Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Optical Communications

Week – IV
Module – IV
Reflection of Waves

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Hello and welcome in this module we will look at reflection of uniform plane wave up on a boundary and then we will talk about the phenomena of total internal reflection okay so when a wave propagates because of you know some reason you have generated an electromagnetic wave it is propagating along a certain direction and in a particular region which we will assume it to be uniform linear and isotropic.

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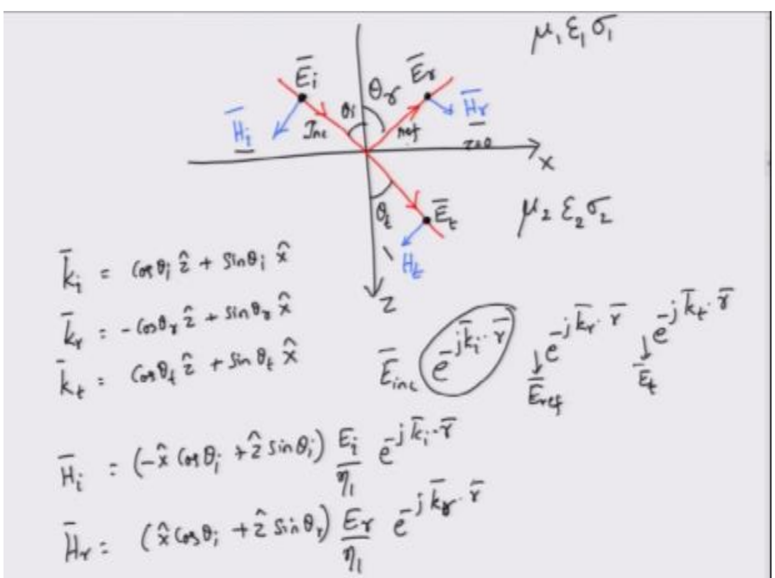

Okay which is characterized by the three material properties $\mu_1\varepsilon_1$ and σ_1 so this wave is propagating happily now it hits a boundary okay so the wave hits a boundary because beyond this boundary there is $\mu_2\varepsilon_2$ and σ_2 you can expect that the electric field and magnetic field here of the wave will be different the magnetic field here okay you might remember actually if you think that electromagnetic wave light is an electromagnetic wave that when light strikes one medium there are two important things that will happen.

Okay assuming that the medium is nice flat and everything okay when a ray of light hits that medium there will be one component of the electro light which is reflected okay so there will reflection phenomenon and a position of the light goes through the second medium which is called as the reflection right so a position of the light goes inside this phenomena is called as refraction okay.

So when a wave hits a boundary you get reflection and you get refraction you have probabably also studied certain relationships between these reflection and refraction right so you must have studied the statement that angle of incidence is equal to angle of reflection or rather angle of reflection is equal to angle of incidence and to find out the angle of refraction you must have used what is called as Snell's law so you have the first Snell’s law which tells you that angle of incidence is equal to angle of reflection then the relationship between angle of transmission and the angle of incidence is related normally in terms of the refractive index of the first medium and the refractive index of the second medium.

This is something that you know let us look at where those law’s are coming from they actually are nothing but matching conditions boundary matching conditions as we will demonstrate now in understanding the reflection of the waves okay we have to defined first certain quantities I will imagine that this my hand is the surface and this the direction in which the incident electromagnetic wave has hit the boundary.

Okay to this my hand which is a surface which separates to medium I can find what is the normal to the surface okay the normal being perpendicular to the surface so in this particular case the normal is along this direction okay so you have a wave which is come in and then there is a normal here, so there is an angle between the direction in which the wave have come and the normal so you if you think of this point as o this point as a this is hitting here oa line represents the direction of the incoming wave and if I consider for this perpendicular line this as n then an represents the normal to that particular point.

Okay and the angle between oa and an is the angle of incidence clearly when the wave is reflected it is going along this particular direction okay and compare to this the direction of the wave which is reflected and the normal there is an angle of reflection below here I have wave which is transmitted into the second region again I have a normal and an angle to the normal giving you the angle of transmission or angle of refraction.

Okay so there are these three angle this hand of mine which determines the boundary right is described by 2 vectors 2 tangential vectors there is one t1 here and a t2 vector this t1 and t2 define a plane which is defining my surface or the boundary okay these are tangential to the surface obviously so these are tangential look at this hand this is 1 direction and my thumb so this is one more so this these two which are perpendicular to each other are defining this plane and this is the plane of interface okay similarly I can actually have a plane which is described by the incident k vector and the normal to the k vector or rather I can have because this can be decomposed into one of the tangential components.

Right so we call this as t1 remember this as t1 and this thumb as t2 so when I look at a wave which is incident in this wave I can decompose this into one component along t1 and 1 component along normal so this normal and this angle t1 they will create 1 plane which is called as the plane of incidence okay so you have a plane of incidence and a plane of interface we have only talked about the direction of the wave what about the direction of the electric magnetic fields.

Well the electric field can be perpendicular and lying in the same plane that is electric field can lie in the plane of incidence but the magnetic field will be lying in the perpendicular scenario right so the magnetic field can be perpendicular that is it will be along this thumb for example or you have the magnetic field lie in the plane of incidence perpendicular to the line oa however the electric field line in that particular case must be perpendicular because we are assuming E, H and K are perpendicular to each other this is the plane wave condition that we are assuming so this is the direction of K this is the electric field which is in the plane of incidence and the magnetic field which is perpendicular, such a situation is called as parallel polarization for a very obvious reason.

It is parallel polarization because electric field is in the plane of incidence if it is the magnetic field which is in the plane of incidence so this is the k vector this is the magnetic field and the electric field was perpendicular to you then this is called as perpendicular polarization or transverse electric polarization and that is the case that we will consider in this particular module the case for EM wave is very similar to the case of TE wave. And I will link that as an exercise for you to fill in, okay.

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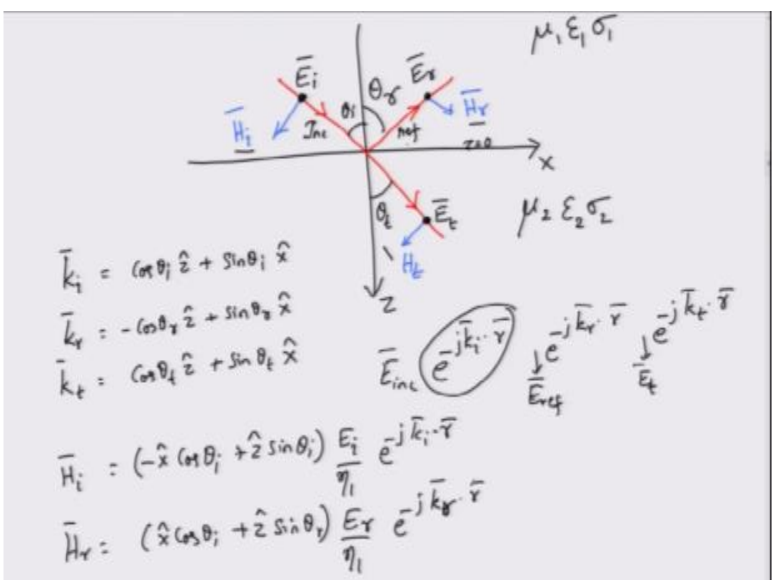

So we have that TE case right which you can see from this figure the wave is propagating in the xz plane okay this incident wave has a direction which is given by this k vector given in the red color okay so this gives you the incident wave vector so this is the incident and the electric field is assume to be along the y direction indicating that it is perpendicular to the pane of incidence, plane of incidence is formed by this red color liner and the black color line, okay. Which is the z axis and the red axis form the plane of incidence, the plane of reflection is formed by the reflected light which is indicated along this or propagating at an angle of θ_r with respect to the normal the normal is along the z axis okay, so this point is z = 0 obviously the interface which is separating the two media, the second media has constant μ_2 ε_2 and σ_2 there is also plane of transmittance which is equal to plane of incidence.

Because the k vector making an angle θ_t will still lie in the same plane of incidence, in fact you can see that all these planes are equal that is plane of incidence = plane of reflectance in a reflection and plane of transmission all these three are the same planes look at the ordination of the magnetic field which are shown by the directions in blue color okay.

So for the incident wave the magnetic field is directed in this particular direction, reflected wave it is directed here and this is the direction for the transmitted wave, does it make sense? Yes, so you have to now think that the electric field is perpendicular and coming to you so this cross H will be going along the k direction.

If the magnetic field is in this way electric field is in the top so this cross H will give you a way which is propagating this way that is the reflected wave and for the transmitted is the same thing, this is electric field this is the direction of the magnetic field so crossing like this will give you the direction of the wave which is transmitted, okay. These are the different ways in which these waves are transmitted.

Now what we want to write down is the expression for the incident wave vector the reflected wave vector and the transmitted wave vector or the propagation constant k, right. What could be this k_i for the incident wave, this is making the certain angle θ_i so there will be the component along +z and +x correct, this is given by Cos θ_i z + Sin θ_i x^ whereas for the reflected wave because it has to propagate along – z direction.

This will be –Cos θ_r z^ + Sin θ_r x ^ for transmission it is Cos θ_t z^ + Sin θ_t x^ it is these $e^{-jki.r}$ $e^{-jkr.r}$ $e^{-jkt.r}$ which describes the phase components for the incident, reflected and transmitted waves that is incident wave will be described by the electric field E incidence and this phase factor this is the reflected wave so it is E_{ref} into this where and this is the transmitted wave which is transmitted times $e^{-jkt.r}$.

So these are the phase factors that we are that we have to attach to the plane wave, okay. What can we write about H_i? Well H_i will have one component along –x direction and one component along x direction correct, so the magnetic field will be given as –x^ Cos θ_i because there will be an angle of 90 - θ + z^ Sin θ_i okay this would be the direction of the magnetic field if E_i is the amplitude of the incident electric field or the wave as an amplitude of $E_{i.}$

Then E_i / η1 will give you the amplitude of this magnetic field, right and there is $E^{-jki,r}$ which is coming from this particular expression okay, and similarly you will have Hr given by x^ Cos θ_i you can see the angles from the figure, z^ sin θ_r E_r / η1 $e^{-j\ kr.r}$ okay this is the reflected electric field,

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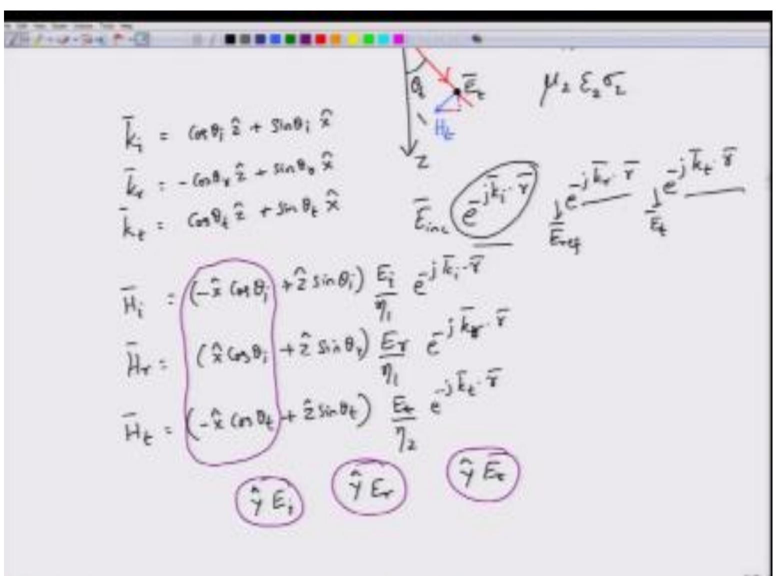

Similarly H_t will be given as $-x^\wedge \cos \theta_t + z^\wedge \sin \theta_t E_t / \eta 2$ now not η1 it is $\eta 2\ e^{-jkt,r}$ okay, what would be the electric fields, electric fields are all directed along y axis incident field has an amplitude Ei reflected field as an amplitude of Er transmitted field has an amplitude of Et okay all of them are directed along the y axis and each of them will carry this particular phase factor okay so this is the lift of electric field components magnetic field components and the k vectors that we were requiring now what do you have to do well you first break up the incident vector into it is tangential and normal components right so when you break up this into tangential and normal component you will see that the tangential component is actually.

The x component correct or rather the – x component in this case if you break up this into tangential decomposition to a tangential and normal component we will see that it is only the ex component which is tangential to this boundary because anything that is along z will be along normal correct finally if you decompose this into normal and tangential again you will see that only the x component or rather – x component that is tangential okay where are we applying the boundary we are applying the boundary at z = 0 and we want to know only the x components of the h fields what about the e components that is electric field components they are all lying along .

Y Y and Y and Y is tangential component for the surface correct z = 0 as a component of x and y sorry the axis x and y so any field which is directed along y will be tangential to that therefore all these three are tangential now you remember what we talked about in the boundary condition we said that tangential E must be continuous tangential E1 or rather tangential components in region one must be equal to tangential components.

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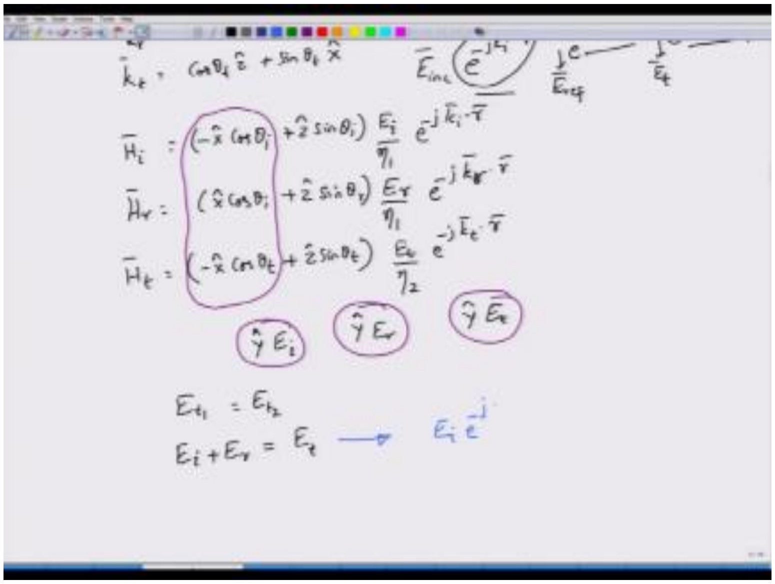

In region to so Et1 = Et2 however what is a total tangential component in region one it is the sum of incident and reflected electric fields correct because both are in the same region so you actually have to write this has Ei + Er = Et is that correct have you written it correctly well there is a small catch here I will tell you what the catch is this equation has we have written is wrong because we forgot the phase factors let us put down the phase factors here so this equation actually has to be written as Ei e – j now let us look at what happens to the phase factor when I consider the position vector r as x x^ + y y^ + 0 x z^.

You might question why a 0 well the answer is obvious because we are assuming that this interface is given by z = 0 and is there a component of the k vector along y no so this components these two will not be useful for as the only tangential component the phase component that you are going to get will be along the x direction and this phase that you get is given by for the incident wave this is given by Sin θ i into x for the reflected field you will get Sin θ r into x for the transmitted you will get Sin θ t into x okay so let us carry these phase factors along with E_i E_r and Et .

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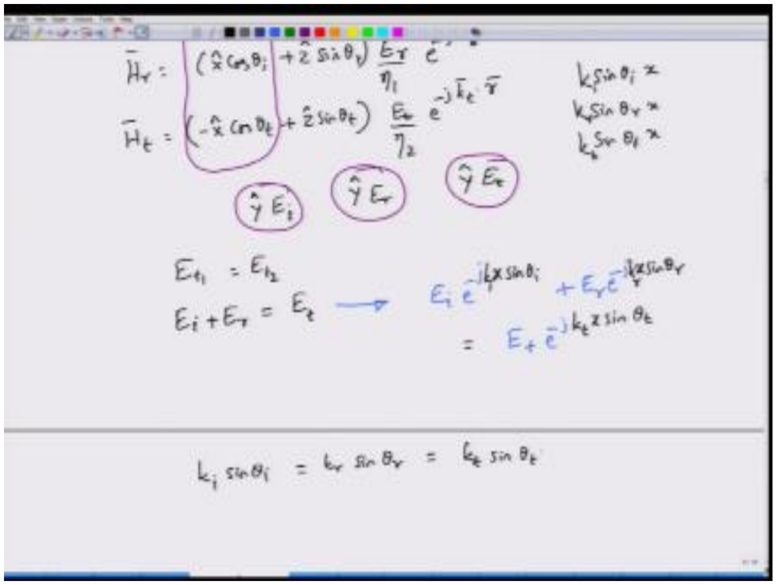

So let us write this as E_i and write the phase factor here e^{-jx} Sin θ i + Er e^{-j} the phase factor is x Sin θ r there is a k here which I forgot to put write so there is k here and this k has to be ki kr and kt so there is actually ki kr did I put a forget to put ki here oh ya here I forgot to put ki here this has to be multiplied by the magnitude of k which is ki for the incident vector kr for the reflected and kt for the transmitted wave vector okay sorry forgot to put that now you can go back and put those equations this must be equal to the electric field tangential component on the other region which is Et e^{-j} the phase factor is kt x Sin θ t in contrast to the boundary condition that we derived in the last module.

Here you have to satisfy the boundary conditions for all points along x right so it is not just at one point that you have to satisfy you have to satisfy this at all points along x so the only way you can satisfy them at all points along x is if you ask ki Sin θ_i be equal to kr Sin θ_r which must be equal to kt Sin θ t okay so this has to be equal to each of those term phase term have to be equal to each other so that no matter what the amplitudes Ei E_r and Etr this equation has to be valid the boundary condition has to be valid okay so in order to make this boundary condition.

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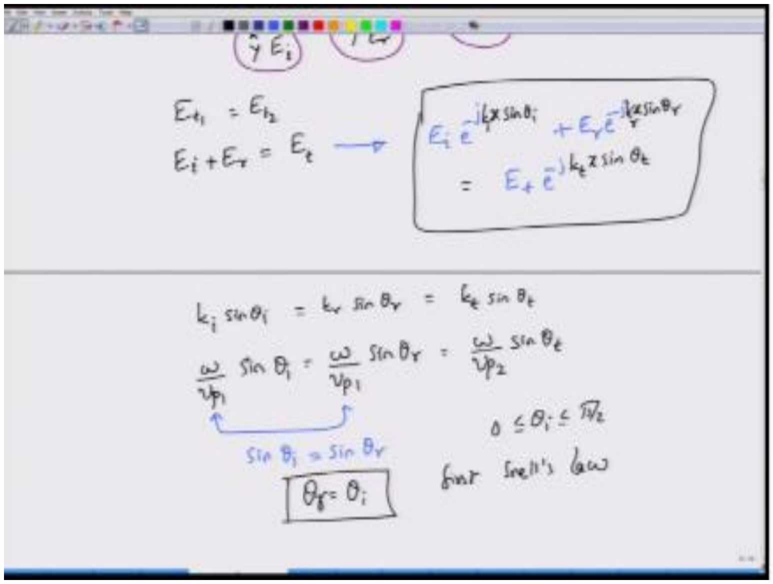

Equal for all values of x for all values of Ei and Er and Et this should be equal however k_i is nothing but ω s by Vp1 which is the phase velocity in medium one Sin θ i = ω by what is the phase velocity for the reflected wave it is the same as the incident wave correct so you have ω by Vp1 Sin θ r which must be equal to $\omega/vp1 \sin\theta_r$ which must be equal to ω/vp2 sin θt.

Now take the first two equations, in the first two equations you can see that ω/vp1 on both sides will cancel, so what you get here is $\sin\theta_i = \sin\theta_r$ and because your θ_i and θ_r are between 0 to π/2 you are looking at two quantities whose signs are equal and the angles are between 0 to π/2. The only condition that will be valid is when $\theta_r = \theta_i$ so this is your First Snell's law, correct First Snell's law which says that angle of reflection is equal to angle of incidence.

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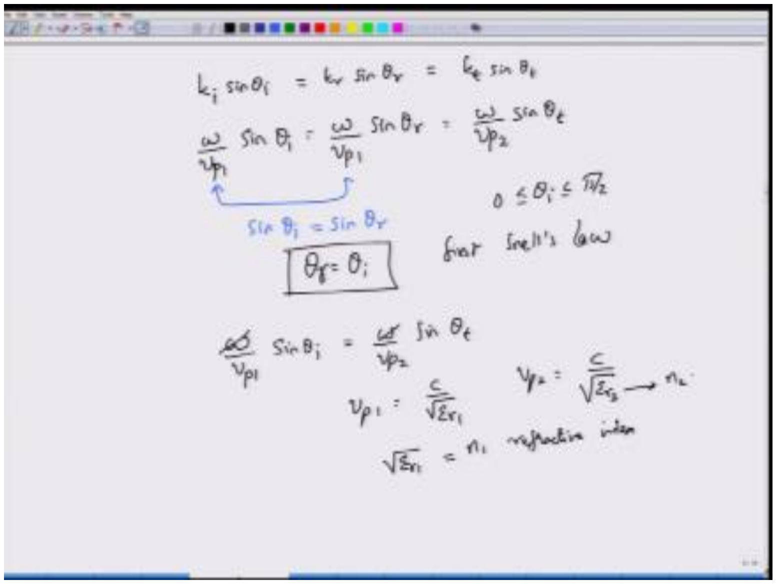

We get the second law, you equate the first k_i sinθ i2 the last term kt sin θt, so you get ω/vp1 sin θ_i must be equal to ω/vp2 sin θ_t ω cancels on both sides since the phase velocity in region one is the velocity of light divided by $\sqrt{\varepsilon r_1}$ and velocity in region 2 is $c/\sqrt{\varepsilon r_2}$ and recognizing that $\sqrt{\varepsilon r_1}$ is actually the refractive index n_1 this is the refractive index n_1, this εr_2 is the refractive index n_2.

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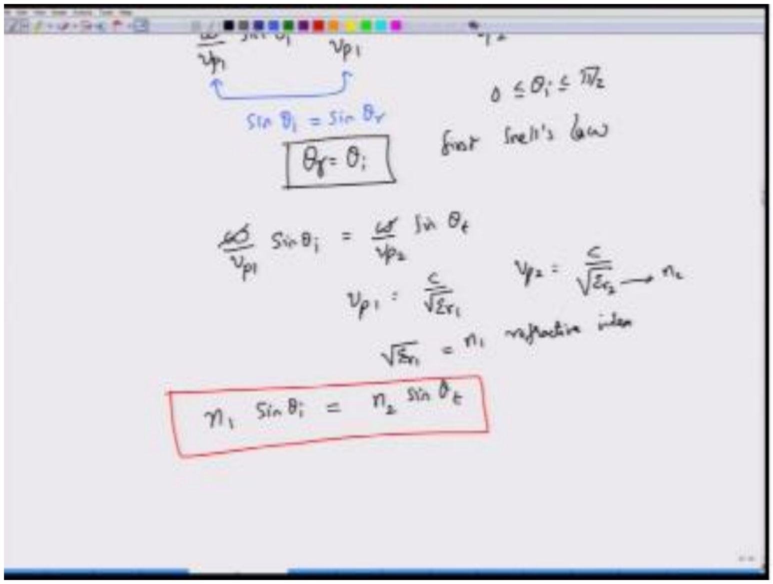

What you get here is sin θ_i which is $n_1 = n_2$ sin θ_t so if you know your boundary conditions you do not have to remember what is Snell's law you can derive Snell's law from this, okay. So this is the second Snell's law.

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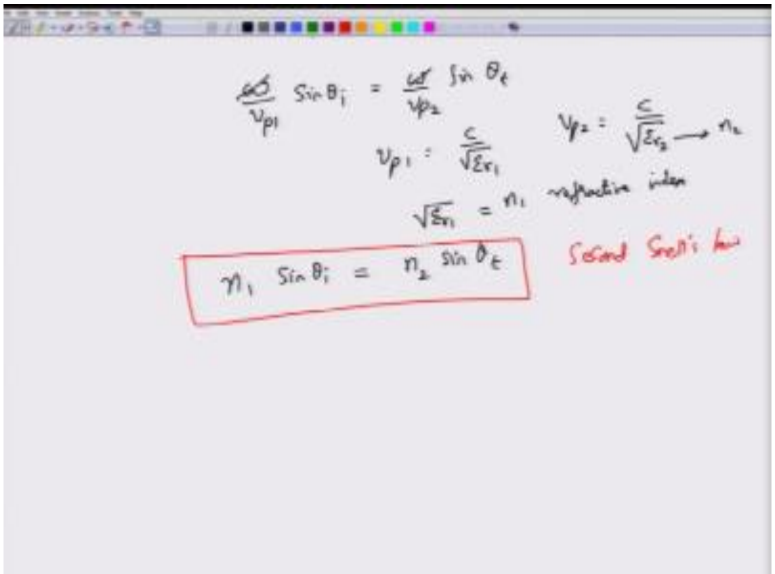

In fact so this law gives you the angle of incidence to angle of refraction or the transmission, okay. Now let us look at the problem is not yet done we have only applied only one boundary condition we need to apply a second boundary condition the second boundary condition is in terms of h fields right.

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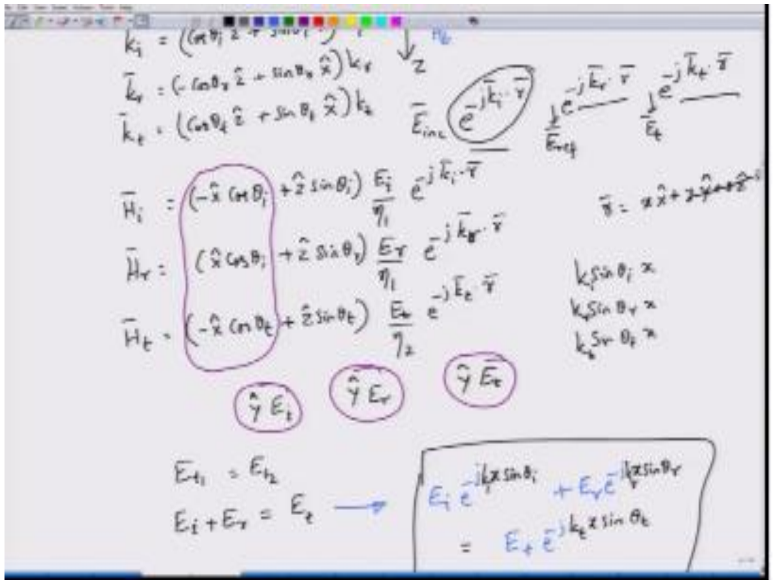

So what was the boundary condition you have to find out what are the tangential components of the h which our $-\hat{x}\cos\theta$ I $\hat{x}\cos\theta$ i-$\hat{x}\cos\theta$ t so I need to hopefully remember this correctly.

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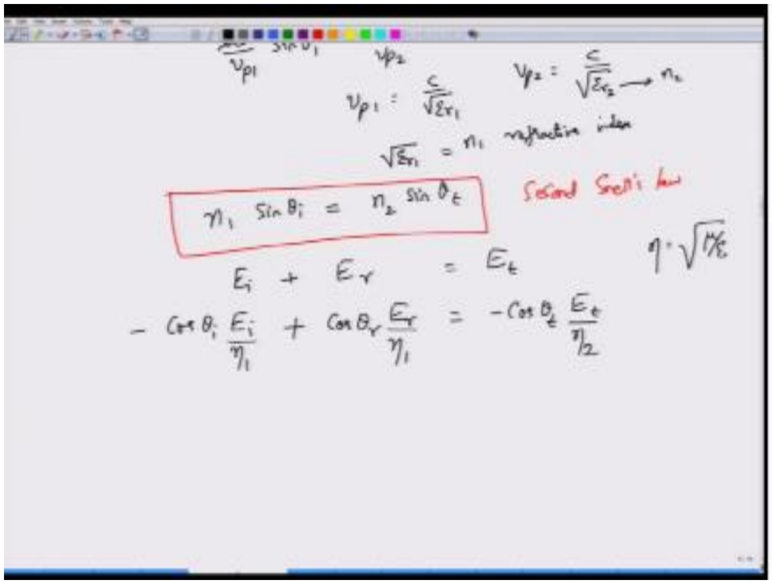

So I will go back and write this so the total tangential component in the first medium is –cos θ iE_i/η1 + cos θ r Er/ η1 which must be equal to –cos θ t Et/ η2 η1 and η2 are the impedances of the medium in region 1 and region 2 so if I have not written it earlier this should be equal to √μ/ε okay now you have two equation one equation is that Ei+ Er = Et the second boundary condition is given by this equation, correct. Now you might ask what happen to the phase factors well, phase factors where your or equal to each other, right.

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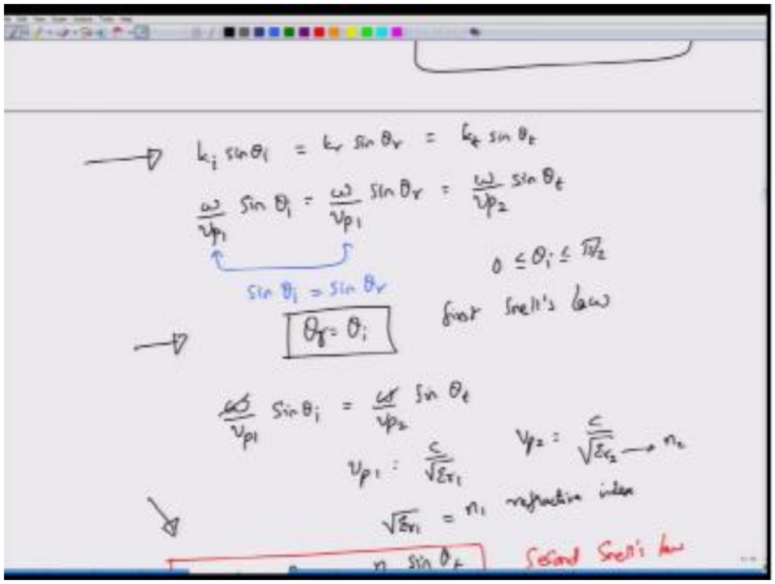

Phase factors where equal because this condition was to be true and because of this condition only we got two Snell's law one Snell's law and another Snell's law. So we do not have to worry about the phase factors for second component.

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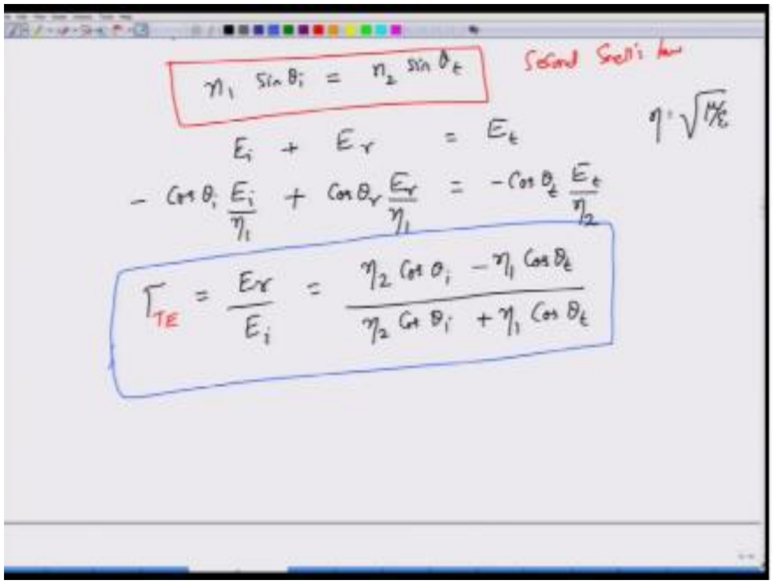

That is for the magnetic field component, now I will leave this as a very simple exercise for you to simplify the equations and obtain the reflection coefficient which is the ratio of reflected electric field amplitude to the incident electric field amplitude for the Te case, that we have considered, and this turns out to be η_2 cos θ_i-η_1 cos θ_t/η_2 cos θ_i+ η_1 cos θ_t this equation this is valid for the TE case, okay is that reflection coefficient that we wanted to derive, this relates the ratio of the electric field in the reflected to the incident electric field.

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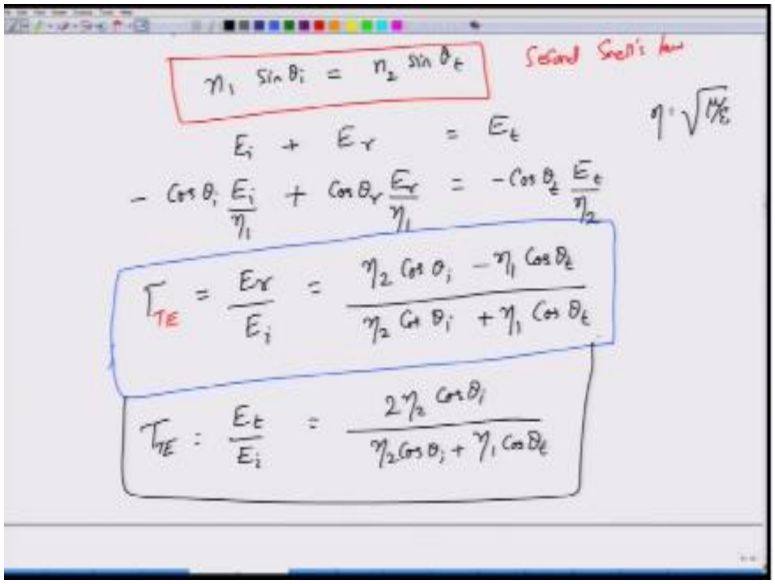

Similarly you can find out what is the transmission coefficient for the TE case which would be the ratio of E_t to E_i and this would be equal to $2\eta_2 \cos \theta_i / \eta_2 \cos \theta_i + \eta_1 \cos \theta_t$ okay, so this would be the transmission coefficient.

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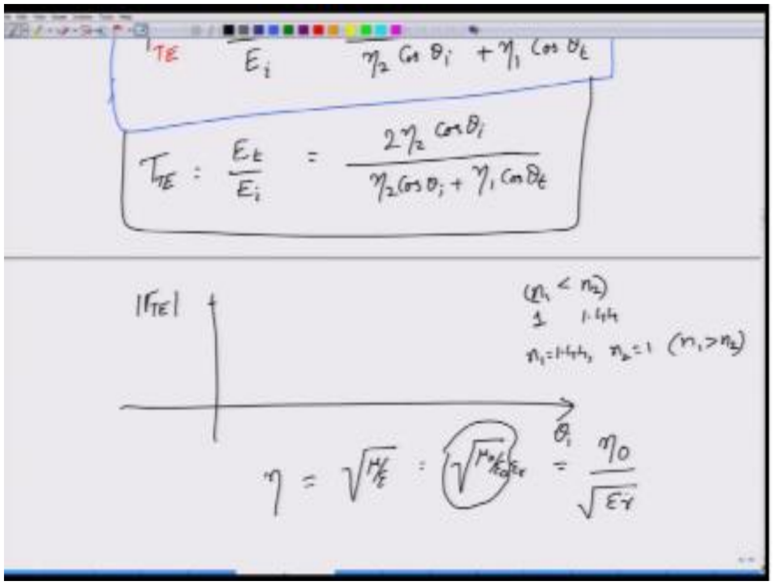

Now if you want to consider two situations and start varying the angle of incidence, okay first let me consider a situation where magnitude of the reflection coefficient that I am going to plot, I will consider to situations, okay one situation I will assume that the medium has a refractive index n_1 is less than n_2, okay this for example could be 1 and n_2 could be 1.44, okay which is silica for us, okay. Then we will interchange the first medium as n_2 and the medium as n_1, okay so this would be n_1=1.44 and n_2=1 for the second case.

In this case obviously n_1 is greater than n_2 your impedance η is given by $\sqrt{}\mu/\varepsilon$ but this can be written as $\mu0/\varepsilon0\varepsilon r$ and this further can be simplified by denoting the $\sqrt{}\mu0/\varepsilon0$ as $\eta0$, so you get $\eta0/\sqrt{}\varepsilon r$.

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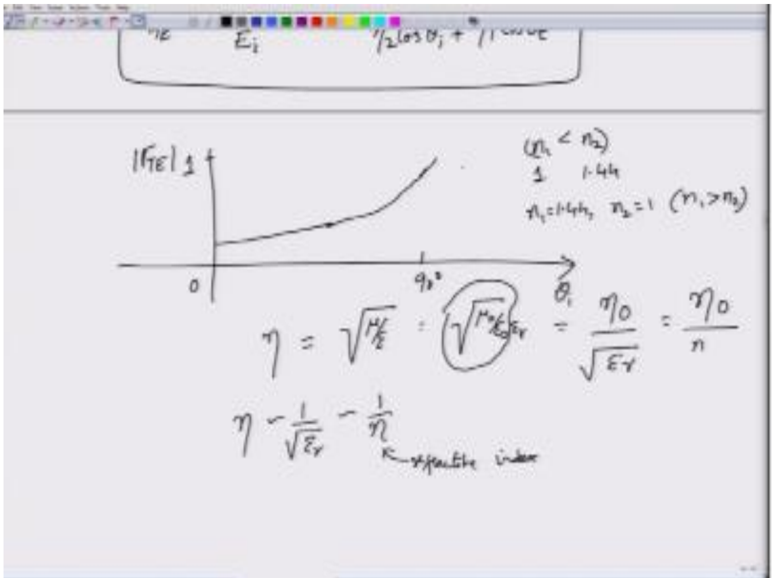

Or this is equal to impedance in the free space divided by refractive index n, so what I want to say is that given a medium having a relative permittivity εr, the impedance is inversely proportional to the √εr or equivalently 1/n where n is the refractive index, so higher the refractive index lower the medium characteristic impedance η, okay. Now consider first the case where n_1 is less than n_2 so your wave is happily coming from here and it goes into silica, okay.

What will happen here is that the transmission the reflection coefficient keeps on increasing slowly at some point which is at θi=0 and maximum you can get is 90^o because that is where you get what is call as the gracing angle, so as it goes slowly and eventually reaches up 1, okay. So it assume topical reaches up to 1 at θ=90^o this is the phenomenon that you might be familiar, so you can actually see like this.

If a ray of light falls perpendicularly I mean normally to the inter phase, it will be reflected back in the same direction. However when the waves comes and hits the surface this way the wave also gets parallel transmitted. This kind of transmission along the axis, okay this is called as gracing angle incidence and transmission from the gracing angle and this is valid as well as for both Te and Tn waves. So that is why at 90^0 the reflection co efficient goes to 1 that is almost entire waves basically gets reflected. You can try this if you want to take a book and then hold it up and then you try to read, you will get maximum reflection that is getting reflected. If there is anything that is printed on the piece of paper you would not be able to read, so this gracing angle is something that is common to Tn and Tm, let us switch the cases now.

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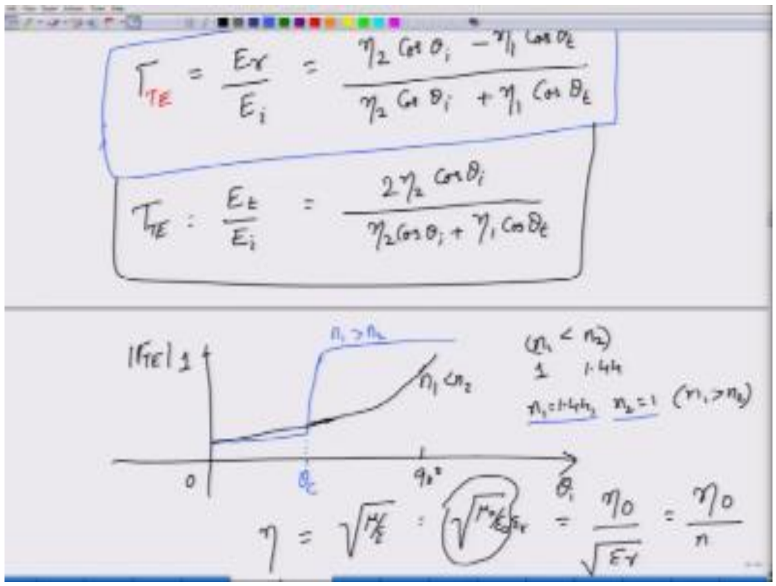

I consider N1 =1.44, N2=1, what happens in this case is that, at θ=0 begins in the same way and there after it kind of goes, and then suddenly goes to actually the small degrees actually which I should point out here, so hold on a minute. So this is your phase when N1 is less than N2. Okay now for the case where N1 >N2 you will see that this would go like this and suddenly hit reflection co efficient equal to 1 but what happens is that there is a sudden change in the reflection co efficient.

This occurred at an angle which we conveniently call as critical angle okay what is this critical angle this critical angle is the angle at which the reflection co efficient becomes = 1 which means that whatever the light that is incidents after this θc will be entirely reflected back this happens only when N1 is> N2 that is the medium has of the first or the incident has a higher reflective index compare to the second medium which is a full over reflective index. That is going from denser to rarer medium. This phenomenon when.

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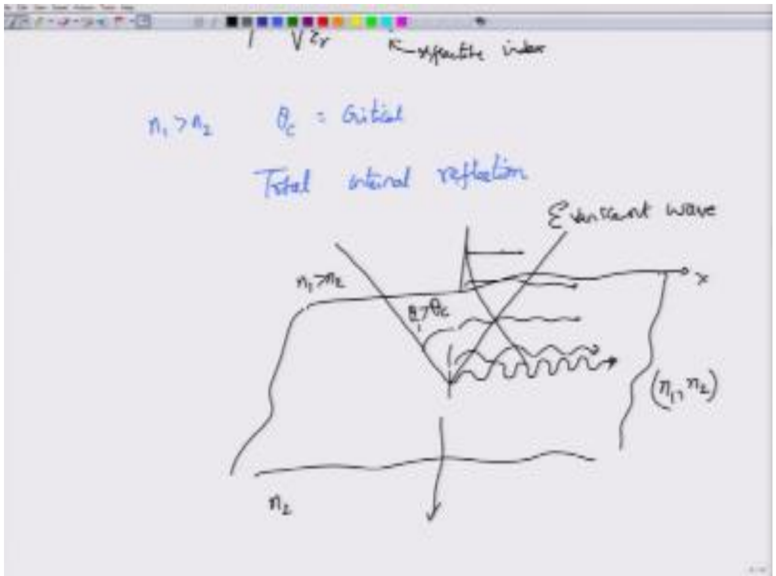

N1 >N2 there is a certain angle call as critical, angle is called as total internal reflection okay this is called a total internal reflection and what happens in this total internal reflection is that if this is the boundary that we are considering okay and this was the normal that we had remember this was one of the axis that we had as the wave is incident from a medium of higher reflective index N1>N2.

This isN2 okay this wave gets reflected totally so this angle must exceed the incidents angle must exceed the critical angle has you have measured okay while this is the simpler picture the electric field amplitudes would not go away what would happen is there would be propagation of the electric field along the axis okay however this amplitude of the electric field D case as you go away from the inter phase from the top to bottom.

You will see that the electric fields amplitudes start form Dk along this way however it is continues to kind of propagate parallel okay it Dk is in amplitude as you go away from the surface but it will be directed will be propagating along the x axis or along with the surface of the boundary, such a phenomenon is called as Evanescent sorry such a way it is called as Evanescent wave okay.

Evanescent waves do not carry any power okay you can show by calculating the electric field and magnetic field that electric field and magnetic field will be 90^0 out of phase with each other and therefore they carry no power however the amplitude of the electric field will Dk okay the rate at which this Dk depends on what is the difference between N1 and N2 if the difference is large then it will Dk faster.

If the difference is small the it will Dk at a lesser way so this is the Evanescent wave whose amplitudes this all decreasing and eventually the amplitude will be equal to very nearly equal to 0 so you can imagine wave along the ocean which are propagating this phenomenon is called as total internal reflection we will talk about modes of optical fiber by talking initially about total internal reflection in the next module. Thank you very much.

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