for this equation is slightly complicated this is given in the text book so let me not write this particular case, okay. So for $\eta=2$ and $\eta=3$ the equation is little complex you can look at the text book for the expression, but what you have to understand here is that, look at the values of u, okay when $\eta=1$ the cutoff condition is j1ua=0 and where will this j1ua=0 will go to its first 0 write when you ua=0 because the j1 function as a function of the argument ua goes like this, right.

So this is the j1 function, j0 on the other hand would go like this, right so the cutoff condition that you are looking for you know you have to at least get to this condition so that the corresponding mode can propagate, okay so that happens at the earliest for the HE11 mode whereas that happens at a later stage you know at that happens at a later stage here for TE01 and TM01 modes, right so for TE01 and TM01 modes the cutoff condition is that they are corresponding value of ua must excite or the parameter ua must excite here, it is the point where so it is this one, okay sorry this one should be the case where you get t0 I and tn 0 1=0 and this happens at a value of 2.405 okay.

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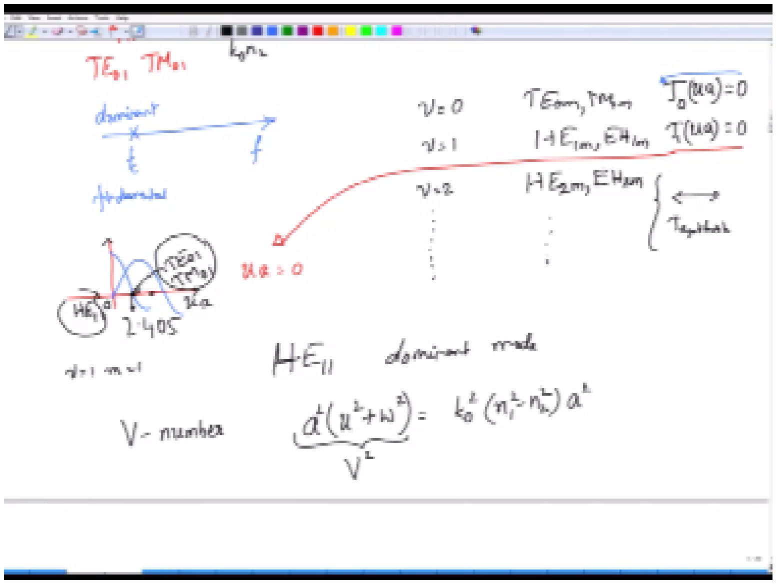


So when your product u x a, exceeds 2.405 or we will actually defined the different parameter but then when this exceeds the value of 2.405 then these two modes begin to propagate. Belong 2.405 rights from 0 to 2.405 the equation that we have written has only one solution which will be for η = 1 m= 1 and the corresponding solution is the so-called fundamental mode which is HE11.

So you see that this is the HE11 mode right which actually begins to propagate as soon as you launch some light in to its so this is the so-called dominant mode actually we defined another quantity which we call as the V number which tells you how tightly this particular mode is propagating okay it tells you the values of V that is required for a particular mode to begin propagating and then it will also tell you how nicely confined this modes E is in to the core depending on this particular parameter.

This is called as the V number and you can obtain V number by starting with this equation $u^2 + w^2$ which is equal to $k0^2$ $n1^2 – n2^2$ you know this is actually $k1^2 – k2^2$ x $a^2$ on both sides okay so you get $a^2$ and then call this equation or this expression has $V^2$.

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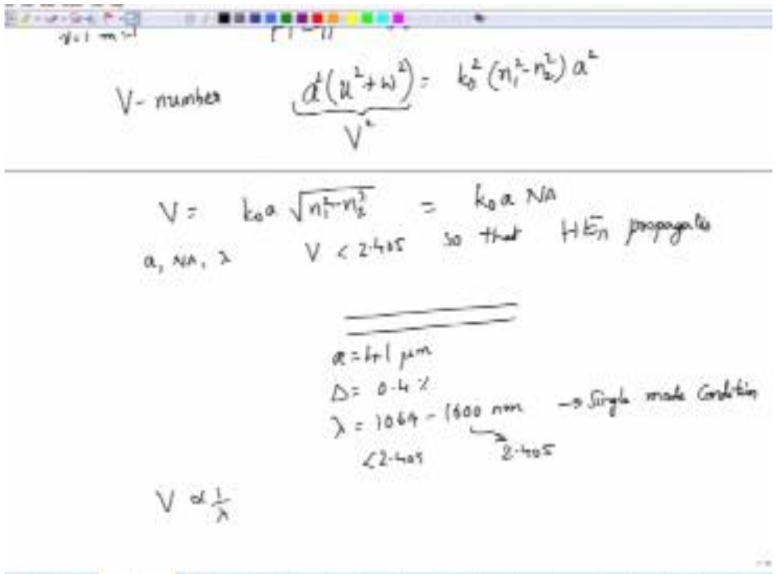


So that you get $v = k_0 a\sqrt{n1^2 - n2^2}$ but I know that this $\sqrt{n1^2 - n2^2}$ is nothing but he numerical aperture so I can write this as $k_0a$ x NA okay, as before if I fix a if I fix numerical aperture and fix λ then for that particular λ V < 2.405 so that only HE11 propagates when you satisfied this condition then we say that the fiber is single mode for this particular wave length okay, if you cannot satisfy this condition then the fiber becomes multi mode.

Now it is interesting suppose I give you a single mode fiber a standard single mode fiber that I will give you which as a of 4.1micron which has a δ which is the refractive index difference right of about 0.4 % I will say that λ is 1064 to 1600 nm I want a single mode condition over this entire band of wave length so I want a single mode condition over here now clearly if I want to satisfy the single mode condition at this wave length my V number has to be less than 2.405 and at this wave length also my V number has to be less than 2.405.

But go back to the expression for the V number V is inversely proportional to λ because V is directly equal to $k_0$ time something $k_0$ is $2\pi/\lambda$ so it is inversely proportional to λ V is inversely proportional to λ.

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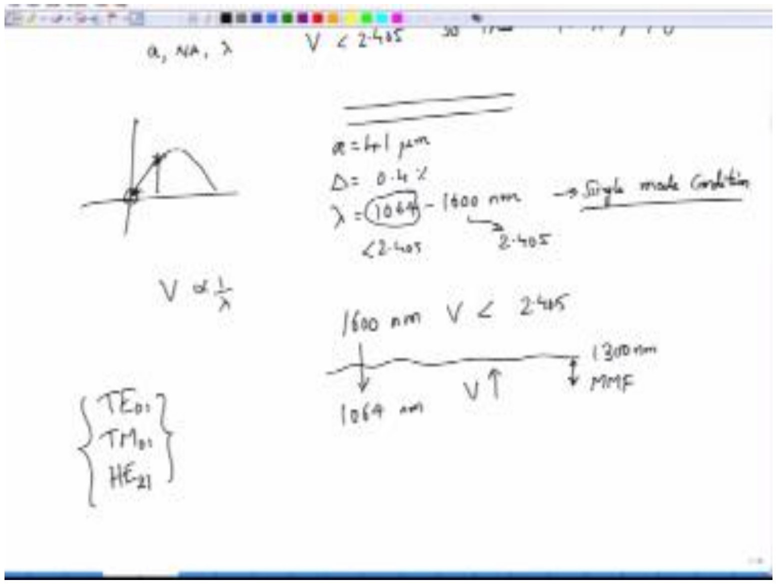


Now if you satisfy the single mode condition at say 1600 and then as you start so initially you might naively think that oh let me satisfy this condition at 1600nm I will ensure that the V number is less than 2.405. So that it is single mode in this wave length for a value that of the radius fixed and numerical aperture. Now as I start decreasing this wave length and reach say 1064 nm what you would see is that V number is actually raise in up because all the other parameters are constant but you are a sorry but your λ wave length has decrease since λ is decreased V number has gone up so at some wave length here which might say 1300nm it might so happen that the V number has actually increased so the beyond this wave length the fiber has become multi mode okay.

It can support higher order modes what are the higher order modes these are t01 tm01 HE 21 and so on so these are three groups which are the first higher order modes they have their propagation constants almost equal to each other. Okay so you need to satisfy if I am designing over a white bandwidth a single mode optical fiber you have to satisfy the single mode condition at the lowest wave length okay.

So if you satisfied this at the lowest wave length then if you increase the wave length no problem because V is anyway go in to decrease of course if you keep increasing the wave length too much then we would have dropped very close to 0 right and then you look at the Bessel function how it performs so this is the Bessel function that you are looking at and if you start decreasing V you are actually moving towards this point.

So would not really get any propagation yes you will get you know single mode condition at very high wave length but the corresponding propagation constant value is so small that it want really propagate at all so if you want to avoid this condition then you have to ensure that there is a reasonable value of the propagation constant.

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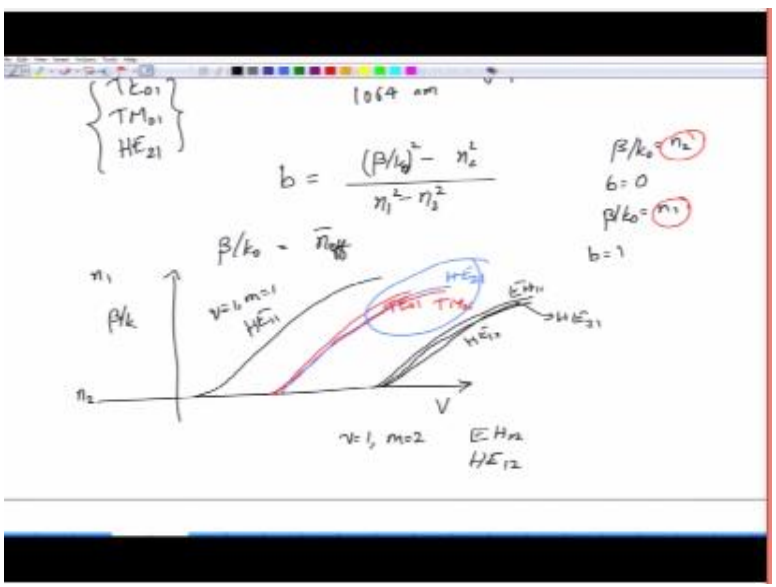


And this condition of propagation constant with respect to wave length is capture by defining a new quantity called as b and b is the normalized propagation constant it is defined as $\beta$ / k $^{2}$ – or other $k_0^{2-}$ $n2^2$ / $n1^2$ – $n2^2$ obviously when $\beta$/k0 = $n_2$ there would not be any propagation because is propagation constant be b = 0 okay when $\beta$/k0 = $n_1$ you get propagation constant b= when which is the maximum possible value that you can have.

Okay so this value that you are looking at which will give you the ratio of some refractive index this fellow β / k0 is sometimes called as the effective index okay sometimes denoted by this bar over it called as the effective index. And people since 1960 is have actually worked on this problem in order to give us the values of this β /k okay between this values of $n_2$ and n1 okay you see that if you plot this fellow b or you plot this β /k as a function of the normalize frequency V.

So this V parametric is called as a normalize frequency V so you will see that for the fundamental mode it basically begins like this and stay sub this way reaching the maximum value of n1 because I have written β/k my minimum value is $n_2$ and the maximum value of this one is n1 okay so as I keep increasing I will reach here the maximum value for the fundamental mode.

And the fundamental mode begins right at almost at 0 okay it will almost begins it 0 or it will it will actually begin at $n_2$ just above $n_2$ actually so if I am plotting this I should be little more careful I should say just above this okay. Just above this, then the next order mode kicks in after certain values of V which is 2.405 okay. So this corresponds to $TE_{01}$ closely associated with that is you know is the next mode which is $TM_{01}$ and in between these two and then going through them is your $HE_{21}$ mode, as I said all these three modes have fairly similar propagation constants, okay, this mode is the fundamental mode HE11, now what should happen?

I should get for this one is ν =1, correct? I got ν=1, m=1, so I suspect ν=1, but m=2, that is I have to get EH $_{12}$,and $HE_{12}$ mode, right and you do get these solutions as the next solutions, okay. However they will have the propagation constant that is determined in this fashion, so this is the $HE_{12}$ solution, and then you also have the $EH_{12}$ solution, or rather in fact you don't get $EH_{12}$ solution, you are at this point you start getting $EH_{11}$ solution, okay, $EH_{11,}EH_{12}$ will come even more later than this one, okay.

So you also have another mode, which is actually $HE_{31}$ mode which is going in-between these two, so there is $HE_{31}$, okay, so you will see that for a corresponding value ν=1, you will get additional terms, okay, but these terms don't always come in one sequence that you might imagine, because these sequences are heavily dependent on the solution of that equation.

But what you have to note here is that until this 2.405 normalized value of V, you only get a single mode propagation, okay, and this would actually be the maximum value of the single mode propagation constant that you can get, because beyond this if you increase "v" number you want to reach this propagation constant but unfortunately you are also going to induce other higher order modes, okay.

So this is what happens now we will go back and look at a different kind of a mode structure, because what we have done so far is that we have not solved it, but we have used the literature results to understands the modes, but turns out that these modes can be considerably simplified, the picture can be considerably simplified if you work to consider what is called as the weekly guided approximation.

What is weekly guided approximation? Well we have been working with this standard single mode fiber values of delta which is around 0.4%, or 0.1%, right this are very small values which means that there is not much of a difference between n1 and n2, or equivalently not much of the difference between k1 and k2, so k1 is almost equivalent to k2, β is almost equivalent two values, because spacing between n1 and n2 this much, just about 0.1% or 0.4%.So in this approximation you can go back to the original equation that we wrote here.

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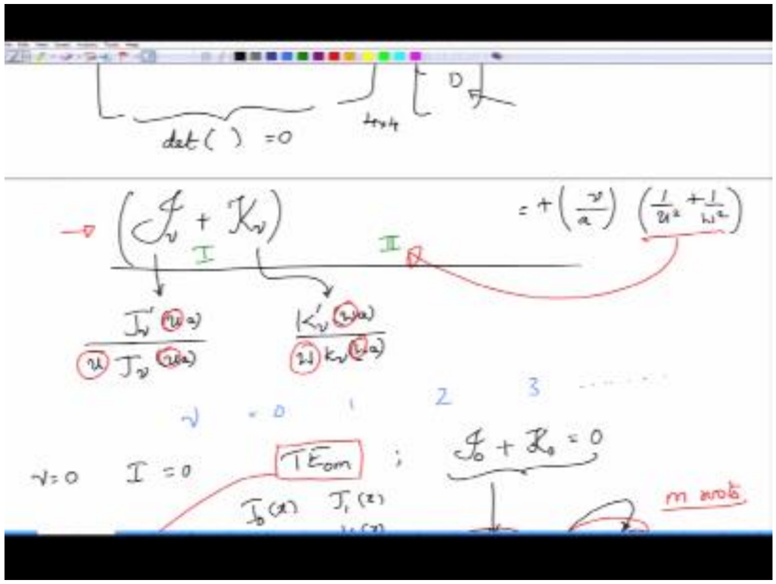


Right so we had this big equation that we wrote here, okay, in this equation you can make some simplifications, you can say that this j ν +k ν is ok ,$k1^2$ is almost equal to $k2^2$ ,so I can remove $k1^2$ here $\kappa 2^2$ here , and there is almost also equal to $\beta^2$ correct? So when delta A is very small, refractive index difference is very small $k1^2$ almost constant so this can go out right? $B=k2^2$ and here I can also remove $\beta^2$ so what I get is only this $\nu/a^{2\cdot}$ $1/v^2+1/w^2$.

Of course here you are left with j ν +k ν, so clearly I can eliminate this one as well, and then put a square here, and I don't want a square here because I want to find out this j ν+kν, right I want to solve this equation, so I can remove the square here and then put a square root on this entire thing.

If I put a square root this 2 which sitting in the power will go away, and this ν/a will also have a power which is 2 it will also go away, except now I get two solutions, I get actually + or - this values, so let me erase all these things around it for you guys to look at the relationship.

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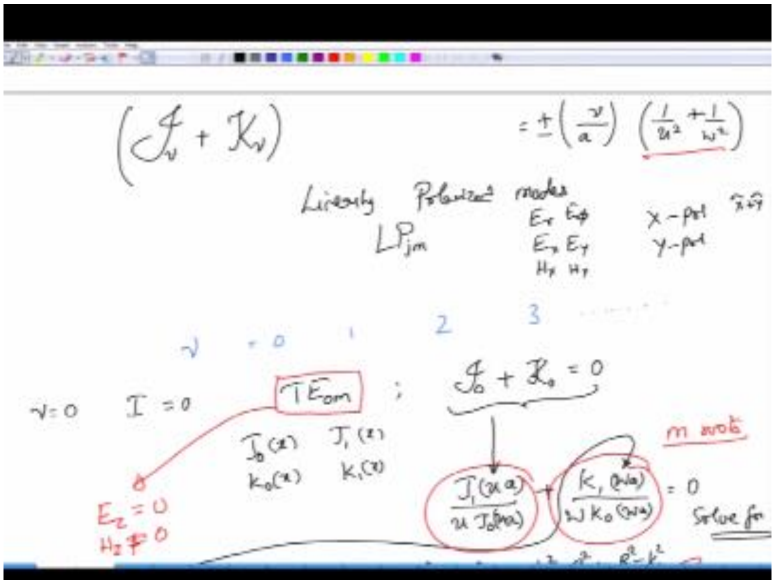

So this is the actual equation that you are going to obtain for what are called as the linearly polarized modes, so these are what are called as the linearly polarized modes, and in this linearly polarized modes you actually write downs this modes as $LP_{JM}$ where j is the different integer that I am going to discuss very shortly.

And the point about the linearly polarized mode is that, you no longer have to think of Er and E ϕ, in the transfers plane, under this linearly polarized condition which applies only when weekly guided approximation is made, instead of Er and Eϕ, you can actually talk about Ex and Ey.

Similarly you can talk about Hx and Hy, now you can talk about x polarized mode , y polarized mode, or in general an x+y polarized mode, that is x had +y had polarized mode, so this is what is called as a linearly polarized mode, okay, as suppose to Er and E ϕ should have turn around the electric fields.

You just get a linearly polarized mode; only under the approximation of the weekly guided condition the refractive index difference must be very small, now how to we build up this equation? The clue comes from this B versus or β/k versus.

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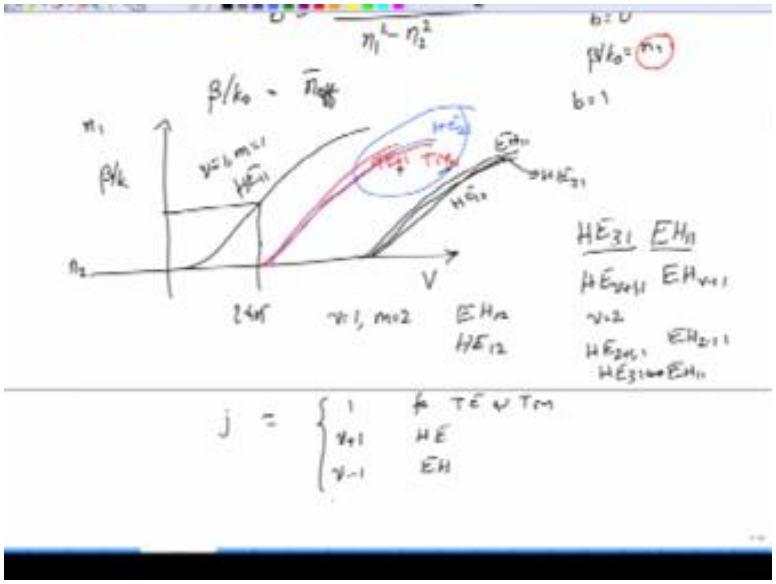

We diagram here right? So you look at these equations over here you see that there are certain components which are group to gather, there is 01 Tn $_{01}$ and HE21, what is ν for $Te_{01}$, $Tm_{01}$ ν =0, but look at the second index here, the first index is always equal to 0 here because for TE tm modes you can only have solution when ν=0, okay.

So since the first index is always equal to 0, I can write down a ν parameter called J which I will define it has 1, for TE and TM modes, you are see that TE is 01 TM 01 ,are actually having 0 all the time, okay, but then I am defining that as j=1, okay so this just a fact of definition but unfortunately this is what we have we following in the literature, so I am going to follow that.

Next what are the other modes that are paired up here, you can actually see that I am not may be I have shown here, you can see that $HE_{31}$ is paired up with $EH_{11}$, now look up the second index here, the second index for both of them is 1 and 1, but what about the first index? The first index for $HE_3$ right ν = 3 here, ν=1 here, if I define j as ν-1 for HE modes, and ν+1 for EH modes, did I get id correctly? So I think HE $_{31}$ is related to $EH_{11.}$

Then you have $HE_{41}$ related to $EH_{21}$ which I am of course not showing in this particular case, but if you consider j= ν-1 and substitute ν =2 here you get $HE_{11,}$ and then you get 2+1 =3, so you get 3+1 so maybe I should switch around the way in which I am writing this right? So I might have to switch around then say ν -1 for HE modes and ν +1 for EH modes ,would that make the sense, so now I have $HE_{12}$ here, and $EH_{11}$ here ,what I am looking at the connection between $HE_{31}$

And $EH_{11}$ ,so let me not write ν =3 here although that is ν, but if I consider the coupling and then say $HE_{\nu+1}$, 1 or another 1 and $EH_{\nu-1}$, if I couple these two modes and set ν=2, I get $HE_{2+1}$, 1 and $EH_{2-1}$ and 1, $HE_{2+1}$ is nothing but $HE_{31}$ and it is nothing but $EH_{11}$, so these are the one which coupled. So I am looking at coupling of $HE_{\nu+1}$ and $EH_{\nu-1}$, so these are the ones which I am looking t coupling and I defined a new parameter j which would actually reflect this particular condition.

So my modes are going to be written as LPjm modes, although we are writing this as LPjm modes, I mean we are writing this LPjm modes, you can actually think of or rather they are the result of combination of $TE_{0m}$, $TM_{0m}$, $HE_{\nu m}$ or rather ν+1m and $EH_{\nu-1m}$ modes. In general if you take this different modes and for a good measure also throw in $HE_{2m}$, so if you put them together right, and then to combine them, then you will end up with the LPjm mode. For example, a fundamental mode in this particular case becomes $LP_{01}$ mode and this $LP_{01}$ mode is actually derived from $HE_{11}$ mode.

In fact $LP_{01}$ is very nearly equal to $HE_{11}$ mode, The next mode that you are going to get is $LP_{1m}$ mode and this is coming from combination of $TE_{0m}$, $TM_{0m}$ and $HE_{2m}$ modes, remember in this equation you had or in this curve you had $TE_{0m}$, $TM_{0m}$ and $HE_{21}$, so if you combine these three different modes you will end up with $LP_{11}$ mode, because $LP_{1m}$ mode is the combination of these three type of mode. So you get, end up with $LP_{11}$ mode, then if you combine this $HE_{31}$ and $EH_{11}$ type of mode together then you will get the modes $LP_{\nu m}$, where ν is greater than or equal to 2. This comes from combining $HE_{\nu+1,m}$ and $EH_{\nu-1,m}$. The fundamental is of course $LP_{01}$ port which will always propagate.

And what is the nature of $LP_{01}$ mode along the radial direction? It would be $J_0$ (ur), you sketch this J0 (ur) as the function of r with respect to a here, so you see that this should be a special function like this and then at r=a and r= -a, -a simply $\phi$=180 degree such as going in this particular direction. So you will see within this core, your electric field pattern are maximum at the center, so you get a maximum electric field at the center, so you can actually close n optical fiber here.

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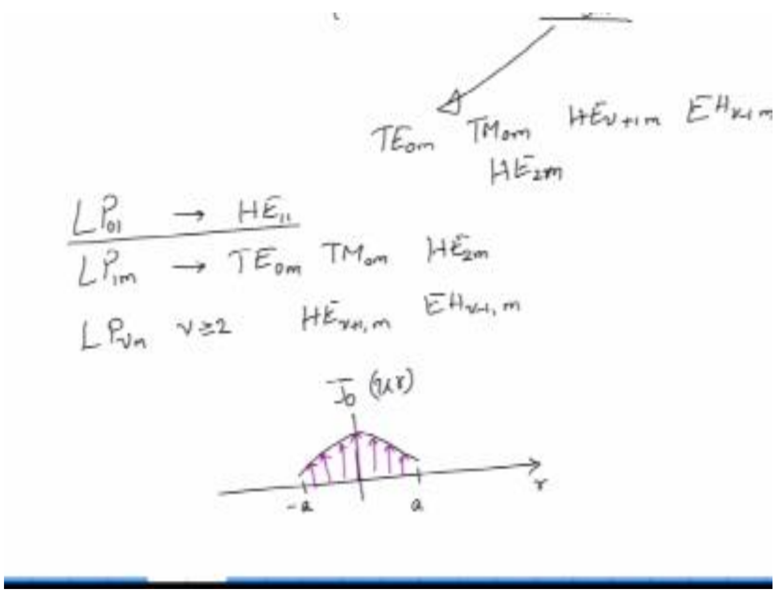


The core and you will see that the electric field lines are actually maximum at the center of the field and there would be k, as you go away from the radial direction but this is the value of the $HE_{11}$ of the $LP_{01}$ mode at the core, beyond this of course you actually expect the field to basically decay. IN the same situation that I have considered you can see that, instead of talking about the special function modes, lightly difficult to handle, you know mathematically, It is an actually nice approximation of this mode as a Gaussian mode.

As a Gaussian mode, it's a nice approximation to this $J_0$ or the special function mode, you can actually check that one by writing a symbol code in mat lab, plot $J_0^2$ (x) over say x is from 0 to 2.405 and similarly plot Gaussian function $e^{-\alpha 2x2/a2}$, so this would be $J_0^2$ (x/a), o if you plot this to

2 and you can adjust the amplitude, you will be able to approximate the special function by a Gaussian function. And you can see here that this Gaussian approximation actually goes slightly beyond your core right, the mode actually is not going to 0 there, it is going slightly you know after the core as well.

So this particular diameter after you obtain, this particular diameter within which, 90% or 99% of the energy is concentrated is called as the mode field diameter. In fact this mode field diameter which is very important when you are coupling light from a light source to the fiber. So this is abbreviated as MFD, there are nice expressions for MFD in terms of V number in terms of some fitting parameter, namely the mode field diameter fitting parameters, you can look at those expressions in the text book.

So this will simply tell you for a given radius a and for n given value of V, how is the width, which is denoted by $w_0$, just to confuse you, how is the width or the half of the mode field parameter so that, the power is concentrated about 99% within this mode field is there, so $w_0/a$ is some function of V and some fitting parameters. This is not theory for this one; it is just a fitting parameter that is obtained from experimentally measuring the mode field.

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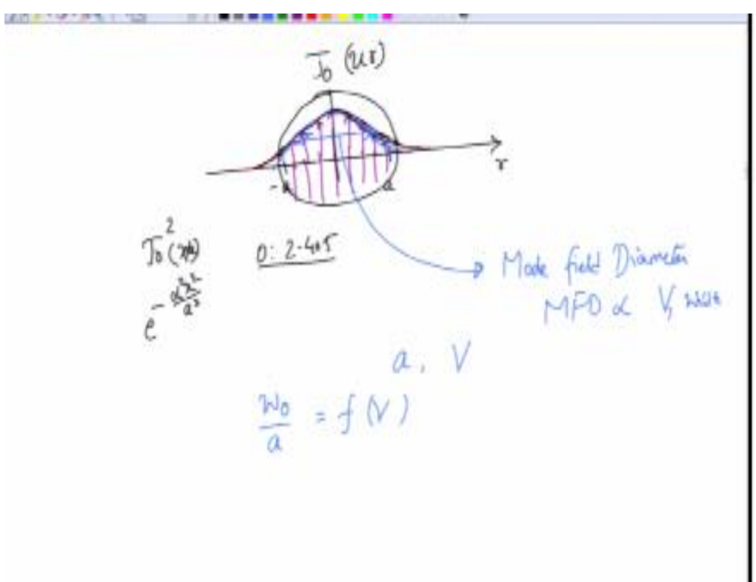

So this is what the lowest order mode and it's mode field diameter is, in terms of however, that you will actually get two types of modes, you will get $LP_{01x}$ and $LP_{01y}$ that is one of th field is directed along x axis and the other is directed along the y axis. Both have the same value of $\beta$, so one of them is called as the horizontal mode and the other on is called as the vertical mode. Here the field lines, here would be in this way, the field lines for the vertical modes would be this way, of course they actually have to curve a little bit as you go out but here again they have to curve a little bit to go out.

But this is the horizontally or thee linearly polarized mode here within the code $LP_{01}$ and the other one is the vertically polarized mode. Although in a nice fiber this  value are the same, in practice these values start to change slightly as you begin to propagate inside the fiber, as you keep travelling around the fiber, these values slightly leading to a dispersion or a delay between these two modes.

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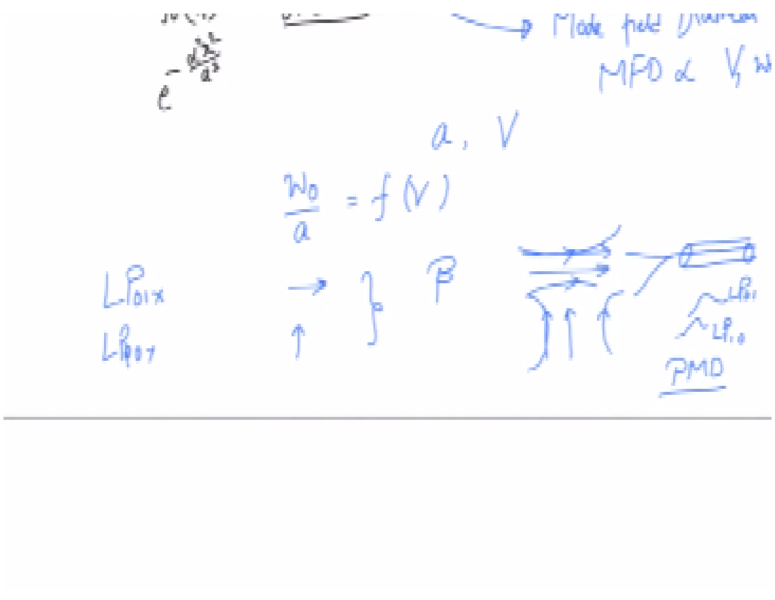


So $LP_{01}$ arrives slightly ahead of $LP_{101}$, that there is a small dispersion and this is called vas polarization mode dispersion, this polarization mode dispersion is a notorious quantity, it is not characterized by the proper theory, that lot of statistical analysis on has to do in order to characterize this polarization mode dispersion, which is one of the main limiting factors in high speed optical communication links.

Especially important at 10 gigabits per second and 40 gigabits per second so we will close our discussion about optical modes here. You may have to look at some assignment problem to understand and, more properties about that and we will begin with a study of pulse propagation inside an optical fiber in the next module. Thank you very much.

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