Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Optical Communications

Week – V
Module – I
Optical fiber-II

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Hello and welcome so in the last module we were looking at the optical fiber we started off with the signal mode fiber and we said very briefly as simplistic picture based on eth geometric optics why total internal reflection is required for the wave to propagate inside the core the objective of the optical fiber is to cause the mode to propagate inside the fiber and that is possible.

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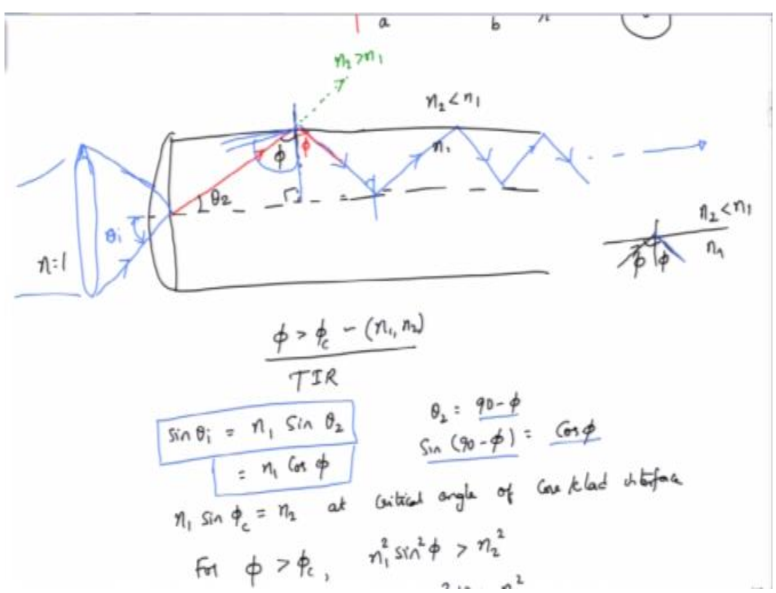

When you shine light on the fiber and you get the light into the core the core cladding interfaced must allow a total internal reflection because when once there is a total internal reflection in the core cladding interface whatever the power that is going from the core will be reflected back into the core itself of course for all angles it will not happen it will only at specific I mean only after a specific angle is reached.

So as you start increasing the angle Ø of the incidence at the core cladding interface you have to reach a certain angle called as critical angle which is a function of n_1 and n_2 in order to basically induced the total internal reflection once your angle of incidence is larger than $Ø_c$ then those angles for at those angle the ray will be reflected back which angle will the ray be reflected back this is something that sometimes students get confused well if you take you know simple interface of refractive index n1 here and n2 which is less than n1 and let us say you send in light at an angle of Ø okay.

Now even without the total internal reflection there is always a reflection happening at the same angle Ø this is the simple reflection because of the interface between two media when total internal reflection happens the reflected ray will be at the same angle Ø which was the transmitted angle this is of course the first Snell's law so when the angle of incidence is Ø the angle of reflection will also be Ø.

For any angle which is greater than the cortical angle assuming that we are at a critical angle for the red curve so for any angle which is greater than the critical angle right your total internal refection works and therefore all the light will be guided in the core okay let us see what rustication does this critical angle plays on the incident angle θi I mean after all we are talking light from a laser source are some other source and then coupling the light from this one so this is basically a lens source which I am using it couple light from for example this could be a laser source.

Okay so this would be the mode coming in from the laser and this as to be couple and focused on to the core what should be the cone of this cone angle of this one what is the maximum cone within which the light coming from the light source will be accepted and guided by the core is called as the acceptance angle or sometimes called as the numerical aperture okay so let us derive an expression for this numerical aperture by writing three laws one law at one Snell's law at the interface of air which is where we are coming in with the light from and going into the core region.

So air core interface will give you one angle I mean one equation which is sin θ_i is equal to n_1 sin θ_2 whet θ_2 is this angle which the ray of light the red one is refracting from the incident blue one so this angle is θ_2 okay however I can relate θ_2 to Ø because θ_2 is not required for me to calculate the critical angle at the core cladding interface what is need is Ø

And I know I that θ_2 and Ø can be related because if I complete this right angle triangle then I know that Ø is given by 90 - θ_2, so or θ_2 is given by (90 – Ø) I can use this expression for θ_2 into this one I know that Sine(90 – any angle Ø is Cos Ø so in place of any one Sin θ_2 I can write the right hand side as n_1Cos Ø, so the left hand side is Sin θ you might ask what happen to the refractive index of A. Well refractive index of A is just equal to 1 therefore I did not show that explicitly, okay. So we have come up to this one we have applied the interface at the air

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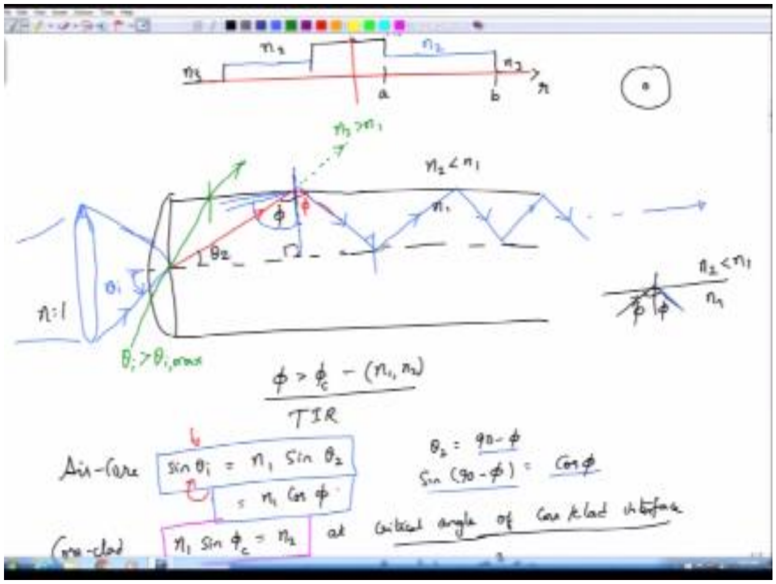

And the core interface so this is basically air Core interface equation, now you have another equation at the core cladding interface right, so this is the equation at the core cladding interface and this equation where we have written $Ø_c$ implies that you are looking at the critical angle of the core cladding interface and critical angle of the core cladding interface is given by Sin $Ø_c$ solving this particular equation.

Sin $Ø_c = n_2/n_1$ remember $n_2 < n_1$ and therefore critical angle is any value between in the first quadrant 0 to Π/2 so let us say this is the critical angle and we hope and expect that all angles of incidence Ø which lie between $Ø_c$ and Π/2 will be guided by the fiber, okay. So this is n1 Sin $Ø_c$ = n_2 will happen at the critical angle for the core cladding interface any angle greater than $Ø_c$ will also tell me that if I go to the same core clad interface equation that n_1^2 Sin^2 Ø.

Ø being the angle of incidence which is you know which we have shown here in this picture, so this is angle of incidence so when Ø > $Ø_c$ then n_1^2 Sin ^{2} Ø > n_2 ^{2} obviously for this reason you do not get the real solution for Ø and hence you get the total internal reflection but other than that this equation is valid for Ø > $Ø_c$ so once your angle of incidence becomes greater than the critical angle.

Then $n_1^2 \sin^2 Ø > n_2^2$ because Sin of the refracted wave here would anyway become equal to Π/2 so this would have been gone but I can relate Sin^2 Ø to Cos^2Ø because Sin^2 Ø is nothing but 1- Cos^2 Ø, so n_1^2 (1- Cos^2 Ø) > n_2^2 for Ø > $Ø_c$ and you know just I rearranging this open up this bracket and then pull $n2^2$ to the left and then you will realize that this equation is equivalent of n_1 Cos Ø being less than $\sqrt{n_1^2 - n_2^2}$, okay.

But I know what is n1 Cos Ø, n_1 Cos Ø is nothing but Sin θ_i therefore I am what I get is Sin $\theta_i < \sqrt{n_1^2 - n_2^2}$ now we have two possibilities one if $n_1^2 - n_2^2$ is much larger than 1 then any angle θ_i is possible, right. Sin θ_i is this value is greater than 1 then any angle this square root fellow is greater than 1 then any angle θ_i will be accepted by the fiber however in practice this is what happens.

In practice as I said the difference between n_1 and n_2 for a single mode fiber is about 0.4% it will change upto 1.5% for non zero dispersion shifted fiber and it will be around 0.6 to 0.8% for a zero dispersion shifted fiber, however for a single mode fiber this is very small and even for any other fiber that is in the current fiber family the difference the refractive index difference $n1^2 - n2^2$ is so small.

Infact It is much less than 1 in that case what happens as you start increasing θ_i so you look at this equation as you start increasing θ_i Sin θ keeps on increasing n1 is fixed what will happen to n2, n2 should you know basically start to increase which means that five starts to decrease and eventually if θi reaches to it is θi max θ2 would have decreased in such a way that sorry θ2 would have increased in such a way that 5 would have decreased below the critical angle and that particular ray would not be transmitted so if you where to send in light ray here okay so sending light ray here then this would hit the core cladding interface.

At an angle which is less than the critical angle and then this light would simply escape out so if this angle θ_i is greater than θi max so what is this θi max this θi max is this cone angle what is the maximum value of θi within which the incident light is actually accepted by the fiber and it is propagated okay.

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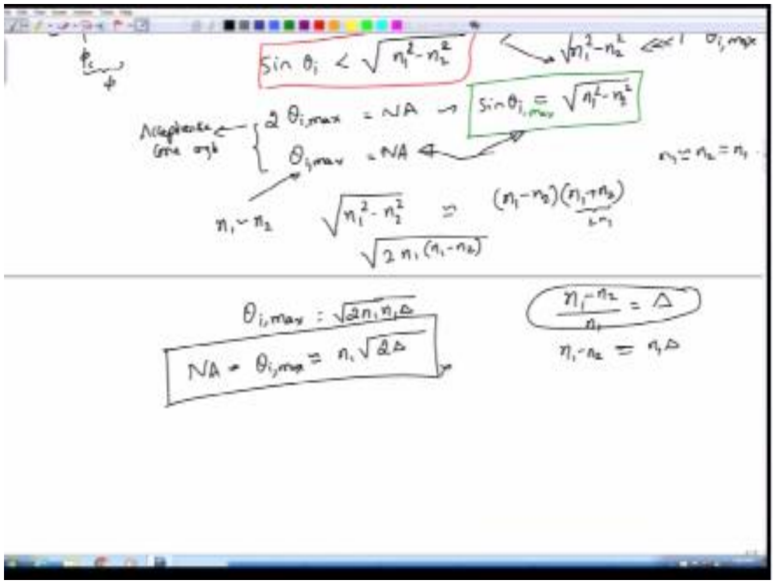

o that is θ_i max and this θ_i max we will reach when Sin θ_i will be equal to square of $n1^2 - n2^2$ okay so this angle which you can obtained after following this particular equation Sin θ_i max is equal to $\sqrt{n1^2 - n2^2}$ twice of that angle two θ_i max is given as is called as numerical aperture in some literature in some other literature θ_i max itself denotes the numerical aperture okay so in some literature this is two times θ_i max in some literature this is just θ_i max you can you have to just look at it how the authors are defining and be clear about one of those okay so we will assume that θ_i max is equal to numerical aperture and call this two θ_i max as the acceptance cone angle okay.

So just to give you the no kind of cone idea will call this as the acceptance cone angle whereas θi max we will call as the numerical aperture because n_1 is close to n_2 the difference between them is very small this $n_1^2 - n_2^2$ and a √ can be written as $n_1 - n_2$ into $n_1 + n_2$ and since n1 is very close to n_2 we will call this as equal to n1 so you will get here as 2 and 1 and this becomes $2n_1$ and then we can call this $n_1 - n_2$ we will call something here once second so this is $n_1 - n_2$ under root so I can pull this n_1 out okay so I have θi max is equal to this follow so I can pull $2n_1$ or under rot here.

And if I define $n_1 - n_2$ by n1 as Δ okay which is the relative refractive index difference then what will happen is $n_1 + n_2$ is approximately $2n_1$ and $n_1 - n_2 / n_1$ is Δ therefore $n_1 - n_2$ is equal to n1 times Δ okay this Δ is the index difference so this becomes $2n_1$, n_1 Δ n_1 is a n_1 x n_1 is a $n1^2$ therefore n1 can come out of the root and what you got is n1 x 2Δ so this is approximately the numerical aperture of the fiber the numerical aperture of the fiber is roughly n1 x $\sqrt{2\Delta}$ of course this approximation you do not have to follow because this is the exact expression for the step index single mode fiber this is the exact expression for the numerical aperture calculate θi max.

From this equation from your calculator once you have calculated that would be the numerical aperture there is no need to really do the approximation this approximation is kind of a anti way for people to just put some quick numbers but other than that you can actually use the real value of $n_1^2 - n_2^2$ and this is just a definition of the index difference.

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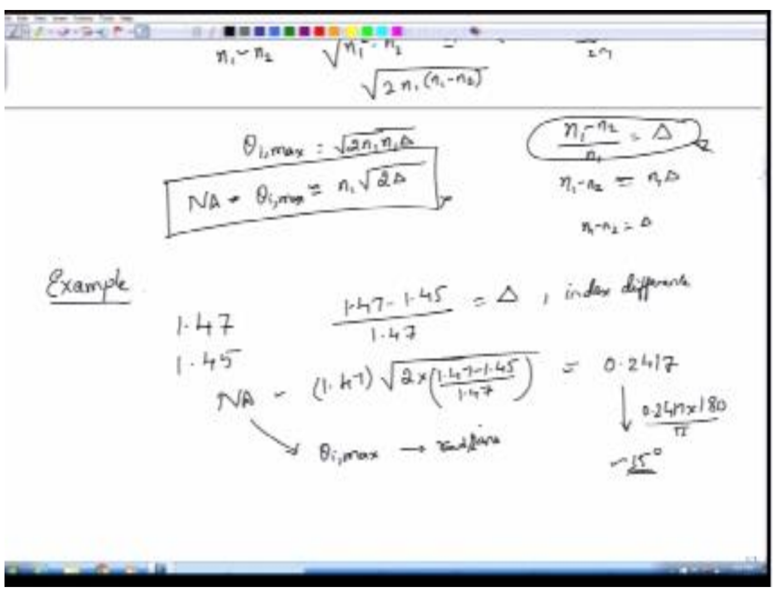

This is definition of the index difference with relative to n_1, you can define just n_1-n_2 as Δ and then you will get a slightly different result here, so you will get $\sqrt{2n_1.\Delta}$ it all depends on how you define, okay. So let us do a quick example here to get an idea of the numbers, let us assume that we are looking at the core refractive index of 1.47 and the cladding refractive index of 1.45 so the core cladding difference will be 1.47-1.45/1.47 this is the value of the index difference Δ, correct.

So this is your index difference Δ you can calculate this, what would be the value for that one and then from here you can calculate what should be the numerical aperture. The numerical aperture is given by 1.47 times √2x this Δ, right which is 1.47-1.45/1.47 and it turns out that this numerical aperture is approximately 0.2417, but remember numerical aperture is nothing but θ_{imax} and this is expressed in radians, okay so we want to convert this 0.2417 into degrees.

If you want to do that one then you need to multiply this one by 180 and divide the whole thing by π, so roughly π is 3 so 3 and 60, 60 into 0.25 so 60/4 this is roughly 15 degrees, of course the actual answer will be slightly different you can calculate that one from your calculator, okay.

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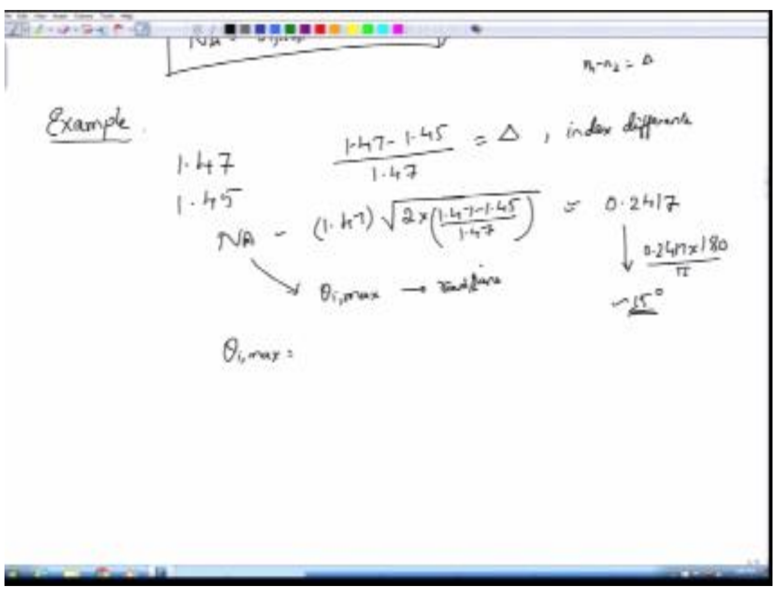

So numerical aperture we have obtained the maximum angle of incidence θ_{imax} so we had defined the numerical aperture as θ_{imax} we will define not θ_{imax} as the.

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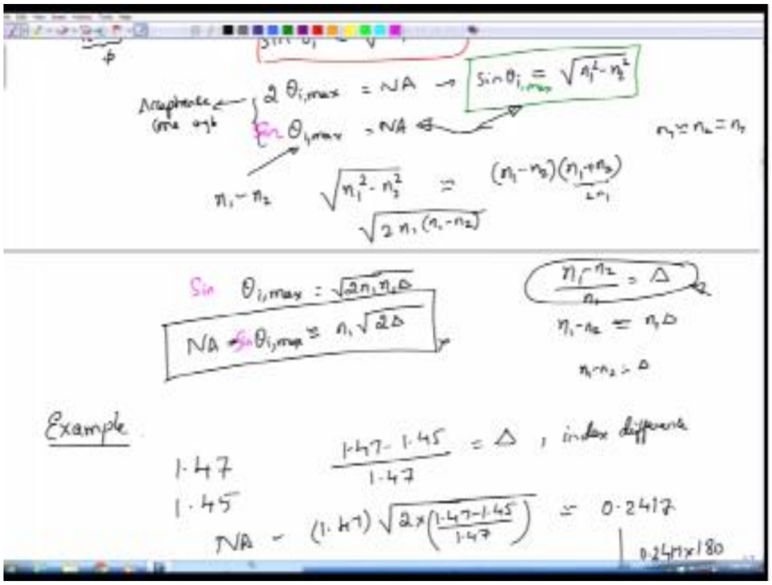

As the numerical aperture but rather sin θ_{imax} has the numerical aperture, so if we do that one then what happens is that this is sin θ_{imax}, sorry so this is sin θ_{imax} a small correction we are not defining θ_{imax} as the numerical aperture but rather sin θ_{imax} is the numerical aperture, so sin θ_{imax} will be $n_1\sqrt{\Delta}$ and you can, so this has to be slightly change now, so this is NA.

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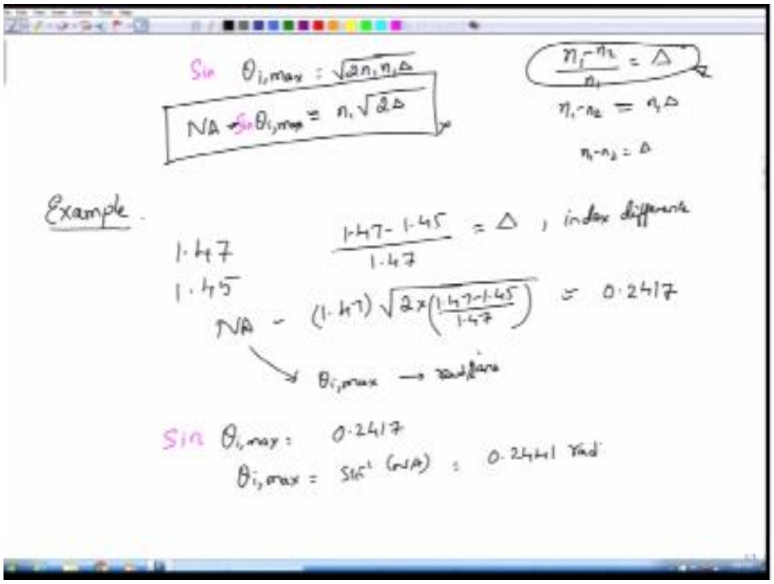

Numerical aperture is still 0.2417 but the acceptance angle is basically sin θ_{imax} which should be equal to the numerical aperture 0.2417 from here you can find out what is θ_{imax} this is sin inverse of numerical aperture which is sin inverse of 0.2417 and this is roughly 0.2441 radiance.

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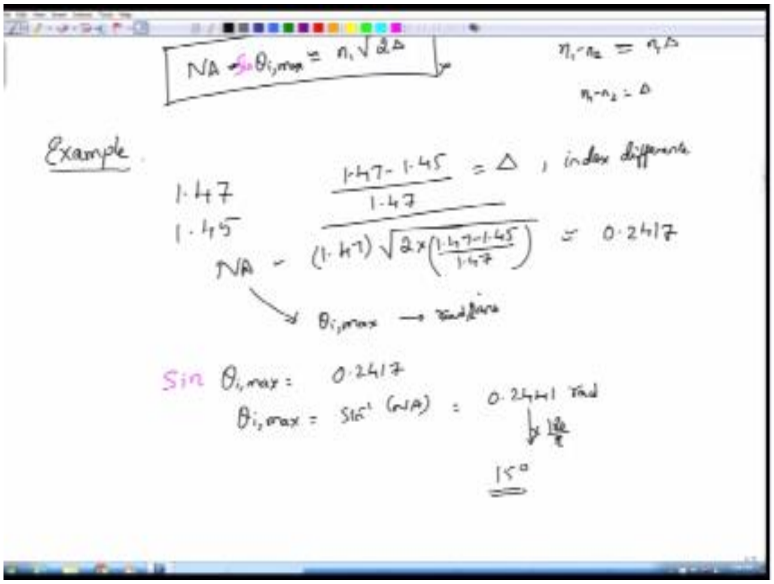

Of course, you can convert this radian into degrees and when you do that by multiplying it by 180 and dividing it by π what you get is roughly 15°, okay. The refractive index difference Δ can be calculated very easily as you cancel already seen this one. Alright, so this was a simple example to show you what is the acceptance angle, so the acceptance angle is roughly 15° on to this side and another 15° on to that side, so if you want to find the lens.

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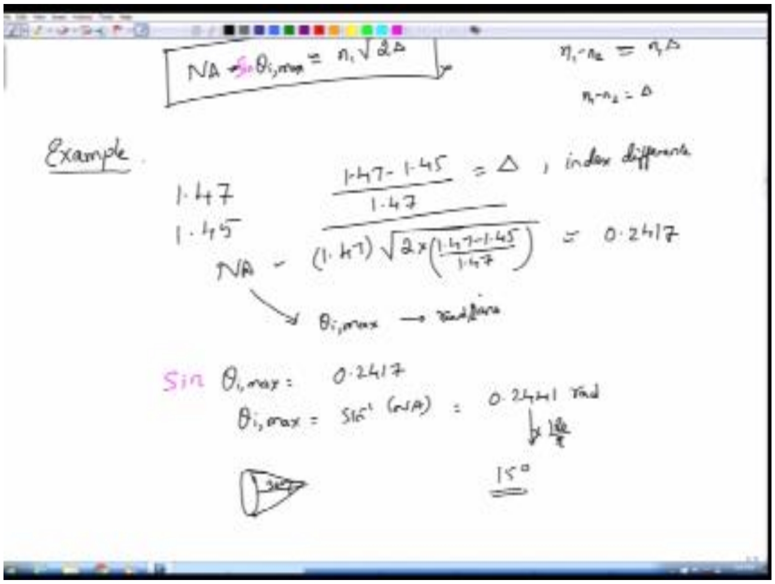

Then this lens has to have an cone angle, okay if you want to put everything into this particular core then the angle of this one should be the cone angle should be 30^o nearly, okay.

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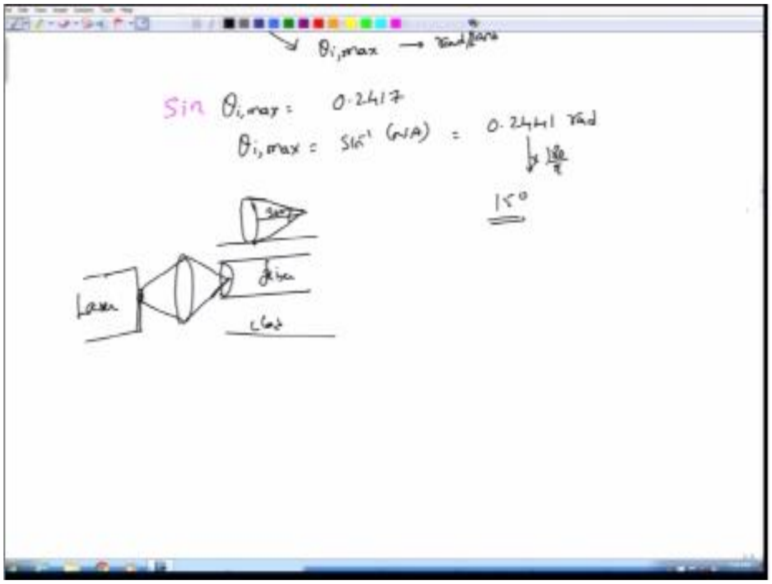

So this calculation is useful when you are coupling light from a laser on to the optical fiber, so you have typically laser diode which would be then meeting light from this side with a small non-zero diversions then you put a lens here in order to make it conversion convergent and then out this one through the fiber okay so this is your fibber core and this would be somewhere here caddie. So this is how you couple light from a laser on to the fiber okay so these calculations are required in order to find the lens power and the lens focal length of the length.

Okay we will look at what is called as multi mode fiber we have told you that multi mode fiber means that it is possible to transmit more than one mode at the same time in fact what happens is that in a multi mode fiber when excite the fiber with or when you couple light in to the multi mode fiber you will see that the fundamental mode or the dominate mode which will begin to propagate right at critical angle will be excited all the time.

Thereafter all the angles within which would not be excited but rather only certain selected angles will be excited.

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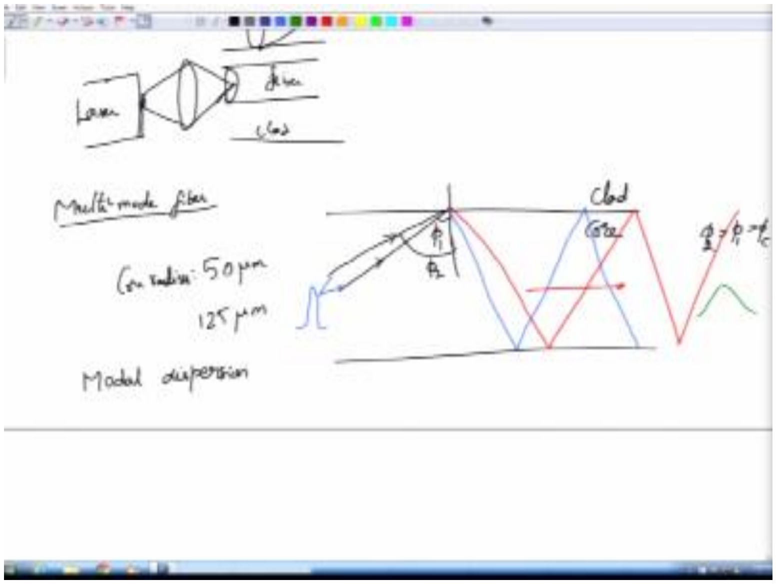

Okay so let us go back to the core cladding inter face here so this is the clad and this is the core and let us say this is where my core cladding inter face looks like and light is coming in here at an angle of say 51 suppose one more ray comes in here at an angle of 52 and we assume that 52>51 and both these angles are greater than the critical angle therefore both raise should get reflected so this get reflected here the second one also gets reflected okay so both are getting reflected the same angles 51 and 52 maybe this figure does not really show you that.

And once again they will be reflected and they would be propagated this way similarly this ray would also be reflected again and then it would be propagated and then this is how this both modes will be propagating okay so this is what happens in a multi mode fiber and this multi mode fiber typically has a larger core radius this is around 50 micron core radius okay and the cladding is till fixed at 125 micron so you can now see that the core region is quite large and cladding to core radius is just about two times.

Whereas for a single mode fiber this cladding to core ratio the diameter ratios or the radius ratios for about 5, now one aspect that becomes very important when you are looking at multi mode fiber is what is called as model dispersion okay. So this model dispersion simply means this suppose I take a pulse of light okay I take a pulse of light and then I launch this light in to the fiber let us say both modes get excited that is light energy goes in to both these modes.

And then these modes will propagate along the fiber you can clearly see that these modes are propagating along the fiber at very different velocities and then when they arrive at the receiver when you put them back together you will see that the pulse has kind of expanded, okay so earlier if you.

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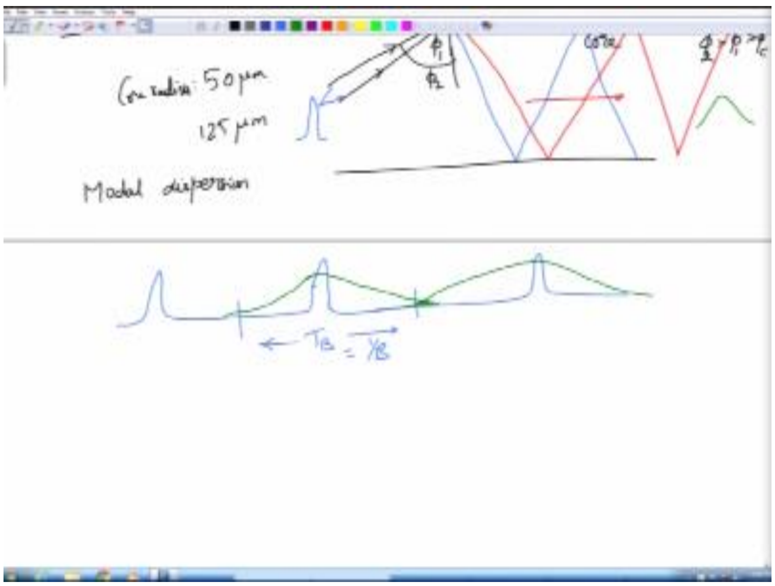

Now what happens is that if you work to send your pulses at a certain rate okay so let us say this is a pulses which are being sent and each pulse carries an information and let us say that time or of between the pulses is T_b which will be equal to 1/B where B is the bit rate of the system. Okay so you are sending out one bit represented by one pulse here so you are sending out each pulse at a rate of 1/b pulses per second.

And then because of dispersion each of these pulses there will be the reduction in the amplitude but more importantly each of this pulses would then be spread out okay, and then there would be one more pulse which is on the side spreading out little bit like this so it is spreading out in this region you know the actual pulse shape will change this is infect slightly better channel, okay, if you got to wait for longer time what would happen is? This fellow would be so wide enough that this it will start occupying the neighboring slots, and therefore you will not be able to send pulses at whatever the rate that you want to send.

Because of this broadening mechanism and this broadening mechanism happens is because of what is called as the model dispersion, sometimes it is called as inter model dispersion, it is dispersion between two different modes of the propagation inside a multimode fiber, let us try and get a very quick estimate for this model dispersion parameter, let us go back to the multimode fiber structure and then draw two extreme cases.

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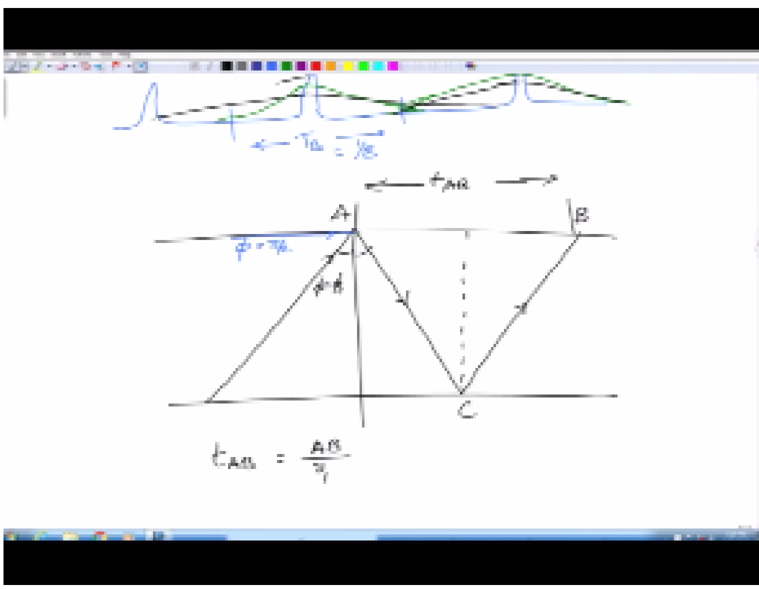

One case is when I have a mode ,which is right at the critical angle incident at the critical angle and one where I have a mode which is incident almost horizontally, okay, so this is when φ=φ/2, and this will; horizontally incident ray of light will actually transmit, okay, I mean it will go back.

The one that is incident at φ=φ c, if it has to reach the same point so let us call these point as point A, okay, and this ray gets reflected of course at the same angel, so we have already assume that this angle φ = φ c, so this comes back to this point let's call this as C and then this again gets reflected here, let us called this as B.

Now if I look at what is the time difference for the ray of light to reach a distance A to B, when your incident this one at an angle φ = φ c and when you have incident this one almost parallel to the interface, remember this was called as the gracing angel of incidence, okay, so because of the symmetry here the total time AB can return as the time taken by the ray to travel from AC and to CB, right so this would be the total time that is taken.

So if you call this as TAB as the time taken from A to B, this would be equal to the distance AB/v (velocity), with which this ray has been travelling, okay, V1 =C1/n1, now here is a catch the distance AB is the longitudinal distance, it is not actually the distance which is you know the transfers distance that you have to consider.

What I mean by this is that, if you look at this ray for example what is this distance that you have to consider? The longitudinal distance that you have to consider is this angel is φ this angel is also φ, this length is say AC, okay, so you have AC cos φ and AC sin φ, right so you have AC cos φ , so AC sin φ, therefore this is AC sin φ, similarly this is BC sin φ, right and AC = BC in our case, so we have the total thing to be about 2AC sin φ , okay.

So you have 2AC sin φ, and if you assume that this is one section of the fiber and then you can think of the fiber as having many such sections and if you want to propagate over a length L, then you would have to propagate by so many sections, so let say N sections propagating or N sections of the fiber that we are taking.

After N section's you are going to get the total distance of L, okay, and time taken for each section will be TAB, therefore the total time taken here will actually get multiply by the number of sections and this becomes N*TAB, right so what is TAB here.

Now, for the overall time length, if you call this as the length of propagation time tl, this is given by number of section N, the length with which this has one section is traverse which is the distance AB and what is the velocity which it was traversed? It is actually c/n1, so that is the velocity v1 so it I divided by v1, so v1 is sorry this is actually n1/c, this has to be n1, here it has to be, in the denominator it has to c and in the numerator it has to n1and then it is not exactly AB right, so you have the total distance as AC sinл BC sinл, AC=BC, therefore it is actually sinл.

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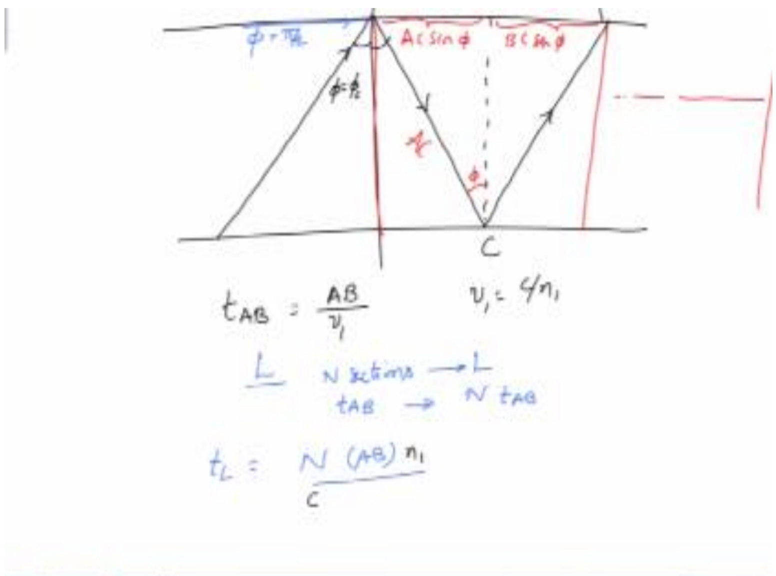

So this length which and since you have taken this two times sinл as one unit section, so for length l there will be total of 2lsinл, so since 2 is one unit cell for me, this would still simply will be equal to in л. O the distance AB is given AC sinϕ+BC sinϕ by symmetric, this is actually equal to 2Ac sinϕ and this would be the distance that you're looking at. So this would be the unit cell distance that you can get, so the time travelled to traverse a fiber of length l is given by this particular expression and since you are looking at n section of AB, n section of AB nothing but l, each section is of length AB.

So n times AB is nothing but l, so you have time taken for any wave, incident is given by Ln1/csinл, now this is interesting to start with the case when ϕ=ϕ, you see which means you are at the maximum value for light to propagate, so this is what you get tlmax and this is given by Ln1/csinл. But I know that n1 sinлc = n2, therefore sin ϕc =n2/n1, so I can substitute for that whether Ln1/c in ϕc is nothing but n2/n1.

So this would be n1 goes to numerator, n1square l by n2c, when will you get the minimum length, so minimum length will happen when ϕ=л/2, at which point sin ϕ=л/2, so you get minimum propagation of time as n1L/c, so this is the minimum length and the pulse is essentially, if you want to assume that all the angles are or at least ϕ=л/2 and ϕ=л/c are excited, then the duration with which the pulse gets stretched equal to tlmax-tlmin and I will leave this an exercise to you to show that this is nothing but n1L/c into n1/n2-1 which I approximately $n1^2$LΔ/Cn2.

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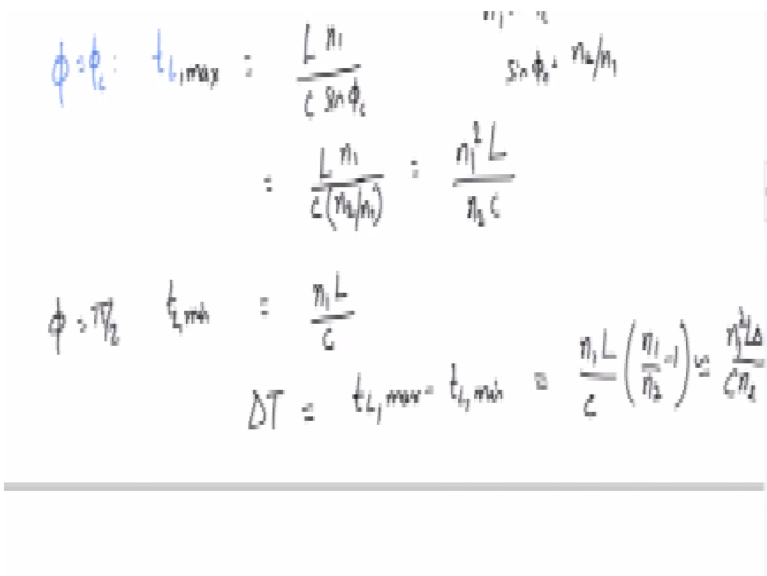

So length of Δt is given by $n1^2$Δ/Cn2 and this is the length over which your pulse actually gets provided. For avoiding this dispersion problem, the pulse providing problem which limits your bit rate, if you want to avoid that, it has to be less than or equal to TB to avoid one pulse cross in over to the other pulse, so you can substitute for Tb as 1/B and this you know and multiply by that by L and rearrange the equations that you get BL is less than or equal to Cn2/ $n1^2$Δ. This product on the left hand side is called as BL product it tells you the bit rate length product, and so if you want that urgent bit rate you have to reduce the length l.

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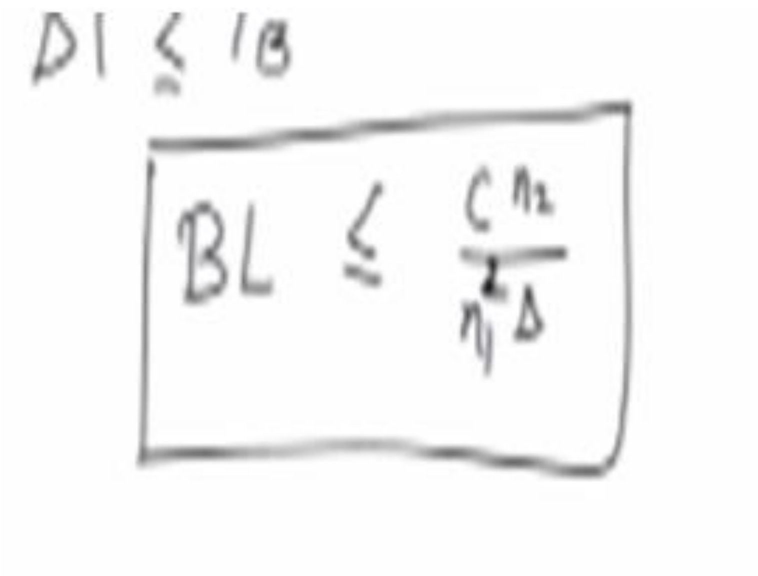

If you want a larger length then you wan t reduce the bit rate, for a given multimode fiber with the parameter n2, n1 and Δ known to you, this would be the limit on the bit rate into distance factor. Thank you very much.

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