Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Optical Communications

Week – V
Module – II
Modes in Optical fiber-I

by
Prof. Pradeep Kumar K
Dept. of Electrical Engineering
IIT Kanpur

Hello and welcome to the online mook for optical communications we will discuss today modes in an optical fiber in the previous module we have discussed how a wave would propagate inside and optical fiber based on the total internal reflection however total internal reflection is based mostly on the or the ideas that we discussed in the previous module where based on geometrical optics something that is quite okay to understand you know initially how the wave would propagate but it does not give you the full picture.

For example if I ask you what pattern of the electric field and magnetic field would exits inside an optical fiber so that I know where to couple the light into the fiber then you will not be able to tell me that or you will have to struggle very hard to tell me that from the geometrical optics techniques that we discussed in the previous module therefore what we will do in this module is to start with Maxwell's equations and solve them for the case of the simplest structure of the optical fiber that we can consider of and then understand what do we mean by electric field and monogenetic field patterns.

These electric field and magnetic field patters which are the solutions of Maxwell's equations you know subject to certain boundary conditions that we are going to impose are called as modes of an optical fiber so you might have heared of this term mode in the context of an howl metallic wave guide you know the rectangular wave guides which have modes of Te and Tm or a parallel plate wave guide which as modes Te and Tm here Te stands for transfers electric and Tm stands for transfers magnetic if you consider a plane wave propagating in free space that would correspond to transfers electric magnetic mode here the pattern of a transfer electromagnetic mode is very simple.

You have electric field lines in this way you know may be my figures are pointing in that way and the magnetic field lines are perpendicular to electric field and this pattern itself it's perpendicular to the direction of the propagation so such a simple patter of electric field and magnetic field right is called a mode of course a free space will support this one a free space will support multiple modes those are the different questions of a mode in fact normally when we talk of free space we do not that of mode there but if you consider a parallel plate wave guide you know something that would be like this but extending all the way to you know to the y direction may be this is my x direction then you will see that there are certain patterns of electric and magnetic fields which we have which we call as modes and only those patters can exist in the mode in the guided condition okay.

So when we say guided mode inside a wave what we mean is that a particular pattern of e and h fields which are propagating along the wave guide for example if you consider a simple.

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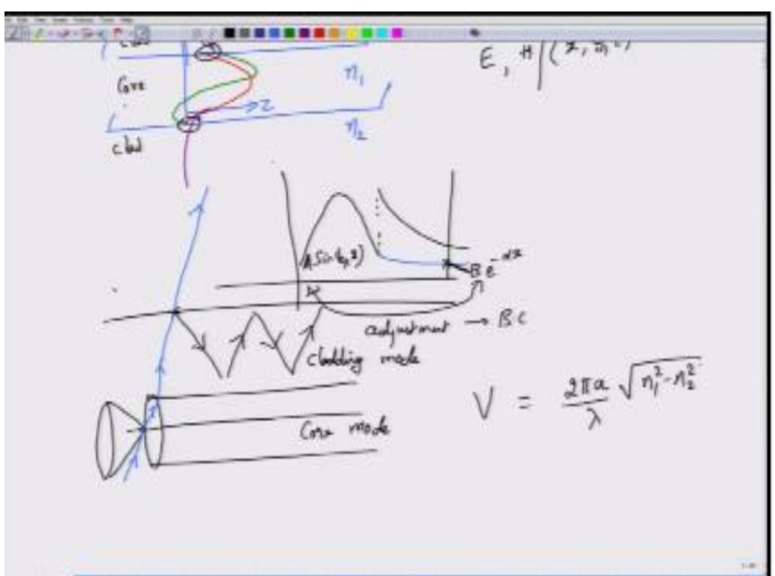

Symmetric slab made of some dielectric so let us say the electric has a refractive index of n1 here and n_2 would be the surrounding refractive index correct so this you can imagine actually as coming in from a certain parallel plate you know like this the kind of the structure that I am actually looking at okay.

So this extends all the way to infinity on this side there is an extension towards infinity on that side as well and this access may be considered as the x axis where as this axis may be considered as the z axis so you want the way to propagate along the z axis okay and for this wave if you solve Maxwell's equation you know going back to the things that you know from microwave engineering or electromagnetic theory you will see that there would be a certain patterns of electric field which I am drawing only for the electric field there are certain pattern for magnetic field as well.

So there are certain pattern for electric field may be that is of this nature or you might have you know a wave that will go like that maybe my pictures are not very nice but you get the idea the first one was a half sine wave the next one is a full sine wave and so on right, so these where the structures that we had seen and then these patterns where associated with TE or TM modes for a parallel plane waveguide we actually call them along with a certain additional index.

We said that if n denotes a number of half cycles then that would be my mode nomenclature I could equally have chosen the number of minima inside and that is sometimes done so if for example n denotes the number of minima then TE_0 would correspond to the fundamental mode wherein there is only maxima at the center and the field never goes to 0, okay. So this is just a mode nomenclature the way you write it is slightly different depending on where you are looking at the literature.

So unfortunately it is not it is although standardized there is not highly standardized as we would like to see, in any case what we have spoken to you about the last 5 minutes is that there exists this patterns of electric field and magnetic field which will be functions of the coordinate system for a symmetric lab waveguide my coordinate systems would be I the Cartesian coordinate system.

And the variables the coordinates would be x, y and z so you have electric field as a function of x, y and z magnetic field also has a function of x, y and z these fields have to satisfy certain boundary condition, so such a thing is called as a mode, for the case of you know the symmetric slab waveguide the actual behavior at the edges here will be slightly different, in fact it would be radically different if you where to say so.

If this material was the pair plates where made up of metals perfect then we know from the boundary condition that the tangential electric field component must go to 0 there however in case of a dielectric to dielectric interface the only constrain that we have is that the tangential components must be continuous across the surface there is nothing like they have to be 0 at the surface.

They just have to be continuous across the surface, so what you see is that although within the guided region or within the region of the slab you will expect a half Sine wave or know 1 Sine wave, 2 Sine wave, 3 Sine waves and so on multiple higher order modes at the edge where this plates are getting where you are meeting the field does not really got o 0 there but rather it extends into the regions surrounding the guided region, right.

So if we call this guided region as core and the surrounding region as clad okay, this is called a symmetric slab waveguide because n2 is you know the same for surrounding the region n1, okay this need not be just an optical fiber in fact this waveguide is actually an integrated optical waveguide circuit, okay. So in integrated optical device such as a laser or a photo detector or you know waveguide.

You will see these waveguide modulator you would see all these type of waveguides, okay. So if you look at the field behavior in the core they would mostly resemble this red and the green cures which stand for half sine wave or a full sine wave half sine wave being the fundamental mode, full sine wave being the next higher order mode or the next mode, okay. At the edge however the fields are not really going to 0.

But they actually extend or rather decay exponentially, okay. So the decay exponentially and it is our job to match the exponential decay outside that is happening you know so this point there is an exponential decay right and to the field which would be there inside the core so the way I have written clearly these are not meeting because at this junction there is a jum or FA so I have to Taylor one of the function amplitudes such that I get a nice extension so this would be the exponential extension right.

So if you call this one as sum Be $^{-\alpha x}$ where α denotes how quickly it is decaying into the cladding okay and if you have this as some sin $k_x x$ okay I am assuming still this slab wave guide so and the amplitude of this one is some a then I have to adjust the amplitudes a and b and this adjustment comes from boundary conditions okay so boundary condition simply stitch the behavior of the wave in the core and in the cladding region now is it always possible that I always get end up with a guided mode that is energy confined in the core actually no if you look at are example in the previous module where we were discussing the concept of acceptance angle.

We said that for a fiber right there exist something called as a acceptance angle show within which if you are light wave is incident then this light wave is guarantee to the guided inside the wave guide however if you choose to in incident your wave at outside this acceptance angle then it would simply propagate out into the cladding so it would not actually be guided but rather it escapes into the cladding however if the cladding where to be truly infinite in extend then this would have continue to radiate all the way to infinity right the energy would be radiated all the way to infinity.

But because the cladding is finite you know at some point the cladding radiate you know diameter has to come in because it is a finite cladding radius so what happens is that this mode one sit hides then kind of reflects back correct because this region is outside is a different refractive index so when the cladding mode hits the clad radius or the clad boundary when it is into reflects back so essentially it gets trapped and propagates in the cladding has what is called as a cladding mode okay what propagates inside the core will of course be core modes okay or modes inside a core.

And there is actually a coupling between the cladding mode and the core mode so if by any chance I have excited the cladding modes then these cladding modes will also propagate along with the core mode of course a different group velocities oh into the fact that the refractive indices are different and the propagation constants are different but never the left they will start to exchange some power oaky.

Which means that the power in the actual core mode will be striped of into the cladding mode so you see that there is actually a power loss in the core mode so for that reason it is necessary not to exit any cladding modes okay so you want to always have modes guided by the core and you want to maximize the power that is contained by the core one simple way of maximizing this power into the core would be to make the radius of the core as large as possible right that way I if I am tapping the exponential at some point.

I will be tapping the exponential at this point if my core area has grown so much then I would be tapping this exponential when it is almost going to 0 right so this is very small the problem with increasing the core edge is that there is a certain parameter called as v parameter okay and it is defined as $2\pi a/\lambda\sqrt{n_1^2-n_2^2}$, okay.

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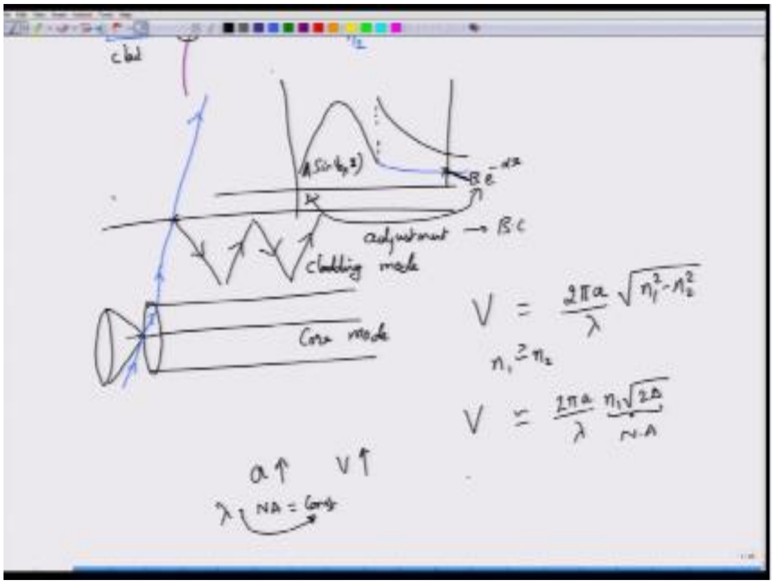

So this $2\pi a/\lambda\sqrt{n_1^2 - n_2^2}$ and under the assumption that n_1 and n_2 are approximately equal of course this has to be greater than n_2, right n_1 corresponding to the core refractive index of the fiber n_2 corresponding to the clad refractive index of the fiber then this V number can be approximately written as $2\pi a/\lambda\ n_1\sqrt{2\Delta}$ or this entire thing is nothing but numerical aperture as we saw in the last class. So if I want to increase a, right so if a increases while keeping the numerical aperture constant and the wave length also to be a constant then what happens V increases, now it turns out that when V is sufficiently large.

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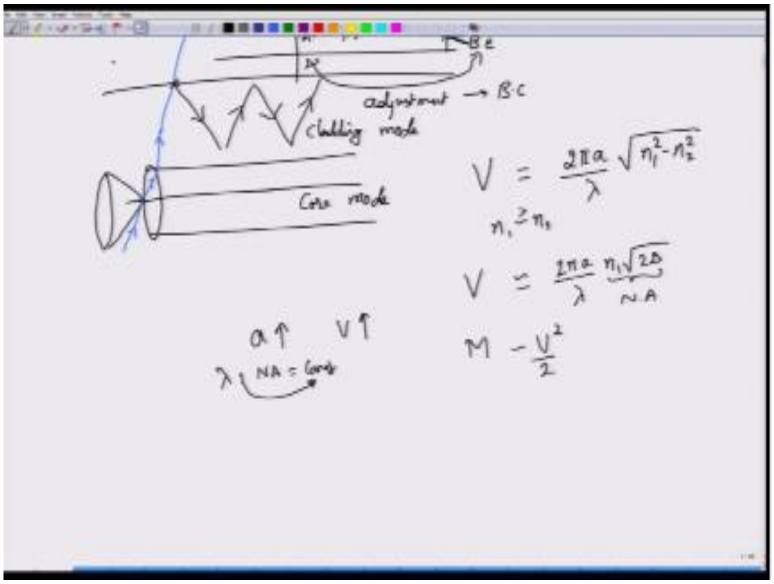

Then the number of modes that the wave guide will support goes as $V^2/2$, okay and what you get is instant of having one more propagating which is what you would ideally want, you would start getting multiple higher order modes so energy is now being eaten away by multiple core modes, just because I had discussed to constrain the optical power inside the core as much as possible when making the area large, right.

So of course the solution would be to then reduce the numerical aperture, reducing the numerical aperture would mean making n_1-n_2 difference to be as small as possible Δ to be as small as possible, right. So you will see that these are the trade of that one has to.

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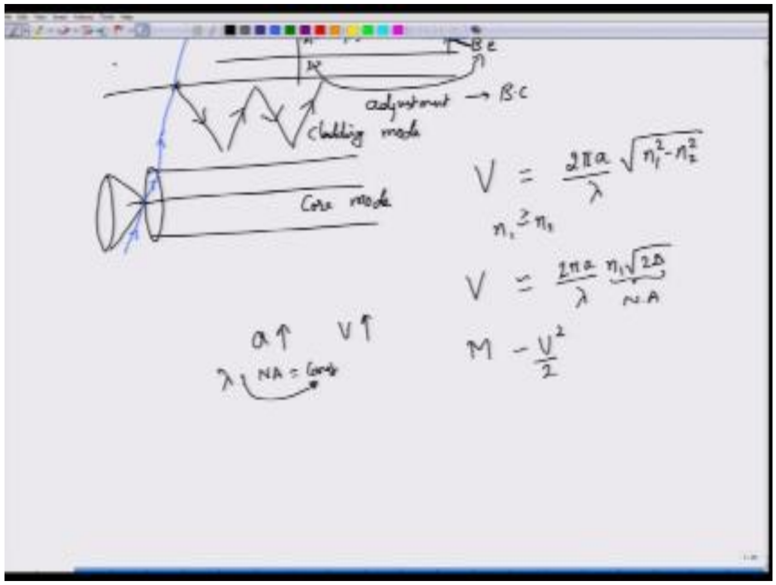

Come with when designing a fiber for communication system for a single mode optical fiber communication system you want no more than one mode to be propagating inside a fiber, so which means that V has to be constrained to a certain values, certain critical value and V should not be allowed to increase beyond that. There is one advantage of having cladding modes and the core modes interacting with each other they are used in realizing what are called as long period gratings, okay.

Where we intentionally per turbo or change the cladding refractive index so as to induce the coupling between cladding and the core modes, and these LPGs are quite widely used as gain equalizers for erbium tope fiber amplifiers, so in most RBM tope fiber amplifiers you will see this LPGs, LPG also have additional uses which we will not get into at this point of time. So coming back to the summary of what we have done, we have looked at the modes and we said that a mode is essentially a pattern of an electric and magnetic fields which satisfies maximum conditions all also satisfies the boundary conditions.

We have also seen that modes inside an optical fiber can be guided modes that is the modes which are in the core and they will be propagating along the axis of the fiber, we have also seen that there could be modes which are radiated in the cladding and actually propagate inside as the cladding modes, there is also another mode called as a leaky mode, a leaky mode happens when the mode is near the cut off, okay we will talk about cut off slightly later but when the mode is near the cut off, right then are the propagation constant for that mode is near cut off then the mode doe not quite become radiating into the cladding but it simply exponentially decays into the cladding, okay.

It propagates slightly along the axis but then while it propagates along the axis its amplitude starts to decay very rapidly, so these are the leaky modes of the optical fiber.

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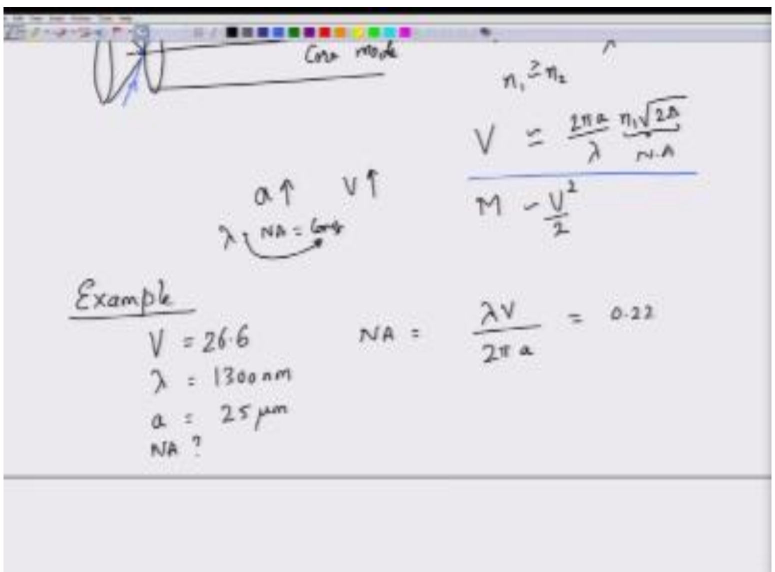

So you can take an example here, something that would illustrate the numbers that are involved so let us suppose that we have selected V=26.6 now this is extremely large number, because the number of modes will be 25^2 nearly, which is around 600 something right so this is quite a large number that you are looking at however this is just for the example seek suppose I assume that λ 1300 nm and I also assume the core radius to be 25 micron you again see that these are not the standard single mode fibers because tin the last module we have seen what are the typical properties I mean what is typical value of a and λ.

Right so can you calculate what is the numerical aperture here I will just solve this example because I know the equation is given by 2πa/λ times numerical aperture so since all the unknown parameter λV and a are given I can invert this equation to find out what is the numerical aperture numerical aperture in the numerator so therefore this must be equal to λV / 2π a, so hopefully that is the correct expression yeah this is the correct expression.

Substituting the values here what you get is a numerical aperture of 0.22 you can see that the numerical aperture is also not small right now let us turn around the problem and I m\leave this as an exercise to you assume that.

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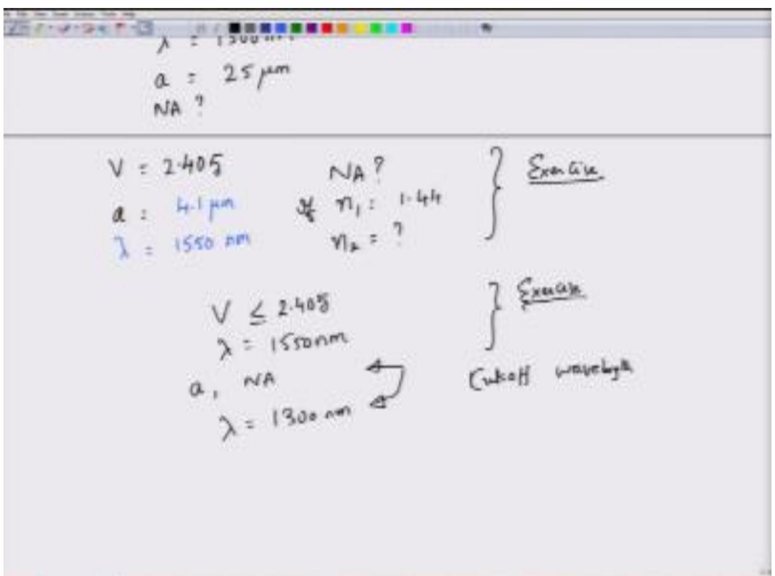

For some reason I want v to be 2.405 you will see why so some time later when we discuss the modes I fix a to 4.1 micron remember a is the radius okay so I fix a to be 4.1 micron and I want the wave to be propagating at 1550 nm, okay so I want the wave to be propagating at 1550 nm. Can you calculate what should be the numerical aperture and if I tell you that n1 is nearly 1.44 can you calculate what should be n2 okay so I will leave this as a exercise problem to you.

The solution will be supplied after the assignments are you know are handed over here to you okay. Let us also give one more exercise here you are going to design a single mode fiber as appose to the earlier problem where n actually has a particular value because all the parameters are known to you, let me say my only constant is that we has to be less than or equal to 2.405 and then λ should be 1550nm okay.

So I will say only these two values and ask you to find out the core radius a find out the numerical aperture clearly there are there is an going to be an infinitive number of choices in this problem correct so there will be an infinite number of choices see whether you will be able to come up with some numbers more importantly I wanted to understand what happens with when you change a when you change numerical aperture okay.

So I wanted to understand that operation okay, finally after you have selected a particular value of radius and numerical aperture let us say I make λ to be 1300 nm okay λ so becomes 1300 nm, see for the selected values of a and numerical aperture whether the wave will be guided here when λ is change to 1300nm okay, this brings up a very important property of cut of wave length okay.

So see if you do this exercise we will supply the solutions later after the assignments are handed over to you. Okay now we need to jump in to Maxwell equations to understand more about what this guided modes are and what kind of pattern are may going to generate inside the fiber as I said it is important for me to know what the pattern is, so if I know that power of the mode that is propagating inside the fiber is 0 at the center and if I have a lens system which is sending light, but the lens is focused at the centre of the fiber, then nothing will be excited because at the centre the power that would be guided by the fiber is 0, so there is a mismatch between what the exciting condition is and what is the power condition there.

On the other hand if I know that the power is maximum at the centre and I have a lens system, which is then focusing light at the centre then this mode can get exited, so if you have to excite a mode you have to excite the electric field and the magnetic field patterns out there, and you can do that only if you know what is the model pattern, or mode pattern.

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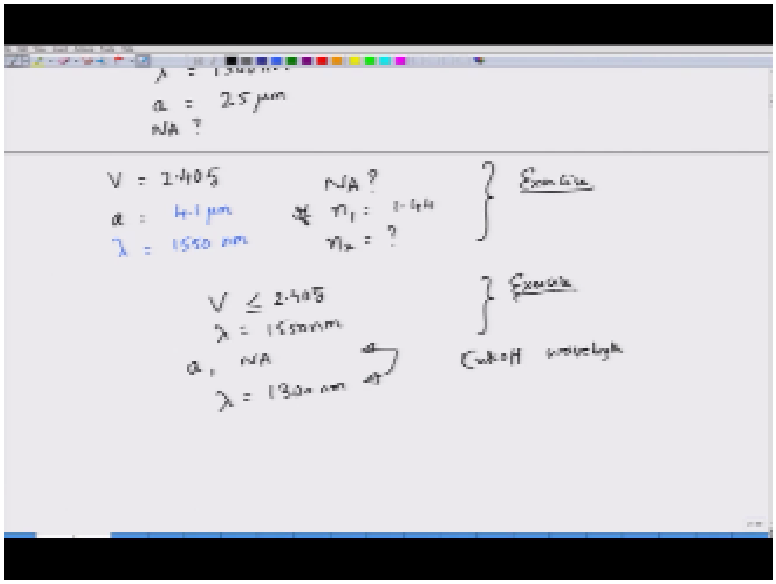

Of the modes which are propagating inside the fiber, and that is what we are going to look at, so in the remainder of the module we will first outline the steps and then give you couple of basic equations, in the next module we will continue to solve the equations, so how to obtain modes?

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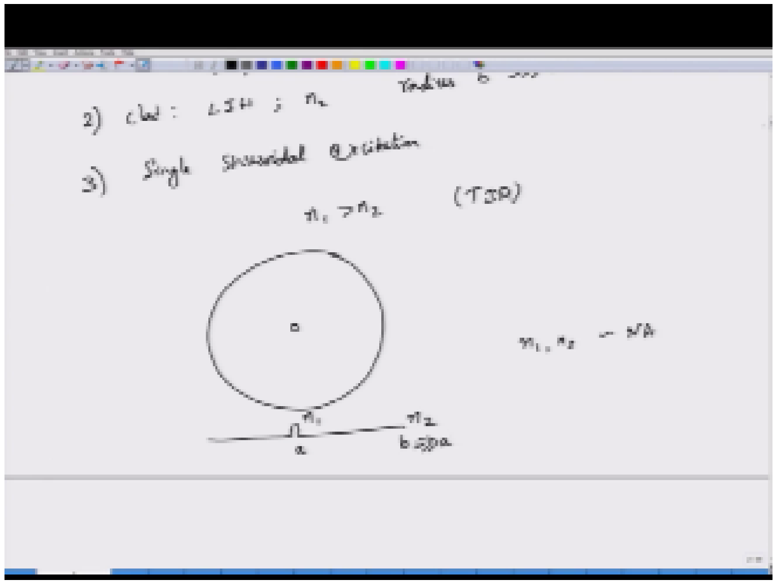

In a general setting this would be a very complicated problem because everything can be changing so that’s going to be a very involved problem to solve fortunately we don’t need to solve such complicated problems, even the simple problem that we are solving is already complicated but we will not introduce additional complexity.

So let us spell out what are the assumptions that we are going to make assumption number. I am going to assume that the core that filled with homogenous dielectric, okay, so it is filled with homogeneous dielectric, it is characterized by a single parameter n1 which is the refractive index, and it is completely independent of XYZ, okay, it is also independent of the direction of the co-ordinate system.

Therefore n1 actually is the result of a linear of the homogeneous isotropic medium. Linear isotropic plus homogeneous dielectric, so you can capture this by saying (LIH) dielectric , we will also assume that the clad is an (LIH) medium, okay characterized by a single parameter n2, and it has the radius which for our purpose radius b, for our purpose so large compare to “a” that you can almost consider this to be infinite.

Okay, however the core has the radius "a" which is definitely finite we will assume a single sinusoidal excitation because the medium is linear any complicated wave form or in the reasonable re complicated waveform can be expressed as sum of sinusoidal, so if I know how the mode behaves for the single sinusoidal frequency then I can put up the solutions from the super info the solution of that and use the techniques of Fourier's theory to obtain the response to the complicated wave form.

So ill assume the single sinusoidal excitation, so we have spelt out the parameters that we are going to consider of course we know that n1>n2 to enable total internal reflection right so this is one additional requirement for a single mode optical mode fiber that we are going to put up for any wave guard that we need to put up which is dielectric wave guide.

If you got to look at the structure itself this would be the core and that would be the cladding and if you look at what is the refractive index, the refractive index distribution would look like this, so this is n1 and this is n2, okay so the refractive index distribution is n1 and n2 and that difference between n1 and n2 is captured by the numerical aperture, okay. So the core has the radius of a the clad has radius of "b" which is much larger then "a", alright

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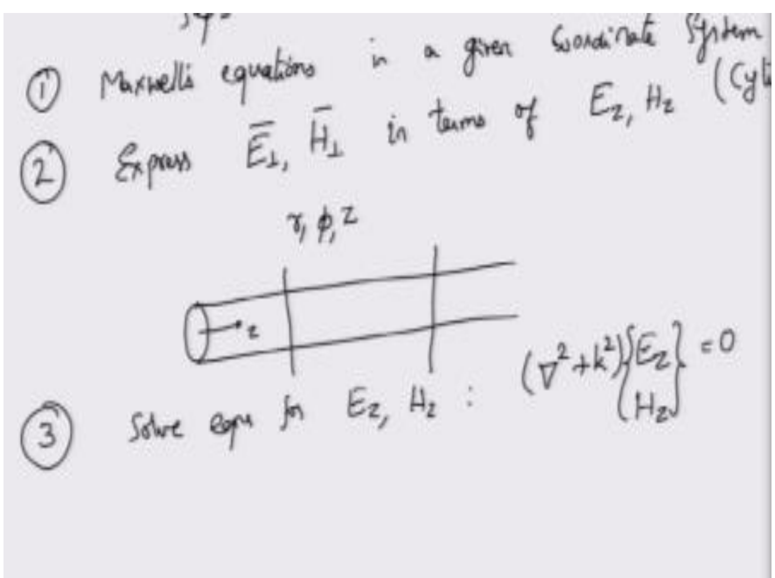

What are the steps that are involved? steps to find the modes ,so we will start with writing Maxwell's equations, we are going to write , in a give co-ordinate system , so we will employ certain co-ordinate system that is easier for us to solve, so we take this co-ordinate system and then write Maxwell's equations, in that co-ordinate system, then what we do we express that perpendicular components of the electric field and perpendicular components of the magnetic fields, in terms of the axial components.

Okay, which we will consider to be Ez and Hz , okay the co-ordinate system we are going to consider will of course be cylindrical because this is the optical fiber that we are looking at and in the cylindrical co ordinates system, you will have r, ϕ and z. And we assume that the fiber is uniform along the z axis, so this is the z axis and it is uniform along the z axis, by which we mean that, if you take a cross section here it would be a perfect circle, if you take cross section here, that would also be a perfect circle, so this is the uniform assumption that we are going to employ.

Once you have done part to, you have expressed E perpendicular and H perpendicular in terms of Ez and Hz; you solve equations for Ez and Hz, most likely you will be solving this equation what is called as the helm wholes equation for Ez and Hz. So you will be solving the helm whole signal, of course we are assuming single sinusoidal, therefore we will be actually be talking about the phaser and not the fields which are functions of both Z and time.

So the time dependency is just thrown out by going into the phaser notations, so all this Ez and Hz are phaser, I hope that you remember what phasers are, once you found Ez and Hz, then you apply boundary conditions. So the result of the equations of corresponding Ez and Hz and then apply boundary conditions to evaluate the arbitrary constants, that are going to come up in the solution, so you have to evaluate those arbitrary constants. There might be many constants that are interested in evaluate, but what you are after is to find this critical parameter known as propagation constant of the guided mode.

You want to know not only how the wave is propagating, but you also need to know, what is the propagation constant of the wave, so that is very import, so what you are after is basically to obtain this critical parameter known as the propagation constant, that's essentially the objective of applying boundary conditions and deriving the necessary equations. Once you have finished step number 4, the obvious step for 5 would be to go back and substitute this Ez and Hz in the expressions for E perpendicular and H perpendicular.

And then obtain that field pattern as well; E perpendicular and H perpendicular are the electric field and the magnetic fields which lye in the plane that is perpendicular to the propagation. So to see the direction of the propagation, let's put this way, this is the direction of propagation which is along the z axis; the plane that would be perpendicular to this in the cylindrical co ordinate system will be the r and ϕ plane. So perpendicular components meet Er, Eϕ, Hr and Hϕ. N contrast to hollow metallic wave guide, you can have a situation where Ez is 0, the corresponding modes are called as te modes, Hz is 0 the corresponding modes are called as tm modes.

In this optical fiber, you rarely get such a chance, actually you have in a very general sense, both electric field tangents longitudinal components, magnetic field longitudinal components, that is Ez as well Hz are not 0, However it might so happen that Ez might be small compared to Hx, in that case we call this as a He mode, we might also have another situation where Ez is larger compared to Hz, then we will call as Eh modes. Sometime s these are called as E waves, sometimes called as H waves as I said the notation's keeps on changing.

So in the next module we are going to solve this modes in the fiber by following the step number, steps which I have given you, step1, step2 and step3 and step 4, so we will look at the solutions in the next module. Thank you very much.

<u>Acknowledgement</u>

Ministry of Human Resource & Development

Prof. Satyaki Roy

Co-ordinator, NPTEL IIT Kanpur

NPTEL Team

Sanjay Pal

Ashish Singh

Badal Pradhan

Tapobrata Das

Ram Chandra

Dilip Tripathi

Manoj Shrivastava

Padam Shukla

Sanjay Mishra

Shubham Rawat

Shikha Gupta

K. K. Mishra

Aradhana Singh

Sweta

Ashutosh Gairola

Dilip Katiyar

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