Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Optical Communications

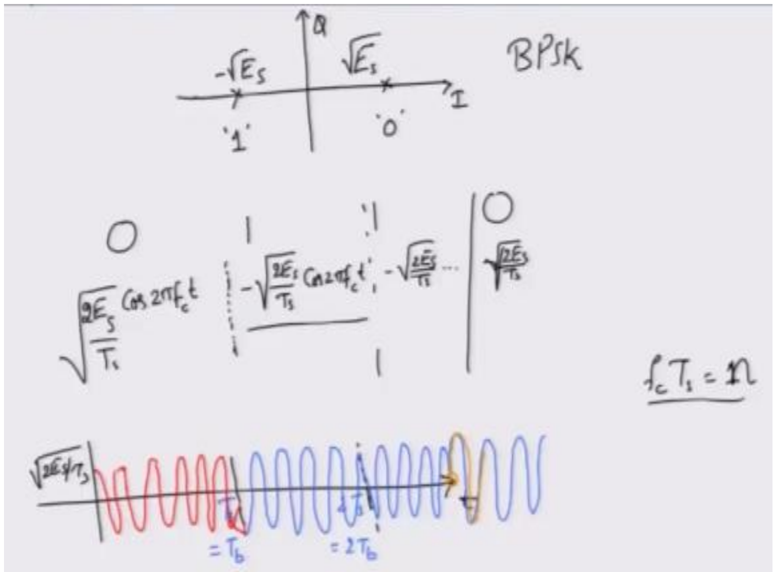
Week – III
Module – III
Digital Modulation-III

by
Prof. Pradeep Kumar K
Dept. of Electrical Engineering
IIT Kanpur

In this module we will be discussing several modulation techniques some additional modulation techniques actually concentrating mainly on quadrature phase shift keying and MRPSK there are in fact lot of other modulation techniques that one can talk about but due to the limited scope of our course we will stop with QPSK and a general discussion MRPSK for some interesting modulation techniques we will have to wait for the end of the course at which point we will again come back to the transmission transmitters.

And then talk about some newer modulation techniques so for now in this module we will simply concentrate on QPSK and MRPSK I will talk about the formulation as well as the optical implementation for these two transmitters okay compare to a binary phase shift keying in which you know you write down the.

(Refer Slide Time: 01:14)



Cancelation point has consisting of 2 points which are separated by a certain distance so let us say this first point is located at $\sqrt{Es}$ the second point here is located at $-\sqrt{Es}$ remember this is the I and this is the Q components right I mean this is the signal space that we are considering this the complex signal space that we are considering and these two points which I have marked by the cross marks corresponds to symbols 0 and 1 I could interchange the symbols from 1 to 0 in this way as well depending on what I am what I am going to assume.

So this is basically left to the discretion of the user whether you want to associate plus $\sqrt{Es}$ to the bit 0 or $-\sqrt{Es}$ to bit 1 in this case of binary phase shift keying the phase of the carrier is keyed or is changed between 0 and π right so I have for let us say if I take this has the mapping that I am considering this has the mapping with 0 and 1 0 map to phase 0 and 1 map to 180 degrees.

Then if I have a sequence that I need to transmit let us say 011 and 0 right the corresponding wave forms that I would like transmit here would be $\sqrt{Es}\cos 2\pi f_c t$ note that there is no phase change here in order to normalize this energies because when you take the energy of this one the energy would have term corresponding to the symbol period so in order to normalize those values we can divide this by √(Ts we can also multiply this one by 2, so that when I take the energy of this wave form the energy will be $\sqrt{(2Es/Ts)^2}$ multiplied by Ts/ 2 right so that when you expand this √ we will see that the energy is given by Es.

Es stand for the symbol energy in which case this is also the bit energy right so in this case the bit energy is equal to symbol energy so because of this reason we will use √Es/ 2s as our amplitude of the carrier okay so in order to transmit one right after this bit or the symbol boundary is ended in this case bit boundary is equal to the symbol boundary because this is a binary phase shift keying so in order transmit a one for the next T duration Ts duration I should transmit -√2Es/ Ts cos2π $f_c t$ again when this ends I should transmit -√Es/Ts cos2π$f_c t$ here is should switch or change the phase from 180 degree to 0 degrees and the transmit.

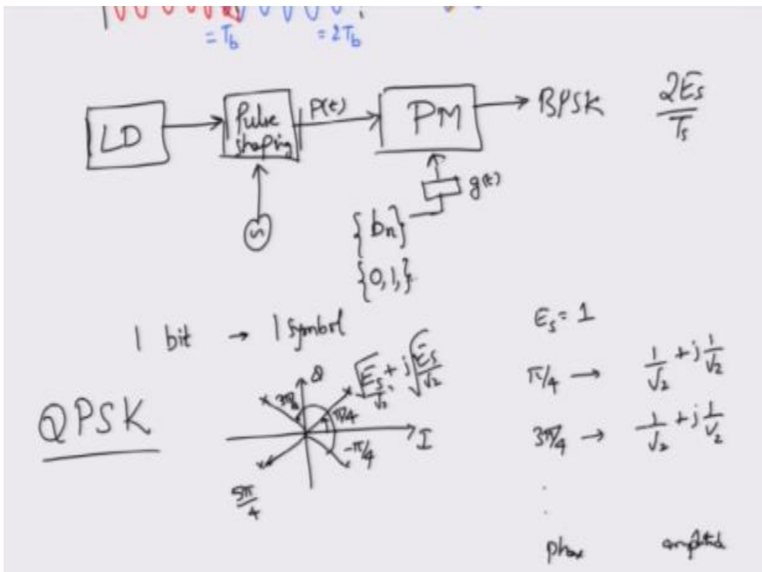
So if you over to look at it as a function of time this is how you would see this so these are my boundaries this is highly exaggerated has I have told you in the last module as well this the carrier is very high frequency but we will write some few cycles in order to show you how the phase changes happening at the boundaries so this Ts this is 2Ts Ts stands for the symbol period in this particular case that is also equal to the bit period.

Because Ts is equal to Tp so to transmit -√2Es/ Ts cos2π $f_c t$ I have to transmit this cosine wave from so please forgive me if my cosine wave forms are not very nicely looking like a cosine wave form it is kind hard to draw it on this part and let us say my cosine wave form which as to have an integral number of multiples here must end at this point, right. I need to satisfy as I told you fc. Ts = n where n is any integer it has to have an integral number of cycles so this is my cosine single representing the bit 0 or the symbol 0, next what I have to transmit is -√2Es/Ts Cos, well this amplitude is √2Es/Ts.

Whereas how would the amplitude how would the signal -√2Es/Ts Cos 2Πfct would change or how would we draw it that needs to be transmitted like this, right. So in need to transmit in tis way, so hopefully I am trying to convey here and this is looking okay, next I have to transmit again a 180 ^{O} so I have4 to switch over and then transmit a 180^{O} right and then I have to transmit a 0^{O}.

So it would actually be something like, okay. So if you notice here what has happened here is if you notice at these points there is a change in the phase so this point the phase goes from 180^{O} to 0^{O}, so this sudden transition of this phase you know from going to 0 to 180 degrees is important because these transitions represent the signal transition or the phase transitions, right corresponding to what sequence you are transmitting. And to implement this using an optical transmitter is quite simple.

(Refer Slide Time: 06:39)



To Implement this one all I need to use is a laser diode I need to have some signal source so I use a laser diode the output of the laser diode can be pulse shaped so I use a pulse shaping MZM so to the pulse shaper I might supply an RF signal so that I get a 33%, 67% or 50% output optical pulse shapes, right. Once I get the optical pulse shapes I just have to use a phase modulator whatever the bit sequence that I am getting here BN which can be a sequence of 01 right.

So it can only take on the values of 0 and 1 so this particular sequence that I am transmitting comes out as BN and this needs to be given as input to the phase modulator, if you wish you can include a electric block here which will do any electrical pulse shaping so let us call this as some g|(t) right and then you have an optical pulse shape which is coming out as a p(t) may be I have switch p(t) and g(t) notations from the earlier module to this module.

It is simply important to note that optical pulses are going in as input to the phase modulator and there is an electrical pulse shaper which will take this B(n) it will also multiply it by a certain constant right you need to multiply this one by $\sqrt{2Es/Ts}$ or you can adjust the amplitude or adjust the power of the laser diode in order to get that power value of 2Es/Ts, okay. So this you can either adjust it by the laser diode or you can adjust it by the bit sequence map multiplication.

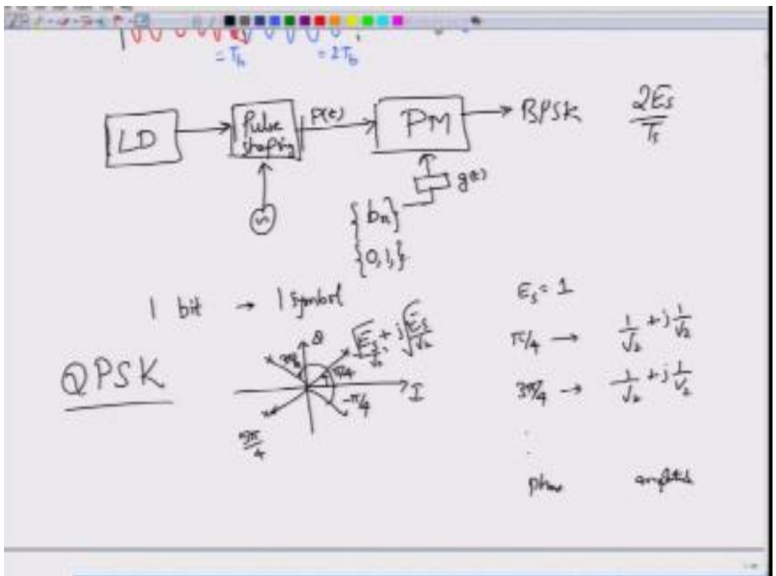
Does not matter whichever way you do, what you get here is the BPSK signal, okay. Now with BPSK signal you are able to transmit one bit for one symbol, right so it occupies a certain bandwidth we will come back to the bandwidth calculation in terms of power spectral density after sometime but if you look at the power spectral density which is having a certain width corresponding to the bandwidth of this particular modulation technique.

You will see that for that given bandwidth you are able to transmit one bit for every symbol that you transmit now this transmission of one bit for every symbol for the given bandwidth seems to be kind of wasteful or in the sense that is in of a minimum that you can do is there a way to increase the spectral efficiency that is can I increase the number of bits per symbol while keeping the same bandwidth.

For example in optical communication systems you have 100 Giga hertz grid, 50 Giga hertz grid sometimes you also have a 25 Giga hertz grid, what this grid means is that from channel to channel you have a both 100 Giga hertz of separation that is each n has a set of bandwidth of 100 Giga hertz this is the two sided bandwidth that we are talking about, so in that 50 Giga hertz for example if you assume.

So if you assume a 50 Giga hertz bandwidth right, so in that 50 Giga hertz bandwidth if you where to use BPSK you will be sending one bit per symbol, okay and if I want to increase a number of bits per symbol while keeping the same 50 Giga hertz bandwidth I cannot use BPSK I have to use a different mechanism in which I am able to send two bits per symbol okay such modulation or such mechanism by which I mean creasing the spectral efficiency that is how many number of bits per symbol for a given bandwidth that is the definition of spectral efficiency so how can I increase spectral efficiency by going to what is called as higher order modulation techniques higher order modulation techniques means that you will be utilizing the phase and the amplitude and combing them in certain ways so have to increase a number of bits per symbol okay one simple.

(Refer Slide Time: 10:36)

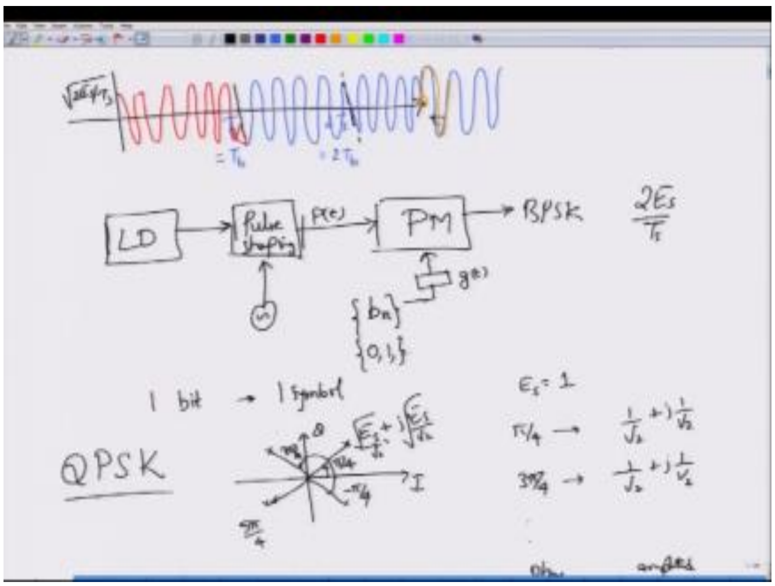


Solution to increase the spectral efficiency is to use what is called as quaternary phase shit key quaternary stands for four so your phase goes through four different values okay there are various ways in which you can arrange them and we will of course look at the standard QPSK format so standard QPSK format is given by the constellation point that would look like this the constellation values are Л/4 then you have 3Л/4 so this is Л/4 this 3Л/4 then you get a suppose 5Л/4 and then finally you get a – Л/4 because 7Л/4 is – Л/4 so if you look at the I and Q axis for this and if you say that this angle is Л/4.

As well as this point here you have chosen to be at EsЛ√2 + J_{es}/√2 okay so this would be the amplitude of the signal that I have chosen that okay or rather I should choose √es/2 id I take es =

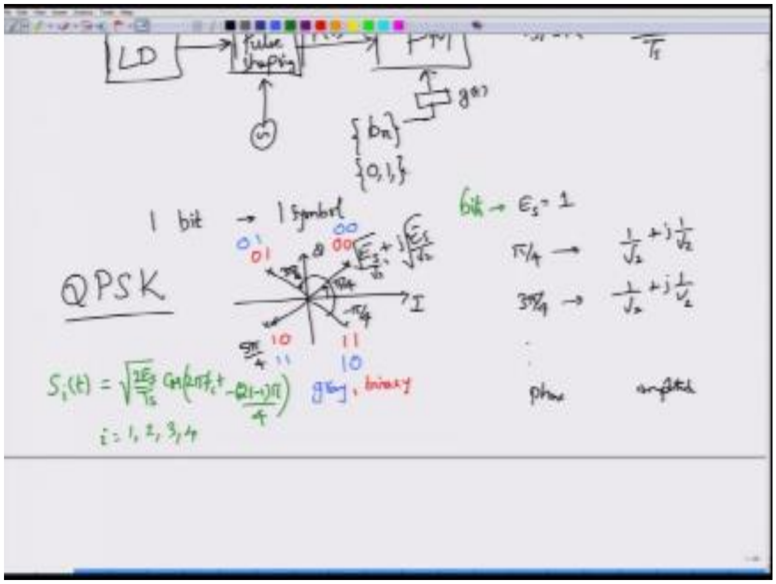
1 some sort of a normalization that I can do then these points the points corresponding to Л/4 angle is located at $1/\sqrt{2} = J_1/\sqrt{2}$ and 3Л/4 is located at $-\ 1/\sqrt{2}+ J_1/\sqrt{2}$ and so on I will leave the other as a exercise for you to complete the phase to amplitude or phase to complex amplitude mapping however there is one thing that we have not yet done remember in the BPS k modulation that we had taken we had talked about.

(Refer Slide Time: 12:23)



First we had a bit or a symbol or rather if you go back to the constellation you had the constellation points and these constellation points which are simply some numbers in the IQ space they had to be mapped into to the corresponding symbols BPSK symbols is equal to bit in this particular case you have four such constellation points and you can actually.

(Refer Slide Time: 12:46)

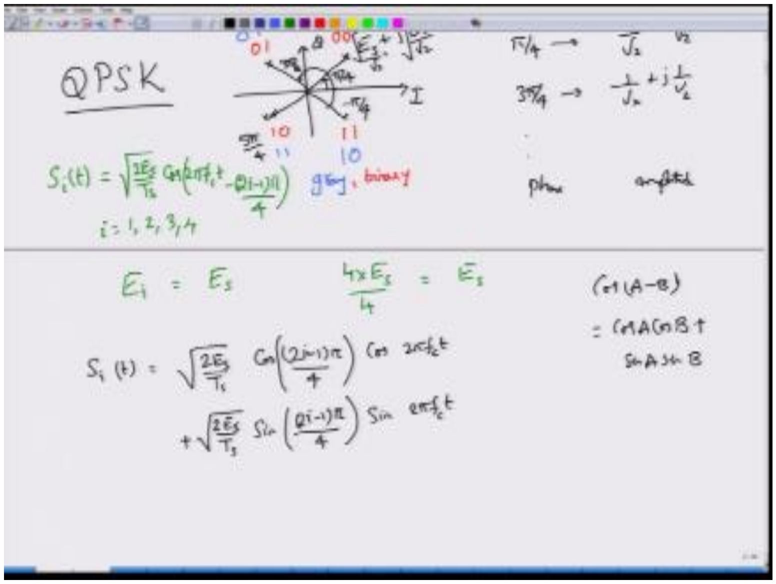


Map these four constellation points to two bits per constellation point correct so I can map this has 0 0, 0 1, 1 0 and 1 1 or if I got to what is called as the gray coded modulation I can map this to 0 0, 0 1 notice here that I will make it as 1 1 and then cost is to 1 0 this blue one which I have written is the gray coded constellation the other one which I have written in the orange color is the simple binary mapping this one is the one which I have written in the oranges binary here in the blue is gray so this is the gray coded model and this gray coded modulation and this is the binary straight binary encoding formulation right.

So what you have done is you had this bits with you and those bits or group of bits which form a symbol so you had symbol for bits, bits or symbol both of them are equivalent and that has been mapped on to the phase, phase has been given by a certain value of the signal that is being carried out right. So what you have done is to take the bits group them into symbols and then map this symbols on to corresponding phase of the carrier so in general for a QPSK signal you can actually write like this the expression for this QPSK signal can be written as the ith way form okay, there are four way forms obviously so this ith way form that you are taking is given by $\sqrt{2E_S/T_S}$ remember why I am module multiplying this one by $\sqrt{2E_S/Ts}$ then you have Cos of $2\pi f_c t$ which is the.

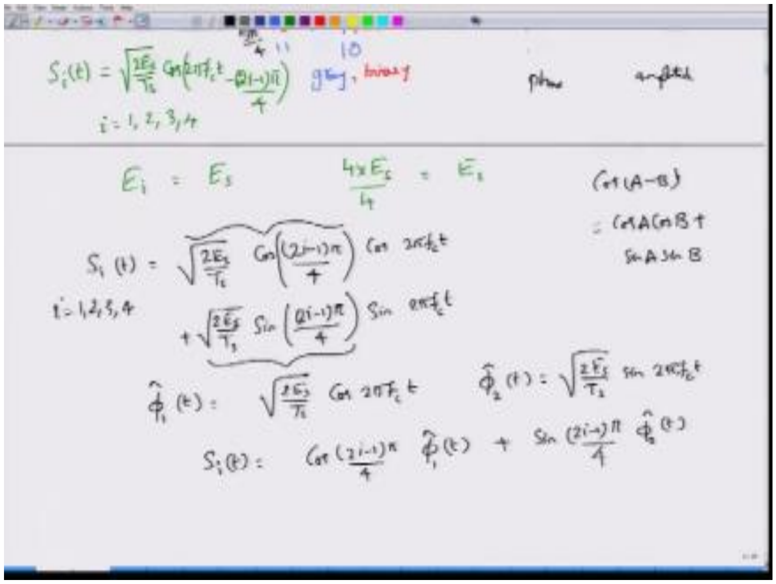
Carrier frequency – 2i – 1 into Л/4 okay where i goes from 1 2 3 and 4 this is not the only way to represent but this seems to be the simplest way for me to represent.

(Refer Slide Time: 14:49)



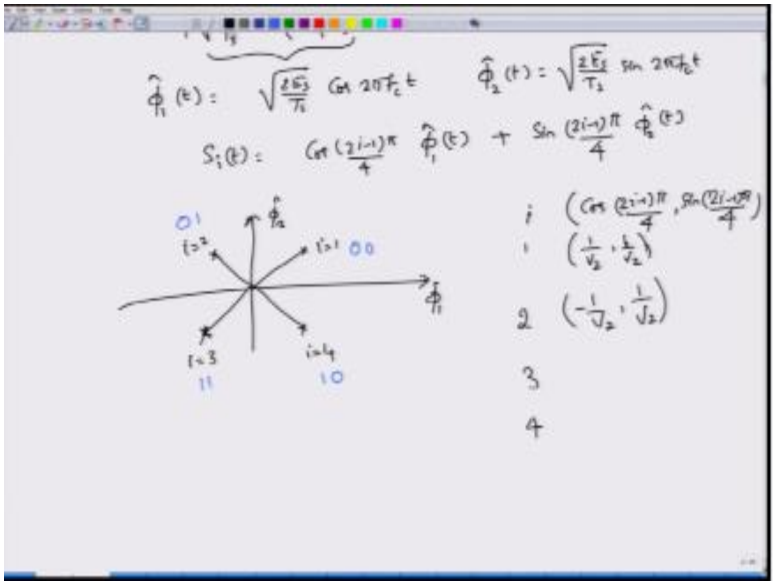
This signal so if you look at this i^{th} way form and calculate what is the energy of the i^{th} signal the energy of the i^{th} signal will be equal to ES how many way forms to be have we have four way forms each carrying an energy of ES and what is the average energy carried by each symbol the average energy carried is equal to E_s, okay. So if you look at the expression for $S_i(t)$ and invoke the formula cos (A-B), right cos (A-B) is given by cosA cos B+sinA sinB and apply that directly to this expression of $S_i(t)$ you can write the i^{th} wave form as $\surd 2E_s/T_s$ cos $(2i-1)\pi/4$ times cos $2\pi f_c t - \surd 2E_s$ or rather $+\surd 2E_s/T_s$ $\sin(2i-1)\pi/4$ sin $2\pi f_c t$ it is that okay.

(Refer Slide Time: 16:07)



So now you can see that you have actually written down $S_i(t)$ where i goes from 1,2,3 and 4 these are the four values you have written down this as some signal multiplied by cos $2\pi f_c t$ and another signal that is multiplied by sin $2\pi f_c t$ and if now denote by $\hat{\phi}_1(t)$ this particular wave form which is cos $2\pi f_c t$ wave form and with $\hat{\phi}_2(t)$ there is $\hat{\phi}_2(t)$ as this wave form $\sqrt{2E_s/T_s}$ sin $2\pi f_c t$ you can clearly see that $\phi_1(t)$ is orthogonal to $\phi_2(t)$ you can easily prove that because f_c is integral multiple of T_s or $1/T_s$, because of this these two are orthogonal to each other and I can write $S_i(t)$ as cos $(2i-1)/4\pi$ multiplied by $\hat{\phi}_1(t)$+ sin $(2i-1)\pi/4$ $\hat{\phi}_2(t)$, okay.

(Refer Slide Time: 17:16)

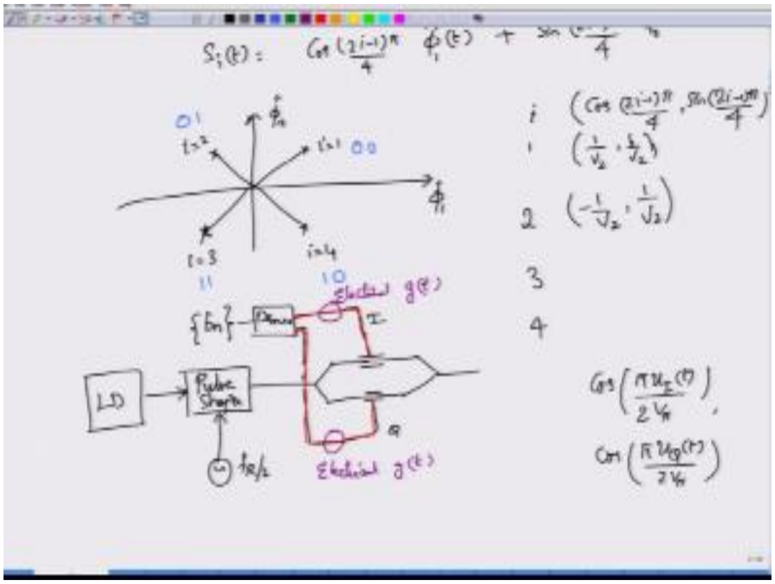


So if you now go to the signal space representation and then call this as the ϕ_1 axis and call this as the ϕ_2 axis, right then I can by changing the value of i, which you say 1,2,3 and 4 I can find out what would be the corresponding points on the signal space, right. So the corresponding points are cos (2i-1)π/4, sin (2i-1)π/4 so is the point on the $\phi_{1,}$ ϕ_2 plain which represent the different symbols, so put i=1 you get 2-1 is 1 so cos π/4 and sin π/4 so you get 1/√2 and 1/√2.

Whereas if you put i=2, you get putting i=2 you cos(4-1) which is 3, 3π/4 cos 3π/4 is in the second quadrant, so this would be -1/√2 and sin 3π/4 is in the second quadrant so it is positive there , so it is 1/√2, you can complete for 3 and 4 obviously, and then you will see that these are nothing but the original constellation points that we had talked about, right. So these are for i=1, i=2, i=3 and i=4 if you wish you can use the straight notation or the grey coded so with grey coded this is 000111 and 10 it is that, okay.

000111 and 10 so this would be i=2, so this would be -1/√2 and +1/√2 you have the grey coded constellation of these for points, right. So this is the standard QPSK format.

(Refer Slide Time: 19:07)



You can realize this standard QPSK format using optical transmitter in a very simple way as usual you have the laser diode and then you have the pulse shaper in the form an MZM to the pulse shaper we can give an Rf signal running at $f_R/2$ which is the data rate by 2, and choosing the appropriate amplitude and the bios in condition then you have your IQ modulator, right. So you have your IQ modulator which consists of two inputs I and Q, right so you have I and Q and this is your IQ modulator that we discussed.

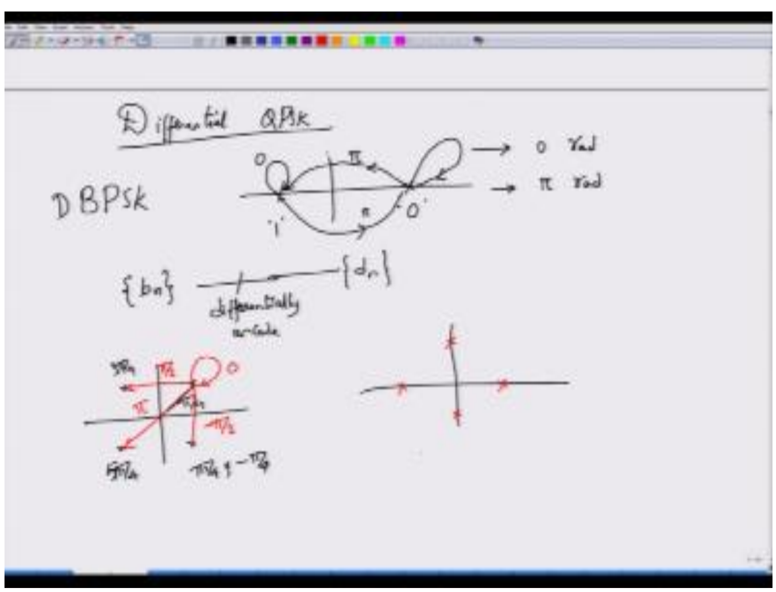
All you have to do is whatever the symbol that you are getting because the call this as the symbol sequence of the bit sequence, so if this is the bit sequence that you are getting you essentially put a demultiplexer over here, okay so put a demultiplexer let me rewrite here correctly, so I put a d multiplexer let me rewrite here correctly so I put it d multiplexer after the symbol or the bit source and then extract the I bits and the q bits so I am try to write it in such a way that I am connecting this I and q okay so I have the I bit here and the q bit connected so please note that this is actually a jump connection her it is not straight connection, so what is the reasoning behind here I know that the corresponding points are cos π $u_i(T)/2V\pi$ and cos π $u_q(t)$ /2V π.

So this is essentially the transfer function of the, i and q modulator right so these are the two points with which it can reach. So by sending the I signal to this and q signal here of course if you want you can also put in a electrical pulse shaper right so you can put in a electrical pulse shaper which is g(t) and also suitably scale up this electrical signal, okay you have to scale you this electrical signal such a way that this is equivalent to cos π or other this one should be equal to cos 2i-1 x π/4 kind of a situation okay.

And this one should be sin so in order to have this sin you have to operate this q modulator by an additional you have to give an additional phase shift of 90^0 to this right so that it becomes cos π ui/2πV π(t) cos or instead of cos it becomes sin π uq(t) /2V π and then put the biasing condition in such a way that this term as π/4 and then you can through the different value for I and then see that using the iq modulator you will able to implement in the standard QPSK modulation.

Okay now there is another way in which I can implement QPSK signal and this implementation is widely use in the 40 GBPS optical communication systems.

(Refer Slide Time: 22:03)



This is called a differential QPSK with differential QPSK what we do here is that instead of encoding the bits in to their corresponding symbols and symbols in to their corresponding phases we encode the phase difference for example let us consider the simple DBPSK right so if you look at the actual PBSK configuration you have a 0 here and a 1 here that is the mapping which have chosen so these are the symbols.

If my present bit is 0 and if my next bit happens to be 0 then I am in the same phase right so I do not have to change the phase so this can be encoded by giving a 0 phase difference or by giving sending out a signal whose phase is 0 radians, if my present bit is 0 and the next bit is 1 there is a transition of 180^0 phase so this transition can be represented by sending out a π shifted carrier signal okay.

If my present bit is 1 and the next bit is 1 then I am in the same phase situation so I represent this one by a 0 phase so I represent this transitions by π similarly I represent this transition by π okay so in order to implement a DBPSK modulation you simply use the same BPSK Modulator except that instead of sending the straight b and sequence the bit sequence you then differentially encode them okay.

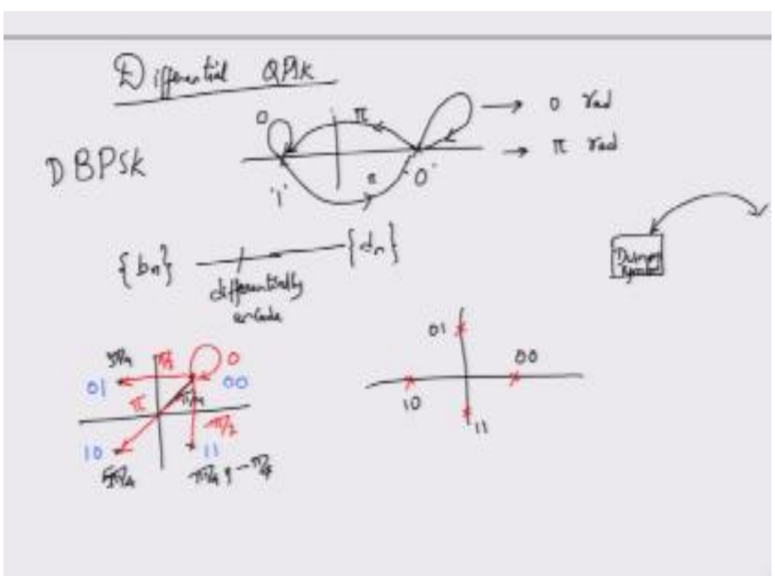
You differentially encode them to generate the differentially encode sequence the details of this you can see in the exercises that we give out for this particular module okay I do not want to go in to the details of that that is not really important for us but you can extend the same idea to differential QPSK as well remember in the QPSK you have 4 different phases right so this is $\pi/4$ you have 3 $\pi/4$ and then 5 $\pi/4$ and then 7 $\pi/4$ or which is equivalent of $-\pi/4$ correct.

So what possible transitions can I have there will be 0 phase transition that is the present group of symbols is 00 or the bits is 00 the next symbol is also 00 so I have represented that by I can represent that by sending out a0 bit this transition from $\pi/4$ to 3 π /4 is 90^0 transition right so I can represent this transition by sending out a $\pi/2$ shifted cosign signal this transition from 5 $\pi/4$ represents a π transition.

And similarly π/4 to – π/4 would represent a - π /2 transition right so if you look at what the differential encoding that I need to do I have 4 different transition point which is 0 phase difference π/2 phase difference π phase difference and finally I have – π/2 phase difference right. So I have four possible combination and if this represents a straight binary coding, 0001 10 and 11 the differential coded bits can also be represented as 0001 10 and 1. And phase not that this 00 in the straight binary of the standard QPSK format denotes different phase, whereas 00 here indicates the same phase difference.

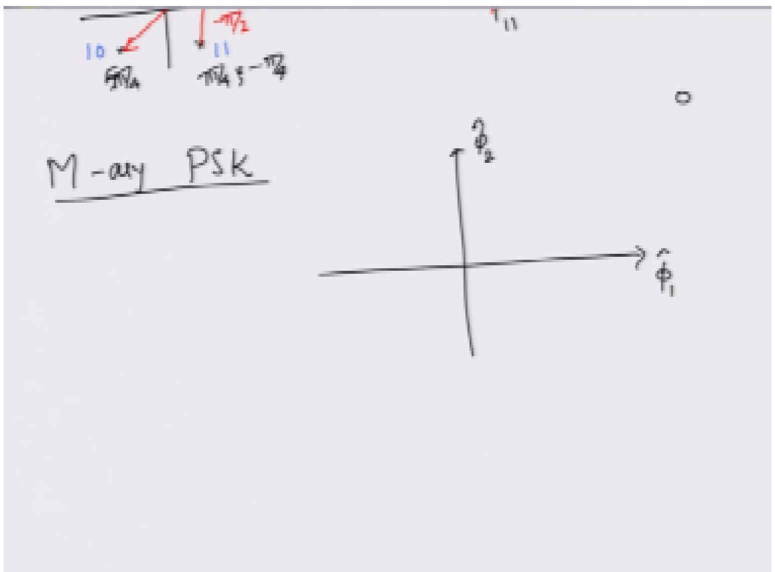
One thing for differential DBPSK or differential QPSK format is that you and to have an initial dummy symbol. So you need to have an initial dummy symbol, this does not really matter because your only looking at phase difference, but this dummy symbol should be known for both transmitter as well a the receiver, so for the DBPSK case let us say the dummy symbol is 0. So if my first bit is 0, the previous bit was also and the dummy bit is 0.

(Refer Slide Time: 26:03)



So in the next case, I can write this s 0 phase, I can even transmit that corresponding slot s 0 phase shift at sequel, so similarly you have to have a dummy symbol, let say the dummy symbol for differential QPK signal is 00. So the ways to implement this one will be the same and we have discussed. One final modulation that w are going to discuss here is called as M –ary PSK, the idea behind this is very simple, you go the constellation diagram over here, so it could be ϕ1 and ϕ2 space or equivalently I and Q space, that is ϕ1 and ϕ2 re the real space, and is the one single complex symbol space.

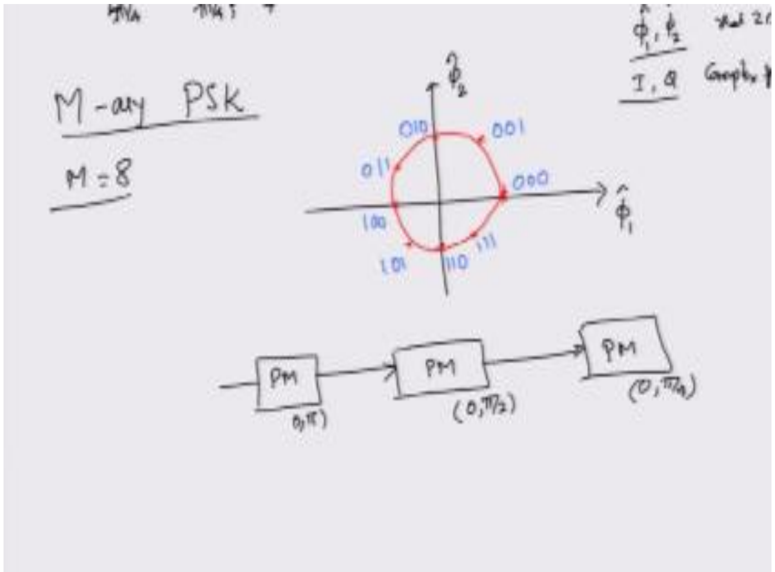
(Refer Slide Time: 26:45)



So ϕ1, ϕ2 is the real 2D space, whereas if I use I and Q that would b one complex space, this space will be complex symbol, this gives me real symbol, these are just a way of representing them. So for n M – ary PSK, consider the simplest case of m=8, what different possible phases that you have or 0, л/4, л/2, this on which I 3 л/4, л this one л/2 and – л/4, sorry they are supposed to be arranged in a nice circle, but I could not draw a circle properly, so I have actually 8n different phases.

At each of these representing a group of 3 symbols, so this is 000 001 010 011 100, this is straight binary configuration 101 110 and 111, how do I implement this, you have the usual conflicts, the laser diode, the pulse shaper but for representing this phases, you can simply use 3 phase modulator. So you will us 1 phase modulator, another phase modulator, this phase modulator will be biased between 0 and л, that is in order to generate 0 and л phase shift, this will give you 0 and л/2 whereas this one will give you 0 and л/4.

(Refer Slide Time: 28:14)



So if you use a bit sequence and then put a de multiplexer over here, and then sending the MSB bit to the first phase modulator, from the middle bit to this one and then send the LSB bit to the last phase modulator, you can see that the total phase that you will obtain in will be a linear combination of all these three and it will then implement this. Thank you.

<u>Acknowledgement</u>

Ministry of Human Resource & Development

Prof. Satyaki Roy

Co-ordinator, NPTEL IIT Kanpur

NPTEL Team

Sanjay Pal

Ashish Singh

Badal Pradhan

Tapobrata Das

Ram Chandra

Dilip Tripathi

Manoj Shrivastava

Padam Shukla

Sanjay Mishra

Shubham Rawat

Shikha Gupta

K. K. Mishra

Aradhana Singh

Sweta

Ashutosh Gairola

Dilip Katiyar

Sharwan

Hari Ram

Bhadra Rao

Puneet Kumar Bajpai

Lalty Dutta

Ajay Kanaujia

Shivendra Kumar Tiwari

an IIT Kanpur Production