Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Optical Communications

Week – III
Module – II
Digital Modulation-II(contd.)

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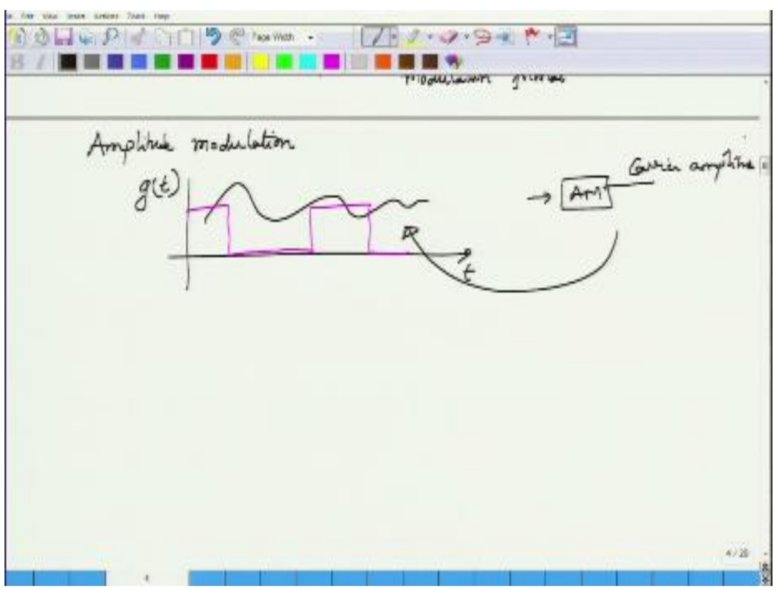

So far we have talked about modulation and everything we have talked about ON OFF keying.

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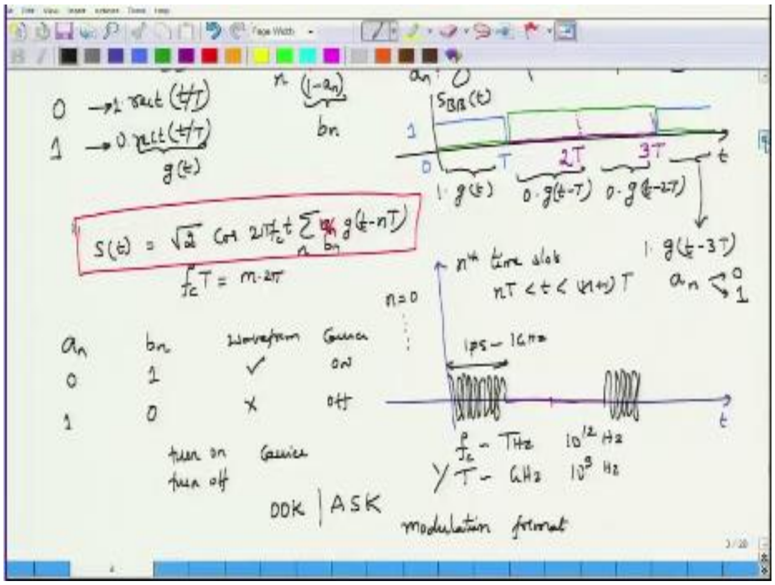

And amplitude shift keying the question is how do I come up with an optical transmitter to basically implement this one right.

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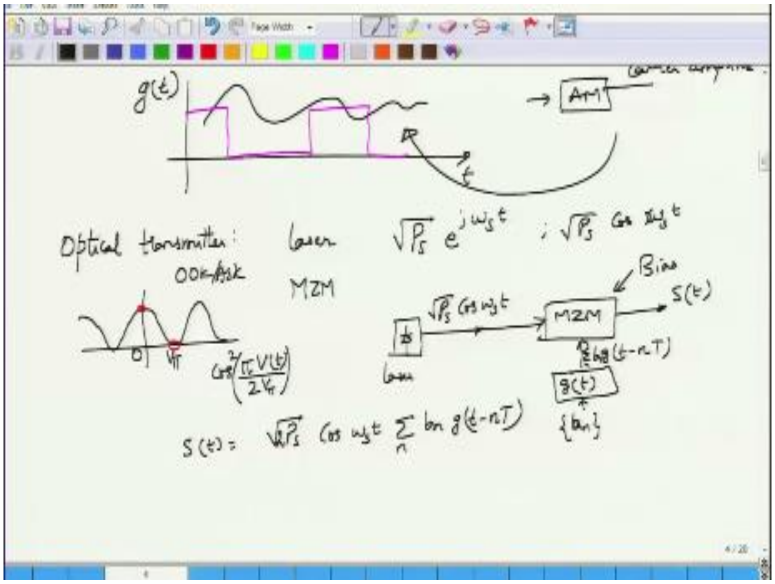

So how do I come up with An optical transmitter to implement this one that would be the question that we are going to answer now what do I need to obtained the optical transmitter what are the elements that I already have discussed I know what a laser is laser is equivalent of a signal generator right it is output will be sum power Ps and it will be $ej^{\omega}s^{t}$ this is the complex way of writing the signal or the complex notation the actual way form that you're going to get from the laser from an idealized laser like this will be $\cos^{\omega}s^{t}$ where $^{\omega}s$ will stand for the signal frequency.

Okay so this is the output of the laser now this output remains you of the carrier part but I don't have the amplitude part that is where we have discussed the optical transmitter called MZM a dual drive mach zehnder modulator we have studied the characteristics of this to briefly recall the power transfer function goes like this it would be $\cos\pi$ v(t) or u(t)/ 2v π sorry $\cos^{2}$ π v(t)/ 2vπ this is the power transfer function for this particular MZM right now if I can somehow operate in only these two regions right.

So I can operate in this region and in this region I will be able to transmit the signal or block the signal so what is signal, signal is the output from the laser so I have a laser okay I have a laser this is my laser which is producing an output which is square root of Ps $cos^{\omega}s^{t}$ so writers right it down square root Ps $cos^{\omega}s^{t}$ and if you take this one connect to an MZM right and the to the MZM you give after shaping the sequence which ever that you want to transmit the sequence is bn this sequence bn goes through the transmit filter g(t) and then goes to the MZM.

This sequence b(n) goes through g(t) gets generates g(t-nT) x bn summed over all n then this should go to the MZM and what you get as the output will be the transmit signal s(t) so this transmit signal s(t) can be written as square root Ps $cos^{\omega}s^{t}$ and then multiplied by this sequence bn g(t-nT) is that okay so this is the signal that you are getting in place of square root 2 you have square root Ps but if you can kind of scale up this square root Ps into square root of 2Ps then everything will be alright.

Okay so this is the implication that we were looking for in the optical transmitter which would correspond to ON OFF keying or amplitude shift keying of course I have not yet completely specified how to construct this modulator because I have not told you how to bays this MZM so I have not told you how to basically complete this ON OFF keying part because I have not told you how to bays the mach zehnder modulator.

Where should I bays the mach zehnder modulator I need to get full transmission and I need to get 0 transmission to get 0 transmission I have to operate this at v π because if I operate this at π π then I have cos^2 π if v(t)vπ, v π cancel from numerator and denominator I get cos^2 π/2 which is equal to 0 if I operate at o I get full transmission okay so I can bays this mach zehnder modulator.

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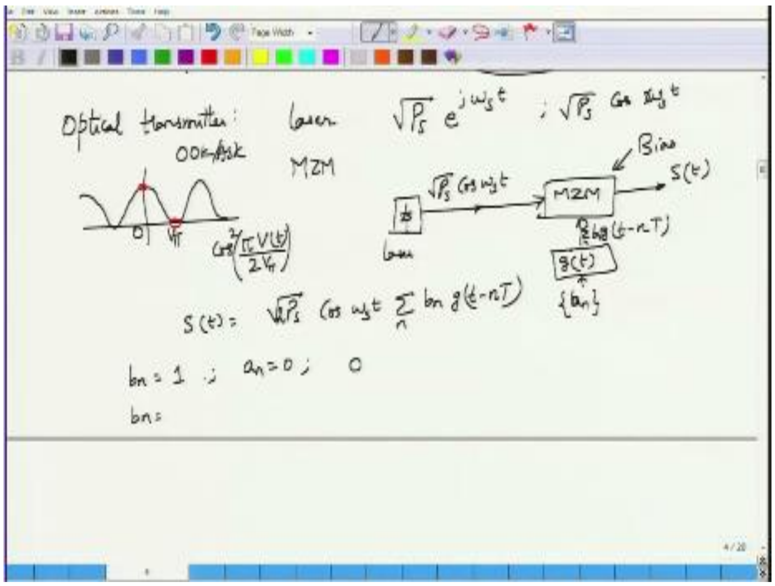

If I get bn is equal to 1 which corresponds to an= 0 right then I have to choose the amplitude of this pulse in such a way that is this amplitude g(t) in such a way that I get full transmission that is I should get 0 if I get bn = so for an=0 I should get a full transmission let me just go back there is small confusion here.

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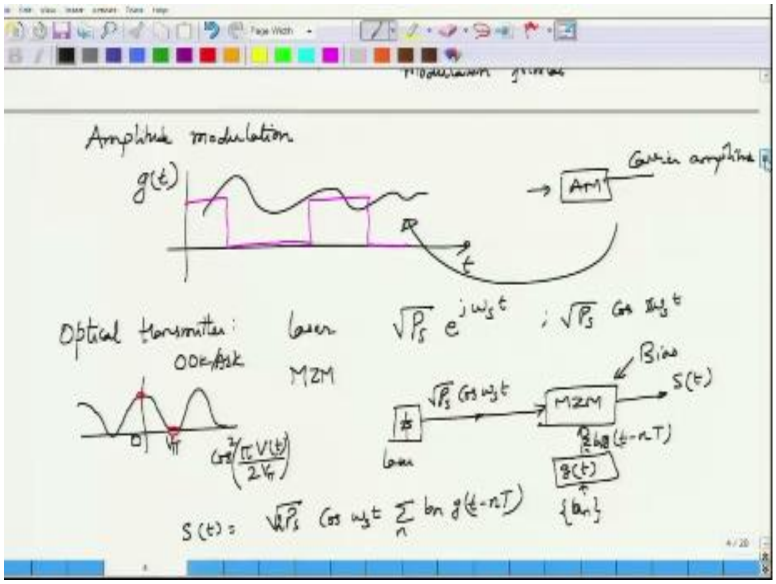

So we have consider an=0 bn we will have a transmission okay so we will have a full transmission a full transmission here.

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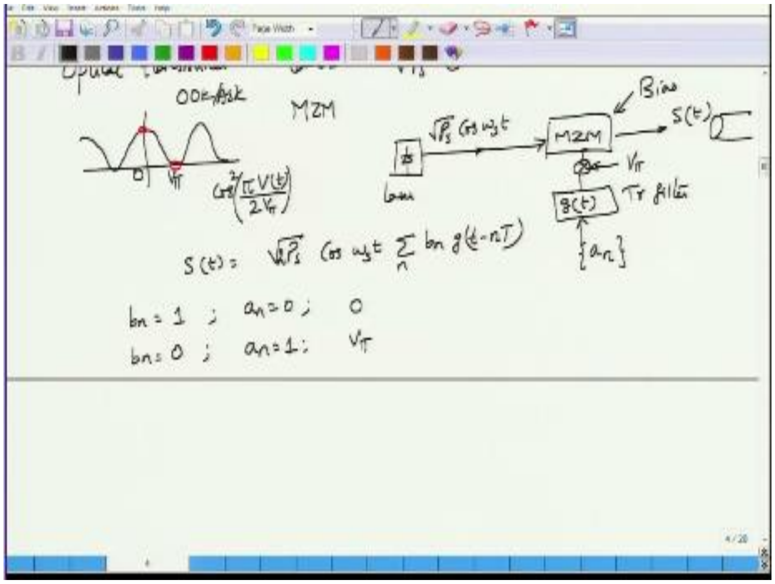

So I need t send in a 0 here for bn equl to 0 which corresponds to an = 1 I need no transmission right so I need to send in an amplitude of v π this simplest way I can do this is instead of connecting this bn right what I can do is I can take this an right I can take this an which is the sequence that already obtaining okay and do not do the convection factor rather I multiply the amplitude of g(t) by v π and then send as input to that transmit filter this that transmit filter or just a pulse shaping filter okay to this instead of sending bn I straight away connect an.

Now you see what happens when an = 0 the output of the filter will be 0 multiplied v π that is I 0 and then goes to the MZM if the input voltage so the MZM is 0 you will be operating in the maximum transmission point or full transmission point and what you get is the connection between laser and the fiber so say for example I put a fiber here then laser will be connected to the fiber here on the other hand when an = 1 the amplitude of the signal going into the MZM will be v π which means that you will now be operating at the minimum transmission point and you get 0 at the output so this is the way in which you can realize this ON OFF keying or amplitude shift keying right now you might also say.

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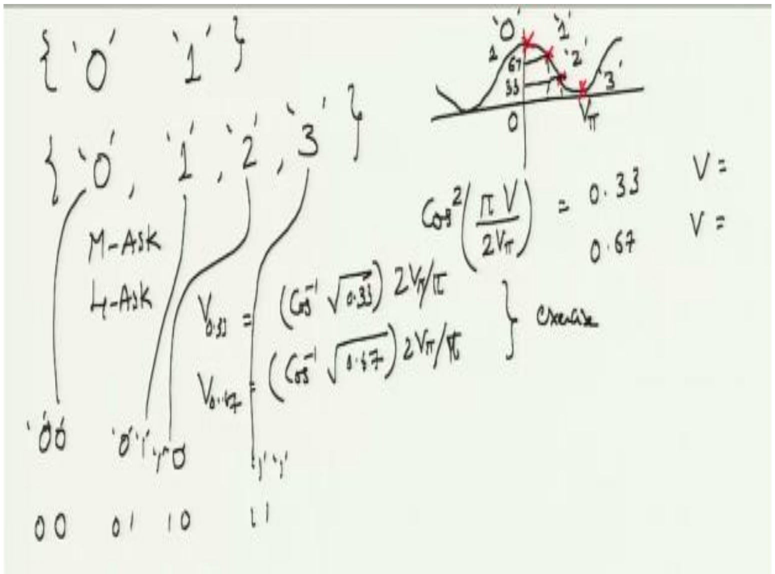

Is there a way to increase more number of amplitudes you know are utilize more number of amplitudes rather than switching between symbols say 0 and 1 can I switch between symbols which we call as symbol 0, symbol 1, symbol2 and symbol 3 there are four symbols now my alphabet includes 4 letters right, the size of my alphabet is 4 essentially and I want to change the amplitudes that would correspond to 4 different levels earlier I was changing the amplitude corresponding to two different levels now I would like to change the amplitude corresponding to 4 different levels, it is certainly possible.

All you have to do is, go to the transfer function of the MZM and then instead of operating at two levels which was corresponding to full transmission and 0 transmission you can introduce additional two levels here, okay. And whenever you get a symbol 0 you transmitted full transmission you get a symbol 1 you transmit this waveform you know or you operate the MZM at this point for symbol 2 you operate here.

And for symbol 3 you can operate here, okay. What would be the corresponding values that you have to verify this is vΠ, this is 0 one way of finding out this one will be 2 I mean one way of finding this is ask how much transmission is required, right? So this is full transmission which is the transfer issue of one this is 0 let us say this 1 should correspond to or let us say this symbol 3 corresponds to 0 transmission.

Symbol 2 corresponds to say 33 percent and then this would be say 67% and then this would be approximately 100%, now how do I obtain 33% transmission? I simply go back to this expression $Cos^2 \Pi v/2v \Pi$ and then say this value should be equal to 0.33 for 33% or 0.67 for 675 and it should be equal to 1 for 1 you know what is the value, so find out what should be v by inverting this relationship, right.

So what you have to find id, $\Pi v/2v\Pi$ must be equal to $\cos^{-1} \sqrt{0.33}$ $\cos^{-1} (\sqrt{0.67})$ right the corresponding value of v will be multiplied by so this entire thing must be multiplied by $2v\Pi/\Pi$ so you need to multiply this one by $2v\Pi/\Pi$ okay. So find out the values corresponding to 0.33 and corresponding to 0.67 transmission ratios, okay. I will leave this as a exercise to you a simple exercise that you can do it by a calculator, okay.

This was in which we have used multiple levels for shifting the amplitude or changing the amplitude is called as M-ASK, okay. It is called as M-ary ASK where M stands for the number of symbols this would be 4 ASK in this example, okay. Now if I want to convert this 0, 1, 2 and 3 now you can think of this as a many English language which has A,B, C and D and for each AB and C I can actually obtain the binary representation, okay.

In terms of 00's and 11's I might for example call this as binary sequence 00 this I would call as 01 this would be 1 and 0 and this would be 1 and 1 please note the inverted commas these are the symbols, what we are essentially trying to do is, taking this larger alphabet and then representing each of the letters in the larger alphabet by this smaller sized alphabet, okay. The corresponding bit sequences could be 00, 01, 10 and 11, okay.

In optical communication finding more than 4 levels of amplitude shift keying is kind of difficult because the symbols start to group very close to each other and when symbols are very close to each other after they have transmit through the channel to separate them out and detect them correctly is a big headache we are going to take about that receiver headaches later on but you

can kind if take it from this one that practically getting more than 4 ASK is quite difficult. Unless you are very rich to invest in very high and products fro optical communication links, okay.
Now we have talked about varying the amplitude now a semisolid carrier also has a phase with respective to some phase and it also has its frequency is there something that we can change in terms of phase and frequency in order to convey the information and the answer is yes you can change the frequency you can change the phase these two corresponds to phase modulation and frequency modulation.

It is not just at that point the you can combine amplitude and phase in general you do not you can combine amplitude, phase and frequency but that is kind of very complicated modulation technique but you can combine amplitude and phase this is called as AMPM combination or sometimes called as or more commonly called as quadrature amplitude modulation you are changing both amplitude as well as phase of the sinusoidal carrier.

Depending on what waveform you want to transmit, okay. So these are all the different modulation techniques which you can play around for a single carrier okay for a single carrier you can change its amplitude, phase, frequency individually or you can change amplitude and phase together to form what is called as quadrature amplitude modulation and finally optical communications allows you one more degree of freedom.

Because electric filed can be polarized into orthogonal ways into perpendicular ways it can be polarized horizontally or it can be polarized vertically you can impose modulation on separate polarized components I can transmit the horizontal polarization into the fiber vertical polarization into the fiber both at the same frequency and have different modulation on both of them, so you have how many degrees of freedom?

You have polarization degree of freedom, you have amplitude, phase, frequency and combined amplitude and phase, okay. So these are some of the advantages of going to optical communication that you will start getting lot of higher degrees of freedom so you get lot of degrees of freedom to implement your modulation techniques, so let us learn a little bit about quadrature amplitude modulation.

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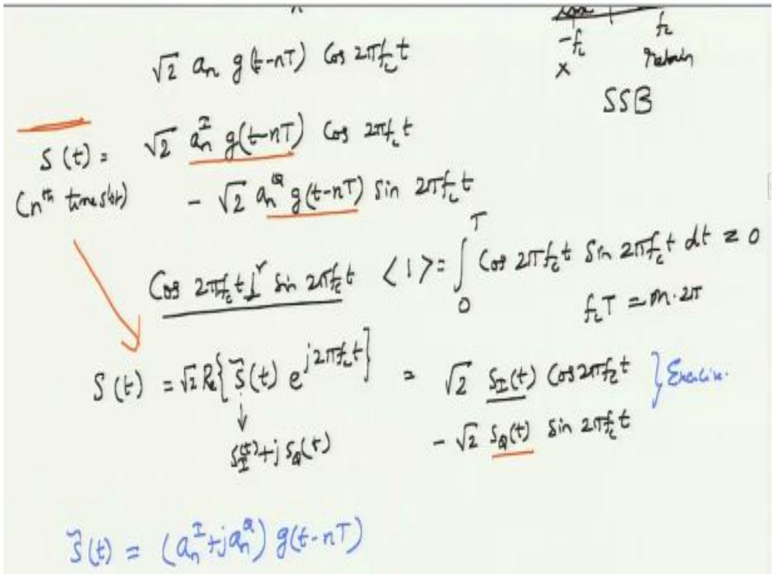

And in order to do that one I would require to introduce to you what is called as signal constellation, okay. I know that this s(t) which we had written down the pass band signal can be written as so I will revert to using a_n rather than b_n okay this mapping was little awkward in the ASK case but it i9s quite straight forward in the AM, PM or the QAM cases, okay. So we will revert to the situation where we where representing the data sequences an right if I now take this.

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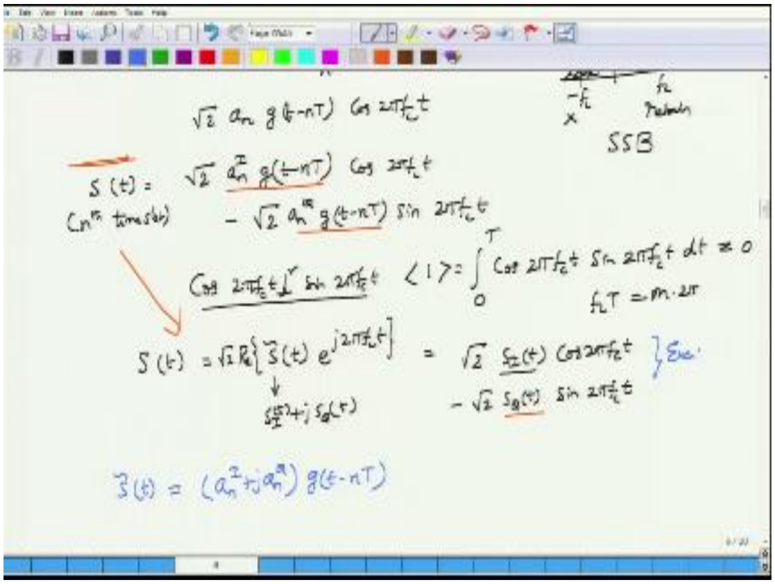

Data √ of data an and then multiply that one to the carrier so I had written down this expression right so n you have an g of t – nT okay if you focus on only symbol at a time then the signal can be written as √ 2 an this is the nth time slot signal so √2 an g of t – nT Cos 2Лfct okay it turns out that if you look at the spectrum of this you will see a spectrum which is centered at fc okay but if you also look at the spectrum this spectrum will be symmetric about fc okay because this spectrum will be symmetric about fc right with this spectral shape determine by this g of t okay.

What you can see is that the information contained in both side band is completely redundant right whatever the upper side band is telling you it is mirror component is the lower side band so why do I have to keep two side bands in order to convene for machine or equivalently I have spectrum centered at fc I have spectrum centered at – fc I do not require both of them to be getting back to the original signal I can in fact multiplied this one by the phase splitter that we talked about whose transfer function was 5 of f in the form of a unit step function.

Eliminate this one and the retain only the upper side band right, so this kind of modulation is called as SSB modulation which is single side band modulation this is use to improve the utilization of band width okay this will be the way in which you can cut down the bandwidth requirement so you have this SSB modulation and entirely different approach is to basically write down f of t in this fashion I will write down and then motivate what I mean by this okay so I am again focusing on only nth time slot okay in fact if you want to find out the overall way form.

Then you have to submit over all possible values of n you know transmission through the from n = 0 to n = infinity for nth time slot of course this can be written as Cos 2Лfct have change the notation here if you observe this is an this a_{ni} because I want to introduce one more signal which is g of t – nT sin 2Лfct what is the advantage of doing this well if you do not have this second term the one that is sin2Лfct your transmitting something at fc okay and your utilizing the spectrum, however if you realize that Cos 2Лfct and sin 2Лfct or actually perpendicular to each other right.

How do I say two way from some perpendicular to each other if their inner product vanishes right so you can take these as an exercise and do it for yourself show that if you were to integrate this, this is the inner product remember this is the inner product of Cos function and sin function seen from the previous module so if you find this inner product and if you satisfy that fc of t is = some integer multiple of 2Л and m can be very large quantity it can be some 20, 000 it can be some 30, 000 does not matter.

As long as this fc of t is very either exact multiple of 2Л or it is very, very large then this inner product essentially is equal to 0 because Cos and sin are phase reversed signals I mean phase shifted signals there areas will cancel out each other and then you essentially get a 0 so in that sense these two are perpendicular signals they do not interfere with each other right if you remember we had written down the real pass band signal so of t in terms of the complex base band signal s/t of t or s~ of t and then we had said if you want to obtain from the complex envelop to the real way from.

You multiplied by $e^{j2Лfct}$ take the real part also attach this √ 2 in order to normalize the energies of pass band signal real pass band signal and the complex base band signal okay. So this is the complex signal this complex signal can be split up into si + jfq of t okay the so called in face and

Quadrature components and then you can write down s of t by after taking the real part into this expression what you get is √2 si of t Cos 2Лfct – √2 sq of t sin 2Л fct okay these si and sq signals will be orthogonal to each other okay now what you can observe from this equation and in this equation is that in place of s of t.

I have a_n^ig of t – nT in place of sq of t I had a_n^qg of t – nT so it is possible for me to combine this expression for s of t in the nth time slot and make it look like this complex envelope or equivalently if I find what is a complex envelope for this signal for this s of t signal the complex envelope will be a_ni + j a_nq times g of t – nT I am assuming that both in fact and Quadrature signals are shape by the same filter g of t okay o this would be the complex base band signal this is the complex base band signal not the real base band signal it is complex because there is a j term sitting here right and as I said this will be the exercise to you we can show that this is indeed true.

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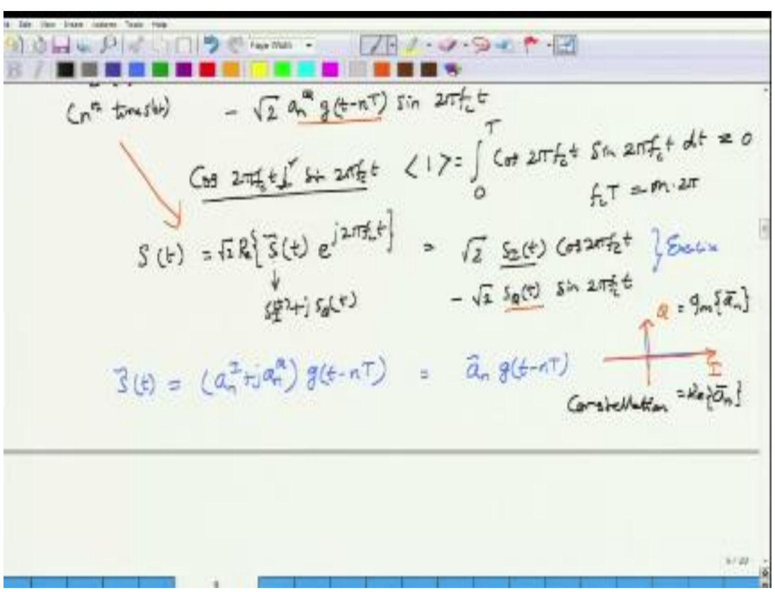

So what we have done is that we can further write this one and call this are the complex symbol an times g of t – nT so the complex base band signal what we have is essentially a complex signal because it is an bar or an~ denotes the complex number and this complex number can plotted in the rectangular coordinate system right I can plot this in this rectangular coordinate system in the I and q access what is the I access I access is nothing but real part of an complex an q is nothing but imaginary part of complex an right such a plot if called as a constellation plot or simply called as constellation or sometimes called as signal constellation okay let us now look at some examples of this constellation and then discuss how can we implement optical transmitter for this okay.

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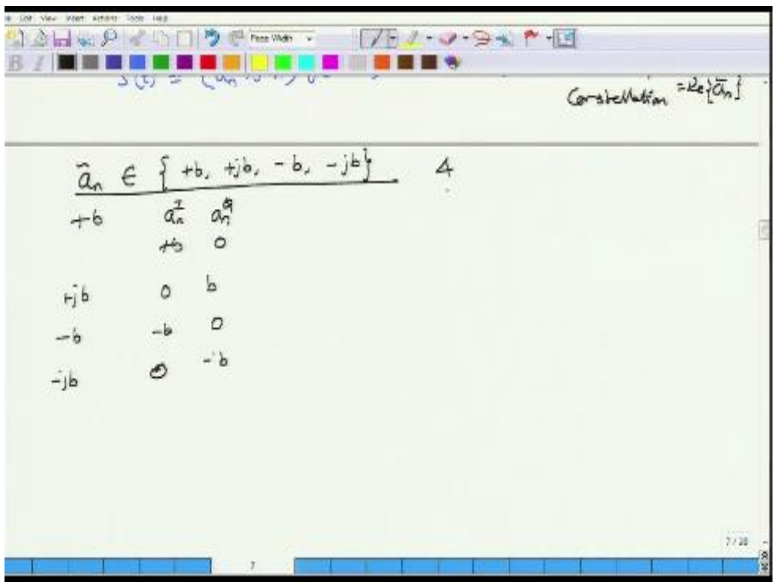

As an example I will consider this complex symbol sequence $\tilde{a}_n$ can be any of these it can be +b, it can be +jb, it can be-b or it can be –jb where b is some number it is a constant, okay. This $\bar{a}_n$ is this particular set it can be any of this numbers, clearly if this is +b then what you are writing here is a_n^I a_n^Q you can write down this a_n^I and a_n^Q for +b if the symbol is +b, right then a_n^I will be +b and a_n^Q will be 0, right for +jb, this would be 0 and this would be +jb for –b this would be –b and this would be 0 and for –jb this is 0 and this is –jb, right.

So the way we have written down here or rather sorry, for a_n^Q we have to remove the references to j here, right so j comes in because a_n^I +ja_n^Q not because j is sitting here, okay. So this would be b, this would be –b, okay is that fine. So you have this complex symbol a_n which can take one of four possible combination, so in other words my alphabet size is 4, okay and let us write down this s(t) here, okay.

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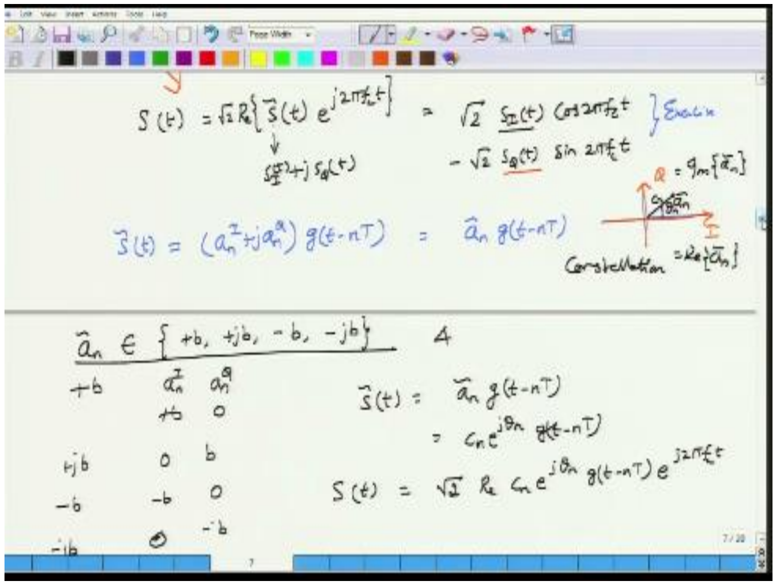

Let us complete what would be the $\bar{s}(t)$ here, since $\bar{s}(t)$ is a_n j(t-nT) there is one more way of writing the same thing, I can convert the rectangular form of a_n into the polar form, for example if I have this as the complex number $\bar{a}_n$ I can specify the length of this arrow or the line segment and specify the angle here θ_n, so I can write this as $c_n e^{j\theta n}$ g(t-nT) substituting this into the expression for s(t) here the real pass band signal what you get is √2 real part of $c_n e^{j\theta n}$ g(t-nT)$e^{j2\pi fct}$, right.

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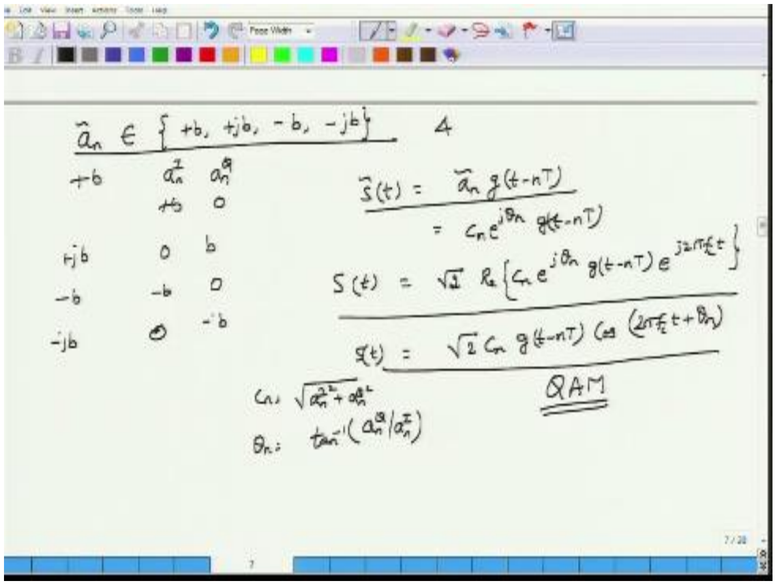

So you can combine this terms here and see that this can be written as $\sqrt{2}c_n$ g(t-nT) $\cos 2\pi f_c t+\theta_n$ what is c_n, c_n is $\sqrt{a_n^{I2}+a_n^{Q2}}$ whereas θ_n is the phase angle which is given by inversed tan of a_n^Q/a_n^I right, this is the simple rectangular to polar conversion formulas that you know from your earlier courses. So what we have our different representation for s(t), if you do not want to refer to the cosin signal or the carrier wave form, you can follow this simple complex base band representation or the complex low pass representation in terms of rectangular or in terms of polar.

You can also represent the same signal in terms of this real part of something or you can simply represent this in the real wave form where in you are changing the phase of the cosin carrier as well as altering the amplitude by multiplying it by c_n, so this form is the general way which shows both amplitude as well as phase being changed, okay.

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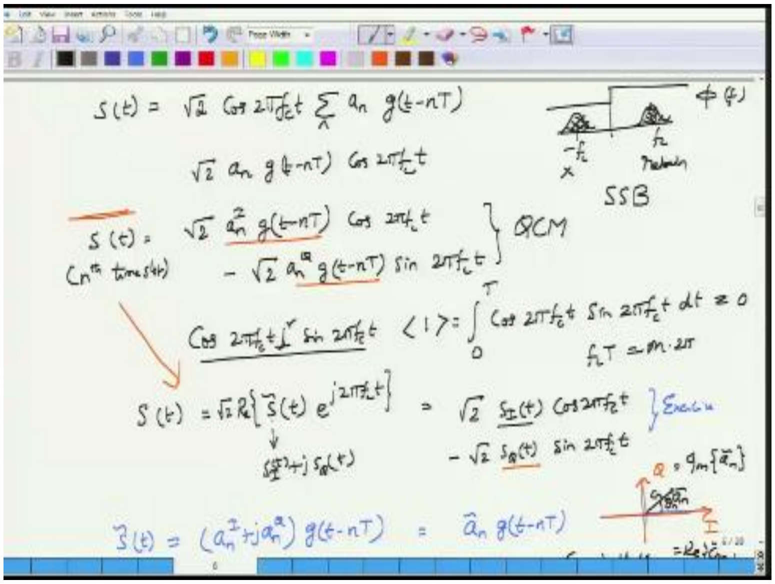

And this sometimes is also called as this way of writing s(t) and realizing this signal s(t) sometimes called as QCM, called as Quadrature Carrier Multiplexing why it is called Quadrature Carrier because one carrier is cos $2\pi f_c t$ the other carrier is sin $2\pi f_c t$ which for historical reasons has been called as quadrature, quadrature denoting 90^o phase difference, okay.

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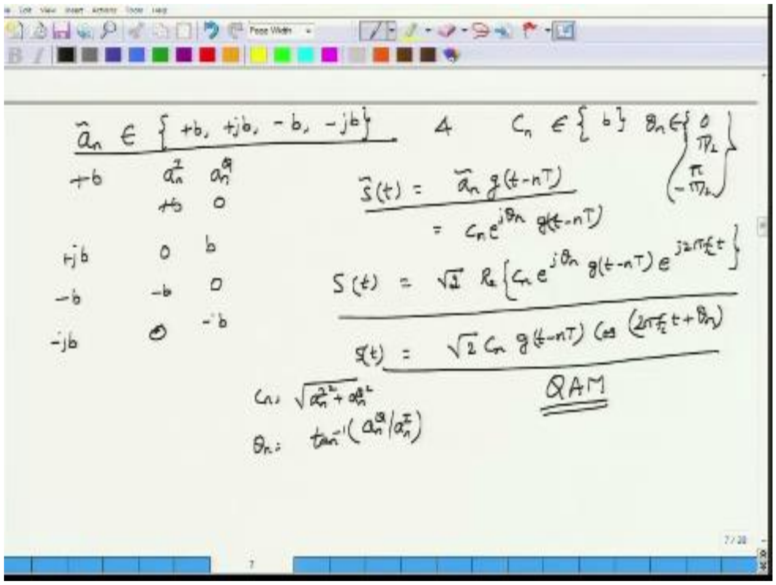

So this is about the expression for QAM let us look at some simple examples of QAM, okay and we were actually started looking at the example we said that the complex symbol sequence a_n can be +b jb -b and –jb, what would be the corresponding sequence c_n for this, c_n sequence will be all bs right, it is a constant, right. However what will happen to θ_n, θ_n will be 0 $\pi/2$ π and $-\pi/2$, right so this would be the sequence θ_n, right.

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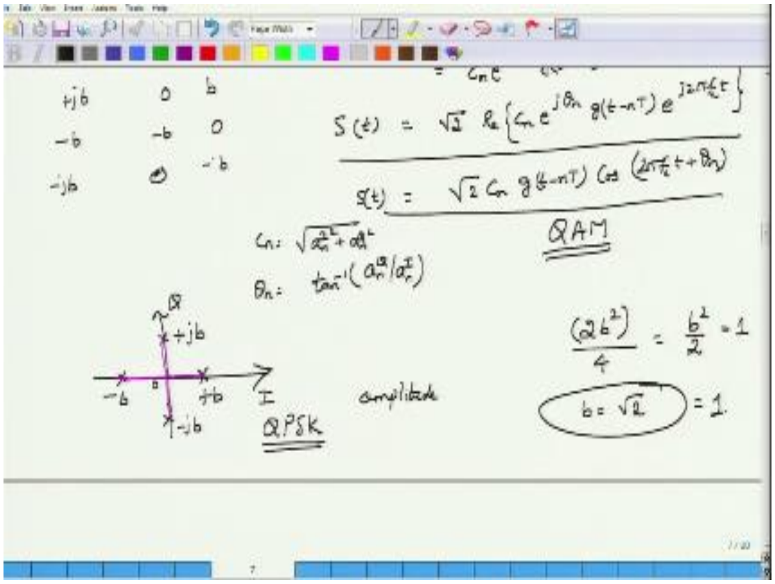

Now if I plot this in the I and Q plain, right this will be the complex I and Q plain +b will be here, this is +b this is +jb, this is –b and this is –jb, okay so this is called as quadrature phase shift keying, actually why it is called as phase shift keying because amplitude here is constant, right. All these points are at the same distance from the origin, all these points are at the same distance from the origin, okay. Furthermore, the way in which you have to choose this b is not arbitrary there is actually a relationship between this amplitude b and the energy of the signal, right.

So back to this s(t) signal here you have √2 c_n g(t-nT) for the quadrature phase shift keying c_n will be constant and it will be equal to b, right. If you evaluate what is the energy of this signal or if you evaluate what is the average energy of the signal you will see that the average energy of the signal will be $2b^2$ because each of them are at the same distance and remember the energy is basically mod of the points square, right.

So these are the four different points in the signal constellation the distance from each of those will be the magnitude square, this is the length squared that we talked about in the last module, right about the geometric representation of the signal. So the energy associated with this one is b^2

or modb^2 this one is modb^2, modb^2 and modb^2 assuming that all these symbols are equally likely to occur and this 2 is because you have a √2 here, okay so the amplitude would actually be √2.b therefore the energy is $2b^2$, okay b is a real number and the since there are four possible symbols and each of the symbols are equally likely to occur we have assumed that one this would be the energy, okay.

So would be $b^2/2$ and if we want to normalize this energy into 1 right we then have to chose b=√2 is that correct? So we have obtains b=√2 or maybe have made a small mistake somewhere because what I was hoping to get is b=1 if I look at the energy here each of these has an energy of $2b^2/4$ anyway so maybe that number √2 is wrong if I have to probilicated 1 but the point stays the same.

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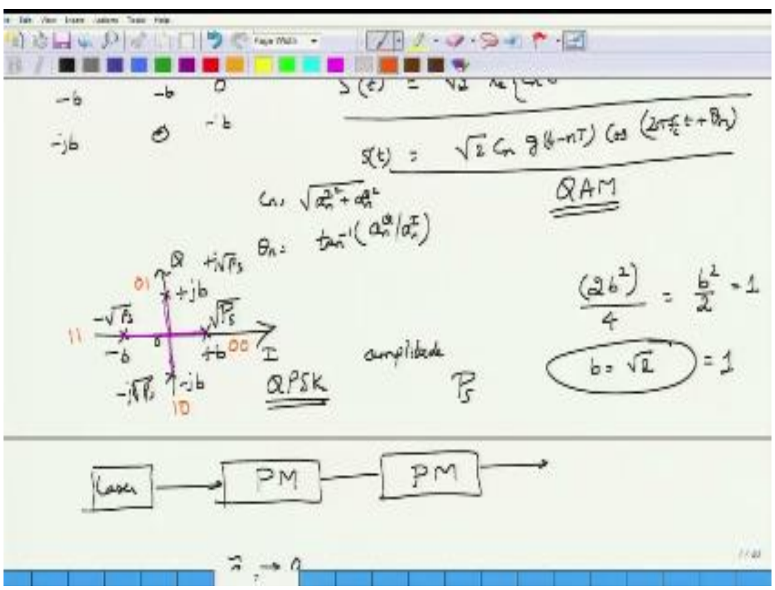

What you have see is that you have to have a maximum energy constraint or equivalently if you say that the laser can only give you 1 mille watt of power right so you constraining the maximum power that the laser can provide you and then say to represent this +b I will use the full power ps in terms of field it becomes √ps this would be +j√ps it is -j√ps this is -j√ps sorry this is -p√ps that is all okay.

So this is -√ps so I have constant power or the maximum power constraint for this, so how do I represent this QPSK signal or how do I realize this QPSK signal using an optical modulator start with the laser which produces your carrier and then pout a phase modulator because this is a QPSK signal you can think of this QPSK signal is actually to BPSK signal okay and then have the output okay I have the sequence in which complex that is coming out here I can split this complex sequence.

Which you know that –bit this components cab actually been written in terms of the corresponding bit sequences well it is say 0001112 and 10this way is called as gray coding consultation so this I mean whenever you transmit this amplitude of +b you are actually transmitting the bit sequence 00 then you have 011 and 10 what you can observe is if you look at the MSB here.

The MSB is this fellow 0 and 1 they are in the opposite side right similarly for the other case also they are in the opposite side.

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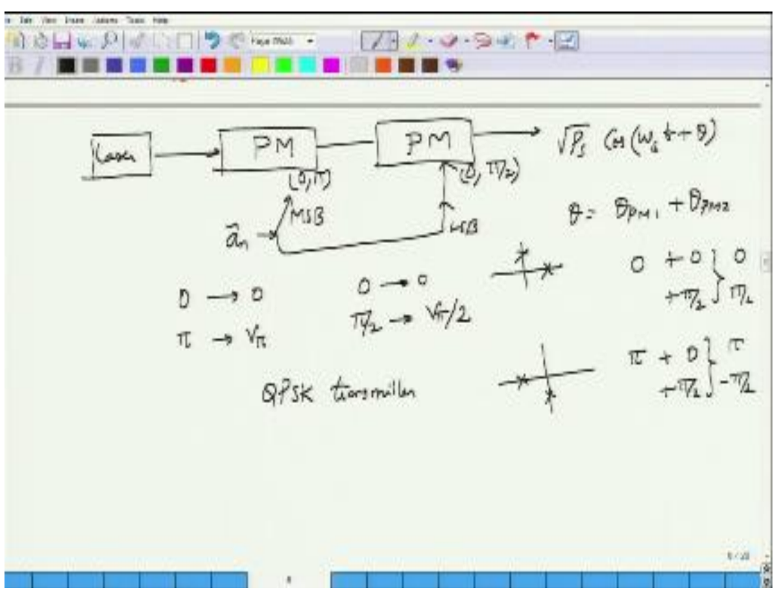

So you can actually employ that fact and whatever that complex symbol a and you are getting you can split this in to 2 portions okay 1 corresponding to the MSB and one corresponding to the LSB so this is the MSB portion and this is the LSB portion you feed this in such a way that the phase modulator is operated in 0 and $\pi/2$ whereas this one is operated as 0 and π okay.

To operate a phase modulator 0 you have to send in 0 rights to operate the modulator at π your amplitude should be $V\pi$ to operate 0 you have 0 to operate $\pi/2$ your amplitude should be $V \pi/2$ so the sequence that you get here the MSB sequence and the LSB sequence you can utilize this I mean you can scale up the sequence values by 0 $V \pi$ or 0 and $V \pi/2$ such that the overall signal what you get is $\sqrt{ps} \cos \Omega$ st $+\theta$ where θ is equal to θ of the first phase modulator and θ of the second phase modulator.

What possible combinations can I have for this well if the MSB bit is 0 then the first modulator produce a 0 the second produce a 0 and $\pi/2$ so this will allow mw a total phase of 0 and $\pi/2$ giving raise top the 2 constellation points okay next if θpm1 produce a π phase shift then I have 2 additional operations I mean 2 additional phases 0 and $\pi/2$ $\pi+ 0$ is π and this is 3 $\pi/2$ which is actually equivalent of $-\pi/2$ right.

I will get these two consultation points with 2 phase modulators I am able to get the all four points and therefore completing the QPSK transmitter, so this is one way of realizing QPSK transmitter we will see one more way of realizing QPSK transmitter in the next class so we will talk about that in the next module. Thank you very much.

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