Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Optical Communications

Week – IV
Module – I
Optical Modulator: Physical Structure

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Welcome to the mook on optical communications today we will be talking about the physical structure of optical modulator in this module the plane for the module is that we will first review wave equation and the talk about wave propagation and the see what is the physical structure of a phase modulator and amplitude modulator are equivalently intensity modulator.

So let us began by looking at a prorogation of an electromagnetic wave we know that light is an elector wave so we would like to understand the propagation and the equation that governs the propagation of the light wave in a given medium okay to do that we need to invoke Maxwell's equations because we picture electromagnetic wave or we picture light as an electromagnetic wave consisting of electric and magnetic field, fields okay.

So these electric and magnetic field satisfy certain set of equations know as Maxwell's equations and we will first very briefly recall Maxwell's equations derive the wave equation consider some very simple and solutions of these wave equating known as plane waves and then use this plane wave idea in order to study the physical structure of wave propagation inside a phase modulator and amplitude modulator optical phase modulator and optical intensity modulator so let us first write down Maxwell's equations.

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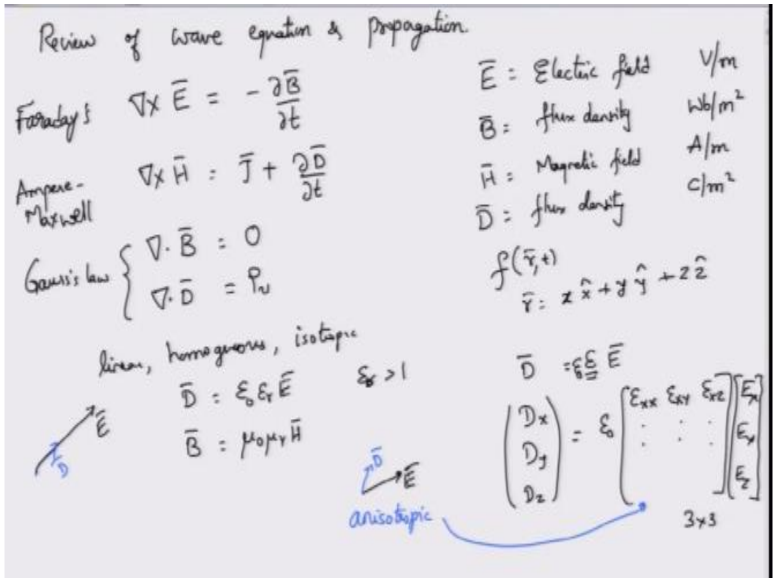

As you probably remember or you can take a look at any text book in under graduate electrometric text book you will discover that there are four Maxwell's equations which are given in terms of these 4 field quantities okay the first equation there is no order to Maxwell's equations so what we are writing is simply that is no by convention or that is something my personal preference so we have what is called as faraday's law which in eth differential form this is called differential form of Maxwell's equations.

So you have curl of electric field $E = - \delta B/ \delta t$ this $\delta/ \delta t$ stands for partial derivative of the magnetic flux density vector B with respect to time okay so E is the electric field or sometimes called as electric field intensity and this is measured in SI units in v/m you have B which is called as the flux density vector this is the magnetic flux density vector and this is measured in wb/m^2 or Tess law and then you have two more equations H is called as magnetic field or sometimes called as magnetic field intensity and this is measured in A/m in the SI units and finally you have flux density which is now electric okay.

This is measured in C/m^2 so these are the 4 field quantities which are functions of both space given by a general position vector r as well as time so these are all functions of both space as well as time this position vector r will depend on which coordinates system you have chosen so in the Cartesian coordinates system this will be given by for any point which is described by the coordinates x, y and z.

This coordinate vector r is went from the origin is given by x, x^ begin the unit vector along the x direction plus component along y and component along z okay so these are called as field quantities which vary both respect to position as well as with respect to time so complete the Maxwell's equations now so you have this equation called as faraday's law okay originally faraday's law was not in this form what we are writing is the law in vector form okay.

Next we have ampere Maxwell law okay so this is ampere was the original person who gave this law but then this law was modified by Maxwell law to introduce his famous displacement current okay so this gives you this is given for curl of edge that is for the magnetic field given by J where J is the conduction current + $\delta D / \delta t$ this d is the electric flux density.

So the time rate of change of this electric flux density plus the conduction current will be equal to the curl of H you have two divergence law one law is $\delta B=0$ okay this there is one more law which relates $\nabla . D = \rho$, ρ being the volume charge density maybe we can write this as ρv so this is volume charge density, okay. These divergence laws are called as Gausses law one for magnetic fields and one for electric fields, right. If you consider a medium which is linear, homogenous and isotropic then this D filed can be related to E filed, okay.

D is given by $\varepsilon_0 \varepsilon_r E$ where ε_0 is the permittivity of the free space ε_r which is greater than 1 is called as the relative permittivity of the medium so if you have a medium with relative permittivity is ε_r then D is related to $\varepsilon_0 \varepsilon_r$ times E so if electric field line is arbitrary directed in this particular direction then the D field will also be directed along the same its amplitude will usually be quite small because ε_0 is a very small number, okay.

Similarly B is related to the magnetic field H by this relationship B is $\mu_0 \mu_r H$ where μ_r is called as the relative permeability of the medium, okay. Again you can see that both B and H are the same direction but they will have different amplitudes if the medium is an-isotropic as many optical elements are okay then D and E are related by a matrix, okay. This E and D are related by a matrix that is to say.

In the rectangular coordinates system the three components of the D vector Dx, Dy and Dz is given by a matrix times the electric field vector E so you have ε_{xx} ε_{xy} ε_{xz} I hope you can fill up the rest of these elements here this will be another 3/3 matrix this will be a 3 x 3 matrix finally you have the electric field components Ex, Ey and Ez clearly you can see that isotropic medium is a very special case of an-isotropic medium.

In an an-isotropic medium all these elements are usually non zero which means that if electric field is directed along this direction which I have taken arbitrarily the flux density D will be directed okay let us not make it perpendicular although it is possible let us keep it slightly general so this D field is usually directed at a angle with respect to electric field, okay. So this is something that is possible for a medium which is called as an-isotropic medium. In an an-isotropic medium D and E are related by this matrix, so many optical elements.

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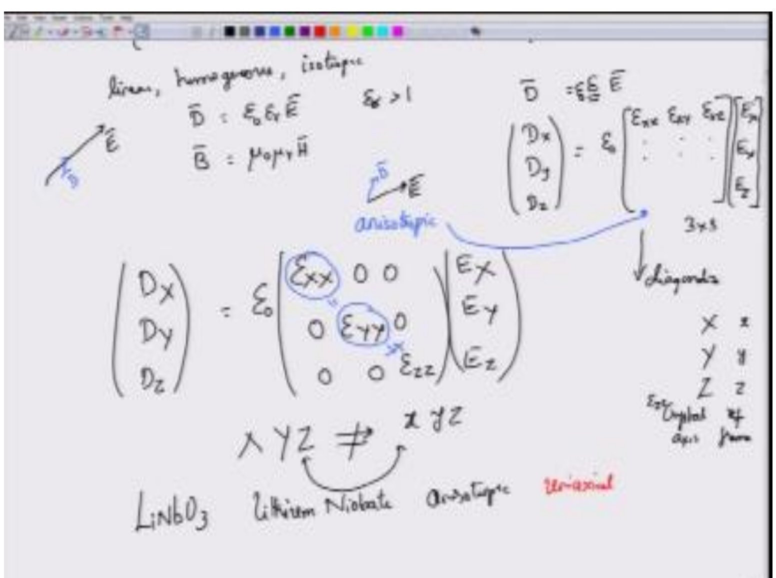

Such as crystals are an-isotropic in nature however not all these elements are no field for such a medium it is possible to actually diagonalise this matrix, okay to diagonalise this matrix and to discover 3 new access which you can call them as X, Y and Z which are not the same as x, y and z these are two different access these are known as crystal access this is your reference frame, okay.

This is the frame of reference for xyz coordinates outside the crystal but within the crystal there exist three very special directions which form the crystal access okay and this crystal access is the result of diagonalising this full matrix ε okay so if you diagonalise the matrix ε you will discover 3 access xyz along which you will find that the D field will be in the same direction as electric field, okay.

So let us make it little more specific, so D_X, D_Y, D_Z is the D field in the xyz or the crystal frame of reference this is equal to some ε_0 which is the permittivity of the free space times ε_{xx} 000 ε_{yy} please note that these are capitals 00 and ε_{zz} okay times you have Ex, Ey and Ez so here you will find that D field and E field are directed in the same direction but this access xyz is usually not the same as the reference coordinates xyz of the frame that is outside of the crystal, okay.

There is a relationship between the two but that relationship is not very important for us. At this point okay we are interested in looking at how the wave is generated and how the wave is propagating, okay but keep in mind that many crystals and especially the one that is most widely used for making modulators this is called as the Lithium Niobate crystal this is the Lithium Niobate crystal okay this is essentially anisotropic which means that if my electric field is directed along x direction it will see a different refractive index if electric field is directed along why it will see a different refractive index so this Lithium Niobate is a anisotropic crystal out of this anisotropic crystal there exist one more special crystal.

Okay if this E_{XX} is equal to E_{YY} but this is not equal to E_{ZZ} okay then such am medium is called as or such a medium which is described by this particular scenario or situation is called as uniaxial materials okay it is called as a Uniaxial material or Uniaxial crystal okay.

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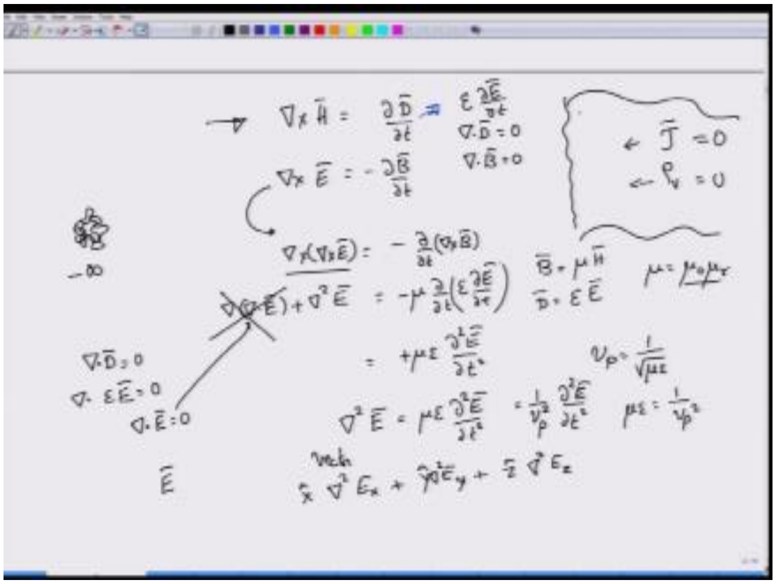

This is about the material properties now let us see how this Maxwell's equation imply that there is an elector magnetic wave and what is the equation that governs that one okay to do that let us assume that source of light or source of electromagnetic wave is located at very far away from the point where we are considering so let say this is located at – infinity and you are looking for fields or waves in this region which is very, very far away from the region very or considering okay so the region where you are looking for the waves is very far away from the region where these waves were originally generated.

So you can imagine a transmitting antenna with an antenna placed at – infinity okay as the antenna generates the electromagnetic wave these electromagnetic waves will propagate through the medium okay and because you are considering the region where there are no currents right there are no free currents in this region I am considering free space for example or a medium of dia electric there are no currents here nor there is any charge no one has sprinkle some charges okay there are no charges sprinkled on any region that we are considering so both these quantities are equal to 0.

Everywhere in the region of way propagation that we are interested in j and row is equal to 0 so what will happen to Maxwell's equation curl of x will be equal to – 𝛿B/ 𝛿T 𝛿.D will be equal to 0 𝛿. B is was any way equal to 0 okay these equations are known as source free Maxwell equations these are known as source free Maxwell equation because the sources namely current and free volume charge density both are equal to 0 okay now what we do is we take the curl of this electric field curl expression so I take curl on both sides obtained – 𝛿/ 𝛿T curl of b here okay.

I have interchanged 𝛿/𝛿D and curl operations so I can write this in this particular fashion this left hand side can be expanded using a certain identity okay I will not show you this vector identity or the proof of this expansion you can find it in any good under graduate electromagnetic text books but for the right hand side I realize that I am actually working with the medium which is simple in the sense that it is non magnetic and having a relationship between B and H described by some μ, μ will of course be equal to μ0 and μr μ0 x μr.

Okay this I am calling this as μ because I do not want to write μ0 μr every time so with that I can μ being a constant can be taken out of this curl operation in fact it can be taken out of this 𝛿/ 𝛿t term so we get – μ 𝛿/ 𝛿t what is left here curl of h but I know what is curl of H curl of H is given by this expression which is 𝛿/ 𝛿t but the right hand side can again this simplified when d is related to electric field.

If D is related to electric field in this particular way then epsilon being a constant okay I am assuming at this point and Isotropic medium okay for an Isotropic medium linear and homogenous of course epsilon is a constant so it can be pulled out of this derivative so this can be written as epsilon 𝛿E/ 𝛿t is that okay so this is how the right hand side would become now you can in place of 𝛿 cross H substitute this epsilon 𝛿E/ 𝛿t so you will have epsilon 𝛿E/ 𝛿t pull epsilon out of this partial derivative.

So you get - μ epsilon and whatever that is remaining is $\delta^2E/\delta t^2$ and we have seen that δ.D=0 because there are no free chargers in the region where we are considering wave propagation, but D is related to electric field so this gives you ε or rather δ.εĒ=0 pulling ε out of this equation gives me δ.Ē=0, okay. So if you substitute this condition into this one then this entire thing goes away gradient of $\delta.\bar{E}$ goes away –sign and –sign will away and what you get is this equation which is called as the wave equation.

$\nabla^2\bar{E}=\mu\varepsilon\ \delta^2\bar{E}/\delta t^2$, you can further introduce the phase velocity of the wave by calling that v_p as $1/\sqrt{\mu\varepsilon}$ so clearly $\mu\varepsilon$ will give you $1/v_p^2$ where v_p is called as the phase velocity of the wave, so this is equal to $1/v_p^2\ \delta^2E/\delta t^2$, okay. Instant of working with a gener5al electric field this left hand side is actually vector laplacian, okay which means that this ∇^2 operation must be applied to x must be applied to the y component, so you will have ∇^2Ey+ you have ∇^2Ez so this vector laplacian must be applied everywhere to the three components that you have.

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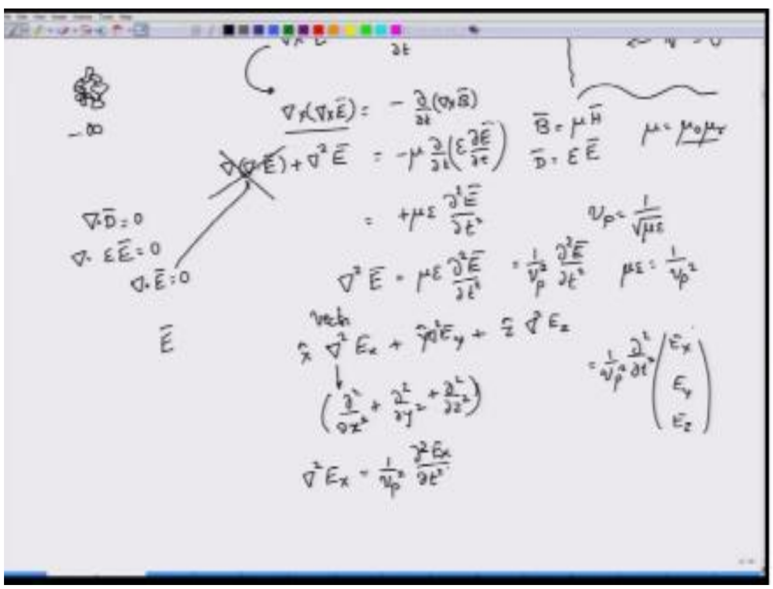

And furthermore, when you apply this ∇^2 each of this ∇^2 is actually $\delta^2/\delta x^2+\delta^2/\ \delta y^2+\ \delta^2/\ \delta z^2$ okay, so this operator must be applied for Ex, must be applied for Ey and must be applied for Ez, when you do that you will actually generate three separate subway equations, one way equation for Ex, one for Ey, one for Ez because the right hand side can also be split up in the same way, you have $1/v_p^2\ \delta^2/\delta t^2$ Ex Ey and Ez, so there are three individual x,y,z components for the electric field to each of which you are taking this $1/v_p^2\ \delta^2/\ \delta t^2$, okay. So what you essentially get is say $\delta^2Ex=1/v_p^2\ \delta^2Ex/\ \delta t^2$, it is that okay.

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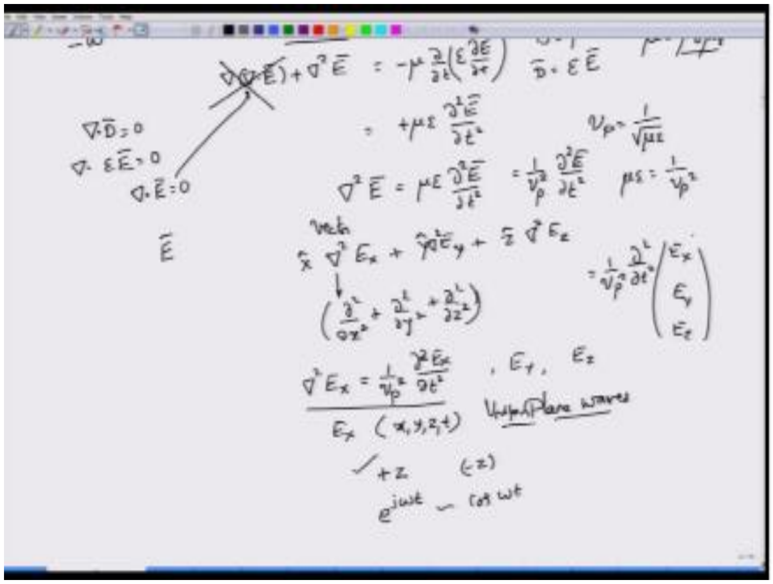

So now you have similarly you can write down one equation for Ey as well as one equation for Ez. In general this equation implies that Ex must be a function of x,y,z as well as time, okay. However, we will consider those waves which are called as uniform plane waves, okay in this uniform plain waves what we have so this must be written as uniform plain waves, what we have is the wave is propagating along +z or it can be propagating along –z but we will stick with wave propagation +z direction, wave is propagating along +z direction in time it varies as $e^{j\omega t}$ or it varies as cos ωt, okay it is a sinusoidal varying wave it is amplitude is varying sinusoidal with respect to time, okay.

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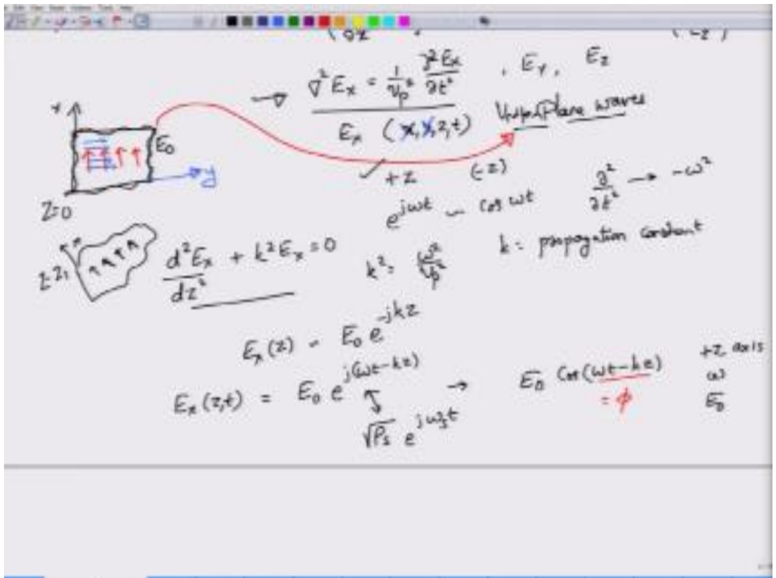

So if I assume these conditions then this equation can be further simplified because this operator $\delta^2/\delta t^2$ becomes $-\omega^2$, okay because $\delta/\delta t$ is $j\omega$ so if you multiply $\delta/\delta t$ or if you take $\delta^2/\delta t^2$ you get $j\omega$ times $j\omega$ that is $-\omega^2$, okay. I can put that into this expression and also demand that this electric field be independent of x it also be independent of y, okay so the wave is characterized by propagation along z in frequency it has a certain frequency ω and then it will be function only of z and time t, okay.

The wave is propagating along z so it is amplitude is expected to change as you go along z axis, okay. Substituting all this the equation can be further simplified you will have $d^2Ex/dz^2+k^2Ex=0$ where k^2 is nothing but ω^2/v_p^2 okay, and this k is called as the propagation constant, so this k is the propagation constant or sometimes called as wave number as well, okay so k^2 is ω^2/v_p^2 actually you will have $-\omega^2/v_p^2$ in this expression, in this equation you will have $-\omega^2/v_p^2$ I have pulled this term into the left hand side and call this as k^2.

This is a simple second order differential equation, remember we want a wave which is propagating therefore, the solution for this Ex as a function of z turns out to be some constant e0 and e-jkz so that the total electric fields which is a function of z as well as tine t will be given by some constant e0 ej Ωt –kz remember I am writing his is the complex form but in the real form this equation is nothing but e0 cos Ω t – kz okay.

This is the equation that is propagating along + z axis it has a frequency Ω it has an amplitude e0, in fact when we wrote down the expression for the field output or the light output from the laser diode we wrote very similar to this right so we said √ps ej Ω st we did not specifically specify that the wave is propagating along a certain axis that was not important at that time but here you can see that this is exactly equal to this way which we are assuming okay.

So far we have use the notion of uniform plan wave but we have not told you that it was uniform plan wave, why is that uniform because if you look at a constant plan z so maybe you look at z=0 plan okay. So in this z=0 plan okay if you look at what is the direction of the electric field the electric field will be directed along x axis so this from bottom tot of this is vertically up arrows are the directions for the x axis.

And no matter where you go along the x axis you will see that the electric field vectors are directed always along x axis. It is amplitude maybe at z=0 something like e0 okay at z= a different plan z one plan the amplitude must be smaller because there will be a term kz therefore the total phase for this one changes the amplitude might be smaller okay but it will be uniform I am sorry I am not drawn it nicely.

But this should be uniform amplitude is small but this amplitude is constant and it is always directed along x axis so this is my x axis her also this is the x axis. You can find out what will be corresponding expression for the magnetic field you will find that the magnetic field must be why directed so that the wave is propagating the long z axis. Okay corresponding to this the magnetic field must be why directed you I am not going to draw that one but you van I mean I going to derive that one you can derive it for yourself and.

If you plot what will be the magnetic field liens the magnetic field lines will be also be constant and they will be directed along the y axis okay they will be directed along y axis here again you can go along the x or y direction the amplitude will not change for a given constant z plan the amplitude will not change that is why this is called as a uniform plan wave, okay. Why is it call plan wave because which is if you look at this expression this $\Omega t - kz$.

And if I said this one to certain value five okay and ask for how does this 5 look like know what is the plan for constant 5 values the plan for constant 5 values or the plans of z itself right so if I say this phase must be equal to 0 which plan how it would look at a time set put t=0 and then you find out where the phase will be equal to 0 so one solution is z= 0 correct, one plan that you will get e will be z=0 right a t= 0.

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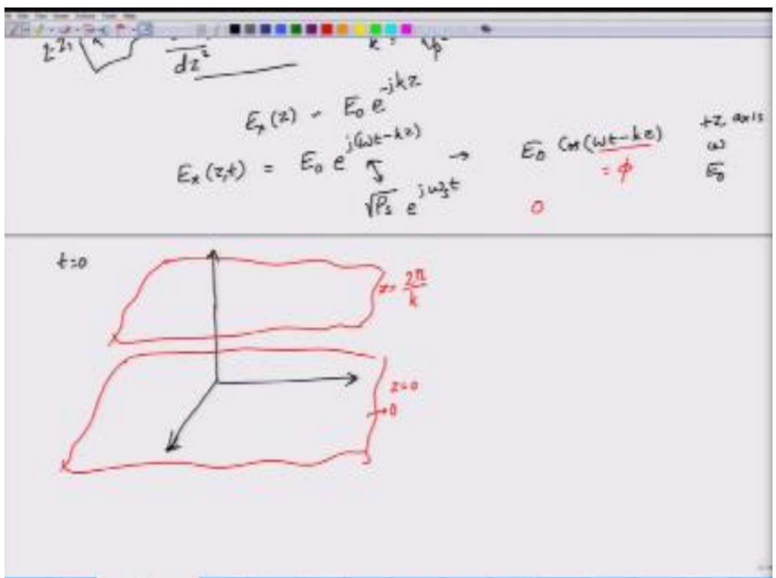

If I ask that the argument of this cos function will be cos k z and ask for constant phase points and if I say 5=0 is the constant phase point one plan that you will get will be the z=0 plan correct so this plan will have along this plan the phase will be constant which will be the other plan actually this plan would go like this right so this is the overall plan that is there maybe we can try writing a better picture for this.

This is your x y and y axis so the constant plan that we were looking for is this z=0 plan so on this plan for this wave the phase will be equal to 0 right is this only plan that we have no you will have one more plan at z=when will this go to 0 again when the argument goes to 2π right so at z = 2π / k right so at z = 0 under z = 2π / k you will have the constant phase and if you look at this is this is actually a plan the entire plan is rectangular in nature right there is no curvature to this right the plan or the points of the constant phase will form a rectangle or they will form a plan in this particular case.

Therefore this is called a uniform plane wave and it is a wave because it is advancing along the z axis. The distance between the two constant phase planes is called the wavelength of the wave and therefore you will see the, wave length is nothing but 2л/k or k=2л/λ.

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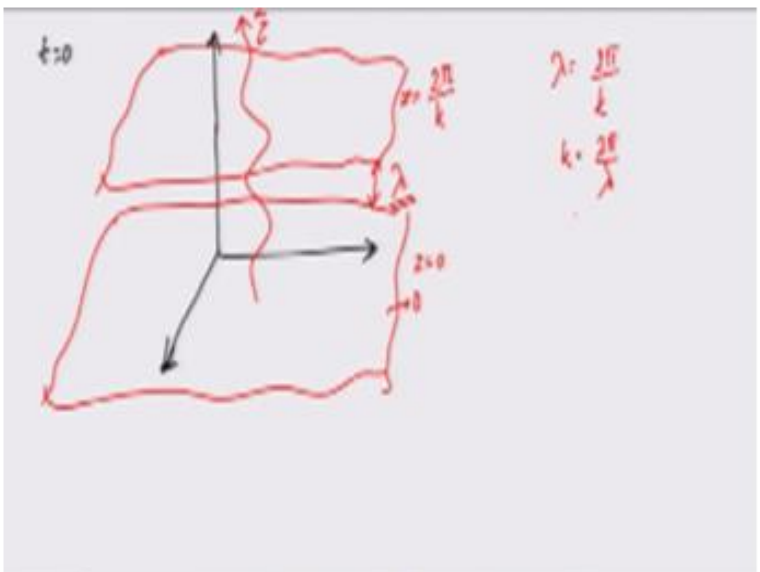

So this was all about a wave which was propagating along particular direction, just to finish of this particular module is the wave propagating along x direction, then it is called as x polarized wave. If the wave is propagating along y it is called as the y polarized wave, possible for you to have in general, both x component as well as the y component not equal to 0 and then the electric field E will be sum of two electric cells,. One will be along x directed, that is x polarized wave, one will be y polarized wave.

Each of these waves, let’s call this say Em is equal to some constant Eom $e^{j(\omega t\text{-}kz)}$, these waves re propagating along the same direction, they might have different phases ϕm, where m is equal to x and y, that is to say Ex = Eox $e^{j(\omega t\text{-}kz)+\phi x}$ Eoy $e^{j(\omega t\text{-}kz)+\phi y}$. Depending on the amplitude Eox and Eoy as well as the phase ϕx and ϕy or rather the same phase difference, ϕy- ϕx which you call it as δ, you will have different polarizations, the ratio of Eoy /Eox is given by tan angle χ, so described by tan(χ)=Eoy/Eox and δ if you define it as ϕy- ϕx, then if tan(χ)=1 that is to say Eoy = Eox, thee amplitudes of the x and y polarized waves are same, but δ= -л/2 or δ= л/2 you will get right circular polarized wave. These dot circles with the dot are right circular polarized wave and this δ= л/2 left circular polarized wave.

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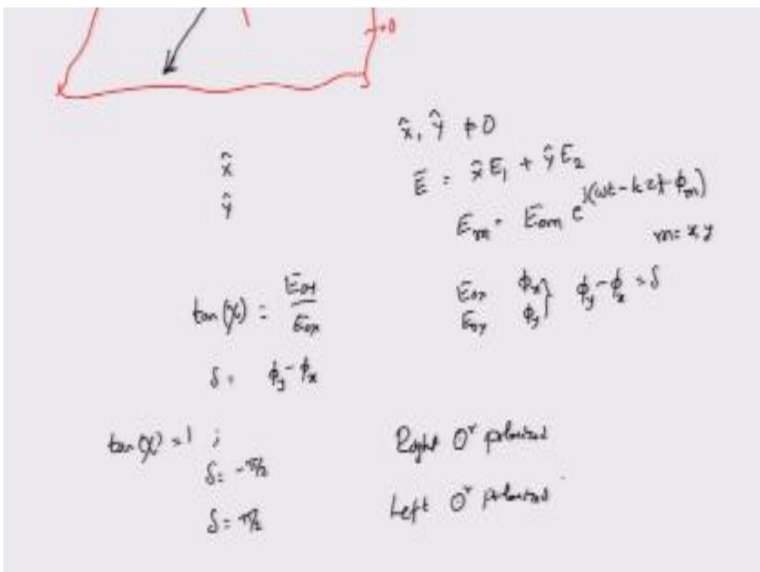

You don’t have to remember which direction the polarization is, but you have to remember that a constant plane, let say z=0 plane, if you look at how in terms of time, the electric field vector, the tipoff electric field vector changes, it will change for a left circularly polarized wav it will change like this and a right circularly polarized wave it will change in this manner. The amplitudes will be equal, one will be anti clock wise, and the other one will be clockwise.

I am sorry I've kind of forgotten which one corresponds to which, this is largely a matter of convention, so the way that I remember is, if I take your right hand side and point it along, the direction of propagation. So this is direction of propagation, if your curling from x to y in a clockwise direction, this is the right hand circularly polarized wave and if you take left hand thumb, and if the thumb points the direction of propagation and your curling from y to x, this will correspond to the left circularly polarized wave. Thank you.

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