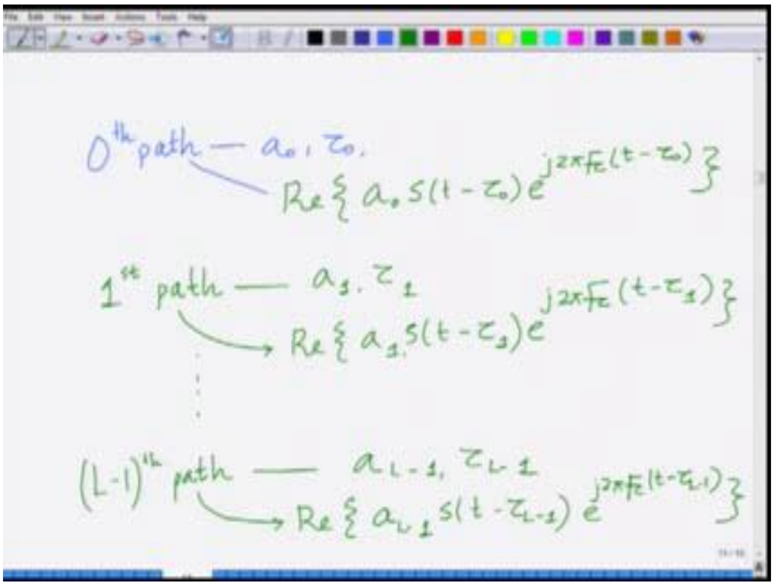
Principles of Modern CDMA/MIMO/OFDM Wireless Communications
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

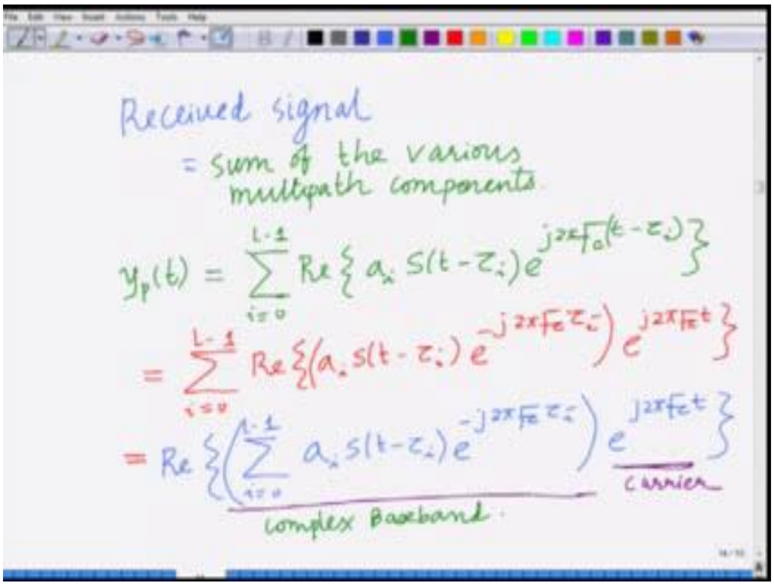
Lecture – 03
Wireless Fading Channel Model

Hello, welcome to another module in this massive open online course on the principles of MIMO, OFDM and CDMA wireless communication systems and we had seen in the previous module there are different multi-path components in the wireless communication channel and therefore, for corresponding to the i-th part the received pass band signal component is $\mathrm{Re}\{ a_i S(t-\tau_i)\, e^{j2\pi f_c(t-\tau_i)}\}$.

(Refer Slide Time: 00:50)



(Refer Slide Time: 00:56)



So, Now, therefore, the superposed signal therefore, the received signal is the sum of these multi-path components, received pass band signal equals the sum of the various multi-path components and therefore what we have is we have

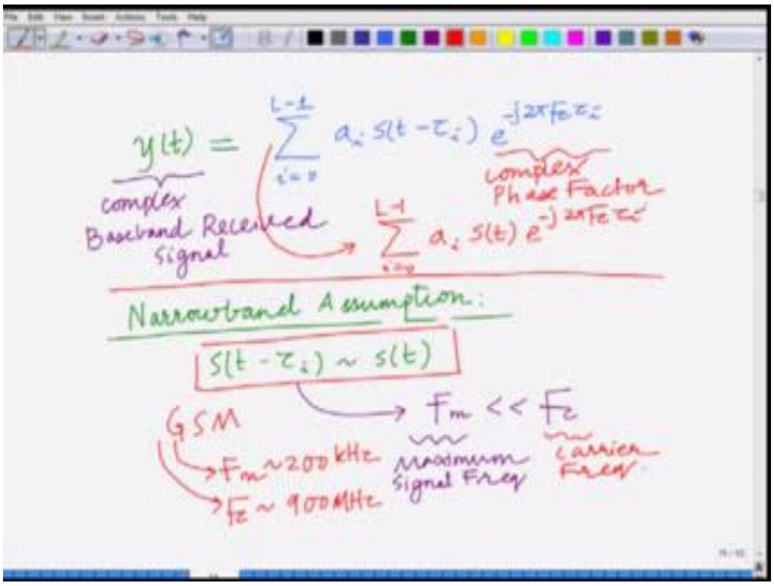
$$Y_p(t) = \sum_{i=0}^{L-1} Re\{ a_i S(t - \tau_i)\, e^{j2\pi f_c (t - \tau_i)}\}$$

$$= \sum_{i=0}^{L-1} Re\{ (a_i S(t - \tau_i)\, e^{-j2\pi f_c \tau_i})\, e^{j2\pi f_c t})\}$$

$$= Re\{ (\sum_{i=0}^{L-1} a_i\, \delta(\tau - \tau_i)\, e^{-j2\pi f_c \tau_i})\, e^{j2\pi f_c t}\}$$

And now if you look at this received pass band signal you can see that this is the carrier component or modulation by the carrier and this is the complex base band part. So, this is the complex, the part that I am indicating over here which is the sum of that attenuated and delayed baseband signal this is the complex received baseband signal.

(Refer Slide Time: 04:05)



And therefore, my baseband received signal if I remove the demodulate the carrier baseband received signal is given as,

$$Y(t) = \sum_{i=0}^{L-1} a_i\, \delta(\tau - \tau_i)\, e^{-j2\pi f_c \tau_i}$$

Therefore, this delay τ_i is giving rise to this complex phase factor you can see this delay τ_i the i-th delay is giving rise to the complex phase factor this is giving rise to the complex phase factor.

What we are going to do now is to further simplify the slightly what we are going to do is we are going to employ what is known as the narrow band assumptions. So, we employ the narrow band assumption for the transmitted signal which says that

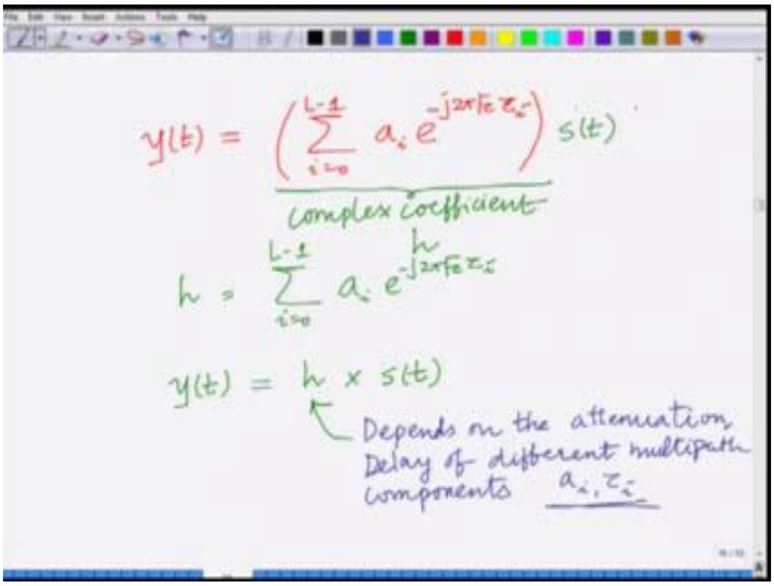
$$s(t - \tau_i) \sim s(t)$$

We can employ this assumption when we have a narrow band signal, what does it mean to say a narrow band signal? Narrow band signal means f_m is significantly less than the carrier frequency that is a maximum frequency of the signal is less than or significantly less than the carrier frequency.

When we have this maximum signal frequency f_m significantly less than the carrier frequency f_c, then we can employ the narrowband assumption for instance when we look at a GSM signal, the GSM signal has a bandwidth of around the maximum frequency is around 200 kilo hertz which is well less than one megahertz, but the carrier frequency fc is 900 megahertz for GSM you have f_m is approximately equal to 200 kilo hertz while fc is approximately equal to 900 megahertz the carrier frequency is approximately around the order of 100 megahertz therefore, the maximum frequency of the signal is much lesser or much lower than the carrier frequency and therefore, one can employ the narrowband assumption in this case where we are approximating this $s(t - \tau_i)$ that is the delayed baseband signal $s(t - \tau_i) \sim s(t)$.

Now, if we employ this assumption that $s(t - \tau_i) \sim s(t)$. Since $s(t - \tau_i) \sim s(t)$, this reduces to $a_i\, S(t)\, e^{-j2\pi f_c \tau_i}$.

(Refer Slide Time: 08:12)



Which means employing the narrowband assumption I have

$$Y(t) = \left(\sum_{i=0}^{L-1} a_i\, e^{-j2\pi f_c \tau_i}\right) s(t)$$

since $S(t)$ can be taken common from this expression and therefore, now you can see this is a complex coefficient let us denote this by H. So, H, I am defining this as,

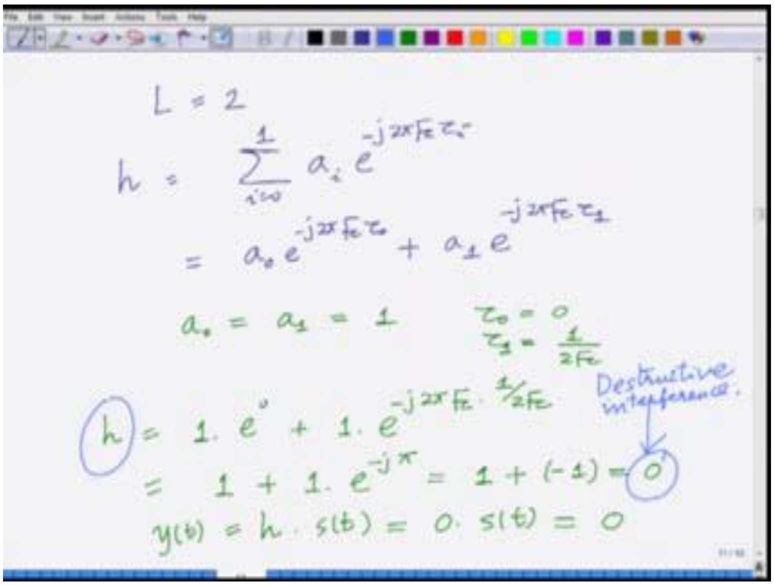
$$h = \sum_{i=0}^{L-1} a_i\, e^{-j2\pi f_c \tau_i}$$

this is my complex coefficient and as a result of this I can conveniently write

$Y(t) = h \times s(t)$

So, this is my complex coefficient. So, $Y(t) = h \times s(t)$ where this complex coefficient depends on the channel, it depends on more precisely the attenuation, delay of different multi-path, different multipath components of the channel precisely depends on all the a_i's and τ_i's. So, this depending on the different multi path components the attenuation factors a_i's and the delays τ_i's of these different multi path channel compliments all right. So, this complex coefficient h depends upon these different channel factors.

(Refer Slide Time: 10:40)



So, now, let us try to understand this channel coefficient better. Let us consider a simple scenario, for $L = 2$, we have h is given as,

$$h = \sum_{i=0}^{1} a_i \; e^{-j2\pi f_c \tau_i}$$

now, what we have is we have the sum of 2 multipath components that is

$$= a_0 e^{-j2\pi f_c \tau_0} + a_1 e^{-j2\pi f_c \tau_1}$$

Now, let us consider a scenario in which $a_0 = a_1 = 1$, let us considered a simple scenario with $a_0 = a_1 = 1$ and $\tau_0 = 1$ and $\tau_1 = \frac{1}{2fc}$. In this scenario what I have is

$h = 1.e^{0} + 1.e^{-j2\pi f_c \frac{1}{2fc}}$

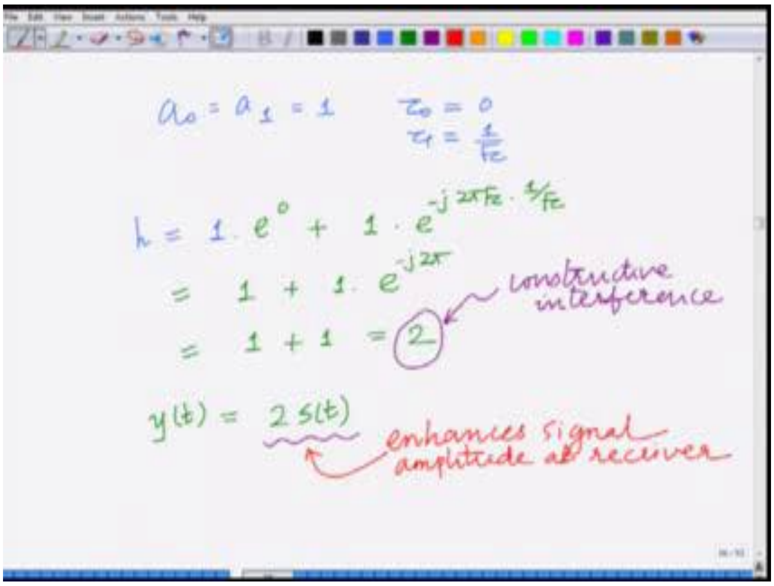
$= 1 + e^{-j\pi}$

$= 1 + (-1)$

$= 0$

So, what we have is h the channel coefficient reduces to 0 and you can see that because both the components have equal amplitude 1 and they have a phase that is exactly the opposite of each other 1 is the phase of 0 the other has a phase of pie and therefore, these components are cancelling each other as a result of these multipath components cancelling each other the channel coefficient h is 0 and therefore, the received signal y, Y(t) which is h times S(t) which is equal to 0 times S(t) equals 0 therefore, the received signal is perfectly cancelled and this is a case of this is an example of destructive interference with these 2 multipath components are perfectly cancelling each other this is a case of destructive interference in which case the received signal is 0. So, in this case this h is 0 this shows destructive this is destructive interference.

(Refer Slide Time: 13:49)



Now, further consider another scenario where again

$a_0 = a_1 = 1$ and $\tau_0 = 0$ and $\tau_1 = \frac{1}{fc}$.

In this case you can see,

$$h = 1.\, e^{0} + 1.\, e^{-j2\pi f_c \frac{1}{fc}}$$

$$= 1 + e^{-2j\pi}$$

$$= 1 + 1$$

$$= 2$$

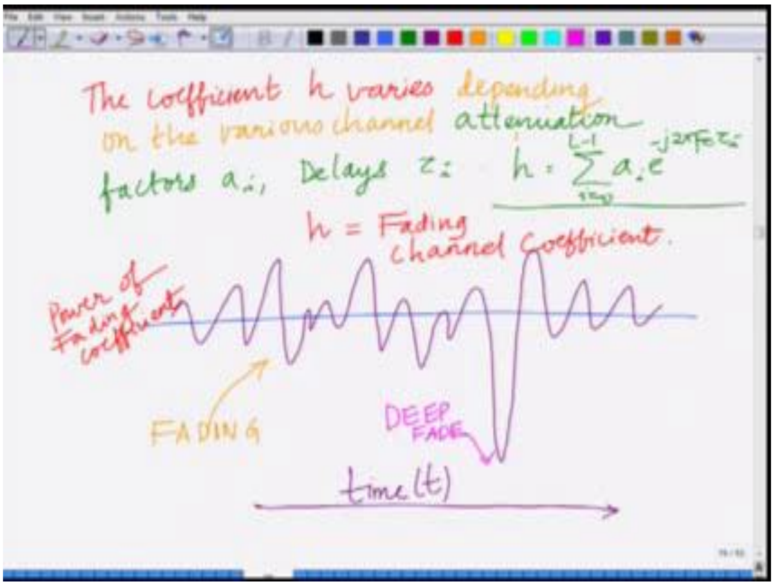
Now you can see since h is too we have

Y(t) = 2. S(t)

That is twice the transmitter signal S(t) and now you can see because these 2 signals are adding up constructively. So, they are adding up to each other constructively. So, this is constructive interference and as a result of this you see enhanced signal amplitude. So, this is basically enhanced signal amplitude. So, this enhances signal amplitude.

So, in these different multipath component they cancel each other then you have destructive interference or with these signal these 2 multiple components are constructively it leads to constructive interference and that enhances the signal amplitude at the receiver and therefore, this coefficient h depends on the attenuation a_i and the delays τ_i it varies between it can go all the way down to 0 with these different signal components are cancelling each other and it can when different signal components add up, it can have a significantly higher magnitude and then it varies between these different magnitude.

(Refer Slide Time: 16:09)



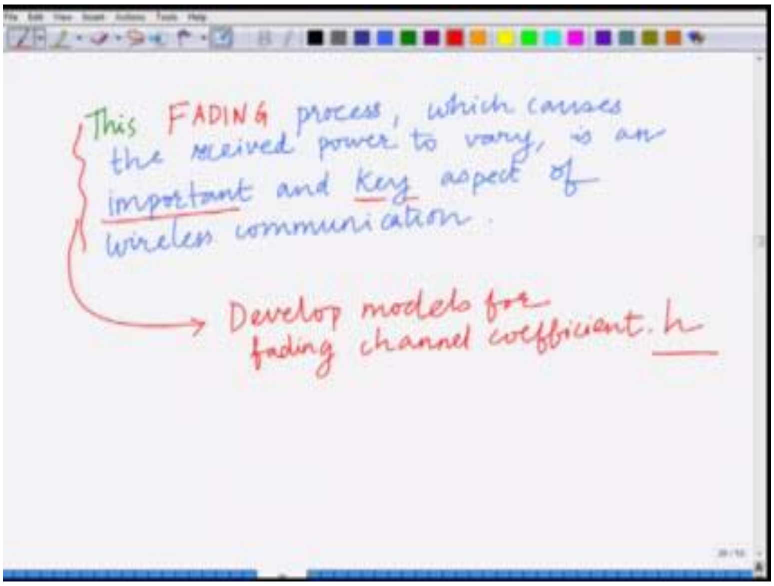
So, the coefficient h varies depending on the various channel amplitude or various channel attenuation factors a_i and the delays τ_i alright and therefore, as these attenuations and delays are changing the channel h, which if you remember the h the expression for h is

$$h = \sum_{i=0}^{1} a_i \; e^{-j2\pi f_c \tau_i}$$

as these various delays a_i and the attenuations τ_i are changing. This channel coefficient h is also changing and it can go all the way from 0 to have a very high magnitude and therefore, if you plot this channel coefficient will have a magnitude that is changing with time that is if you are looking at the time and if you are looking at the average the power of the fading coefficient the power of the magnitude square power is. So, this fading coefficient is changing with time and this changing this process that is why this complex channel coefficient, because magnitude is varying with time is changing with time, is known as the fading channel coefficient.

This process is known as a fading process and this is known as the h is known as the fading and this is an important aspect of wireless communication systems this is known as the fading channel coefficient. So, this process is known as fading and this is known as h is known as the fading channel coefficient. And of course, as you can see here sometimes when the paths perfectly cancel each other the signal amplitude or the received signal amplitude goes all the way can go all the way up to 0 go to very low amplitude as it is represented by this stuff here in the power of the fading coefficient this is represented this particular point is known as a deep fade event. And this has a significant impact on the performance of a wireless communication system we're going to look at all these aspects in subsequent modules as we proceed further and we will see that this such deep fade events have a profound impact on the performance of the wireless communication system. Therefore, it is the fading therefore, this fading process which results from the multipath wireless communication environment.

(Refer Slide Time: 19:48)



So, this fading process is basically which causes the receive power to vary is important and key aspect of a wireless communication system.

So, this is a very important aspect and has a significant impact it significantly impact the nature of wireless communication, because as the channel coefficient is varying the received signal power is varying and as the received signal power is varying the quality of the received signal at the receiver is changing. And this is unlike and this is arising remember because of the multipath propagation environment in which these different signal component with different delays and different phases are super imposing and this is leading to constructive and destructive interference. And this channel coefficient basically captures this constructive and destructive interference properties of the wireless communication channel and this is significantly different from the wire line channel where there is no multi-part propagation therefore, this fading coefficient has a significant impact on the nature of wireless communication and it has a key role to play and we are going to spend a significant amount of time trying to understand its properties, try to understand its impact and trying to understand how are the different technologies, how are the different solution, what are the different solutions that have been proposed to overcome the adverse impacts of this fading wireless communication fading wireless channel coefficient which is a key feature of wireless communication systems in general.

So, with the subsequent modules we are going to develop models for this fading channel coefficient. So, in subsequent modules what we are going to do is we are going to develop models for this. We are going to develop models for this fading channel coefficient h and based on this model for this fading channel coefficient or in other words the model for the wireless channel; we are going to understand the various properties and performance of the wireless communication systems.

So, we will stop this module here and we will continue in subsequent modules.

Thank you very much.