

Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Optical Communications

Week – IV
Module – II
Propagation of Electromagnetic wave

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Hello and welcome in this module we will look at propagation of electromagnetic wave or light inside phase modulator and then we will also if possible we have time we will also look at propagation of wave in a amplitude modulator that is to say we consider the physical basis of phase modulator and amplitude modulator okay we have seen that an electromagnetic wave light is an electromagnetic wave and this electric field or a magnetic field of the electromagnetic.

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Handwritten notes on a whiteboard showing electromagnetic wave propagation in an anisotropic medium. The notes include equations for electric field components, wave vector components, phase velocity, and wave number.

Top left equations:

$$\begin{aligned} \vec{E}_x &= \hat{x} E_0 e^{j(\omega t - kz)} \\ \vec{E}_y &= \hat{y} E_0 e^{j(\omega t - kz)} \end{aligned}$$

Top right equations:

$$\begin{aligned} E_x &\rightarrow k_x \neq \\ E_y &\rightarrow k_y \end{aligned}$$

Diagram of an anisotropic medium:

A diagram showing a medium between $z=0$ and $z=L$. The medium is labeled "Anisotropic". Inside the medium, the electric field components are given as E_x with $k_x = k_0 n_x$ and E_y with $k_y = k_0 n_y$. The wave vector k is shown pointing into the medium.

Bottom left equations:

$$\begin{aligned} \vec{E}_x &= \hat{x} A e^{j(\omega t - k_0 n_x L)} \\ \vec{E}_y &= \hat{y} A e^{j(\omega t - k_0 n_y L)} \end{aligned}$$

Bottom right equations:

$$\begin{aligned} v_p &= \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{\epsilon_r}} \\ c &= \frac{1}{\sqrt{\mu_0\epsilon_0}} \\ k &= \left(\frac{\omega}{c}\right) n \\ k &= k_0 n \\ &e^{j(\omega t - kx)} \end{aligned}$$

Wave which is propagating along a particular direction so let us say propagating along the next polarized wave which means its electric field will be oriented along the x direction and it will have an amplitude of E_0 and it will have a certain frequency ω propagation constant k_z okay so this particular electric field corresponds to a propagating wave where the wave is propagating along x direction.

I also know that if there was one more electric field so this is x electric field if I had one more electric field which is polarized along the y direction then that electric field can also have a different amplitude it assume that it will have the same frequency because we want to consider that scenario and then it will have certain k_x and k_y right so it will have a certain k vector or the propagation constant and it will be propagating along set these equations hold when the medium is linear homogeneous and isotropic.

Now let us assume that the medium is linear and homogeneous but let us now assume that the medium is an isotropic okay if we assume that the medium is an isotropic then what happens the corresponding E_x component will see a propagation constant which is k_x E_y we will see a propagation constant which is k_y and this k_x and k_y are not generally equal okay so this is the result of propagation through an isotropic medium so we will not look at an isotropic medium form now we will receive this treatment of an isotropic medium later.

But then what you can imagine is that if I take an isotropic medium I will not go into details but I am trying to motivate the phase modulation to you if I take an isotropic medium of some length L okay and then I have a wave which has both E_x component as well as E_y component okay and it is propagating along the z axis.

So you have both E_x as well as E_y components right and then the wave is propagating along this is the E_y component this is the E_x component the wave is propagating along the z direction okay so at $z = 0$ let us assume that both amplitudes $E_{0x} = E_{0y}$ okay so this is $E_{0x} = E_{0y}$ let us assume that at this stage both amplitudes are the same they have the same frequency they are propagating along the z axis.

So this is at $z = 0$ this is at $z = L$ now because this is an x polarized wave it will see a propagation constant k_x okay and I know that the relationship between k_x , ω and v_p correct so k_x is given by ω / v_p phase velocity v_p is given by $1 / \sqrt{\mu \epsilon}$ okay this can be written in rewritten as $c / \sqrt{\epsilon_r}$ where c is the speed of light in vacuum given by $1 / \sqrt{\mu_0 \epsilon_0}$ and ϵ_r is the relative permittivity of the medium okay this $\sqrt{\epsilon_r}$ in optical literature or in fact in literature is actually denoted by n standing for refractive index of the medium from this relationship I can go back and write.

So I have $k = \omega / c \times n$ right and then go to k_x and k_y , so $k = \omega / c \times n$ this ω / c is a number that is determined by the frequency and speed of light in vacuum or in free space, right. So that is denoted by k_0 so k in general is $k_0 \times n$ where n is a refractive index of the medium, okay.

Now if the medium is an-isotropic then the refractive index n or equivalently the permittivity ϵ will be different for an x-polarized wave and it will be different for a y polarized wave so if you are considering the x component then this x component must see $\sqrt{\epsilon_{rx}}$ whereas the y component or the y polarized wave will see $\sqrt{\epsilon_{ry}}$ equivalently x polarized wave will see n_x , y polarized wave will see n_y , okay.

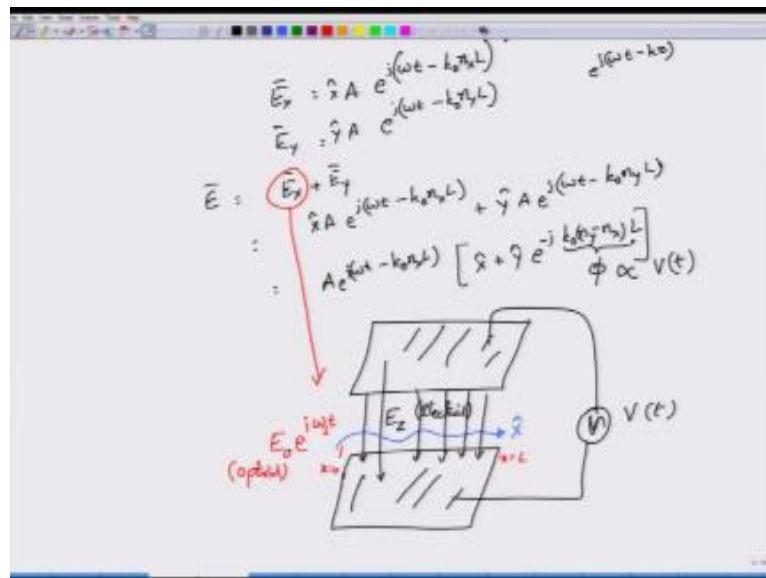
So the corresponding propagation constants k_x will become for the x and y polarized wave they will become $k_x = k_0 n_x$ and $k_y = k_0 n_y$ is that okay, now at $z = l$ these waves have now propagated, propagated a distance of l so what will happen to the electric field E_{ox} ? Well E_{ox} or the x component electric field it will propagate like this $E_{ox} e^{j\omega t - k_0 n_x l}$ and what is the distance it has propagated? L , why this l ?

Because you have an equation which says $e^{j\omega t - kz}$ where z would be the distance of the propagation, right so z is at any point along the space y here I am considering the plane which is $z = l$ so this would be $e^{j\omega t - k_0 n_x l}$ sorry this j is for the whole thing, similarly the y field would also arrive we had said that the amplitudes of these two are equal so we can stick to that condition and call the common amplitude as A .

So A stands for the common amplitude of the x field or the x wave and for the y polarized wave you have the amplitude A and ωt is the same but then its propagation term will be different

because K vector will be different I is the same but k vector is $k_0 \times n$ by for the y polarized wave is that okay, so this has to be written sorry with x and y ^ because I have vector quantities on the right hand side.

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So these are the x wave x polarized wave and y polarized wave which are propagating here, initially this total wave was a combination of x and y polarized wave correct, initially it was linearly polarized because x component wave was equal y component amplitude was equal they were both in phase with respect to each other because they were coming from a homogenous linear isotropic medium.

Now once they have entered on an isotropic medium and their refractive indices are different right because one is x polarized wave the other one is y polarized wave their refractive index is are different their propagation constants are different and when they exist the an-isotropic medium you will see that there is essentially a phase difference between the two because the total electric field can be written as $E_x + E_y$ which is $\hat{x} A e^{j\omega t - k_0 n_1 x} + \hat{y} A e^{j\omega t - k_0 n_1 x}$ you can take this $A e^{j\omega t}$ as a common factor.

And in fact you can take this $Ae^{j\omega t - k_0 n x L}$ as a common factor, okay and then what you are left here will be $x^i + y^j e^{j}$ or other $-j k_0 n y - n x L$ this term $k_0 n y - n x L$ will give rise to a phase shift between the two components okay, so what you have seen is that, there is a phase shift between the two components and if for some way if I can make this phase shift be a function of the external applied voltage then I have obtained phase modulation, correct.

This would actually become amplitude modulation because this phase modulation gets converted into amplitude change as well if instead of having two components I take only one component okay I take only one component that is say E_x term or the E_x x-polarized wave which has an electric field component E_x . So if I take only this term pass it through a medium not an-isotropic medium this time I mean the medium could be an isotropic but you do not want just an-isotropic in this case you want to have a medium which will relate.

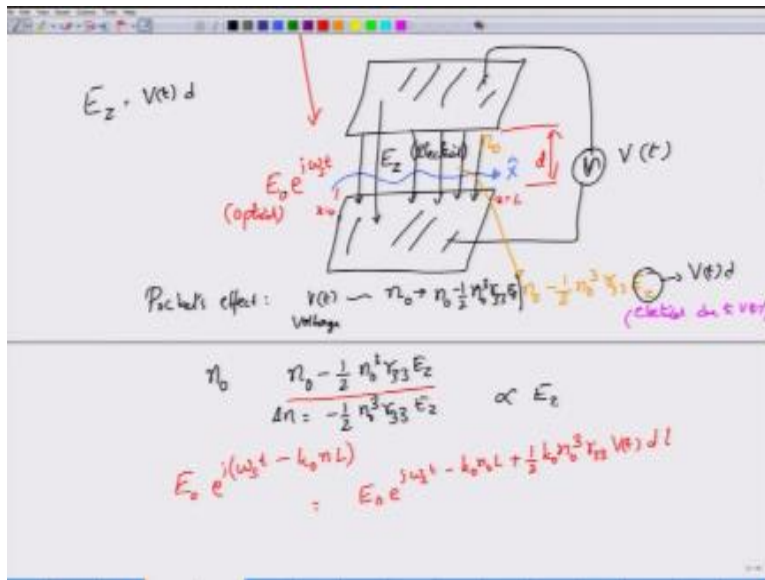
The phases shift to the applied voltage okay so let say I have a certain medium which is you know this particular medium I will call this direction as the z axis and call this as the electric field E_z these are conductors okay these conductors I will maintain a certain voltage here so I will maintain a certain voltage V of t which is a function of time in this particular case I know that for two conductors.

If I maintain a voltage difference V then I will generate an electric field inside right this electric field is directed along z axis and the wave is now propagating instead of z axis the wave is propagating along x axis it has a certain initial amplitude it has a certain amplitude initially that is outside this particular region so these electric field lines are directed from the top to the bottom they are uniform they are assuming that the plates are very wide area and their placed very close to each other so the electric field is uniform down there okay, so this is E_0 which is the amplitude at the outside of the region where the electric z exist.

Please note that this is a wave optical wave so this is optical wave where as this black one which I have written is the electrical signal or the field that is generated by the electrical signal or the voltage v of t okay so just before this region is there you can consider you can write down what is the electric field for the wave optical wave that would be $E_0 e^{j\omega t}$ or we can write this as $e^{j\omega t}$

this can be directed along some axis but this is propagating along z axis okay so this optical wave is propagating along x axis okay so this plane is $x = 0$ this plane will be $x = L$ so what will be the.

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Electric field at the output region that is as the optical wave propagates through it is subjected to certain electric field okay and what happens is that this electric field will change the refractive index of the medium okay the refractive index of the medium before the application of the voltage v of t is n_0 but after you have applied the voltage the refractive index inside this region that is between $x = 0$ to $x = L$ becomes $n_0 - \frac{1}{2} n_0^3 r_{33} E_z$ where E_z , E_z is the electric signal okay electrical due to v of t .

So there is this electric field which is generated because of the applied voltage v of t okay and that electric field E_z will affect this particular refractive index okay, since E and V are related for a plane capacity type of a situation that we have considered E_z is given by v of t \times d where v of d is the slowly varying function of time which we have assumed so you can substitute for E_z and write this as v of t \times d what you have just absorbed is what is called as Pochal's effect and what is Pochal effect implies that you can apply external voltages so this voltage or the potential difference and then change the optical properties.

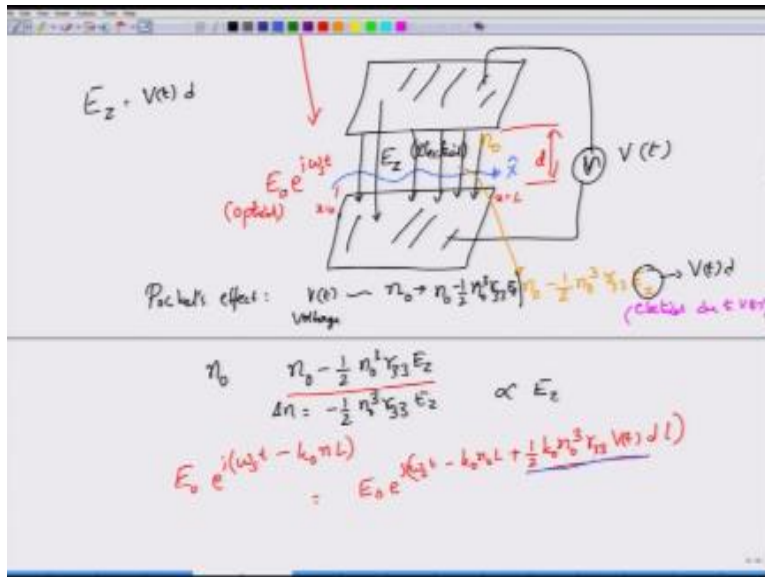
That is you can change the refractive index from n_0 to some $n_0 - \frac{1}{2} n_0^3 r_{33} E_z$ okay E_z being the result of applying voltage v of t so Pochal's effect implies that you take voltage which is the voltage that you can apply and then change the optical refractive index okay this change in the refractive index which is $\frac{1}{2} n_0^3 r_{33} E_z$ will be linear in electric field correct initially the refractive index was n_0 after you have applied it has become $n_0 - \frac{1}{2} n_0^3 r_{33} E_z$ right so the change in the refractive index is given by $\Delta n = - \frac{1}{2} n_0^3 r_{33} E_z$ this change is some constant times E_z .

So this is called as linear Pochal's effect okay so this is the linear Pochal's effect so what is this effects implied to as is that the electric field of the optical wave after propagating through this medium when it comes out at $x = L$ will become E_0 it is amplitude has not changed $e^{j\omega t - k_0 n x}$ L correct n is the refractive index but I know that the refractive index n is given by $n_0 - \frac{1}{2} n_0^3 r_{33} E_z$ whatever this expression, correct. So I can rewrite this as $E_0 e^{j\omega t - k_0 n_0 L + \frac{1}{2} k_0 n_0^3 r_{33} v(t) dL}$ where d is the separation between these two plates, okay. So this particular element on which we have deposited two metal electrodes and applying a voltage $v(t)$ between them causing the change in the refractive index is coming from lithium niobate, okay so this material is lithium niobate in fact this is the crystal it would not look exactly like this it will actually be in the wave guide formulation.

So you have to actually make an optical wave guide such that you have a substrate, okay so you can imagine that this is a substrate that I have and on this substrate I put two metal plates, okay and then put a voltage between the two the light will enter from here this particular region so the light will enter in this region the electric field because of the voltage that we have applied is going through in this way and then as the wave propagates this $x=0$ and as the wave propagates, propagates and comes out at $x=L$ its phase has changed, okay.

And the phase change would have been just $k_0 n_0 L$ if the electric field was not applied but because we have applied an electric field or we have applied a voltage difference the phase will be different.

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And in fact the phase is actually a function of time now, right the phase is now a function of time, this is very crucial. If $v(t)=0$ this corresponds to having applied no voltage and hence no electric field so this term would be vanishing, so there is just a constant phase shift there is nothing you can do just happily go home without constructing a phase modulator.

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The whiteboard contains the following handwritten text and equations:

Faculté d'optique
Voltage (circled in pink)
(calcul de Δn)

$$\Delta n = \frac{n_0 - \frac{1}{2} n_0^3 \chi_{33} E_z}{-\frac{1}{2} n_0^3 \chi_{33} E_z} \propto E_z$$
$$E_0 e^{i(\omega_0 t - k_0 n_0 L)} = E_0 e^{i(\omega_0 t - k_0 n_0 L + \frac{1}{2} k_0 n_0^3 \chi_{33} V(t) d)}$$

But now you have a situation where your phase is varying with time because you have applied a time varying voltage $v(t)$ and managed to introduce a phase shift into the optical wave itself by applying a voltage $v(t)$ you generate an electric field this electric field will influence the refractive index and that refractive index change when the electric, when the optical wave observes it will lead to a phase difference, okay or a phase shift which is now changing with time.

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Handwritten notes on a whiteboard:

$$n_0 \quad \frac{n_0 - \frac{1}{2} n_0^3 r_{33} E_z}{\Delta n = -\frac{1}{2} n_0^3 r_{33} E_z} \propto E_z$$

$$E_0 e^{j(\omega t - k_0 n_0 L)} = E_0 e^{j(\omega t - k_0 n_0 L + \frac{1}{2} k_0 n_0^3 r_{33} V(t) d / L)}$$

Half-wave Voltage $V_H = \frac{\lambda_0 d}{n_0^3 r_{33} L} \quad \frac{c}{f_0}$

$\frac{\lambda}{2} \rightarrow \pi$

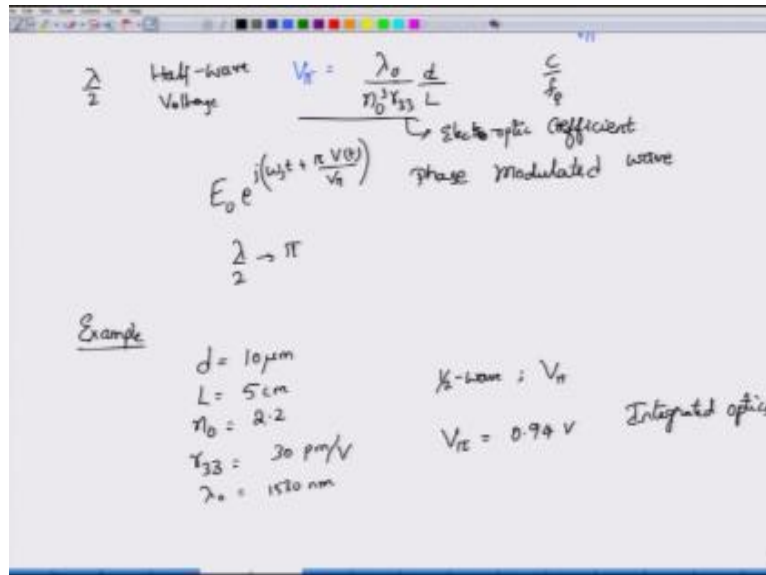
Electro optic coefficient r_{33}

phase modulated wave

So I will remove this constant phase term not very important for us and I will simplify this remaining part, okay so I will write this one as $v(t)/v\pi$, okay so I will write this as or rather I will write this as $\pi v(t)/v\pi$ and then I will leave this as a simple exercise for you to find out that this $v\pi$ is given as $\lambda/n_0^3 r_{33} L$ or rather d/L , L is the length of the phase modulator d is the distance between the electrodes that we have taken λ_0 actually λ_0 is the free space wave length, free space wave length is given by c/f_0 or f whatever the frequency that we have applied so our frequency was fs.

And r_{33} is called as the electro optic coefficient, this is called as electro optic coefficient, okay. So this $v\pi$ is called as the half wave voltage because your output which is now written as $e^{j\omega t + \pi v(t)/v\pi}$ right this is now a phase modulated wave, correct so this is a phase modulated wave, right and when $v(t)=v\pi$ the total phase shift will be equal to π , so that is why this $v\pi$ is called as half wave voltage $v\pi$ is called as half wave voltage, you might wonder why there is this half wave, half wave simply means $\lambda/2$ and we know that $\lambda/2$ corresponds to a phase shift of π , because λ corresponds to a phase shift of 2π by $\lambda/2$ corresponds to a phase shift of π .

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So this is called as half wave voltage let us do a simple example here, okay to calculate some numbers so let us assume that the thickness d is about 10 micron and let us say the length of the modulator is 5 cm, let us say in the absence of any input voltage n_0 is 2.2 the poical coefficient r_{33} which is the electro optic coefficient or the poical cell coefficient is given by 30pm/ v that is if you are apply one volt you will r_{33} of 30pm okay you can calculate what is the half wave voltage for this that is you can calculate what V_{π} for this one, substituting the values so I have to give you what is λ , so let say λ is 1530 nana meters okay.

Substituting this in to the expression what you will find is that V_{π} is given by around toughly about 0.94 volts okay. So calculate this by writing the formula $\lambda_0 / n_0^3 r_{33} d / l$ right okay so you can use this equation and then you can find out that V_{π} is very small. In fact this is the magic of integrated optics you can by a lithium have weight crystal separately which will of this particular bulk region and you know this size crystal or smaller crystal and then put some electros from the top and the bottom apply of voltage externally.

Okay but because this D the thickness is very large right so the V_{π} voltage will also be very large because it would be directly proportional to d so V_{π} is very large for a crystal that you can

purchase and then deposit certain electrets and then apply a voltage when in integrated form you're the separation between the two electrets is drastically reduce it is just about 10 to 50 micron and then the length is around 5 to 10 cm and then your $V\pi$ is very manageable for you get in market phase modulator with a $V\pi$ of about 3 to 4 volt peak rf signal okay or 3 to 4 volt voltages.

So $V\pi$ is just about 3 or 4 volts which is very small compare to about 100 volt $V\pi$ that you will have you will normally get with a crystal operating alone, okay.

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Handwritten notes on a whiteboard:

$$E_0 e^{i(kz - \omega t)}$$

$$\frac{\Delta}{2} \rightarrow \pi$$

Example

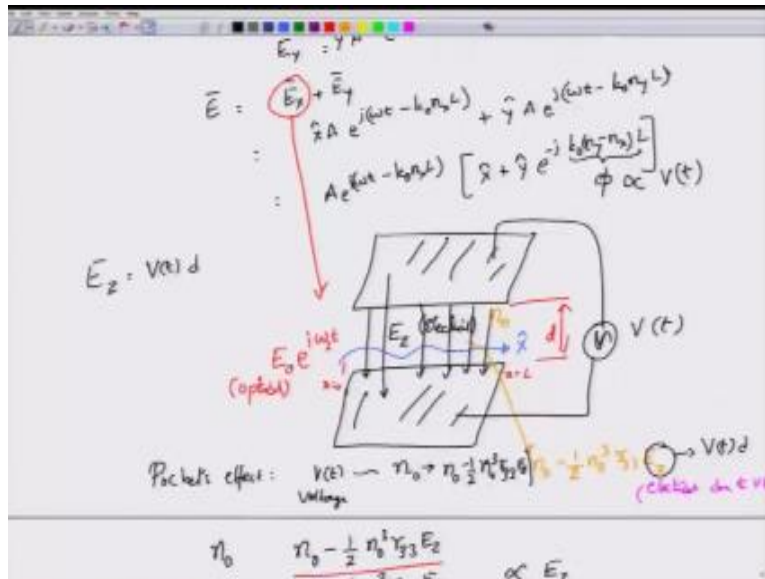
$d = 10 \mu\text{m}$
 $L = 5 \text{ cm}$
 $n_0 = 2.2$
 $r_{33} = 30 \text{ pm/V}$
 $\lambda_0 = 1570 \text{ nm}$

$\frac{1}{2}\text{-wave} : V_{\pi}$
 $V_{\pi} = 0.94 \text{ V}$

Integrated optics

So what we have seen is that it is possible to talk about the phase modulator by having electro optic crystal okay.

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And then putting them in the integrated optical frame work one last point this crystal is usually as I said lithium Moabite crystal and the way which we have indicted here is that the lithium they have it crystal is cut along the y axis okay it is cut along the y axis the wav is propagating along the x axis and the electric field is indicated along the z axis this is called as a y cut lithium Moabite it is the most important configuration for lithium Moabite optical phase modulators.

In fact all optical modulator are may dote of lithium Moabites commercial optical modulators which are operating for optical communications are may dote of lithium Moabite modulators they are all usually y cut and you have x propagating wave but please remember what is y cut what is x cut and what is z cut is the crystal frame of reference not our frame of reference okay so one has to perform some crystallographic studies in order to locate the principle plans and then cut your crystal along that particular plan.

So this is the y principle plan that you will have to locate and then cut along that particular plan of the crystal.

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$$\vec{E} = \hat{x} A e^{i(\omega t - k_x x)} + \hat{y} A e^{i(\omega t - k_y y)}$$

$$\vdots$$

$$A_z(x - k_x x) \left[2 + \gamma e^{-i \frac{k_x (x - k_x x) L}{\phi_{oc}}} V(t) \right]$$

$$E_z = V(t) d$$

$$\text{LiNbO}_3$$

$$E_0 e^{i\omega t}$$
 (optical)

$$E_z$$

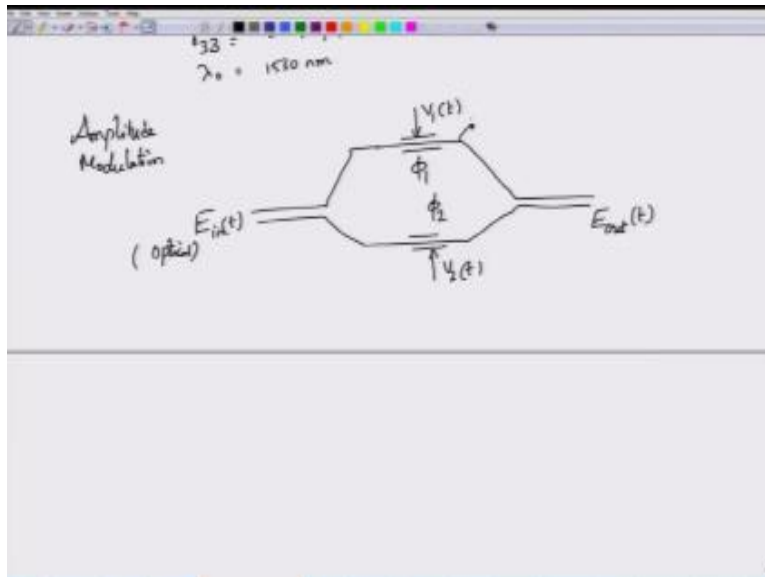
$$V(t)$$

Pockel's effect: $V(t) \rightarrow n_0 + n_0 \frac{1}{2} r_{33}^2 E_z^2$

$$n_0 \quad \frac{n_0 - \frac{1}{2} n_0^3 r_{33}^2 E_z^2}{dn = -\frac{1}{2} n_0^3 r_{33}^2 E_z^2} \propto E_z$$

So what we have seen is that if you just take such a lithium Moabite crystal and apply a voltage we will be able to induce a phase shift and if the voltage is that you have applied is veering with time you have actually managed to obtained phase modulation right.

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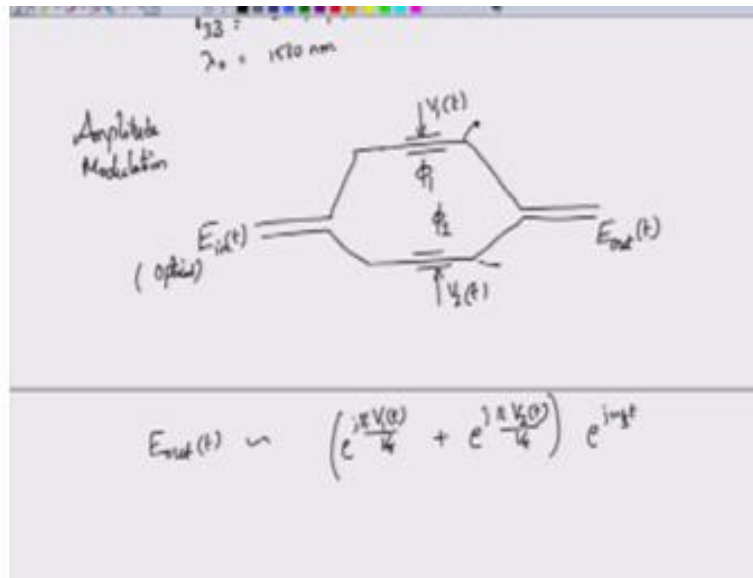
Now there is also amplitude modulation correct how do I obtain in amplitude modulation well we have already seen this amplitude modulations structure so you take a wave guide and then split this wave guide into two, portions right and then you put 2 electrodes here right each of these arms by themselves are phase modulators right so you can apply a voltage $v_1(t)$ you can apply a voltage $v_2(t)$ to this one.

And then combine them into one waveguide I should have combined them into a waveguide type so each of them can be combined into a waveguide now if you supply a wave optical wave its electric field is $e_{in}(t)$ you get an optical wave at the output whose electric field is $e_{out}(t)$ you have applied $v_1(t)$ so you cause a certain phase shift ϕ_1 in the upper arm you apply $v_2(t)$ you cause a certain phase shift ϕ_2 in the lower arm.

And then you combine the light intensity over here right so when you do that I am not going to derive this but you should be able to derive it as an exercise because you know how this can be modeled as a 50/50 splitter you know how to obtain the electric field here because this is simply a phase-modulated wave this is one more phase modulator wave and then you know how to combine them, so when you do all that you will see that $E_{out}(t)$ will have

components which is $e^{j\pi V_1(t)/V_\pi} + e^{j\pi V_2(t)/V_\pi}$, and there will also be $e^{j\omega t}$ term, there will be some additional amplitudes here, that I don't want to really worry about it.

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So there will be some additional factors constant that we can eliminate, now what we normally do is, we eliminate this $e^{j\pi V_1(t)/V_\pi}$, that is we take this as common and end up here as $e^{j\pi V_2(t)/V_\pi}$. Then we also take this $V_2 - V_1$ outside, so that I have $e^{j\pi V_1(t)/V_\pi}$, then there is also $e^{j\pi/2 V_\pi}$, $V_2(t) - V_1(t)$, this is all common and inside what you get is $\cos \pi V_2(t) - V_1(t)/V_\pi$. So you can see that by changing the operating point, by choosing $V_2 - V_1$ difference, I can choose or I can make into different biasing points for the operation of the amplitude modular operation.

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Handwritten notes showing the derivation of the output signal and power for a modulator. The notes include:

- Top line: $e^{-j\frac{\pi}{4}}$
- Second line: $e^{j\frac{\pi}{4}} e^{j\frac{\pi}{2V_{\pi}} [V_2(t) - V_1(t)]} \cos\left[\frac{\pi}{V_{\pi}} [V_2(t) - V_1(t)]\right]$
- Third line: $\text{bias} = V_{\pi}/2 \rightarrow \text{Amplitude}$
- Fourth line: $E_{out}(t) = E_{in}(t) \sin\left(\frac{m(t)\pi}{V_{\pi}}\right) = \frac{E_{in}(t) m(t)\pi}{V_{\pi}}$
- Fifth line: $P_{out}(t) \sim m^2(t) \text{ DSBSC}$

We have seen that when the bias is such that $V_{\pi}/2$ when the bias at $V_{\pi}/2$, you will end up having amplitude modulation, because you know we have seen this earlier, the output amplitude E_{out} will be equal to input amplitude $E_{in}(t)$ times sin of whatever the message signal that you're transmitting times π/V_{π} . And for small values of $m(t)$, it is very nearly $E_{in}(t) m(t) \pi/V_{\pi}$. So this amplitude is getting modulated into $E_{in}(t)$, or this is basically an amplitude modulated wave, you can bias this one at different regions.

So this is said you can bias at V_{π} and in that case your output power will be very nearly equal to m^2 of t , this corresponds to what is called a double side band separate carrier modulation. So these things I will give as an exercise, because it's simply coming up with together, but one thing I would like to talk about before we close the module is this expression. So if you look at this expression what you see is that, you can forget that this is the common factor, but then you will see that there is that this particular term.

This term which gives the overall phase shift, you have to combine both terms, you will get $e^{j\pi V_1(t) + V_2(t)/2V_{\pi}}$, this term which is the sum of two signals, $\pi/2V_{\pi}$ can lead to what is called as chirping, because V is changing the incoming modulating signal which is $E_0 e^{j\omega t}$, will pick up this

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