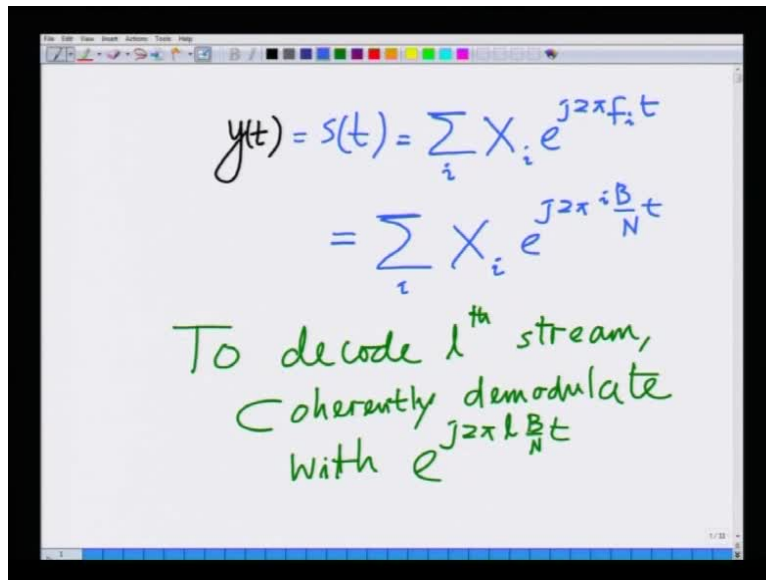


Advanced 3G and 4G Wireless Communication
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Lecture - 29
OFDM Schematic and Cyclic Prefix

Hello, welcome to another lecture in the course on 3G 4G wireless communication systems.

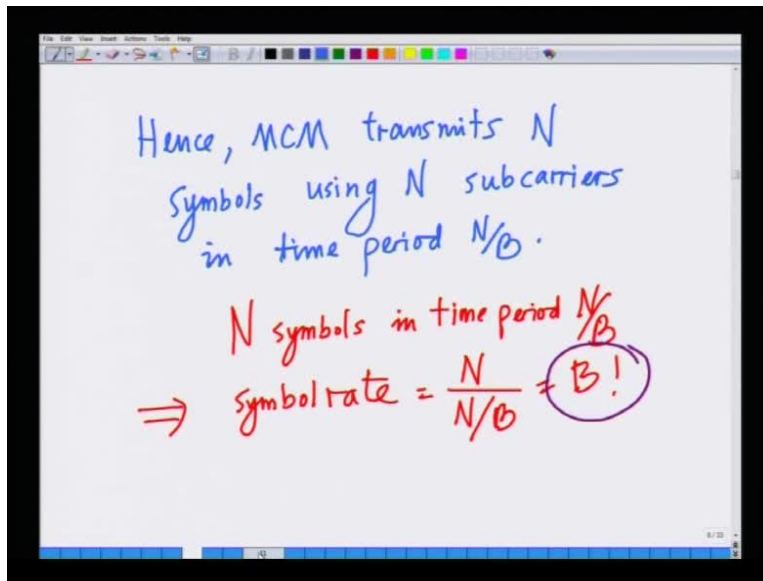
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The image shows a handwritten derivation on a whiteboard. The first line is $y(t) = s(t) = \sum_i X_i e^{j2\pi f_i t}$. The second line is $= \sum_i X_i e^{j2\pi i \frac{B}{N} t}$. Below this, in green ink, it says: "To decode l^{th} stream, Coherently demodulate with $e^{j2\pi l \frac{B}{N} t}$ ".

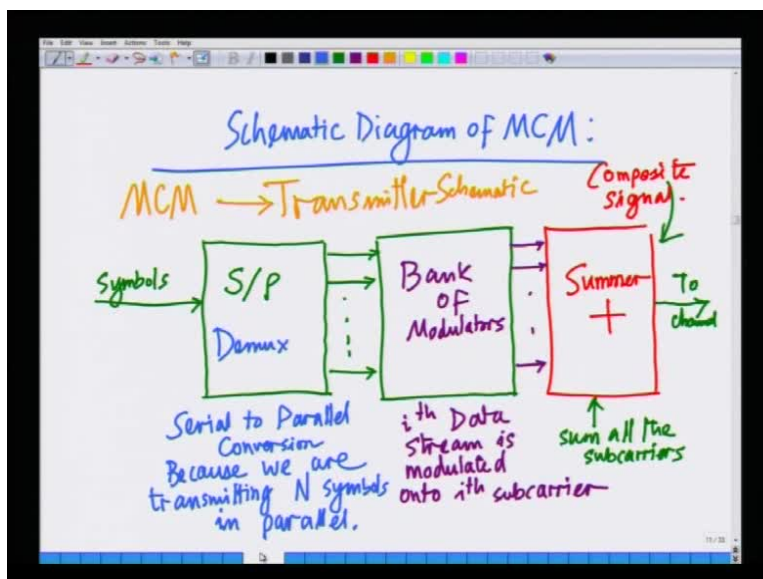
In the last lecture, we had completed our, we were discussing multicarrier modulation. And we said that after modulating the transmission simplex or multiple subcarriers and transmitting this composite signal, so the communication channel; we said that each subcarrier or the data symbols on each subcarrier can be coherently demodulated by employing the subcarriers $e^{j2\pi l \frac{B}{N} t}$. That is I am demodulating the symbol that is transmitting on the l^{th} subcarrier using coherent demodulation or demodulation using the l^{th} subcarrier.

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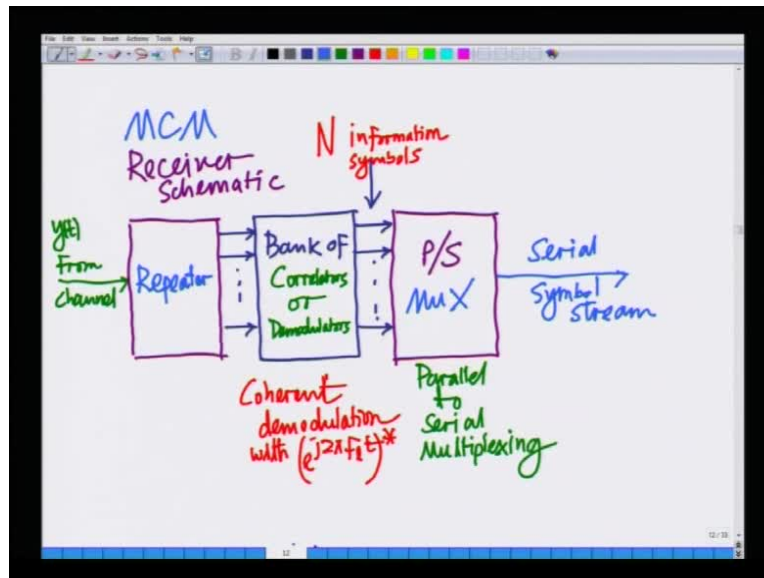


Further, we said that multicarrier modulation is transmitting N times N information symbols in parallel over a time period of N over B , remember the time period also increased to N over B , hence the net it transmit N symbols in N over B , which essentially means it is transmitting one information symbol in time period 1 over B giving it a data rate of B symbols per second, which essentially the same as that of a single carrier system, that is what we said last time.

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We also looked at the schematic of this multicarrier system modulation system, that is a schematic at the transmitter, and we also looked at the schematic of the receiver over the architecture schematic of the receiver. Further we also said she is both the multicarrier modulation system, and the single carrier system and transmitting symbols at the rate of B.

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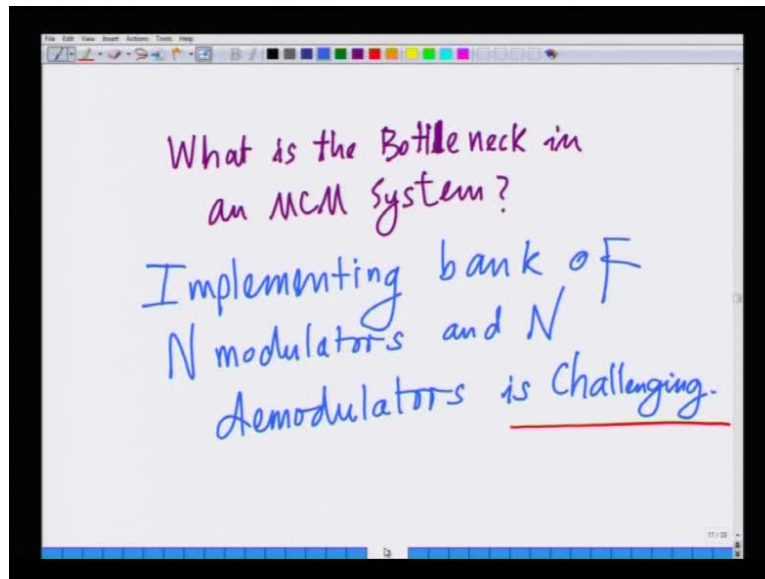
$$\begin{aligned} B &= 1024 \text{ KHz} \\ N &= 256 \text{ subcarriers} \\ \text{Bandwidth of subcarrier} \\ &= B/N = \frac{1024}{256} = 4 \text{ KHz} \\ 4 \text{ KHz} &\ll B_c \sim 200-300 \text{ KHz} \end{aligned}$$

The advantage of a multicarrier system is essentially by dividing this wide band into multiple subcarrier of narrowband, we are insuring that this system is does not subject is not subject to enter symbol interference, because this bandwidth of each subcarrier is much less than

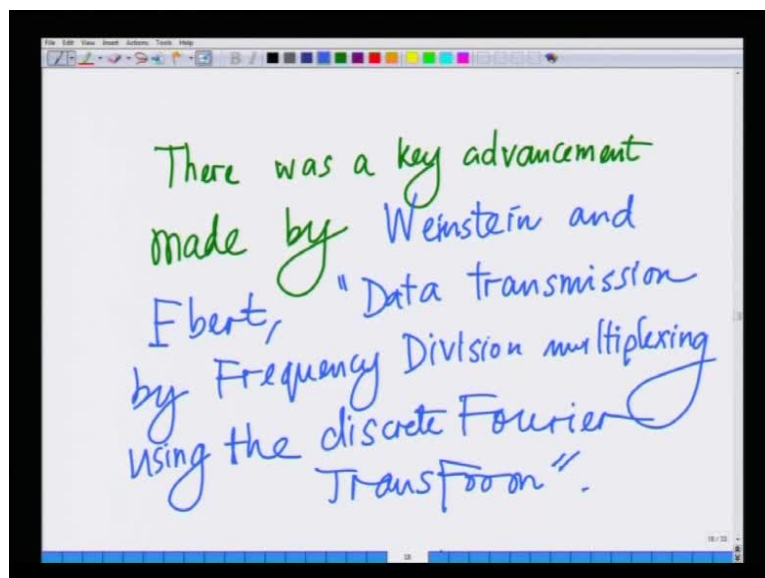
coherent band coherence band thus by insuring that the band width of the subcarrier is much less than the coherence band width.

We are essentially removing the effect of inter symbol interference, which is a greatly simplifies receiver design in wireless, especially broadband wireless communication systems.

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And further we also saw there is there is one problem with multicarrier modulation, which is essentially that it requires N modulators over the subcarrier and N demodulators, which was

essentially solved by in engineers idea by Weinstein and Ebert, who proposed a novel transmission scheme, which were discussing which is related to considering the FFT.

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Handwritten notes on a whiteboard:

Composite MCM signal $s(t) = \sum_i X_i e^{j2\pi i \frac{B}{N} t}$

Consider the u^{th} sample
 $t = uT_s = \frac{u}{B}$

$s(uT_s) = x(u) = \sum_i X_i e^{j2\pi i \frac{B}{N} \cdot \frac{u}{B}}$
 $= \sum_i X_i e^{j2\pi \frac{iu}{N}}$

So, what we said was we consider since the signal is band limited to B instead of transmitting the signal directly, we will sample it at the nyquist rate is corresponding to B samples per seconds, which means sampling rate of 1 over sampling time interval of 1 over B.

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Handwritten notes on a whiteboard:

$X(u) = \sum_i X_i e^{j2\pi \frac{iu}{N}}$

Samples of MCM signal.

DFT of information symbols
 $X(0) X(1) \dots X(N-1)$

And consider the sample signal, and we saw that this sample signal can be generated but, nothing but the I by nothing but, the IFFT nothing but, the IFFT or the IDFT of the transmission symbols. So let me so today let me illustrate that idea further.

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Handwritten notes on a whiteboard showing the MCM composite transmit signal equation and its components:

$$S(t) = \sum X_i e^{j 2\pi \frac{B}{N} t}$$

Annotations:

- $S(t)$: MCM composite transmit signal.
- X_i : i^{th} stream
- $e^{j 2\pi \frac{B}{N} t}$: i^{th} subcarrier
- $T_s = \frac{1}{B}$
- $u T_s = \frac{u}{B}$: time instant of the u^{th} sample.

So, we considered a signal $s(t)$ this is remember the MCM transmission signal MCM composite transmit signal. This is given as we said $X_i e^{j 2\pi \frac{B}{N} t}$. This is the information symbol the i^{th} stream and this is the i^{th} subcarrier. What I can do I can sample this at rate B , which means sample time T_s equals or sample interval T_s equals 1 over B . Let me consider the u^{th} sample or the time corresponding to the u^{th} is nothing but, $u T_s$, which is u over B ; this we say is the time instant of the u^{th} sample, all right, this we say the time instant of the u^{th} sample.

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The image shows a digital whiteboard with the following handwritten equations and annotations:

$$x(u) = s(uT_s) = s(u/b)$$

$$= \sum_i X_i e^{j2\pi i \frac{B}{N} \frac{u}{B}}$$

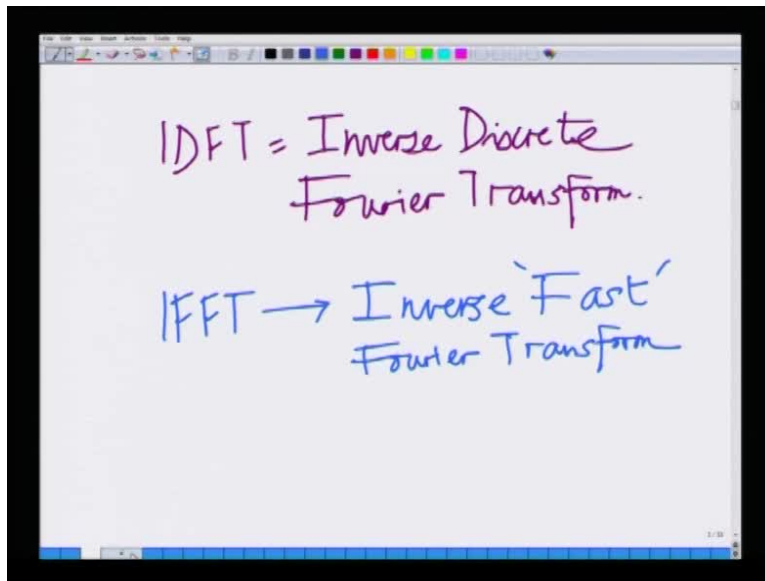
Below this, a blue arrow points from the text "IDFT of transmission symbols:" to the second equation:

$$= \sum_i X_i e^{j2\pi i \frac{u}{N}}$$

That u th sample x of u is nothing but, s of uT_s which is equal to s of u over b , that is nothing but, summation over i X_i into the power of $j 2\pi i B$ by N over T but, T is nothing but, u by B , this you can clearly see is summation over i X_i $e^{j 2\pi i u}$ over N . And this is nothing but, this is the IDFT this is the IDFT of the transmission.

And this is a very important result, because it says I do not need these N modulators to generate the composite signal I can generate a sampled version of that signal sampled at rate B , which is the nyquist rate by essentially computing the IDFT and there is a very fast algorithm to compute the IDFT, which is simply the IFFT or the inverse fast Fourier transform of the symbols.

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Hence, one can compute the IDFT, let me for the let me just simply write our what IDFT stands for IDFT equals the inverse discrete Fourier transform. And in practice IDFT is computed using IFFT, which is essentially the inverse fast Fourier which is essentially which is essentially a fast algorithm to compute the inverse fast the IDFT of the inverse discrete Fourier transform, all right.

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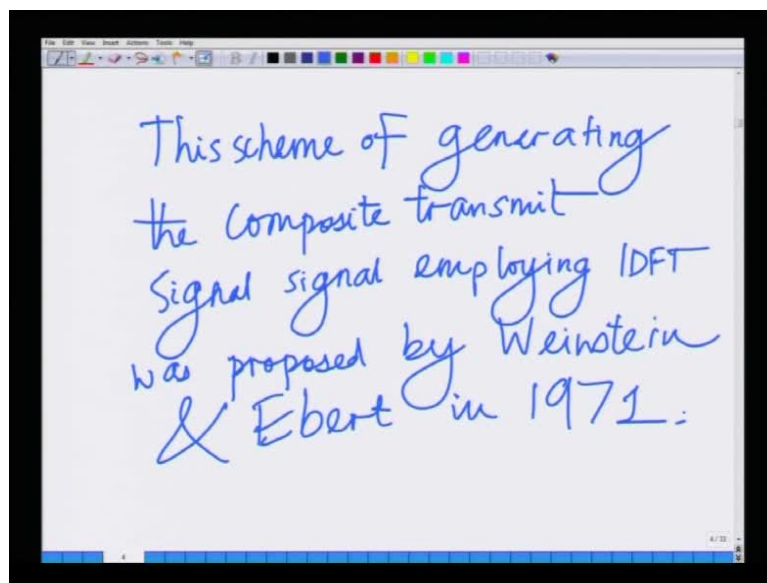
The image shows a whiteboard with handwritten equations in green and blue ink. The top equation is $x(u) = s(uT_s) = s(u/B)$ in green. Below it is $= \sum_i X_i e^{j2\pi i \frac{B}{N} \frac{u}{B}}$ in green. To the left of this equation, the text "IDFT of transmission symbols:" is written in blue, with an arrow pointing to the next equation: $= \sum_i X_i e^{j2\pi \frac{iu}{N}}$ in blue.

So in practice is what it means essentially is that from the structure given here, I can take my transmission trans signal symbols to be transmitted, that is the X_0, X_1 up to X_{N-1} of

$N - 1$, and compute a simple IDFT of this simple or the IFFT of this symbols to generate transmit signal and there is no need to use the N modulators for the N subcarrier which greatly which as we saw employing N modulators greatly in greatly increases the complexity of this system.

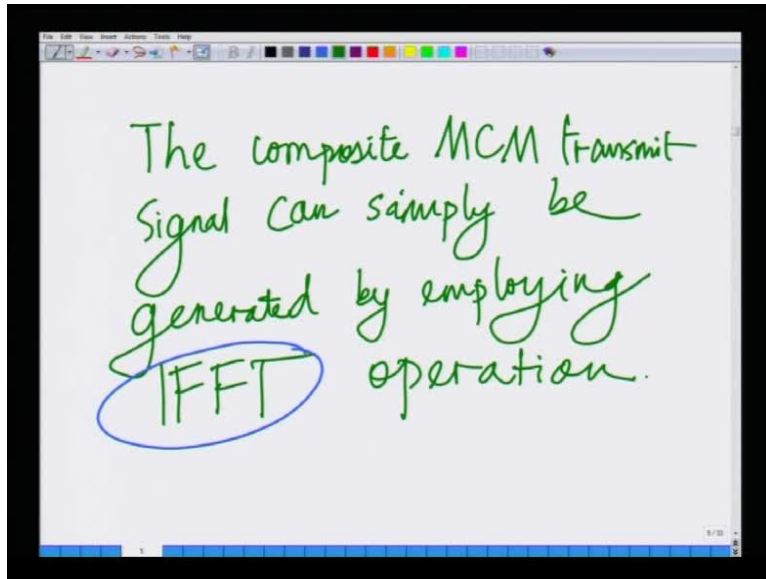
So, this scheme this is the scheme so this scheme that is to generate the signal by computing the IFFT instead of the N modulators who was proposed by Weinstein and Ebert, this was the scheme that was proposed by Weinstein and Ebert in the 1971 paper.

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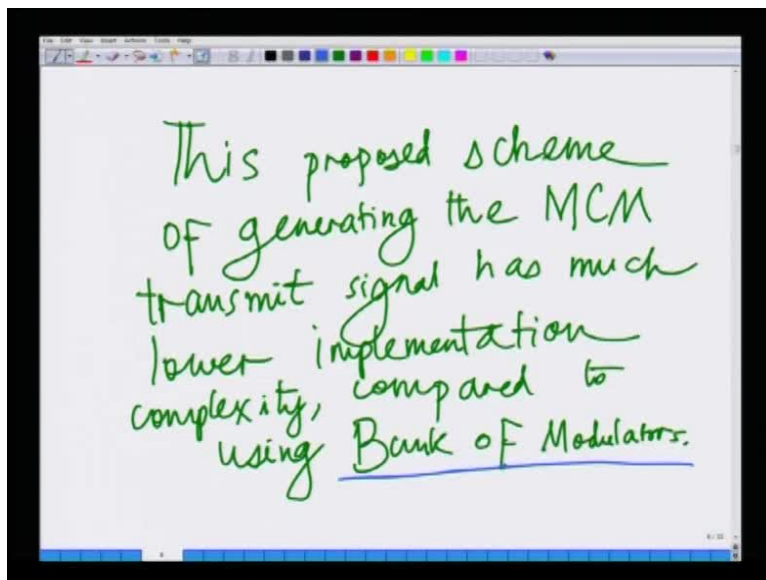
So, this scheme of generating the composite transmit signal employing IDFT was proposed by Weinstein and Ebert in 1971. In fact we also talked about the paper in which this novel came was proposed in, in the last lecture, we I mentioned the reference of the paper in which this novel scheme to generate the multicarrier modulators signal was proposed by Weinstein and Ebert. What it is says is instead of generating instead of using N modulators the composite MCM transmit signal can simply be generate by N generated by N IFFT operation.

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So, the composite MCM transmit signal can simply be generated by employing by employing an IFFT and this is the key an IFFT operation, which means there is no need to use the N modulators 1 for each subcarrier, and that greatly simplifies transmitter design.

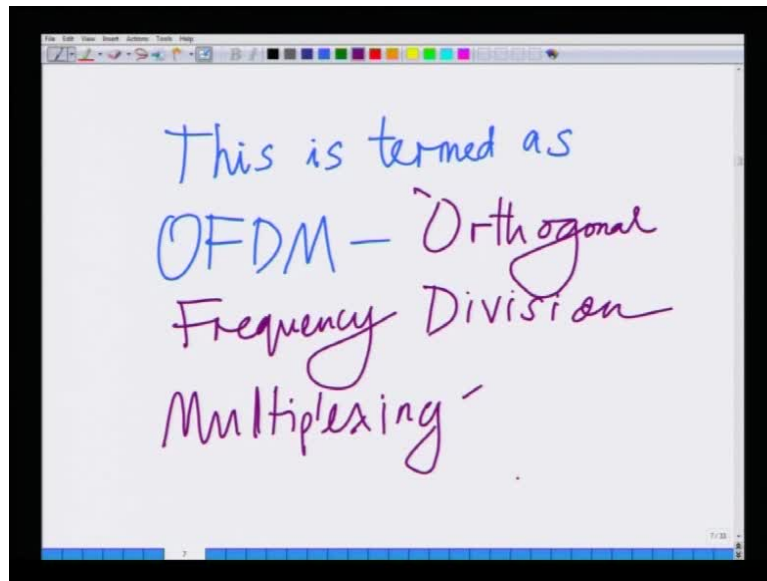
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And this scheme was of course, proposed and this scheme this proposed scheme of generating that transmit signal by using IFFT as much lower computational complexity, then using much lower implementation complexity, then using the N modulators or which is the bank of modulators of generating the MCM transmit signal as much lower implementation

complexity, compared to using the bank of this so this has much lower complexity compare to using the bank of modulators, thus significantly enhancing the appeal of the system, and significantly enhancing the realize ability the practical realize ability of this system based on multicarrier modulations. And this system multicarrier modulation with the signal transmission replaces by IDFT; this has a name this is known as OFDM, all right.

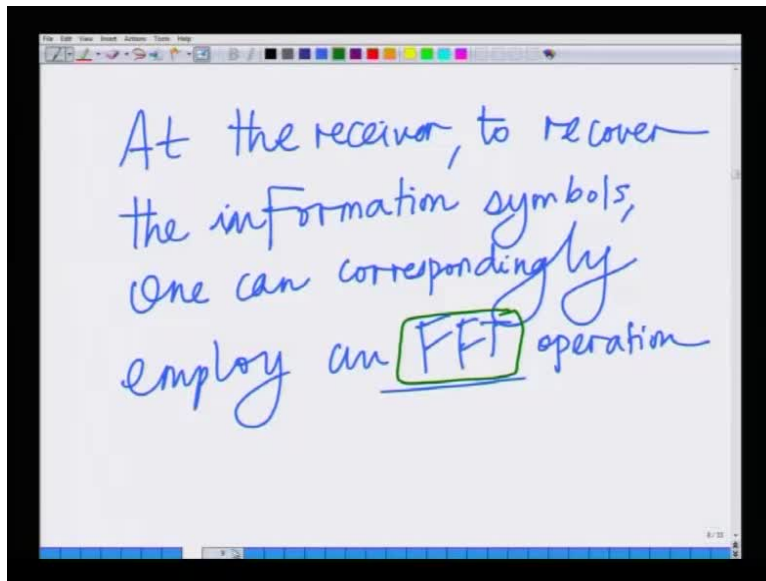
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So, this MCM with signal generation done by IFFT operation is termed as OFDM, so this is this is termed as OFDM which as we said is the key technology for next generation wireless communication, 4G, such as l t y max, and Wi lan and it stands for orthogonal it is stand for orthogonal frequency division multiplexing, and this stands for orthogonal frequency division multiplexing and I said this is essentially multicarrier modulation with the signal transmission signal generation replaced by the IDFT or in practice the IFFT operation, which is the fast operation implemented on a DSP chip in a modern wireless communication system, all right.

And now, what can be done the receiver, receiver instead of using the bank of modulators, I can simply use the inverse of the IFFT for operation which is the FFT operation.

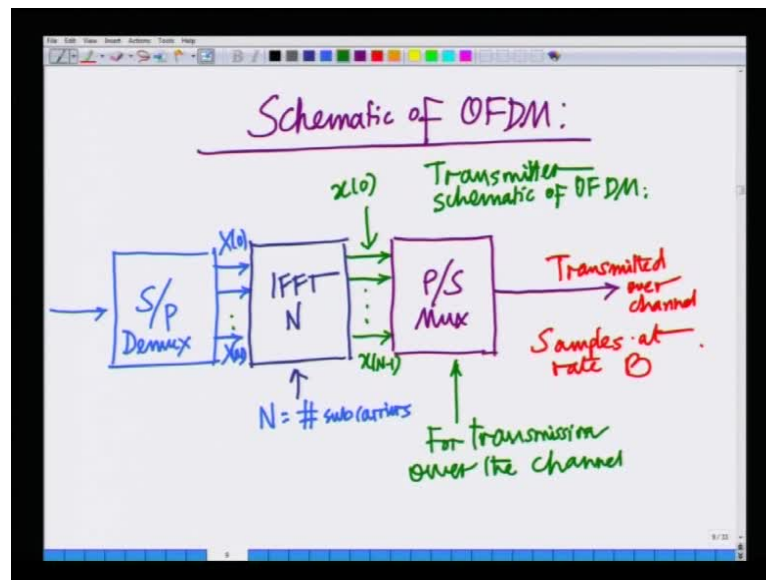
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So at the receiver, now to recover the information symbols one can correspondingly one can correspondingly employ an FFT all right. So, instead of the bank of demodulators at the receiver, now you can work with FFT to recover the transmitted symbol, so at the transmitter the bank of modulators is replaced by an IFFT at the receiver the bank of demodulators is replaced by FFT, that greatly simplifies transceiver design and wireless communication, and that is the essentially the genius contribution of Weinstein and Ebert towards realizing this multicarrier system, and now you can get all the gains of the multicarrier modulated system, the advantages which is essentially removing the effect of inter symbol interference. As we are going to show next...

So before we proceed further, let me give you a simple schematic of this multi multicarrier modulation system, similar to of the OFDM systems similar to the multicarrier modulation system. So, what I am going to do here, similar to what we have done earlier I am going to draw the transmitter and receiver schematic that is the architectural the transmitter and receiver to illustrate how the transmission, and detection of symbol system ok.

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So, we have so let us say we have the schematic of OFDM I have a similar to what I have in a multicarrier system I am to similar to parallel d de multiplexing, this is a d multiplexing operation remember I have a serial stream I have to converted into parallel to modulate toward the N subcarrier. So, I am basically converted into a parallel block for modulation except, now I am no longer going to use the bank of modulators I am going to use the simple IFFT operation.

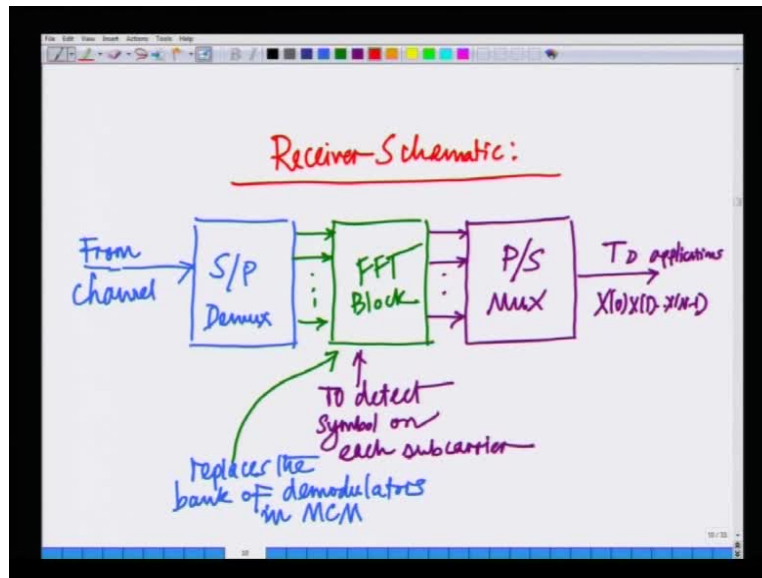
So, now I am going to simply use instead of a bank of modulators I am simply going to use an IFFT of size N , where N equals number of subcarriers, ok. So, the symbols X_0 to X_{N-1} are going into the IFFT, that is what is going to into IFFT are the actual constellation symbols, these are the constellation symbols such as b, p, s, k, q, p, s, k the digital symbols.

What comes out of the IFFT are the samples, which are to be transmitted and which correspond to the signals, so I get the samples to be transmitted, and now I do the inverse operation that is convert this parallel to severe or I multiplex them for so to convert it into a stream, which can be transmitted all the channel I convert the parallel to serial stream; this can now be transmitted this is for transmission over the channel.

This signal can then be transmitted over the channel, so this is $X_0 X_1 \dots X_{N-1}$, this is transmitted over the channel. So, let me write this over here. Remember, these are samples at sample returns each samples occupies sample occupies an interval of t sample which is essentially which is essentially $1/B$. So, samples at rate B that is sample are transmitted

rate B , that is B sample per second or essentially what we also say is the sample time or the sample interval is 1 over B . So, this corresponds to samples transmitted at the rate B corresponds to sample interval of 1 over B . Now, this is the transmitter schematic.

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How does the corresponding receiver schematic look, so at the receiver once I receive the information from the channels, so I am receiving the serial stream from the channel. I compute a first perform again similarly, I perform the serial to parallel demux operation, that is the demux this because remember I have to pass this to the FFT block. So, demux this and then I pass this to the FFT block, this block perform the inverse of IFFT, if you remember FFT is the inverse of the IFFT, and hence this helps recover or detect symbols at the receiver.

This is to detect symbol on each subcarrier, and remember this replaces the bank of demodulators, that is the coherent demodulation with every subcarrier that we have illustrate it earlier. This replaces the bank of demodulators in a multicarrier modulated system. Once, I do this once I finish with this what remains then is to take this parallel information symbols and covert them back into a serial stream, so that they can be forwarded to the higher layers to the necessary application.

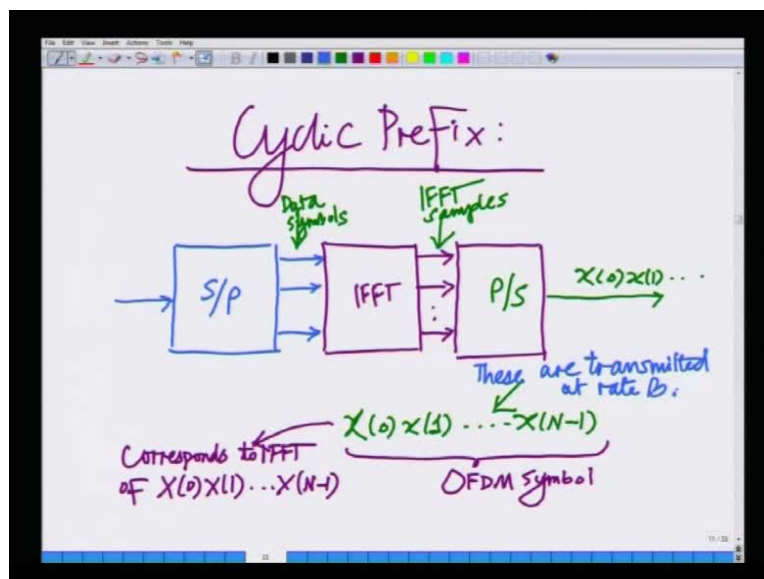
So this is again a parallel or serial or parallel to serial or a multiplexing operation to application, all right. So, the parallel serial this again converts the parallel block into a serial streams at which point I have my X_0, X_1, X_{N-1} which are passed on all right. And

this is an OFDM receiver schematic this is the receiver schematic, so this is the current receiver schematic that we have ok.

So what is the novel technique that has been proposed in the context of multi MCM. The novel technique is a essentially to replace the generation replace the generation using bank of modulators by an IFFT, which greatly enhances the practical realizability of that system. In fact it drastically enhances the practical realize ability, because implementing an IFFT is much more simpliment simple of that implementing a bank of modulators corresponding to each subcarrier, that is the great ideas which has helped me OFDM possible.

There is one more small modification to OFDM, because OFDM is current in it is current form that we have proposed is not yet usable because of the problem of inter symbol interference that exist in the wireless channel. So the next modification which essentially is the slight modification to this scheme is what we going to talk x not talk about next, the system as the cyclic prefix or the cyclic prefix.

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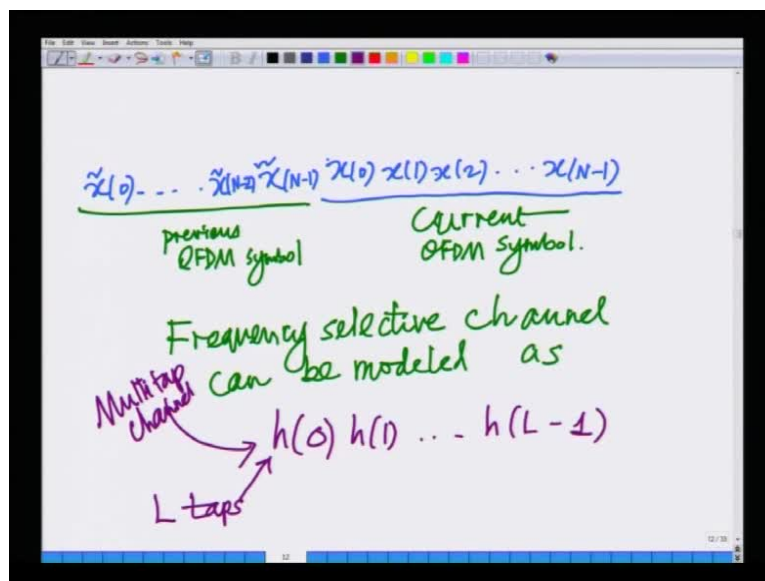
This is termed as a cyclic prefix in an OFDM system. What is this essentially mean... Let us see what we have looked at before, let us consider the system so far I have in OFDM system, that is as we have seen serial to parallel this is the incoming information symbol stream I have something that converts from serial to parallel I have converted it into parallel, then I employ my IFFT operation at which I used to have a parallel IFFT.

Then I convert this into back into a serial stream this is as we said a multiplexing operation, at which point I have the samples that I am that I am going to transmit. This in fact is known as 1 OFDM symbols, this group of x_0, x_1, \dots, x_{N-1} which corresponds to the samples of capital X_0, X_1, \dots, X_{N-1} which is the which are the block of N information symbols is known as 1 OFDM symbol ok.

So I have 1 OFDM symbol which essentially consists of ... This is termed as an OFDM symbol, this essentially corresponds to this corresponds to FFT of x_0, x_1, \dots, x_{N-1} up to x_{N-1} , so I take a block of capital N symbols 1 to $N-1$, compute that FFT or compute their IFFT I am sorry this corresponds to the IFFT, and then transmit this samples, this samples collectively form 1 OFDM symbol. We transmit 1 OFDM symbol after 1 OFDM symbol after 1 OFDM symbols. So, now we have 1 OFDM each OFDM symbol comprising of N samples transmitted successively, ok. So that is the transmission scheme that we have so far. So at this point I have the data symbols, and that this point I have the IFFT samples ok.

So I have the IFFT samples. These are transmitted of course, as we said these are transmitted at rate at these are transmitted at rate B or time interval by B . So I will say these are transmitted these are transmitted at rate B ok. So, now let me go to illustrate what happens during one such transmission.

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Let us say I am considering one symbol x_0, x_1, x_2 up to x_{N-1} these are the samples corresponding to 1 OFDM symbol. And let say the previous OFDM symbol also has some

samples these samples are \tilde{x}_{N-1} , \tilde{x}_{N-2} , \tilde{x}_0 . So, these are the previous OFDM symbol, this is the current OFDM symbol. So I have a block of N samples corresponding to the current OFDM symbol, I have a block of N samples corresponding to the previous OFDM symbol.

Now, what we want to do is we want to consider frequency selective channel, similar to what we have seen earlier. So, now remember we considered the frequency selective channel in the context of CDMA also where we illustrated the performance with respect to rake receiver, and so on.

And there we said that the frequency selective CDMA system it can be modeled as a frequency selective channel can be modeled as a multi path channel, it can be some it can be modeled similar to a FIR digital filter. So, I will choose the taps of this multi path frequency selective channel as follows. The frequency selective the frequency selective channel can be modeled as multi tap channel, where taps h_0 , h_1 up to h_{L-1} , so it just to remind you again this is a multi this is a multi tap channel, and it has 0 to $L-1$ that is L taps all right.

A let me say this is the model of a frequency selective channel I am transmitting my OFDM symbol block across this frequency selective channel, and I want to see what kind of a signal is arriving at the receiver and is there any problem with the signal, that is arriving at the receiver. So, now I pass this 2 OFDM symbols in fact big OFDM symbol of duration N over B through this multi path channel.

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First symbol corresponding to current OFDM symbol block is,

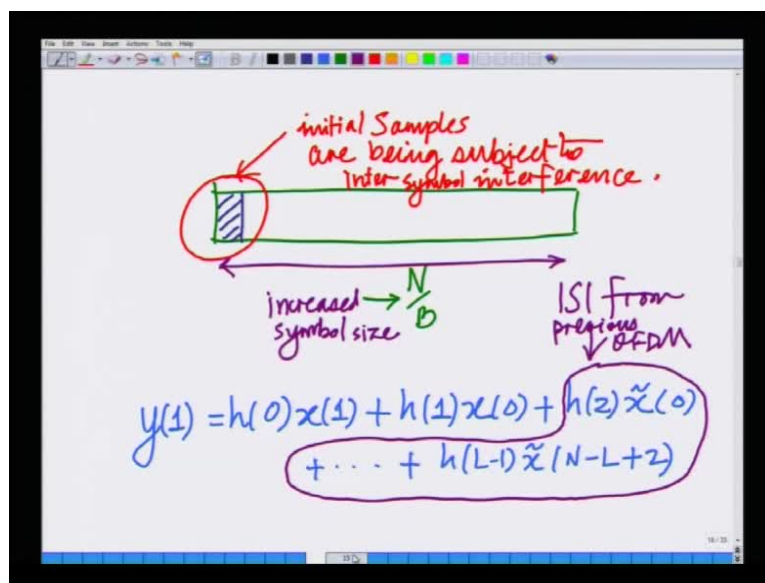
$$y(0) = h(0)x(0) + h(1)\tilde{x}(N-1) + h(2)\tilde{x}(N-2) + \dots + h(L-1)\tilde{x}(N-L+1)$$

From previous OFDM Symbol.

Let us look at the first symbol that is received, so the first symbol corresponding to the current OFDM block OFDM symbol block, the first the first symbol is y_0 equals $h_0 x_0$, that is x_0 times h_0 the first sample. But this is not the only component that is present there is remember this is multi path frequency selective channel, which means there is interest channel interference, there is h_1 times \tilde{x}_{N-1} .

Remember the previous sample corresponding to x_0 as \tilde{x}_{N-1} . Hence, I will have an \tilde{x}_{N-1} plus $h_2 \tilde{x}_{N-2}$ plus $h_{L-1} \tilde{x}_{N-L+1}$, in fact corresponding to the current sample x_0 or sample x_0 in the current symbol there are there is interference from $L-1$ pass sample. So this actually this whole component here that is in the circle area this is from the previous OFDM symbol. So this \tilde{x} this is from... So what we have done is we have increased this OFDM symbol size, that is what we had earlier we had a transmission time of $1/B$. Now we are increased the transmission time $2/B$ to N/B .

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So, now we have an increased OFDM symbol size, this is transmission times N/B this is increased OFDM symbol size increased symbol size corresponding to transmission time N/B , that is I made I am making this symbol essentially look like something with a symbol time that is much larger than the delay spread of the channel. Remember, that is the essential idea behind inter symbol interference, if the delay spread is much smaller than the symbol

time, or the symbol time is much larger than the delay spread then the inter symbol interference is can be neglected.

However, now there is inter symbol interference however small it is it is in the initial part of the period. So, the initial samples in this OFDM symbols are being subject to inter symbol interference, so initial samples initial samples are being subject to inter ok. The initial samples in this system are being subject to inter symbol interference. So, in fact let us continue with that example, let me also illustrate what is happening with the second received symbol, that is y_1 just to cement this idea when I received y_1 , y_1 is nothing but, now you can also see it is a $x_0 x_1$ plus $h_1 x_0$, until here it is fine this is the same OFDM symbol.

But now when you look at h_2 I have $h_2 x_0$ plus plus $h_1 x_{N-L+1}$ minus $h_1 x_{N-L+2}$ all right. And this portion here again corresponds to the inter symbol interference part ok. So this portion here, if you look at this portion here again corresponds to the inter symbol interference ISI from previous.

Remember not the previous symbol as in the case of narrowband system this is from the previous OFDM symbol, there is the big difference because this OFDM symbol is much larger it consists of N samples. So, the there is the effect has been decreased by increasing this OFDM symbol size however, there is still this effect of inter symbol interference in the initial part, all right. So, OFDM as it stands now still suffers from that inter symbol interference part for the initial part of this large symbol.

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Cyclic Prefix:

$x(N-L+1) \cdot x(N-L+2) \cdot \dots \cdot x(N-1)$ (Cyclic prefix)
 $x(0) \cdot x(1) \cdot \dots \cdot x(N-1)$ (all samples belong to current OFDM symbols)

$$y(0) = h(0)x(0) + h(1)x(N-1) + \dots + \dots + h(N-1)x(N-L+1)$$

So, what is the solution to overcome this problem we consider a simple transmission scheme, in which what we are going to do is we are going to take the transmitted samples x_0, x_1, \dots, x_{N-1} . Now, instead of transmitting this directly what we are going to do is we are going to take a small portion of this we are going to take a portion of this comprising of some symbols, and we are going to repeat that portion here as follows I am going to prefix this with $x_{N-1}, x_{N-2}, \dots, x_{N-L+1}$ ok.

So this is essentially a prefix, let me name this first this is a prefix, and there is a cycle here that is whatever I have here the cycled back to the end hence this is known as cyclic this is known as cyclic prefix, all right. Cyclic prefix is nothing but, I take the OFDM sample, OFDM symbol that is to take some sample of OFDM symbol move back, and and prefix the OFDM samples or OFDM symbol with these samples which are towards the end, this is a cyclic this is cyclic in nature it is a prefix this is known as a cyclic prefix that is over here to going to consider, now which is the cyclic or the cyclic prefix over OFDM. What is the advantage of the cyclic prefix over the OFDM how does the transmission of the cyclic prefix effect effects effects this procedure of OFDM that is what we are going to look now. The received signal y in this scenario is given as follows.

Now, let us look at y_1, y_2, \dots, y_L across the same inter symbol interference ISI channel multi multi tap ISI channel with taps h_0, h_1, \dots, h_{L-1} is given as h_0 or I am sorry let us look at y_0, y_1, \dots, y_L is given as $h_0 x_0$ plus now h_1 and to look at this previous symbol to x_0 is nothing but, x_{N-1} . So this is $h_1 x_{N-1}$ plus $h_{N-1} x_{N-L+1}$. So, now because we are not directly transmitting the two OFDM symbols next to each other instead we are cyclically taking some of the last sample of this OFDM symbol, and prefixing these samples to the current OFDM symbol.

I have reduced the I have removed the impact of inter symbol interference, and this OFDM symbol all right. So, now you can look at this part all the symbols belong to the say all the samples y_0 belong all the samples belong to same OFDM belong to the current OFDM symbol. So, all samples belong to current all the samples belong to the current OFDM symbol.

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$$y(1) = h(0)x(1) + h(1)x(0) + \dots + h(N-1)x(N-L+2)$$

$$\vdots$$

$$y(N-1) = h(0)x(N-1) + h(1)x(N-2) + \dots + h(L-1)x(N-L)$$

Hence, this is now you can similarly look at y_1 , let me also write y_1 equals $h_0 x_1$ plus $h_1 x_0$ plus plus $h_{N-1} x_{N-1}$ plus 2 so on, so forth. In fact you can look at the $N-1$ symbol which is $h_0 x_{N-1}$ plus $h_1 x_{N-2}$ plus so on and so forth. Hence, I have removed the effect of inter symbol interference ok. That is the first thing I wanted to observe in fact can write this is $h_{L-1} x_{N-L}$, ok. I can write this as $h_{L-1} x_{N-L}$ ok.

So, now you can see essentially that what I have done here is essentially I have removed by prefixing this OFDM transmitted symbol this large symbol of samples x_0, x_1, x_{N-1} .

(Refer Slide Time: 42:16)

$$y(1) = h(0)x(1) + h(1)x(0) + \dots + h(N-1)x(N-L+2)$$

$$\vdots$$

$$y(N-1) = h(0)x(N-1) + h(1)x(N-2) + \dots + h(L-1)x(N-L)$$

(Refer Slide Time: 42:18)

Cyclic Prefix:

$x(N-L+1) \dots x(N-1)$ Cyclic prefix

$x(0) x(1) \dots x(N-1)$ all samples belong to current OFDM Symbols.

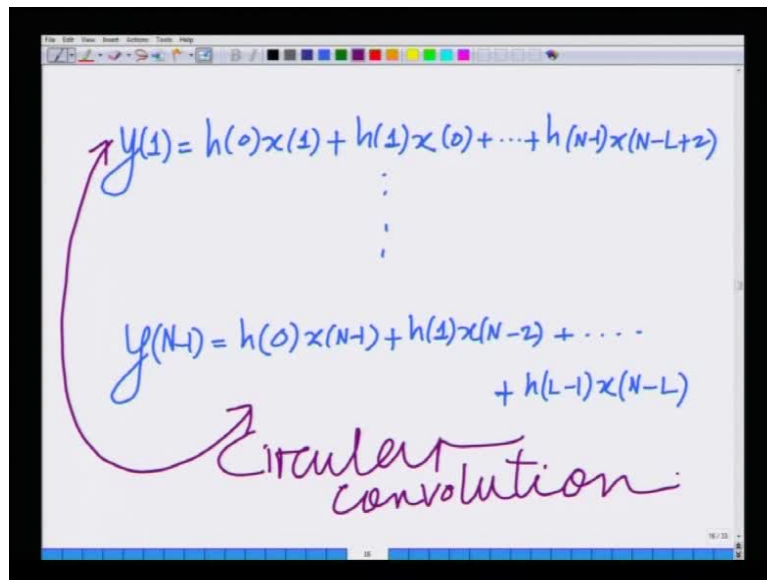
$$y(0) = h(0)x(0) + h(1)x(N-1) + \dots + \dots + h(N-1)x(N-L+1)$$

This samples I have removed the effect of the inter symbol interference in other words nowhere is the inter symbol interference arising from samples of the previous OFDM symbol, which was which was remember symbol $x(N-1)$ $x(N-2)$, by prefixing this now I have restricted the inter symbol interference to the same OFDM symbol. And more importantly observe look at the nature of the convolution that we have right now.

Remember I am transmitting sequence or symbol to this LTI system hence the output is the convolution of the of this transmitted samples with the channel filter but, look at the nature of

the convolution. It is $y[0]$ equals $h[0]x[0]$ plus $h[1]x[N-1]$, it is as if I am wrapping this $x[0]$ to $x[N-1]$ around this filter, and most electrical engineering students will be familiar with this type of convolution this is not a linear convolution any more, in fact this is a circular convolution.

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The image shows a digital whiteboard with handwritten equations in blue ink. The first equation is $y[1] = h[0]x[1] + h[1]x[0] + \dots + h[N-1]x[N-L+2]$. Below it are three vertical dots. The second equation is $y[N-1] = h[0]x[N-1] + h[1]x[N-2] + \dots + h[L-1]x[N-L]$. A purple arrow starts from the word 'Circular' in the text 'Circular convolution' and points to the $x[0]$ term in the first equation. The text 'Circular convolution' is written in purple cursive.

So this convolution which has a very unique nature this is in fact a circular convolution, because look at this it is like I am taking the filter and $h[0]$ I have it $h[0]$ but, $h[1]$ I have it $x[N-1]$ $h[2]$ I have it $x[N-2]$ and so on. And progressively I am taking this filter and circularly shifting it towards the transmit sequence. Hence, this is nothing but a circular convolution, so what this is this sounds an interesting structure this has a circular this is a circular convolution, so this is a circular convolution.

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$$\begin{aligned}
 & \text{Received samples } [y(0) \ y(1) \ \dots \ y(N-1)] \\
 & = [h(0) \ h(1) \ \dots \ h(L-1)] \text{ Multitap Channel Models the Intersymbol interference channel.} \\
 & \quad \otimes [x(0) \ x(1) \ \dots \ x(N-1)] \\
 & \quad \text{N samples of the current OFDM symbol generated after the IFFT operation.}
 \end{aligned}$$

Which means now my output of samples received sample y_0 y_1 y_{N-1} is nothing but, the channel filter h_0 h_1 h_{L-1} appropriately 0 padded, because they have to be same length circularly convolve with x_0 x_1 x_{N-1} . This is the these are the received samples ok. y_0 y_1 y_{N-1} are the received samples, this is the multi tapped channel filter or this is the multi this is the ISI channel or this is the multi tapped channel, which is a model of a inter symbol which models the inter symbol interference channel.

This models the this models the inter symbol interference channel, and this x_0 x_1 up to x_{N-1} this is nothing but, this is the these are the N samples of the transmitted OFDM symbol generated after the IFFT operation, remember we are still performing the IFFT operation. These are the N samples of the current OFDM symbol of the... So, these are the N samples of the current OFDM symbol block let with the generated by the IFFT of the transmitted modulated symbols ,which is capital X_0 , capital X_1 , capital X_{N-1} let me remind you of that.

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$$y = h \otimes x$$

This is possible because of addition of cyclic prefix

$$Y(k) = H(k)X(k)$$

k^{th} subcarrier

N point DFT of y

N point DFT of h (after zero padding)

Modulated Information Symbols.

So, now I can write this y equals h circular convolution with h and this is possible because of addition of this is possible, because addition of the cyclic prefix and that is the key this is possible because of the addition of the cyclic prefix, which is the essential key that is the key. And, now once I have this we all well know as electrical engineering students that if y is the circular convolution of x h and x , I can write the DFT of Y capital Y as H times DFT of X or instant or what I mean is Y_k which is across the k^{th} frequency in the IDFT in the DFT equals H or in the DFT, so if I take the DFT of Y that is nothing but, Y_k equals H_k times X_k , this is the main result that we want to show.

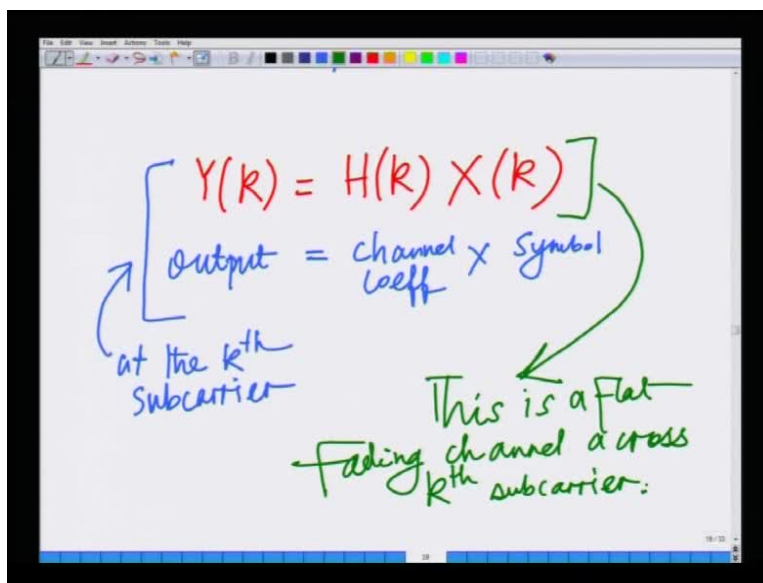
Let, me again define each of the terms in this y_k this is nothing but, in fact let me make it additional clear this is the N point DFT of y , this is the N point DFT of H after zero padding of course, that is assume that is implicit let me write that also clear, because after zero padding, because it has only h_0 up to h_{l-1} after zero padding. And X_k is nothing but, the DFT of the transmitted OFDM samples, but the transmitted OFDM samples are themselves the IFFT or the IDFT of the capital X X_0 X_1 up to x_{N-1} .

Hence, DFT of IFFT gives me the same original transmitted symbol. So these are nothing but, the modulated and by modulation I mean a digital modulators that is p s k q p s k or quamp modulated information symbol. What we are saying is by performing in N point DFT, because by the addition of the cyclic prefix, this has been converted into a into a into a circular convolution I can perform when N point DFT, and what I have is each symbol s_k is

multiplied by h_k which is now the channel coefficient corresponding to that subcarrier and that is received at eh_k the DFT point of the output received samples y_0 up to y_{N-1} .

And, now if you see and so this is the symbol transmitted on the k th subcarrier, this k corresponds to a this k is nothing but, the k th subcarrier this k is nothing but, the k th frequency point of the DFT, this is nothing but, the k th subcarrier. And what we are saying is the symbol transmitted in the k th subcarrier is essentially the H_k the coefficient of the k th subcarrier, and that is the received symbol.

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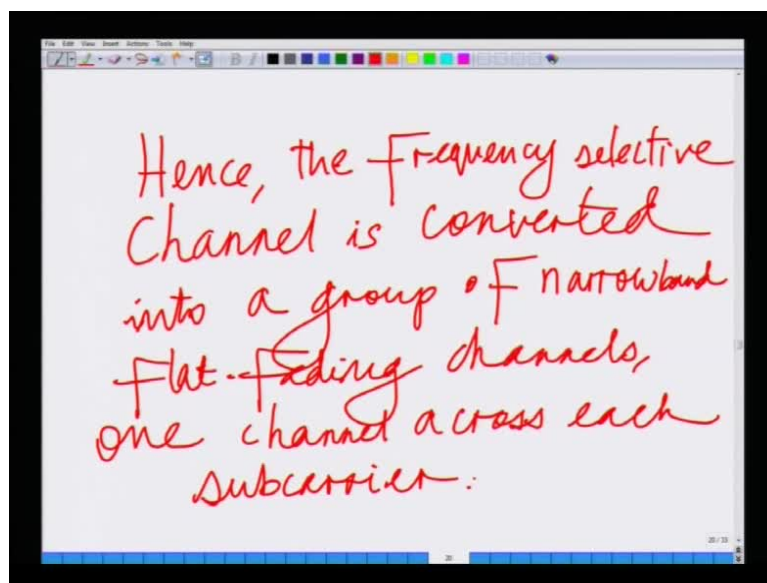
The image shows a whiteboard with handwritten text and an equation. At the top, the equation $Y(k) = H(k) X(k)$ is written in red. Below it, in blue, is the text "Output = channel coeff x Symbol". A blue arrow points from this text to the equation. Below the equation, in green, is the text "This is a flat fading channel across k th subcarrier:". A green arrow points from this text to the equation. To the left of the equation, in blue, is the text "at the k th subcarrier" with a blue arrow pointing to the equation.

So we have Y_k equals H_k into X_k on the k th subcarrier, and this we say is nothing but, a flat fading channel. Why this is a flat fading look at this this is the symbol on the k th carrier, this is the channel coefficient, so we write output equals channel coefficient times symbol. So I have nothing but, the symbol and this is at the k th subcarrier, and this is remember at every subcarriers, so this is the this is the k th subcarrier.

And the output of the k th subcarrier is nothing but, the symbol transmitted on the k th subcarrier times the channel coefficient. Hence, what we have is we have removed the effect of inter symbol interference across this subcarrier, all right. Because, if you look at this that is what our main motivation was work for using MCM or OFDM that is to convert this wide band channel, which is a frequency selective channel into a flat fading channel by removing the effect of inter symbol interference.

And this precisely illustrates that effect, where symbol X_k into channel coefficient H_k equals Y_k that is nothing but, this is nothing but, a flat fading channel. So, let me again clarify that this is the this is a flat fading channel across this is the flat fading channel across the k th subcarrier ok. Hence, though OFDM symbol OFDM hence addition of this cyclic prefix, this is the complete OFDM system, hence OFDM effectively converts a wide band frequency selective into a collection of parallel narrowband flat fading channels, thus removes the effect of inter symbol interference, and because it uses the simple IFFT, FFT based designs it is also a very low in implementation complexity.

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Hence, what we have here is hence the frequency selective channel is converted into a group of a group, remember we have now N narrow flat narrowband flat fading across each subcarriers, it is converted into group of narrowband flat fading channel one channel across each subcarrier, it is converted into hence a frequency selective channel is converted into a group of narrowband flat fading channels with one subcarrier one channel across each subcarrier, that is the big advantage of OFDM. For instance what you can clearly see what the advantage of OFDM is for instant consider that used a single carrier system.

(Refer Slide Time: 54:13)

Single carrier: Heavy inter-symbol interference.

$x(0) x(1) \dots x(N-1)$
actual symbols

$y(n) = h(0)x(n) + h(1)x(n-1) + \dots + h(L-1)x(n-L+1)$

Let us take the let's go back to our single carrier system. If you use a single carrier system, then what we have is x_0, x_1, x_{N-1} and so on. These are the actual symbols there is no IFFT FFT operation inverse hence, these samples are nothing but, the actual symbols these are the samples these are also actual modulated symbols in a single carrier system. Now if I look at the reception at the receiver, that is y_0 equals $h_0 x_0$ plus $h_1 x_1$ or let me write this or let us look at it symbol N x_N plus $h_N x_{N-1}$ plus $h_{L-1} x_{N-L+1}$, and you can see all these symbols experienced inter symbol interference, and there is heavy inter symbol interference all right.

So, single carrier system there is heavy so the greatest advantage of OFDM is that it has very low complementation complexity, and plus it converts a frequency selective channel into multiple flat fading parallel flat fading channels, and removes the effect of inter symbol interference, there by greatly enhancing a practical realizability of such system, and caving the way for realizing broadband wireless communication system in four three in 4G wireless communication system in next generation wireless networks. So with this we will conclude the lecture today, and we will discuss other properties of OFDM starting from this point in the next lecture.

Thank you.