Principles of Modern CDMA/MIMO/OFDM Wireless Communications
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture – 06
Bit Error Rate (BER) Performance

Hello everyone. Welcome to another module in this Massive Open Online Course on Principles of MIMO CDMA and OFDM Wireless Communications. And, today we are going to talk about the characterization of the performance of wireless communication systems.

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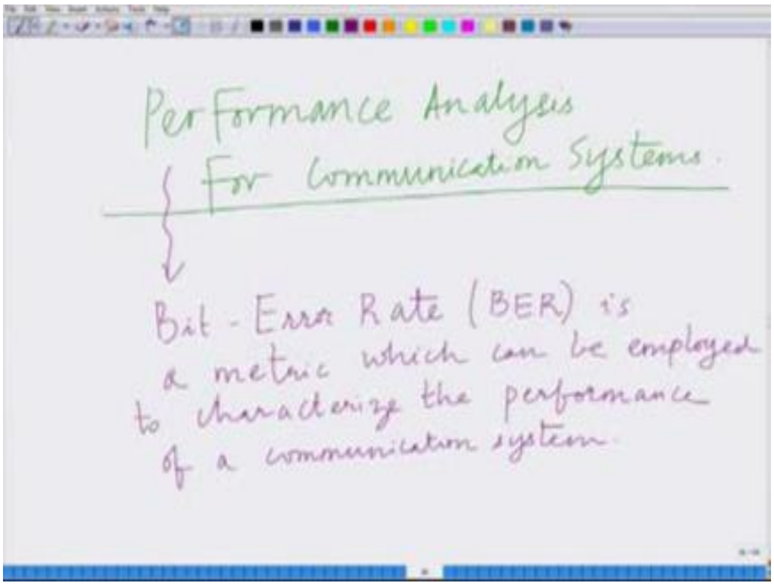

So, we talk about the performance analysis that is, how to characterize the performance analysis for communication systems and how to characterize the performance of a communication system. There are many matrixes to characterize the performance of communication system. But frequently one of the most convenient and one of the most informative matrixes is the Bit Error Rate of a communication system.

So, we are going to characterize the performance of a communication system in terms of the Bit Error Rate, and we are going to describe what this Bit Error Rate is shortly. So, we are going to employ the following metric. So, Bit Error Rate is often abbreviated as, BER is a metric is in fact; the convenient metric which can be employed to characterize the performance of a communication system. So, Bit Error Rate is the metric, which can be employed conveniently to characterize the performance of a communication system. So, what is this Bit Error Rate? Well, as you know in the communication system, we transmit information bits from the transmitter to the receiver and these information bits can be resulted as a digital stream of binary information symbols?

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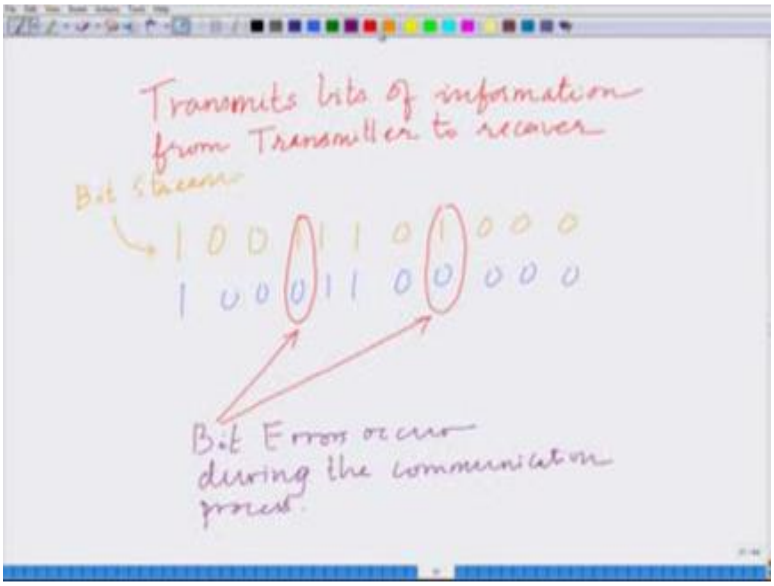

So, in a communication system 1 transmits information bits such as, for instance, 1 transmits bits of information from the transmitter to receiver these are binary information symbols for instance, 1 0 0 1 1 1 0 1 0 0 0 this is the possible stream of information bits, this is also known as the bit stream. This is also known as a bit stream which is transmitted from the transmitter to the receiver. However, all the transmitter bits are not received correctly by the receiver frequently there are errors during the reception of these bits.

So, there are bit errors for instance, corresponding to this transmit stream one might receive at the receiver the transmit stream might be decoded as 1 0 0 0 1 1 0 0 0 0 0. So, we can clearly see that, this bit 1 transmitted bit has changed to a 0 at the receiver and again this bit 1 here has changed to a 0, so there are bit errors. So, bit errors occur during the transmission some of the bits are received in error. So, these bits are in error and bit errors occurred during the communication, during this communication process and the Bit Error Rate is that the average rate at which these bit errors are occurred during the communication process is termed as, Bit Error Rate.

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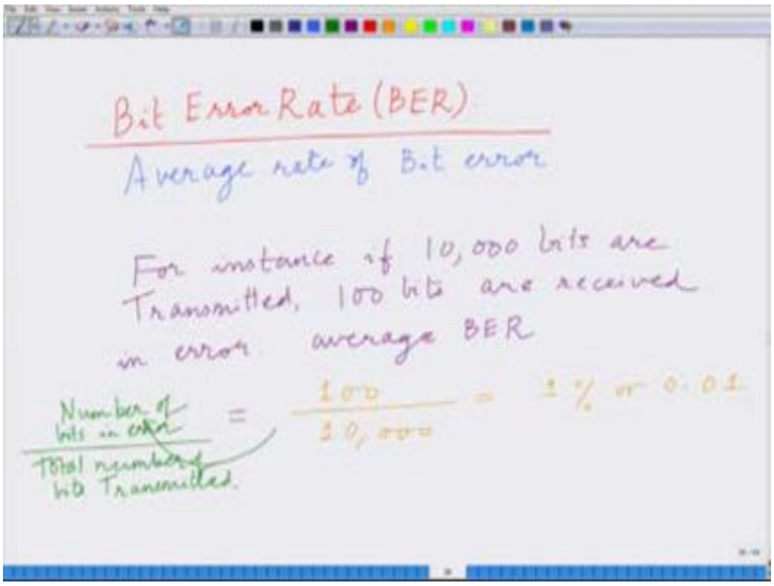

So, Bit Error Rate or BER is basically average rate, this is the average rate, average rate of bit error for a particular communication system. For instance, if 10000 bits are transmitted if between the transmitter and receiver for instance; if 10000 bits are transmitted and out of which 100 bits are received in error then we say the average Bit Error Rate, average Bit Error Rate equals the number of bits in error that is; 100 divided by the total number of bits transmitted.

Therefore, this is equal to 1 percent that is;

$$\frac{100}{10000} = 1\%$$

or we say a Bit Error Rate of 0.01, we say that out of 10000 bits if 100 bits are received in error, we say that the average Bit Error Rate is the number of bits in error divided by the total number of bits transmitted that is in this case example; it is 100 divided by 10000 which is a Bit Error Rate of 10 of 1 percent or 0.01.

So, this is the Bit Error Rate is number of bits in error divided by total number of bits transmitted which is in this case, 1 percent or 0.01 and also frequently. Since these bits, the transmitted bits over the channel and the received bits are random quantities, the Bit Error Rate can also be expressed as a probability. Since, these are random quantities the transmitted bits the Bit Error Rate is frequently expressed as a probability this is known as the probability of bit error.

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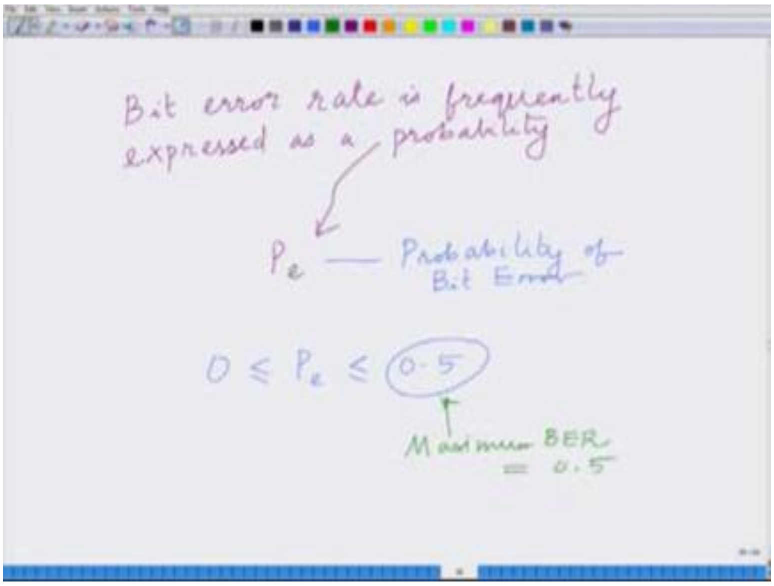

So, the Bit Error Rate is frequently, as a probability that is, it is denoted by the quantity Pe and this quantity Pe is termed as the probability this quantity Pe is termed as, the probability of bit error, this is the probability with which a received bit is going to be in error. And normally, probability is between 0 and 1, but for the probability of bit error this is between 0 and 0.5. We are going to see this shortly the probability of bit error always lies between 0 and 0.5, that is the maximum possible Bit Error Rate that is maximum possible BER is 0.5.

So, error probability of bit error is the probability with which a received bit is in error and this quantity lies between 0 and 0.5.

$$0 \leq P_e \leq 0.5$$

And also another important aspect to keep in mind when analyse the performance as a communication system, there is although we think of a communication system as transmitting a stream of information bits these digital information bits that is 0's and 1's are not directly transmitted over the channel, but rather these information bits are modulated prior to transmission over the channel.

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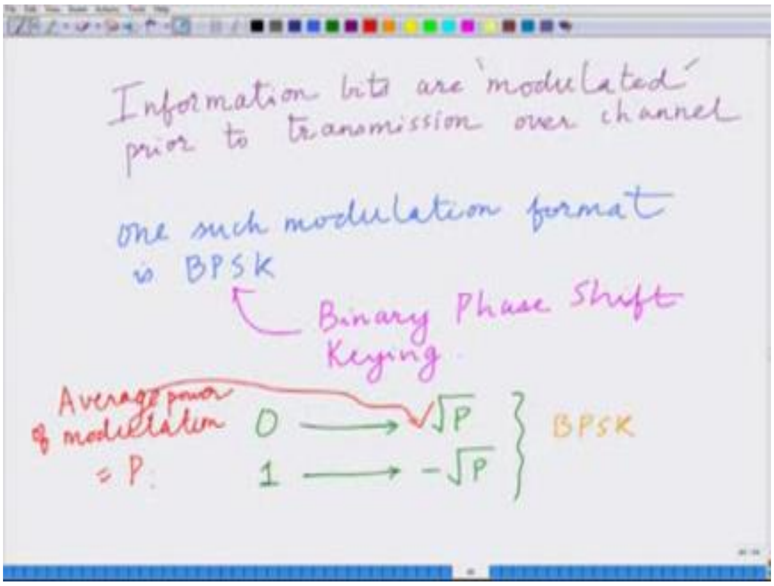

So, the information bits are, modulated I mean these are modulated prior to the these information, bits are modulated prior to the transmission channel over the channel, and one such modulation format or one such digital modulation format because we are going to talk about digital communication system is BPSK. And BPSK stands for as many of you might be familiar, BPSK stands for binary phase shift keying in which the information symbol 0 is modulated as the amplitude level $\sqrt{P}$. The information symbol 1 is modulated as an amplitude level $-\sqrt{P}$. So, we have 2 voltage levels $\sqrt{P}$ and $-\sqrt{P}$. So, there are 2 phases the phase of $\sqrt{P}$is 0 and the square phase of $-\sqrt{P}$ is 180 degrees.

So, as you can see there are 2 phases in this modulation scheme therefore, it is known as binary phase shift key transmission. This is a digital modulation format and this is known as this is basically, known as BPSK, modulation format and we are considering the value P that is $\sqrt{P}$ and $-\sqrt{P}$.

Since, we are setting the average power to P. Therefore, the average power of this modulation format, so since we are transmitting the amplitude $\sqrt{P}$ or $-\sqrt{P}$, the average power is the square of the amplitude and therefore, average power of this transmission format is P with BPSK modulated symbols of $\sqrt{P}$ and $-\sqrt{P}$ and these amplitudes these modulated symbols are subsequently transmitted over the channel and at the receiver, since, we receive a continuous voltage level corresponding to the transmission of $\sqrt{P}$ and $-\sqrt{P}$ we have to design appropriate detector at the receiver and 1 such simple detector is the following thing looking at the symmetry in the transmitted symbols, if they receive symbol is greater than 0 then the received symbol is greater than 0; then we map it to 0 because, we say it correspond to a positive voltage level if the received symbol is less than 0; that means, it corresponds to a negative voltage level then we map it to 1.

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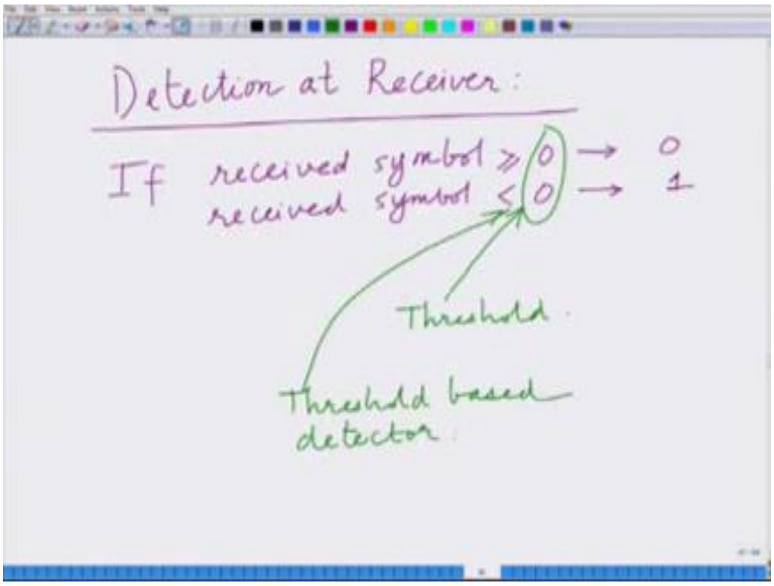

So, a simple detector at the receiver, if received symbol is greater than or equal to 0, then we map it to the binary information symbol 0. If on the other hand, received symbol is less than 0 we map it to the binary information symbol 1 that is; 0 and 1 are the binary information symbol. If received symbol is greater than or equal to 0 we will map it to 0, it is less than 0 we will map it to 1. Therefore, this is also 0 can be also thought of as a threshold this 0 is basically the threshold of this detector and this is a threshold based detection process.

This is a threshold based simple detector for a binary phase shift keying base digital communication system and this is also known as threshold base detector, that is a received voltage level is greater than or equal to 0 we map it to the binary information symbol 0, if it is less than 0 then we map it to the binary information symbol 1 and now we have to consider the various channel models. So, let us start by considering the performance of a simple digital communication system without fading that is a simple communication system without any multi path propagation that is a communication system with a wire line or a wire between the transmitter and the receiver.

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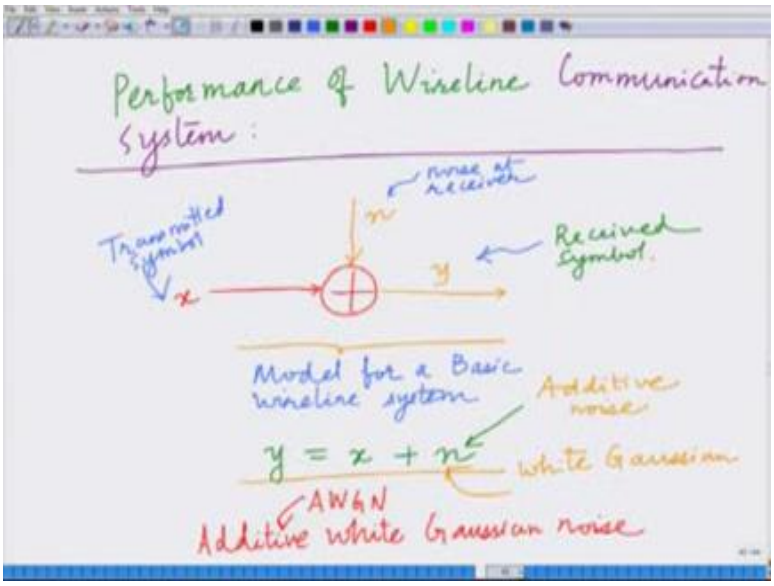

So, let us start by modelling the performance of a simple wire line communication system without any fading. So, let us start by modelling the performance of a wire line and to model the performance of a wire line communication system I can consider a simple system with a transmitter symbol x. This symbol is the transmitted symbol at the receiver, of course there is going to be the corrupt corruptive influence of noise. So, I have the edition of the noise at the receiver and I have the received symbol which is y, which is the output of the received symbol. We are considering a wire line communication system and this system can be modelled.

So, what we are developing is, this is the simple model for a wire line communication system, which is x is the transmitted symbol, n is the noise at the receiver and y is the received symbol y is the received symbol. So, we can express this system as

$$y = x + n$$

So, this is the simple model for this communication system where y is the received symbol which is equal to x which is the transmitted symbol plus n which is the noise at the receiver. This noise is the first thing you can see this noise is additive in nature this noise is additive noise.

Further, if this noise is white Gaussian, the noise process is a white Gaussian noise process, then this system is known as an Additive white Gaussian noise channel. Then this system is known as, AWGN channel which represents Additive white Gaussian noise. So, the noise is additive in nature if in addition to that the noise process is white Gaussian that, it has a Gaussian probability density of function then this noise is known as Additive white Gaussian noise.

And the system model is known as an Additive white Gaussian noise channel, which is abbreviated as AWGN. This is the abbreviation that is frequently employed this is an AWGN channel and further, we consider this noise to have a Gaussian probability density of function remember, Gaussian probability density function is given as the Gaussian PDF.

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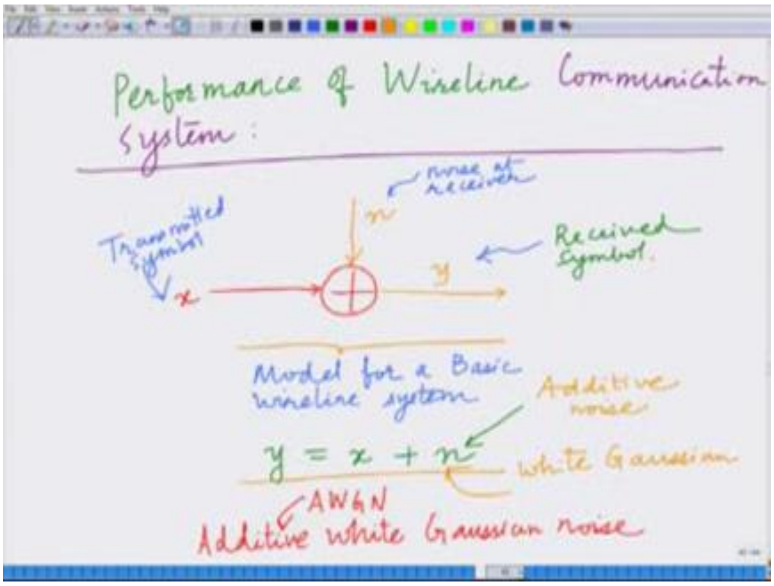

That is the noise PDF which I can denote by $F_N(n)$ is given as

$$F_N(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{n^2}{2\sigma^2}}$$

where we are assuming this noise is Gaussian which means 0, variance σ^2. So, we are assuming noise ~~to be~~ have mean equal 0 and variance, that is the noise variance equal to σ^2 or in other words, it also says that the noise power is σ^2. Therefore, we are considering Gaussian noise which has a Gaussian probability density function with means 0 and variance σ^2 that is noise power is σ^2.

Therefore, the probability density function of the noise is

$$F_N(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{n^2}{2\sigma^2}}$$

This is the Gaussian probability density function and to refresh your memory we have already seen the shape of Gaussian probability density function it is a bell shaped curve which is given as follows. This is the mean, which in this case is 0 and the spread of this Gaussian is related to the variance that is it is proportional to the variance that is σ^2.

As the variance σ^2 increases the spread of the Gaussian curve increases as variance increases. Let me write that down as variance increases the spread of this Gaussian also increases, correct. And therefore, now we are considering such a channel which is an additive white Gaussian noise channel where the noise PDF following this Gaussian a distribution. And now what we would like to do is, we would like to consider the transmission of information symbols over this Additive white Gaussian noise channel and analyse the Bit Error Rate performance corresponding to the transmission of this BPSK modulated symbols over this additive white Gaussian noise channel.

So, I think this is a good point what we have done, so far is we have considered we developed a model for the transmission, a model for the modulation, that is BPSK modulation and model for the channel that is our Additive white Gaussian channel and in the next module. We are going to analyse this communication system to develop expressions for the bit error performance of this communication system.

Thank you very much.