Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Optical Communications

Week – IV
Module – III
Review of EM Theory

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Hello and welcome in this module we will first review some basic concepts from electromagnetic theory which is required for us to study the wave propagation inside an optical fiber then we will look at behavior of this electromagnetic waves at boundaries we will look at the phenomena of total internal reflection which forms the basis for construction optical wave guides, so let us began by talking about electromagnetic theory we describe electromagnetic fields.

There are 4 electromagnetic fields which are primary concerned to us these electromagnetic fields are called as electric field.

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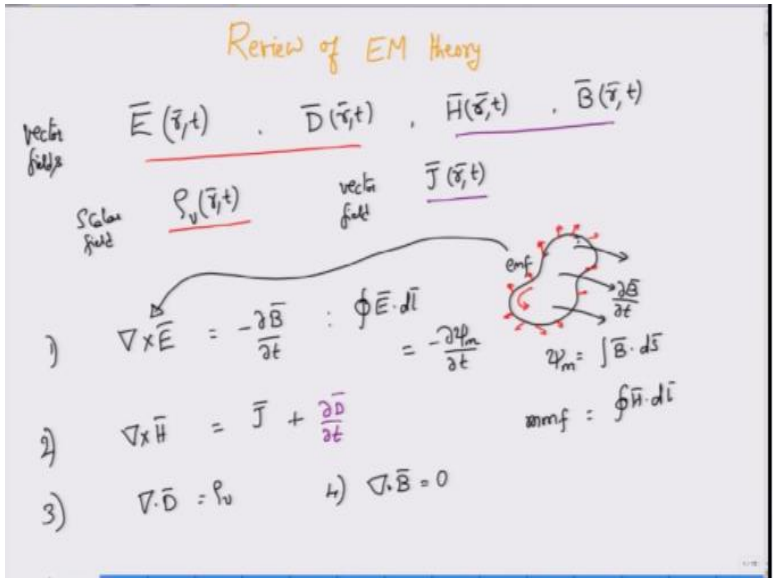

E which is a function of both position in space as well as time in general this electric field E is sometimes called as electric field intensity and you have other field which is called as D which is electric flux density which will again be a function of both position as well as time you have H which is magnetic field or sometimes called as magnetic field intensity which will also be functions of time and finally you have magnetic flux density which is a function of both position as well as times.

These quantities are functions of positions which are represented by vectors r position vectors r and at each position as well as with respective to times these quantities change and these quantities themselves vectors therefore these are all called vector fields okay the source for these vector fields E, D, H and B are the charges and currents the charges are specified as functions of position and they can also change with respect to time.

So if the amount of charge in give region is changing then it means that there will be some current flow that is happening so this charge density being a scalar quantity we usually represent this one by the symbol σ and the most general charge is distribution that you can imagine is the volume charge density this two will be a function of both space and time however this one is a scalar field.

Why is this called a scalar field because at each position and at each time the quantity that you get is a scalar charge density is a scalar and you have another vector field okay this vector field is the current density vector which will also be a function of space and time now for a long time it was though that the electric phenomena which can be described by these quantities was kind of different from these three quantities which are closely associated with magnetic phenomena.

So for a long time people studied electric phenomena separately magnetic phenomena separately but then it was shown that these two phenomena are you know interrelated by experimental work of faraday and this complete electromagnetic model was given by Maxwell building up on all earlier experimental researches in electricity and magnetism so today if we talk about electromagnetic theory or electromagnetic model we start with Maxwell's equations which are applicable to all electromagnetic phenomena occurring at macroscopic scale that is occurring at scales which are considerably larger than the atomic scales okay.

And there is of course a corresponding theory for behavior of electrons and protons in terms of electromagnetic theory but at those distance one has to supplement electromagnetic theory by quantum theory as well however for most macroscopic phenomena we study electromagnetic theory based on Maxwell's equations, Maxwell's equations we customarily number them there are 4 Maxwell's equations these equations relate E, D,H, B

Okay and there is one more equation which the conservation of charge or the continuity of the current which relates the two equations σ and J let me write down Maxwell's equations and I will briefly explain what these are I would be unfortunately not able to completely talk ablaut Maxwell equation or electromagnetic theory in detail because the scope of this course is not that much how ever there are excellent textbooks which talk about electromagnetic theory at the end of this module I would recommend two of them you can look at those text books to fill in any gabs that we are going introduce.

As I said there are 4 Maxwell's equations there is no definite order to this equations however we write I mean this is my personal preference so I start with two curl equations and two divergence equations come later, so the first curl equation is called as faradays law, it states that if you consider a loop or a conductor and there is a magnetic flux which is changing magnetic flux being given by integral of the magnetic flux density, so if this magnetic flux is changing with respect to time okay then there will be a EMF induced around the conductor.

And this induced EMF is further related to this integral of E. dl the way I am writing this equations are what are called as differential forms I will just write down the corresponding integral form for you guys to appreciate this law better, okay. So this integral of electric filed so there will be some electric field okay, this electric field if you integrate the only the electric field component which is tangential.

To this loop will contribute to this integral because if the line integral around this particular path so if you look at this line integral of E this gives you the EMF and this EMF generated must be equal to total change or time rate of change of magnetic flux ψ_m where magnetic flux is given by integrating the flux density vector over the surface which is bounded by this particular conducting loop, alright.

The second equation says very similar things except it applies to magnetic fields just as we have defined electro motive force or EMF one can define MMF okay and this MMF is given by integral of H. dl if I am looking at a closed loop then this would be the closed loop MMF okay, so the corresponding quantity in the differential form is curl of H or in the point form is curl of H this was originally thought to be just the current density vector J, okay.

It was originally thought that this curl or the magnitude motive force MMF is only because of the current density J however later Maxwell added a very important term known as displacement current density, okay. Time displacement current density is a time rate of change of electric flux density D, D is the electric flux density so the time rate change of this flux density D electric flux density D will act like current and it will continue the current.

Wherever in those regions conduction current is not possible, conduction current being the J vector, okay. There are two additional equations one is $\nabla . D = \rho$ v it simply says that the source for the D field is the volume charged density and you have the 4^{th} equation which says that $\nabla . B = 0$ it does not mean that there is no force of magnetic fields it just means that magnetic field lines or the magnetic flux lines B are you know occur in the closed group configuration.

There is no start or the end point there is nothing like a magnetic charge okay which can generate magnetic fields just like there is an electric charge an electric charge will generate electric field lines which you know if it is appositive charge yo would have seen that the field lines are all drawn like coming out of the charge if it is a negative charge then the field lines would all be coming in however for the case of magneti9c field lines the sources are all currents.

Which means that the lines the electric where the magnetic field lines will always be occurring in continuous loops so these are four Maxwell's equations these Maxwell's equations have to be supplemented by a few additional relationships these supplementary equations are normally the material descriptors.

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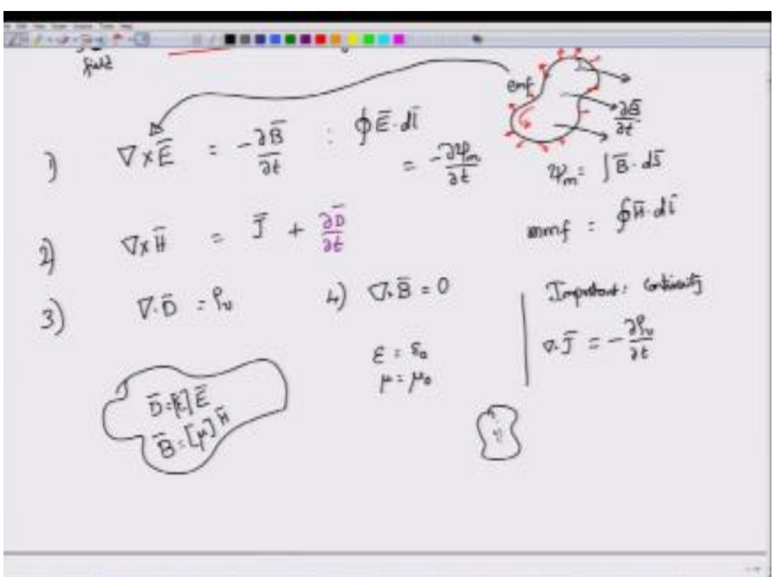

That is to say if I consider a certain material medium okay and if I want to represent what is D here what is E here Maxwell's equations does not directly tell you that okay rather than that this relationship must be given in terms of the constitutive parameters of the medium, so these parameters are the permittivity ε and permeability μ which relates D inside a material medium to electric field E.

Which relates B inside a material medium to field H, okay in the free space of course $\varepsilon = \varepsilon_{0},$ $\mu = \mu_0$ these are known a s free space permittivity and free space permeability, okay. Finally there is also one important equation if this equation is called as the continuity equation this continuity equation relates the charge density ρ or rather it relates the time rate of change of the charge density ρ to the current density J, okay.

So it means that in a closed loop you know if you consider the close surface in this surface if the amount of charge is getting reduce right, then obviously those charges must be coming out of the surface so if the charges come out of the surface they would be constituting the current density j okay so it simply says that the amount of current that your are getting from a closed surface must be equal to the time rate of change of the charge density inside or the time rate of change of the charge inside.

Okay so these are Maxwell's equations now we will look at the wave equation okay we have already looked wave equation the previous module so we will not spend too much time there I will just write down what is the wave equation however we will write down the solution of the wave equation and then examine what happens as these waves propagate and hit a particular boundary okay so we will see what happens to that case the wave equation has derived in the previous module.

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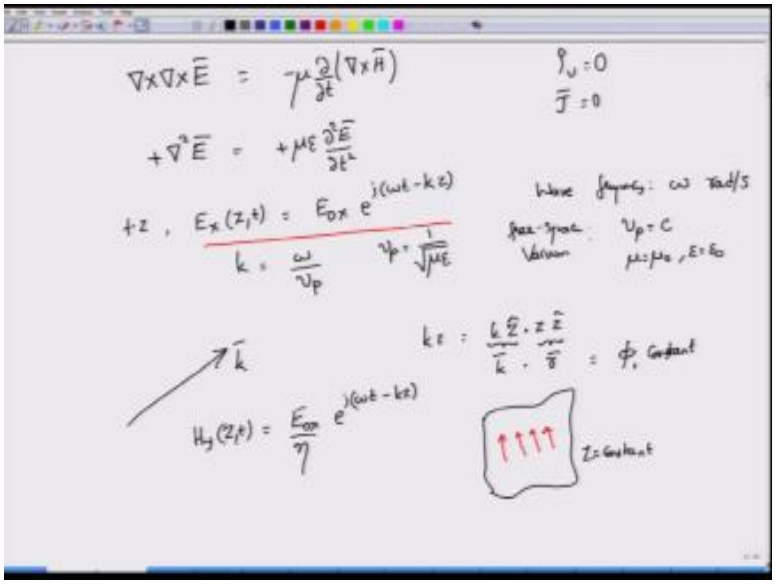

You may want to check up on the previous module that w have derived is obtained by first in the curl of the faradize law and then invoking the second equation usually we are not applying this wave equations slide at the source itself your applying at a region which is far away from the source so which means that your applying them in the region where there are no sources that is both the charge density row as well as the current density vector j = 0 so the equations are considerably simplified we will also assume that the medium is linear homogenous and isotropic okay.

So in such a medium b = μ x H so I can write down in place B I can write down H and since I am taking curl I can interchange that δ/ δt operation and curl operation okay simplifying the left hand side I get δ2E and the right hand side or rather I should get – δ^2E the right hand side I will get as – μ epsilon δ^2E/ δt^2 cancelling of the – and + and you can remove this one okay now one solutions for this is to assume that waves are propagating along z direction okay of the waves are propagating along z direction and we take only that Ex component is 0 then the solution for this one will be of wave.

Whose electric field is a function of only z and time and this electric field is given by some easy row which is the constant so we cam may be write down this as E_{ox} and then you have $e^{j\omega t}$ – k x z where we have assume that this is a wave whose frequency is given by ω so this wave frequency is given by ω this angular frequency so this is radiance per/ sec okay and this k vector is related to ω for this case it is given by $k = \omega/V_p$ where V_p is the phase velocity of the wave.

Okay it is the velocity with wish to constant phase point for this plane wave would be propagating so Vp is = 1/√ μ epsilon for free space or vacuum actually for free space or vacuum In fact this phase velocity is equal to the speed of light because μ will be equal to μ_0 and epsilon is equal to $epsilon_0$ okay t5hese of course is not the only way you can have a wave in face you can have a wave along the political direction which is described by the propagation vector k.

Okay in the previous case the propagation vector was directed along the z access because I can write k_z as k x z^. z x z^ this k x z^ could have given me the vector k right and z x z^ is the nothing but the position vector on a plane okay so the position vector is r so what you are doing is the dot product of k.r and wherever this k.r = sum 5 and this 5 is a constant these are the constant phase planes okay these constant phase p[lanes in the case of this wave solution r plane okay more over in the plane which is z = constant.

The orientation of the electric field so suppose this is the plane that I am considering which is z 0 constant plane okay in this the orientation of the electric field will be along say x direction with the certain amplitude and that amplitude does not change so the amplitude is essentially uniform over this plane the corresponding H field will be H along y so that e cross H will point in the direction of the wave propagation or the energy propagation and the amplitude of this magnetic field is actually reduced by a factor η compared to the electric field.

However the frequency of this wave would be the same and this wave also be propagating along the z axis in the sense that a complete electromagnetic wave or light because we will consider light as electromagnetic wave will be propagating along the z axis, it will have an electric field which is directed along the x axis.

And the corresponding direction for the magnetic field is along y axis, so that x cross y will give you the direction of the wave propagation, okay. Now as I was saying in general it is possible that the wave is propagating in any general axis, okay so it might be propagating along a particular direction so we describe this direction by giving a direction vector or we giving a propagation vector, okay.

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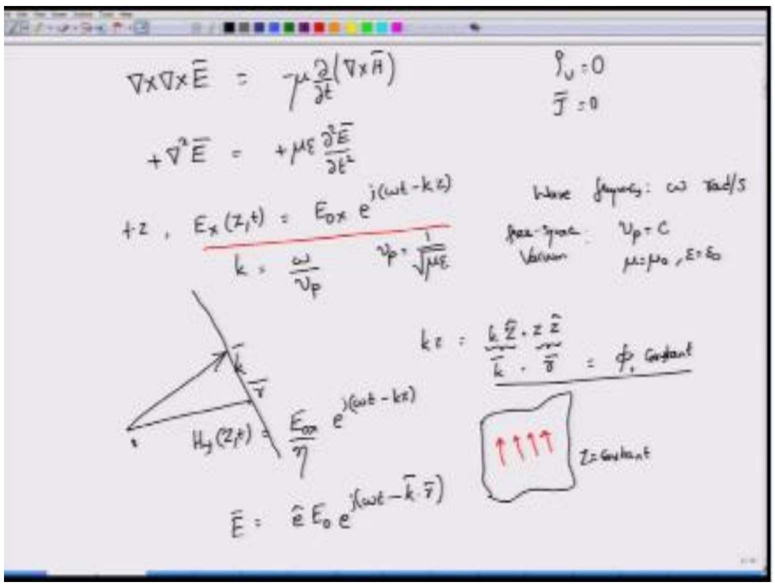

This propagation vector I have indicated here in this diagram as k, okay and if I consider a plain okay, such that on this plain the position vector is given by r then this k.r represents all those points along which the phase is constant and then we actually write down the electromagnetic wave on this plain by writing this as, so assuming that this electric field is directed along some direction e, okay and has certain amplitude E_0 and has a frequency ω. However, it is propagating in a general direction k therefore you write this as k.r, okay.

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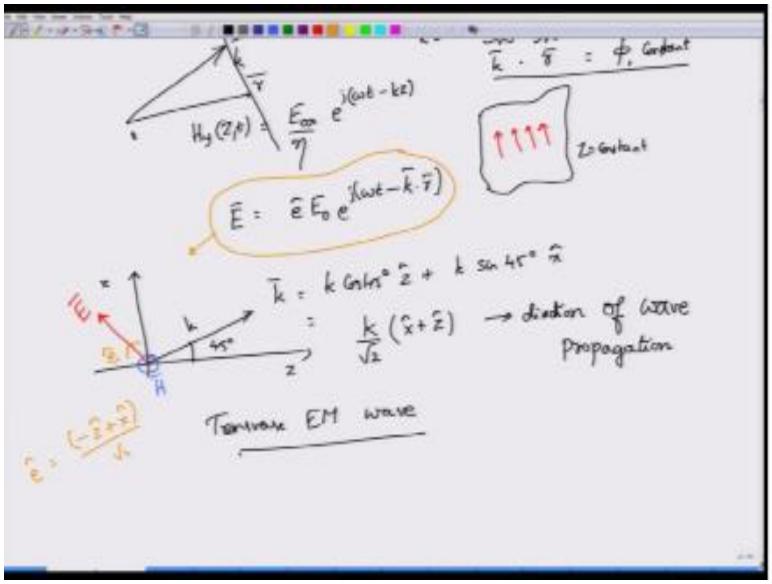

As an example, consider wave which is propagating along the z and x plain it is propagating at an angle of 45^{o} with respect to the z axis, okay. So what will be my k vector, k vector will be assuming that the magnitude of the k vector is given by k, okay the vector k will be given by magnitude of k times cos 45^{o} along z axis plus k sin 45^{o} along x axis, right so this is basically $k/\sqrt{2}$ and $\hat{x}+\hat{z}$ this describes the direction of wave propagation this describes the direction of wave propagation.

So what would be the direction of the electric field, well the electric field has to be perpendicular to this one so because we are assuming plain waves the electric field has to be perpendicular and let us say this is the direction of the electric field. What will happen to the magnetic field well, magnetic field has to be perpendicular to both k vector as well as the e vector that is both propagation direction as well as the electric field vector, in order to form a uniform plain wave, okay.

So this particular wave which is given by plain of constant phases in the form of a plain and a certain electric field which is perpendicular to k and uniform in that plain and magnetic field which is perpendicular to both electric field and k vector and it is also uniform in the plain transverse plain is called as transverse electromagnetic wave, okay.

So this is called as transverse electromagnetic wave, so this is the wave that would be described by you know the electric field for such a wave can be described by this particular equation, where e will be the unit vector along the direction e such that e is perpendicular to the direction for k, okay. In our coordinate system that is x and y coordinate system this electric field e will have two components, right what is the unit vector along e, it will have a component along –z direction because you can decompose its electric field into –z and +x component.

Therefore, the unit vector along the electric field will be along $-\hat{z}+\hat{x}$ in this case it kind of is 45° therefore this could be $1/\sqrt{2}$ but otherwise it will be a certain number, but the important point is the direction of the unit vector e is along x-$\hat{z}$/x-$\hat{z}$ that is the essential point by $\sqrt{2}$ is because we are assuming 45°, okay. So this is all about uniform plain waves and they propagate I understand that I have been rather quick in talking to you about these waves, okay

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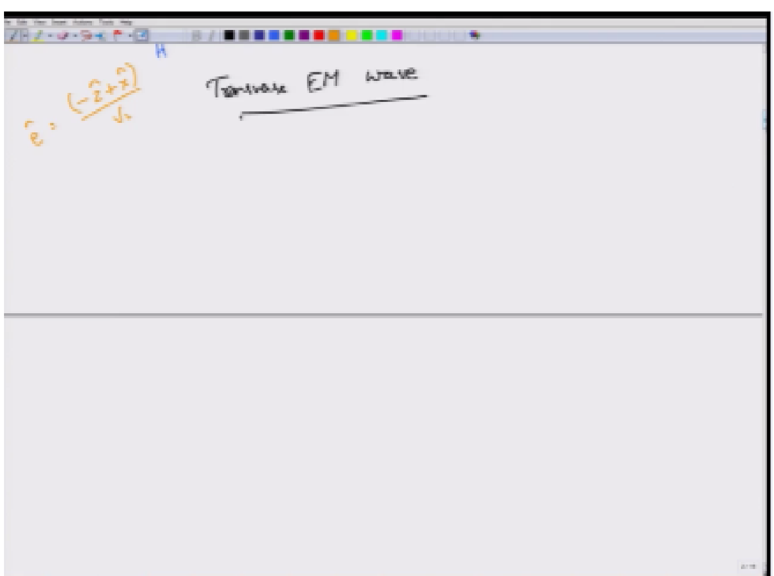

However, as I said the goal is to get to optical fiber propagation as soon as possible so we will talk this review of wave equation now and then talk about what happens when such a wave actually hits a particular boundary that is it goes from one medium to an entirely different medium, so you can imagine that there are 4 vector quantities or vector field quantities e d b and h and we have to individually understand what happens when the electric field e of a certain wave on a certain.

This one it is a boundary beyond this one the entire region is characterize by certain parameters μ of silent and a different μ have silent and σ which is the conductivity and below this is characterize by μ ε and σ which is different so you have a region one so I am maybe not able to completely show you because my hand is kind of finite and limited but you imagine that my hand is stretching all the way to infinity okay.

So it is like a plane which is stretching all way to infinity and there is an electric field which is propagating okay in some direction does not matter but the electric field e is in this direction okay so electric field e in region one which is satisfying max wells equation is given by this particular direction now the question is what will happen to this electric field so if there is an electric field in this direction now you know arbiter I am taking it.

In the second region is there a relationship between this electric field and this electric field similarly is there a relationship between this d field and d field in the second region right similarly is there a relationship between b and h in one region to b and h ion other region. So in order to understand these relationships among this vector field quantities it is necessary to briefly review the notion of boundary conditions.

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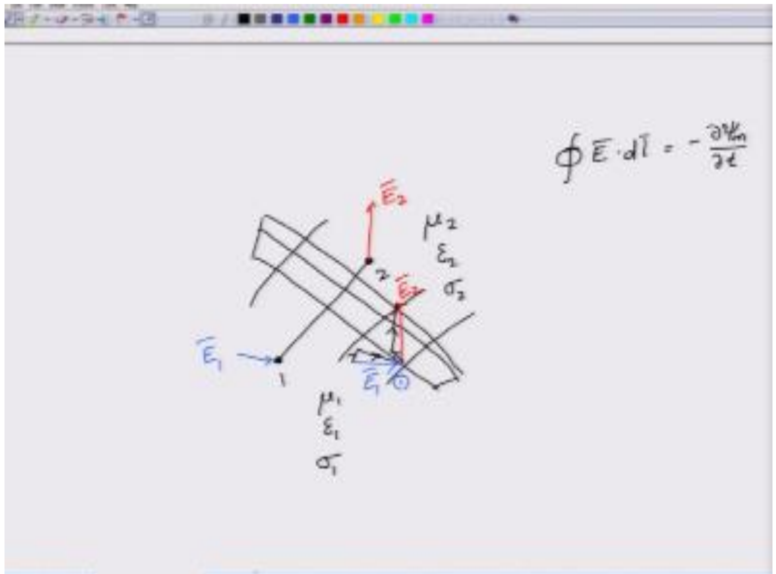

Boundary conditions come about in this way there are two point I have 1 I have point 2 and these 2 should be join by a straight line okay now when I have a line I actually also have be a plan which separates these two correct so I can actually have a plane which separates these 2 quantity is this is the notion my plane which is separating point 1 and point 2 point one is in region which is characterize by μ1 ε1 and if necessary σ1 which is the conductivity.

Point two is in the region which is μ2 ε2 and σ2 there is an electric field here let us say this is directed along this direction and there is an electric field which is directed along this direction e1 okay. Now these are shown to be very far away from the two boundaries but what we are interested is what happens is this electric field approach as this boundary right so at this point what would be the electric field.

It would be parallel or it would actually be the same electric field even which I have written, so this is the point one I am considering for simplicity I started by showing points which are faraway but I want to understand what is happening on the boundary therefore I will consider this points I has close to the boundary as possible. What will happen to the electric field e2 the electric field e2 will also come out in this particular direction okay?

Now is there a relationship between e1 and e2 first observe this electric field can be written down in terms of its tangential and normal components right similarly this can be written in terms of its let me try and write down this way, so this can written in terms of its tangential and normal components. Now one of the equations that we have at our hand is the equation which says integral of E.dl =-del magnetic field by del t.

So this is the equation that I have unfortunately if you look back at max well equation this is the only equation that I have for a electric field e, so whatever I have to do I have to use only this equation to understand how the total electric field is changing now that problem is not very obvious at all because now let us try and evaluate this equation to the points which are separated by the boundary.

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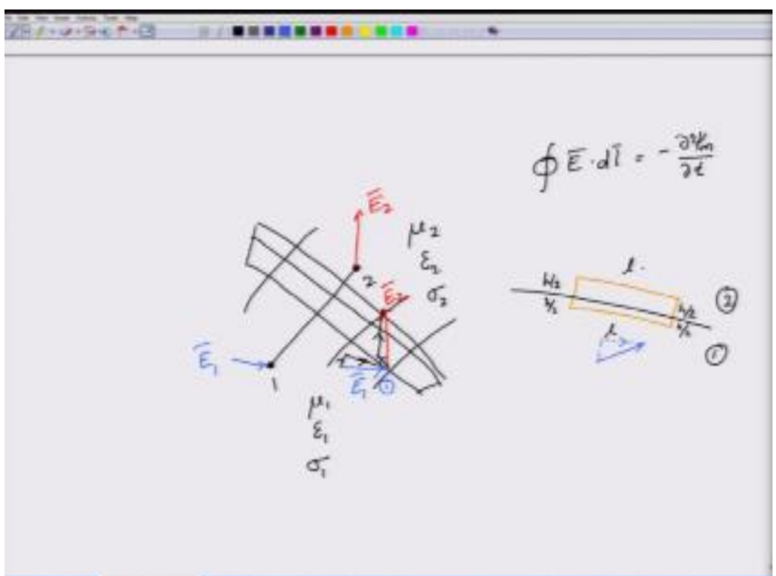

Let us re draw this boundary situation that we have this is the boundary that I have so I am going to assume a loop whose height is very small all though I am showing that is height is pretty large okay, again I made a mistake this height should have been symmetrical but anyway does not really matter okay so this height is h/21 here is also h/2 h/2 the length of this loop is l okay right so this is the region 1 this is region 2.

Now if you apply electric field E.dl expression to this one okay if you apply that condition what yu have is you had an electric field which was directed in this way right so you resolved this in to 2 parts that is normal and tangent and while going round this loop let’s pick direction for the loop, let us say that the loop is to traversed in this particular way. So as I am going through this loop this portion of the loop lies in region 1 and this portion lies in region 2.

There are two portions which are lying so you can label these segments as 1, 2, 3 and 4 segments, so what is the left hand side when you evaluate to, the segment 1, well whatever the tangential component that you have obtained, which et us call it as Et1 times l will be the result of the first segment. The integral of electric field on this line will be Et1l. What can you say about the integral along segment 2? Well integral along segment two has to broken up into two parts, here you have En1h/2 and top here you will have En2h/2.

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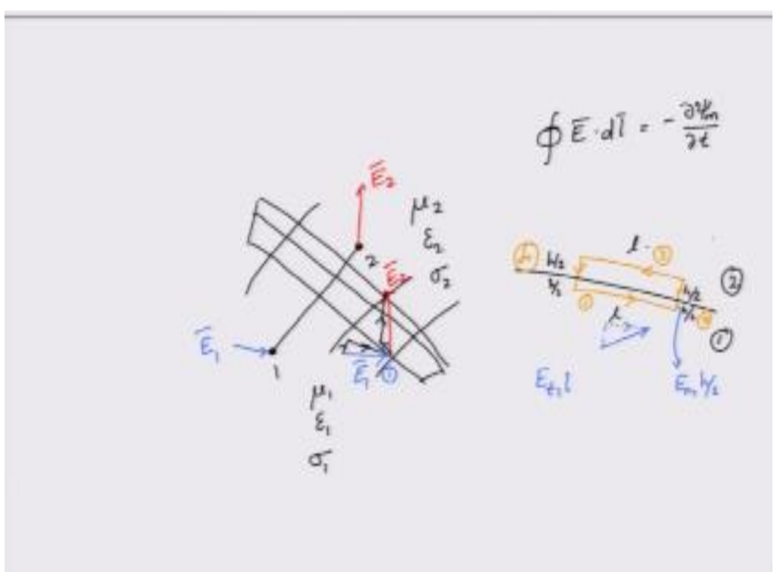

What is the component at this stage, at this segment, fourth segment; it is –En1h/2, because these two re in the opposite direction and the electric field along this segment, half of the segments per this half of the segment are the same. So these two will cancel with each other and don't have to worry about the segments 3 and 4. For a similar reason electric field here will be En2h/2, the direction will be En2h/2 or the component will be En2h/2, here you have electric field En2, however this En2 into h/2 will cancel with these other En2h/2.

So you don't have to worry about the segments, the vertical segments, and look to only the horizontal segments. For the horizontal segments you have the contribution from the third segment as Et2 because it is the tangential component of the electric field region 2 times l, this is the left hand side. What would be the right hand side? Let us keep this –δ/ δt a its is and realize that this magnetic flux density is given by integral of d.ds, assuming that this loop is uniform, what you get is the component of B times the area, the area is given by lh and let's assume that it is only the normal component of the B, that is contributed to the right hand side.

Now we look at left hand side and the right hand side, so you have Et1-Et2 into l is equal to this one, l cancel from both sides, now that we do is we reduce the hype of this loop. Take limit of h going to 0 and because Bn has to be finite, it is the field quality which has to be finite, this entire right hand side goes to 0 and what will end up ,is having Et1=Et2 approximately two regions, similarly you can show that the normal component En1-En2 should equal to the surface charge intensity that would exist, if at all the medium is dielectric and not, you will similarly have Ht1-Ht2 is equal to the surface current density or the sheet current density and finally you have Bn1 – Bn2 is equal to 0.

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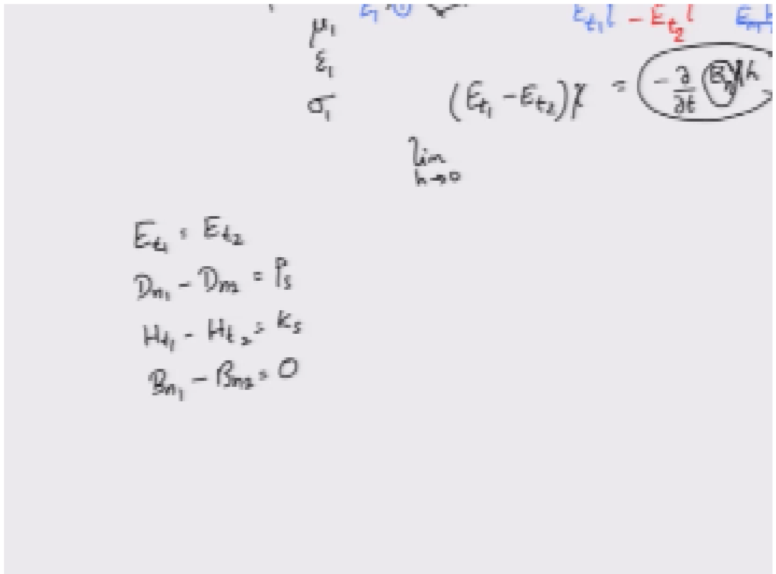

We will stop here and look at the reflection and the transmission of the wave in the next module. Thank you very much.

<u>Acknowledgement</u>

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