Principles of Modern CDMA/MIMO/OFDM Wireless Communications
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture - 05
Rayleigh Fading Channel

Welcome to another module in this MOOC on Wireless Communication. Let us now continue with our discussion on that the probability density function or the modelling of the fading channel coefficient h.

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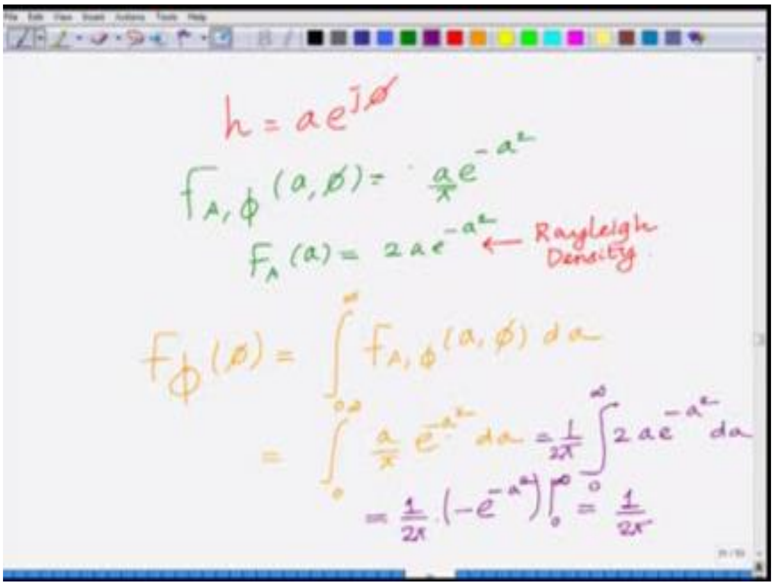

We said that, if the fading channel coefficient $h = a\, e^{j\phi}$. Then the joint distribution is given as :

$$F_{A,\phi}(a,\phi) = \frac{a}{\pi} e^{-a^2}$$

We had already found the marginal density with respect to the channel coefficient A; that is

$$F_A(a) = 2a\, e^{-a^2}$$

This we said is the Rayleigh distribution or the Rayleigh density.

Let us now find the distribution of the phase; that is

$F_{\phi}(\phi)$, the phase component and for this; I can integrate the joint density with respect to the amplitude a and the amplitude lies between 0 to ∞, because the amplitude is always positive.

$$F_{\phi}(\phi) = \int_0^\infty F_{A,\phi}(a,\phi)\, da$$

Now, if you look at this integral;

$$= \int_0^\infty \frac{a}{\pi} e^{-a^2}\, da$$

$$= \frac{1}{2\pi} \int_0^\infty 2ae^{-a^2}\, da$$

$$= \frac{1}{2\pi} \left(-e^{-a^2}\right)\Big|_0^\infty$$

$$= \frac{1}{2\pi}$$

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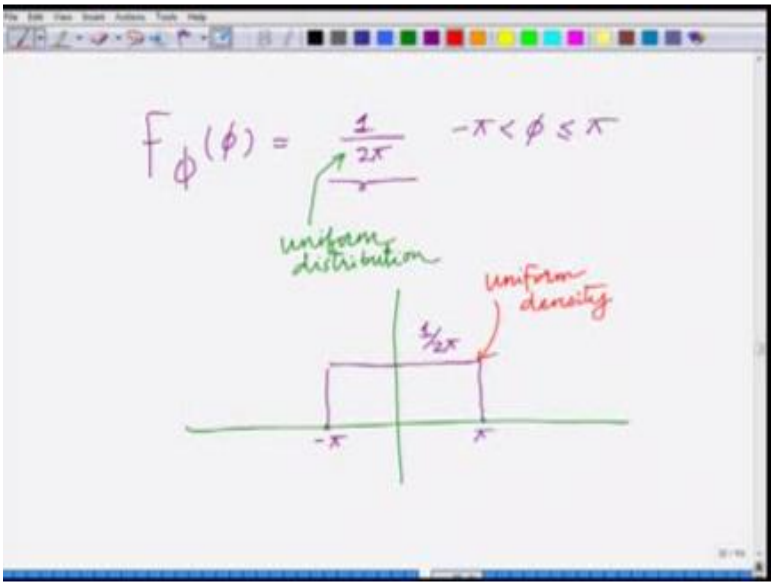

Therefore, the distribution of the phase we have

$F_ø(ø) = \frac{1}{2\pi}$ $\quad -\pi < ø \leq \pi$

Which means this is a constant and this is the uniform density. If you look this is the uniform distribution. Therefore, if you have the limits $-\pi < ø \leq \pi$; this as the height $\frac{1}{2\pi}$. And this as we said, is the uniform probability. The phase is distributed uniformly between the limits $(-\pi, \pi]$.

This is a uniform distribution and therefore we have derived both the marginal density; that is the distribution of both the amplitude of the Rayleigh fading channel coefficient.

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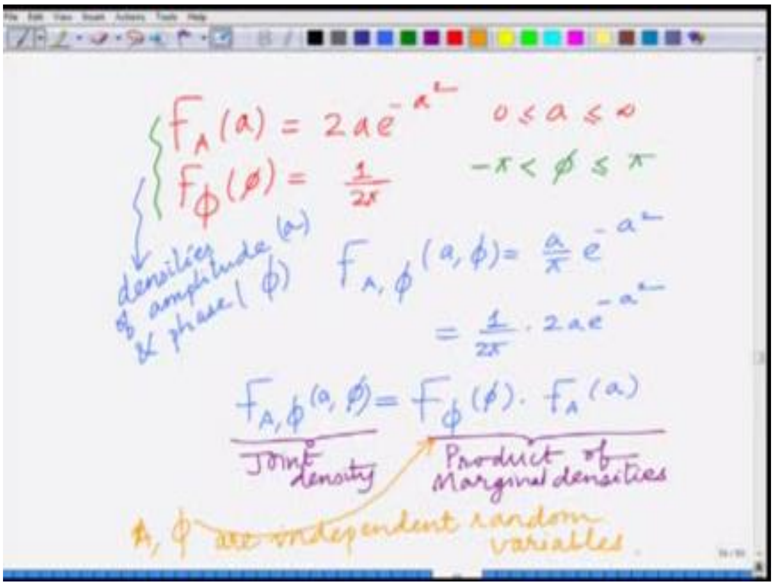

And the phase of the Rayleigh fading channel coefficient and therefore,

$F_A(a) = 2a\, e^{-a^2}$ $\quad 0 \leq a \leq \infty$

$F_ø(ø) = \frac{1}{2\pi}$ $\quad -\pi < ø \leq \pi$

And therefore, now we have these are the distributions, the densities of the amplitude and the phase; that is the amplitude a and phase ø of the wireless channel. These can now be used to characterized, derive various properties of the wireless channel.

Further, before we proceed further. Let us look at one other interesting point. If we look at the joint distribution of the amplitude and the phase; remember the joint distribution is given as

$$F_{A,\o}(a,\o) = \frac{a}{\pi} e^{-a^2}$$

$$= \frac{1}{2\pi} . 2a\, e^{-a^2}$$

$$F_{A,\o}(a,\o) = F_{\o}(\o) . F_A(a)$$

What we have is that the joint density with respect to the phase and amplitude is equal to product of the marginal densities with respect to the amplitude and the phase. The joint density is equal to the product of the marginal densities therefore; this implies that the amplitude and the phase are independent random variable. The amplitude and phase of the Rayleigh fading channels are independent random variable. This means that, A , ø the amplitude and phase are independent. And these densities can be used to derive valuable properties of the fading channel for instance.

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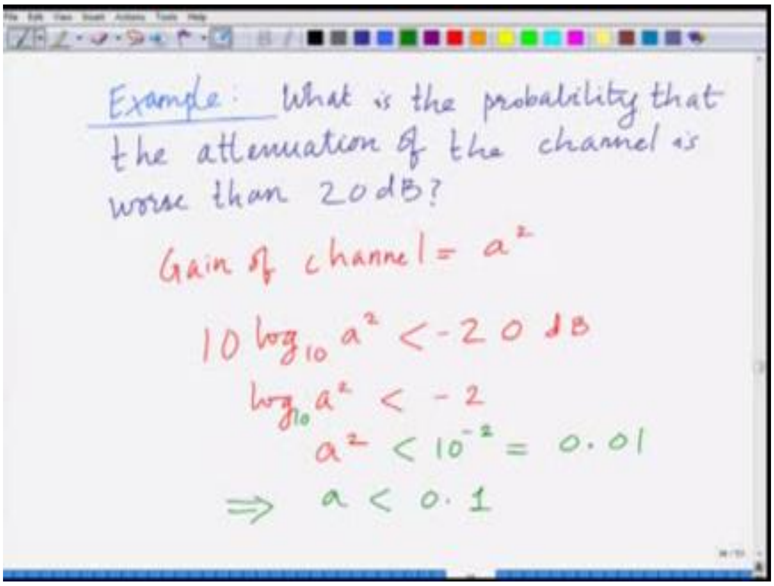

Let us look at the following example to understand this better. What is the probability? That attenuation of the channel is worse than 20 dB.

Let us ask the following question. What is the probability that the attenuation of the channel is worse than 20 dB? Let us look at the following example. What is the probability that the attenuation of channel is worse than 20 dB? What we are asking is; what is the probability? Let us look at the gain attenuation of the wireless channel or the gain of the wireless channel.

Gain of channel = a^2

What we are asking, if the attenuation is worse than 20 dB. It means

$$10 \log_{10} a^2 < -20 \text{ dB}$$

Attenuation is 20 dB implies the gain of the received signal, the power gain of the received signal minus 20 dB or lower.

Attenuation is worse than 20 dB, which means

$$\log_{10} a^2 < -2$$

$$a^2 < 0.01$$

$$a < 0.1$$

The gain of the channel is attenuation is worse than minus 20 dB if the amplitude of the channel is less than 0.1.

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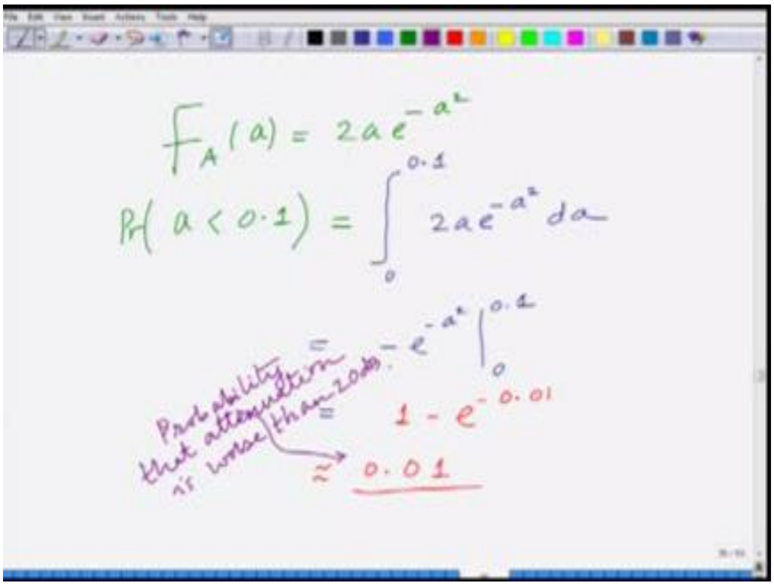

And now, we know that the distribution of the amplitude is

$$F_A(a) = 2a\, e^{-a^2}$$

Therefore; the probability the amplitude is less than 0.1 is basically equal to the density integrated between the limits 0 to 0.1. This is

$$P(a < 0.1) = \int_0^{0.1} 2ae^{-a^2}\, da$$

$$= -e^{-a^2}\Big|_0^{0.1}$$

$$= 1 - e^{-0.01}$$

$$\sim 0.01$$

The probability, that attenuation or probability that attenuation of the channel is worse than 20 dB is 0.01. This is the probability. What is this? This quantity that we calculated here, this is the probability that that attenuation of channel wireless channel worse than 20 dB that is 0.01. As we have seen this probability densities or this model, that we have developed for the fading channel coefficient, that is joint distribution of its amplitude and phase. That is individual distribution of the amplitude and phase components are very important tools. They help us characterized the channel statistically and derive valuable inferences, derive valuable statistical properties of the channel from these distributions that amplitude and phase components.

And hence, these are also going to be important, when we look at characterizing the performance of the wireless channel in various scenarios. This concludes this module and we will continue in the subsequent module.

Thank you very much.