

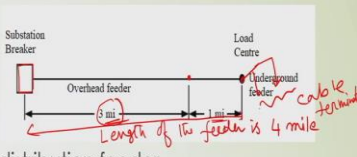
Operation and Planning of Power Distribution Systems
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Lecture - 17
Numerical problems on reliability evaluation

In today's lecture I will basically finish this reliability aspect or reliability assessment of power distribution networks. And I will start with a practical problem of a typical distribution network to assess the reliability of that particular network ok.

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68 Example...



✓ The Figure shows a 4 mile long distribution feeder. Approximately 1 mile of the feeder has been built underground, while the rest of the feeder is overhead. The underground feeder has two termination points. Two faults per circuit-mile for the overhead section and one fault per circuit-mile for the underground section of the feeder have been recorded in the last 10 years. The annual cable termination fault rate is given as 0.5% per cable termination. Furthermore, based on past experience, it is known that the average repair times for the overhead section, underground section, and each cable termination are 3, 30, and 3 h, respectively.

Ref.: T. Gonen. Electric Power Distribution System Engineering

Handwritten notes: Difference between (λ, τ) for overhead section and underground cable

So, look at this example ok. This example is very simple example, we have a substation at this point, and this box is representing a substation or substation breaker ok. And there is a primary feeder, there is a primary feeder and this feeder is of 4 miles of length. So, length of the feeder is of 4 mile; length of the feeder is 4 mile out of which some part is overhead feeder this 3 mile is, some part is overhead feeder and some part is underground cable.

So, overhead section is of 3 mile length and underground section is of 1 mile length ok. And it is also mentioned that there are two cable terminations, there are two cable terminations after this particular feeder end. So, let me go through this problem as a whole. So, the figure shows 4 mile long distribution feeder, approximately 1 mile of the feeder has been built underground ok.

Underground feeder is made of cable ok, you know that overhead feeder is made of overhead line conductors and underground feeder is made of cable ok. So, approximately 1 mile of feeder has been built underground while rest of the feeder is overhead ok. The underground feeder has two termination points as I made, these are the cable terminations ok. Now, it is mentioned that two faults per circuit mile for overhead section and one fault per circuit mile for underground section of the feeder have been recorded in last 10 years ok.

So, from this one need to extract failure rate of this overhead section and underground section look at this sentence again, two faults per circuit mile for overhead section, and one fault per circuit mile for the underground section of the feeder have been recorded in the last 10 years ok. The annual cable termination fault rate is given as 0.5 percent per cable termination. Furthermore, based on the past experience, it is known that the average repair times for the overhead section, underground section, and each cable termination are 3, 30 and 3 hours respectively ok.

So, this is the data given to you. What are the data given to you? Number 1, is failure rate of the overhead section of the feeder ok. What is that failure rate? It is two faults per circuit mile ok; now how we will get failure rate from this information we will see ok. Similarly, there is one fault per circuit mile is recorded in last 10 years. So, failure rate is normally determined as a unit of number of faults per year ok and here the failure rate is given as failure you know recording data is given as or recorded failure data is given as two faults per circuit-mile per circuit mile per unit length.

In fact, for this overhead section; and one fault per circuit-mile or one fault per unit length for the underground cable ok. And also based upon experience, it is seen that the average repair time for this overhead section is 3 hour, underground section is 30 hour and each cable termination is of 3 hour. There is an important point that you can look at that is the difference between the failure rate and repair duration that is λ and r .

We represent this failure rate by λ and repair duration; average repair duration you can say that is \bar{r} for overhead section and underground cable. What is the difference? First of all this failure rate of overhead section is two faults per circuit-mile whereas, that failure rate of this you know that underground cable is one fault per circuit-mile. So,

overhead section is having twice of the failure rate as the underground cable ok. And the repair duration of the overhead line is 3 hours and underground cable is 30 hours.

So, it is because that because we have this overhead line conductors, its maintenance or repair time is less, but since it is a cable which is in a laid underground the repair duration is higher. So, failure rate of this overhead line although it is higher, but its repair duration is lower. And for this cable it is vice versa that is failure rate is less, but repair duration is high. So, this is the one of the important observation that you can make which is obvious.

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Example...

This is an example of 4 components which are repairable and are connected in series

Substation Breaker

Overhead feeder 3 mi

Underground feeder 1 mi

Load Centre

Using the given information, determine the following:

- a. Total annual fault rate of the feeder $\lambda_{feeder} = \lambda_{O/H} + \lambda_{U/G} + \lambda_{C1} + \lambda_{C2}$
- b. Average annual fault restoration time of the feeder in hours (r_{sys})
- c. Unavailability of the feeder
- d. Availability of the feeder

Ref.: T. Gonen. Electric Power Distribution System Engineering

$\lambda_{feeder} = \lambda_{O/H} + \lambda_{U/G} + \lambda_{C1} + \lambda_{C2}$

Now, using this information you are supposed to determine the total annual failure rate or total annual fault rate of the feeder i.e., question number a, question number b is average annual fault restoration time of the feeder. And question number c is unavailability of the feeder, and question number d is the availability of the feeder. This problem I took from Gonen's book ok, and this once you got this unavailability, availability is just 1 minus that unavailability.

So, fraction of time it is unavailable which is equal to that 1 minus fraction of time it was available ok. So, this two can be easily found out, once we can find out at least one ok. Now, first of all we need to find out total annual fault rate of the feeder. So, to determine this we have learned so many formula and we know that how to determine that failure

rate of a composite system. So, from that idea we have to determine that what would be the failure rate of the composite system.

Now, here we have three different systems or four different systems which are connected in series, one is that overhead section. So, suppose overhead section I represent by OH overhead section, this is one component. Another component is underground cable that is UG section underground section; another component is cable termination 1; another component is cable termination 2.

So, these 4 components and they are in series. So, this is an example of 4 components which are repairable. So, 4 components which are repairable; which are repairable are connected in series, are connected in series ok. So, in order to find out this composite or system failure rate, so let me go back and check that what expression is applicable for that.

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Reliability of repairable components in series

- Hence, the *mean time to failure* of a given series system with n components can be expressed as:

$$\overline{m}_{sys} = \frac{1}{1/\overline{m}_1 + 1/\overline{m}_2 + \dots + 1/\overline{m}_n}$$
- Hence, the *mean time to failure* of a given series system with n components can be expressed as:

$\lambda_{sys} = \lambda_1 + \lambda_2 + \lambda_3 + \dots + \lambda_n$
- Similarly, the *mean time to repair* for an n -component series system is:

$\overline{r}_{sys} = \frac{\lambda_1 \overline{r}_1 + \lambda_2 \overline{r}_2 + \lambda_3 \overline{r}_3 + \dots + \lambda_n \overline{r}_n}{\lambda_{sys}}$

$\approx \frac{\sum \lambda_i \cdot \overline{r}_i}{\sum \lambda_i}$

So, if we go back and we see that when we have repairable components connected in series the overall failure rate of the system is sum total of the individual failure rate of the components ok. So, this expression will be applicable to determine the overall failure rate of that feeder ok. So, here we have 4 systems; that means, n is equal to 4 ok, so; that means, we know that in order to find this first solution or answer of the first questions we know that λ_{sys} , that is failure rate of this overall system.

Or let us write it as failure rate of the feeder is equal to lambda of the overhead section plus lambda of underground cable underground section plus lambda of first cable termination C T 1 and lambda of C T 2. Since both are identical because as it is mentioned that this failure rates of both the cable are same. So, it is mentioned that the annual cable termination fault rate is given as 0.5 percent per cable termination ok. So, both the cable have same failure; suppose the cable terminations have same failure rate ok, are of same failure rate.

So, this lambda C T 1 and lambda C T 2 are equal. So, we can replace it as lambda feeder is equal to lambda of overhead section plus lambda of underground section plus lambda of C 1 cable termination and you multiply with 2 because both the cable terminations are identical ok.

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70 Solution...

✓ a. Total annual fault rate of the feeder is

$$\lambda_{FDR} = \sum_{i=1}^3 \lambda_i = \lambda_{OH} + \lambda_{UG} + 2\lambda_{CT}$$

Where

λ_{OH} is the total annual fault rate of overhead section of feeder

λ_{UG} is the total annual fault rate of underground section of feeder

λ_{CT} is the total annual fault rate of cable terminations

$\lambda_{OH} = \left(\frac{2}{10}\right) \times 3$

$\lambda_{UG} = \left(\frac{1}{10}\right) \times 1$

$\lambda_{CT} = 0.005$

$$\lambda_{FDR} = 3\left(\frac{2}{10}\right) + 1\left(\frac{1}{10}\right) + 2(0.005)$$

$$= 0.71 \text{ faults/year}$$

So, we write it in the next page that this here OH represents this total annual fail fault rate of the overhead section, this lambda UG represents that total annual fault rate and of the underground section and lambda CT is representing total annual fault rate of the cable terminations ok. Now, if we know these values of this lambda OH, lambda UG and lambda CT, we can find out the total annual failure rate or fault rate of the feeder ok.

Now, it is mentioned that the overhead section suffers from 2 faults per circuit-mile in 10 years ok. So, we can find out overhead section failure rate as 2 faults per 10 years. So, in

1 year it is 2 divided by 10 faults. So, this is per circuit mile multiplied by its length. Its length is given as 3 mile length of the overhead section is given as 3 mile.

So, these multiplied by 3. Similarly this lambda underground section is suffering from 1 fault per 10 years multiplied by its length that is 1 mile ok, because this fault is recorded per circuit-mile, per circuit length. Similarly, this cable termination lambda CT we can find out as 0.5 percent, so which is equal to 0.005 ok. So, you put all these values over here, we will get this overall failure rate of the feeder is 0.71 faults per year, this is what the unit of the fault rate ok. So, first question is solved so we got this total annual fault rate of the feeder.

Now, second is average annual fault restoration time of the feeder in hours ok. So, average annual restoration fault time means it is equal to r_{sys} ok. So, that represents this average annual fault restoration time or fault repair time ok. Now how to determine this r_{sys} for having this 4 subsystem or 4 components which are in series and which are repairable.

So, let me go back again and check the formula. We already determine this r_{sys} is equal to this. So, which represents summation of this $\lambda_i r_i$ divided by summation of λ_i ok. So, where λ_i is basically the failure rate of the i th component or i th subsystem, r_i represent; r_i represents that repair time or average repair time of the i th component. So, once we know this entire thing we can find out this mean time to repair or mean restoration time ok alright.

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72 Solution...

However, the average annual fault restoration time of the feeder is:

$$\bar{r}_{FDR} = \frac{\sum_{i=1}^3 \lambda_i \bar{r}_i}{\sum_{i=1}^3 \lambda_i}$$

$$\bar{r}_{FDR} = \frac{(l_{OH} \lambda_{OH})(\bar{r}_{OH}) + (l_{UG} \lambda_{UG})(\bar{r}_{UG}) + (2\lambda_{CT})(\bar{r}_{CT})}{\lambda_{FDR}}$$

$$= \frac{(3 \times 0.2)(3) + (1 \times 0.1)(30) + (2 \times 0.005)(3)}{0.71} = \frac{4.83}{0.71}$$

=6.8 hr

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72 Solution...

However, the average annual fault restoration time of the feeder is:

$$\bar{r}_{FDR} = \frac{\sum_{i=1}^3 \lambda_i \bar{r}_i}{\sum_{i=1}^3 \lambda_i}$$

$$\bar{r}_{FDR} = \frac{(l_{OH} \lambda_{OH})(\bar{r}_{OH}) + (l_{UG} \lambda_{UG})(\bar{r}_{UG}) + (2\lambda_{CT})(\bar{r}_{CT})}{\lambda_{FDR}}$$

$$= \frac{(3 \times 0.2)(3) + (1 \times 0.1)(30) + (2 \times 0.005)(3)}{0.71} = \frac{4.83}{0.71}$$

MTTR for the feeder = 6.8 hr

Handwritten notes:
 λ_i for o/h section \rightarrow length of o/h section (miles)
 λ_{UG} for u/g section \rightarrow length of u/g section (miles)
 λ_{CT} for CT section \rightarrow length of CT section (miles)
 λ_{FDR} is the sum of all failure rates.

So, in order to solve this question number 1, So, let us find out this formula, this formula is directly applicable that average annual fault restoration time is $\lambda_i r_i$, here we have 4 sub system, but two systems are identical. So, they considered these 3 and we multiplied with factor two with this λ_{CT} . So, ok and this summation of this λ_i already you determined in the last slide and we also know the individual failure rates of the system.

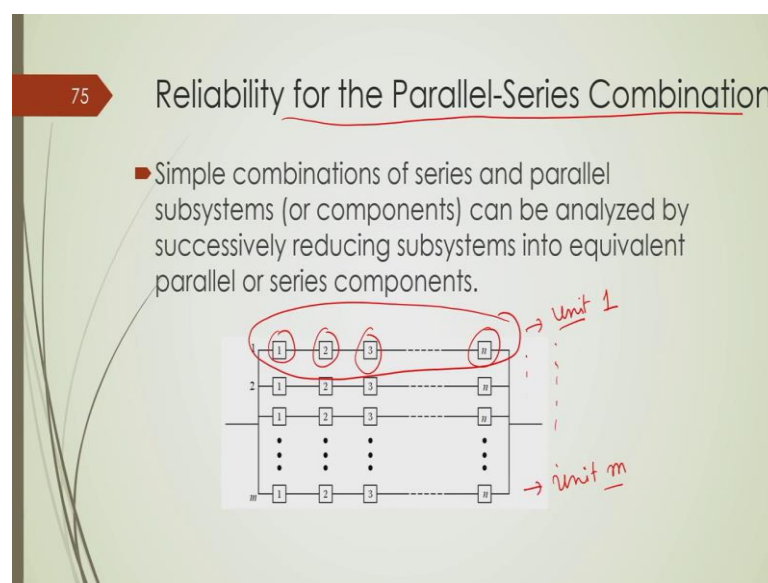
So, which is determined as this number of faults divided by total time period is recorded multiplied by its length. So, this one is basically OH basically representing the length of the overhead section, length of overhead section. Similarly, 1 UG is basically representing length of underground section and for this problem both are in mile; both are in mile ok.

Now, for cable termination you know that it is λ_{CT} multiplied by r_{CT} and since there are two identical cable termination having identical failure rate and repair duration so we can multiply it by factor 2 ok. So, once you put all these values in fact, this basically represents 1 OH multiplied by λ_{OH} is basically represent λ_i for overhead section. I write this overhead in short O by H ok.

And similarly, this is basically representing λ_i for underground section ok. So, do not be confused with this. This is basically representing that failure rate of the overhead section and failure rate of the underground section. since this number of faults recorded per unit length, we multiply this length in order to find out the failure rate of the overhead section and failure rate of the underground section ok.

So, you put this in this all these values over here and you got, we got this annual average restoration time or mean time to repair this is basically nothing but mean time to repair for the feeder ok, so that many hour ok.

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Now, we will determine that reliability for a parallel series combination ok. So, as I said in a practical power system, there may be a several components out of which some are connected in series and some are connected in parallel.

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76 Reliability for the Parallel-Series Combination

- The equivalent reliability of the system with m parallel paths of n components each can be expressed as:

$$R_{sys} = 1 - (1 - R^n)^m$$

Where
 R_{sys} is the equivalent reliability of system
 R^n is the equivalent reliability of a path
 R is the reliability of a component
 n is the total number of components in a path
 m is the total number of paths

Assumption: Reliability of all components is identical (R)

So, this we will make a series parallel combination ok. So, let us consider we have here n number of equipments which are connected in series and this will constitute 1 unit, this is one unit and there are. So, this is unit 1 and there are you know m number of such units. So, we have n number of subsystems they are connected in series and which constitute 1 unit of the system.

And they are identical or maybe different, but there are m number of units such unit are connected in parallel ok. So, for this we can find out this overall reliability of the system. So, assuming that individual here we are assuming that R is the reliability of the component. So, here the assumption is reliability of all components identical and it is represented by R ok.

So, once you consider that it may so happen that this components reliability are different if the components are components themselves are different the reliability would be different. But if we assume the reliability of individual components are identical meaning that all components are identical. So, for a one particular unit this is a series combination of n number of such units connected in series.

So, for this particular unit the overall reliability will be R to the power n because they are connected in series. So, we have such kind of you know such, we have m number of such kind of systems where individual reliability of all these units are identical that is R to the power n . Now as we know that when we have such kind of parallel combination; such kind of parallel combination of m number the such units and for parallel combination we know that unreliability function follow this product rule, that is 1 minus R to the power n will follow this product rule.

So, we have m number of units. So, overall unreliability will be 1 minus R to the power n to the power m ok. So, they will follow this chain rule or product rule ok. So, this gives this unreliability of the overall system and if you subtract with 1 then you will get the reliability of the composite system ok.

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Reliability for the Series-Parallel Combination

- The equivalent reliability of the system of n series units (or banks) with m parallel components in each unit (or bank) can be expressed as:

$$R_{sys} = [1 - (1 - R)^m]^n$$

Assumption: All components are identical

Unit 1 Unit 2 ... Unit n

Where

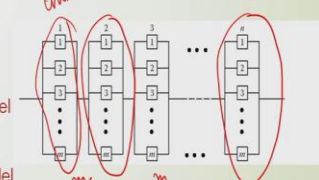
R_{sys} is the equivalent reliability of system

$1 - (1 - R)^m$ is the equivalent reliability of a parallel unit (or bank)

R is the reliability of a component

m is the total number of components in a parallel unit (or bank)

n is the total number of units (or banks)



Handwritten notes below diagram:

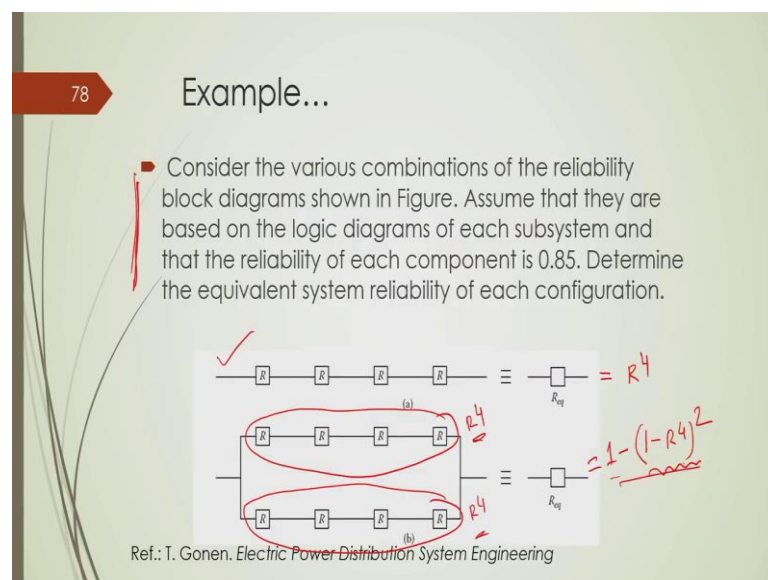
$Q_1 = (1-R)^m$ $Q_2 = (1-R)^m$...

$R_{sys} = [R_1 = 1 - (1-R)^m]^n$

Now, similarly we may have m number of identical units connected in parallel and we have n number of such units. So, this is unit 1, this is unit 2 and we have n number of such unit this is unit n ok, this is unit n . Now, since they are in parallel and here also we assume that all subsystems all components are identical. So, here also our assumption is all components are identical. So, they have identical reliability ok. So, since we have m number of components in parallel and the unreliability function will follow this product rule.

So, we know that $1 - R$ which would be the unreliability of individual component. They will follow this product rule. So, this to the power m will give you the unreliability of the individual system that is Q_1 . So, this will be Q_2 ; will be equal to $1 - R$ to the power m and so on. Now, as we know that for n number units connected in series their reliability will follow this product rule. So, if Q_1 is that much, so R_1 will be equal to $1 - R$ to the power m and this will be same for n number of units. So, overall this reliability of the system will be R_{sys} will be equal to this to the power n which is written over here ok. So, this is how we can determine the composite reliability for a system having n number of subsystem which are connected in series parallel combinations. Here we have in multiplied by n number of units. They are connected in series parallel fashion.

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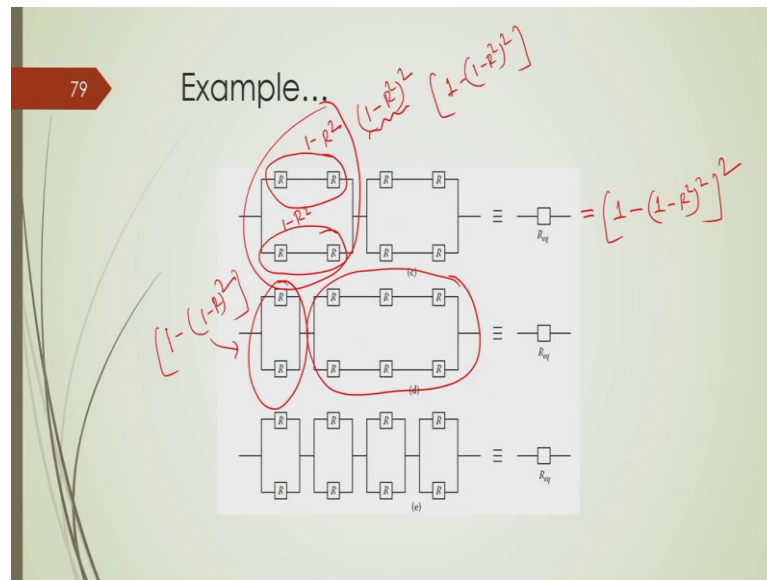


Now, we have some examples, this is easy example only you need to determine that composite reliability or equivalent reliability of several subsystems connected in different fashion. For example, here 4 units are connected in series. So, composite reliability $R_{equivalent}$ will be equal to R to the power n , sorry here n is equal to 4. So, we can directly write as R to the power 4 ok.

Similarly, here this reliability level of this will be R to the power 4 and this will be also R to the power 4 ok and they are in parallel. So, they are equivalent, they will follow this product rule of their unreliability function. So, unreliability function will be $1 - R$ to

the power 4 and that square will be total unreliability function of that system. So, reliability will be R equivalent will be equal to 1 minus this ok. So, this is the unreliability of this both this unit and so reliability will be that much.

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Similarly, all other ok and it is very simple to determine ok.

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80 Solution...

- (a) The equivalent system reliability for the series system is:

$$R_{eq} = R_{sys} = \prod_{i=1}^4 R_i = \underline{(0.85)^4 = 0.5220}$$
- (b) The equivalent system reliability for the parallel-series system is:

$$R_{eq} = \underline{1 - (1 - R^4)^2} = 1 - [1 - (0.85)^4]^2 = 0.7715$$

So, first one is R to the power 4, second one is as I said 1 minus 1 minus R to the power 4 square. So, this so third will be this example will be here the reliability is R square here the reliability is R square and they will follow this product rule of their unreliability

function that is 1 minus R square ok. So, overall unreliability of this unit will be equal to 1 minus R square, square.

So, this will be total unreliability of this unit ok. Now, reliability will be 1 minus 1 minus R square, square. Now, we have such kind of two units and connected in series. So, overall R equivalent will be equal to 1 minus 1 minus R square, square that is square ok.

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Solution...

■ (c) The equivalent system reliability for the mixed-parallel system is:

$$R_{eq} = [1 - (1 - R^2)^2] [1 - (1 - R^2)^2]$$

$$= [1 - (1 - 0.85^2)^2] [1 - (1 - 0.85^2)^2]$$

$$= 0.8519 \quad \checkmark$$

■ (d) The equivalent system reliability for the mixed-parallel system is:

$$R_{eq} = [1 - (1 - R)^2] [1 - (1 - R^3)^2]$$

$$= [1 - (1 - 0.85)^2] [1 - (1 - 0.85^3)^2]$$

$$= 0.8320 \quad \checkmark$$

So, this is probably determined over here. So, this is 1 minus 1 minus R square square ok alright. So, similarly we determine this; similarly we can also determine the other. So, it is not difficult at all. So, if you try you can find out, here the overall composite reliability will be equal to 1 minus R square ok. So, 1 minus R is basically unreliability of that unit.

And since we have two units in parallel, so 1 minus 1 minus R square will be the reliability of that unit ok, this has to be multiplied with the reliability with this unit. So, you can find out. So, as I said, you can find out there should be as bracket here which is missing yeah ok.

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Solution...

- (e) The equivalent system reliability for the series-parallel system from Equation is:

$$R_{eq} = [1 - (1 - R)^2]^4$$

$$= [1 - (1 - 0.85)^2]^4$$

$$= 0.9130$$

So, in that way you can find out and also you can find out for any combinations for example, here or whatever combinations ok.

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Example...

- Assume that a system has five components, namely, A, B, C, D, and E, as shown in Figure, and that each component has different reliability as indicated in the figure. Determine the following:
- a. The equivalent system reliability.
- b. If the equivalent system reliability is desired to be at least 0.8, or 80%, design a system configuration to meet this system requirement by using each of the five components at least once.

Good (medium) Very Good reliability level poor

✓ ✓

Ref.: T. Gonen. Electric Power Distribution System Engineering

Now we have another example; we have another example. This example says that we have five components A, B, C, D and E they are connected in series combination like this, A, B, C, D, E ok. And their individual reliability value is also given R A is equal to given 0.8, R B is equal to given 0.95, R C is equal to given 0.99, R D is 0.9 and R E is 0.65.

Then find out the first question; find out the equivalent system reliability. Find out the equivalent system reliability which is not difficult as you know, since they are there are 5 components and all components are in series combination. So, R equivalent will be equal to R A multiplied by R B multiplied by R C multiplied by R D multiplied by R E.

So, whatever we will get that will give this equivalent system reliability ok. Now main interesting point is question number b, which says that if the equivalent system reliability is desired to be at least 0.8 or 80 percent, then design a system configuration to meet the system meet the meet this system requirement by using each of the five components at least one ok, by using each of the five components at least one. So, so let me check that what is the equivalent system reliability value we get, that is by multiplying this 0.8, 0.95, 0.99, 0.9, and 0.65.

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84 Solution...

► (a) The equivalent system reliability is:

$$R_{eq} = \prod_{i=1}^5 R_i$$
$$= (0.80)(0.95)(0.99)(0.90)(0.65)$$
$$= 0.4402 \text{ or } 44.02\%$$

We get this value as 0.44 ok or 44.02 percent. If you multiply this individual reliability level of all these five units or five subsystems then the composite reliability is coming out to be this ok. If you look at this value then you see we have two components, one is having reliability is 0.65 another is reliability is 0.8 because of these two the composite reliability become very less ok. Because see other three units having higher reliability value 0.95, 0.99, 0.9.

So, their reliability values are pretty high, but since that unit consists of 5 subsystems connected in series and out of which one component is having very poor reliability level that is 0.65. That composite reliability level is become very very less as compared to these three units ok and this unit is also having moderate reliability level. So, what I mean to say is out of these five units we have at least three units B, C, D they are having very good reliability level.

But we have unit that is unit E which is very poor reliability level or we can write is poor reliability level and we have another unit A which is having good or moderate at reliability level; good or moderate reliability level. But because they are connected in series, so composite reliability is even less than the my unit which is having the poor reliability level which is having the least reliability level that is 0.44; which way I have in fact, explained when I talked about this series units or a number of units connected in series.

Now, because of that this overall reliability level of the system becomes poor. So, what the second question is, we need to improve this reliability level of the composite system at least 80 percent, which is equal to this component at least using one component at least one time because they have individual functionalities ok. Now, in practical power systems we may encounter this problem; and how do we solve this problem or how do we come out from this problem?

Suppose I have in fact, if you visit any power generation stations or power generation units you will see there are many components, many auxiliary motors, many auxiliary switches, many auxiliaries' circuit breakers and a number of devices which are required to produce the power ok. Now, all these components are very important, if one component is down the overall we would not generate any power.

That is why if you visit any power generating station you will see for a certain task if we need one component ok. Generally we have multiple backup components so that if that component fails other backup component can be used to keep the work intact ok, keep the main functionality of the system intact ok. So, here also you have one of the ways to improve this composite reliability of a system having n number of systems connected in series is to use multiple units for those component which are more prone of fault which are having a less reliability level ok.

And that is exactly done in order to achieve this composite reliability level.

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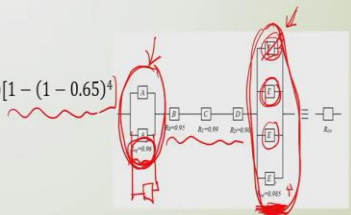
Solution...

(b) In general, the best way of improving the overall system reliability is to protect the less-reliable components with parallel components. Therefore, since the relatively less-reliable components are A and E, these can be backed by parallel redundancy as shown in Figure. Therefore, the new equivalent system reliability becomes

$$R_{sys} = \prod_{i=1}^5 R_i$$

$$= [1 - (1 - 0.80)^2](0.95)(0.99)(0.90)[1 - (1 - 0.65)^4]$$

$$= 0.8004 \text{ or } 80.04\%$$



So, here as you see we have out of this 5 components as I categorised them these 3 components B, C, D having very good reliability level. So, their failure rate is very less. So, we do not bother about their failure, but since these two units R A and means a component A and component E particularly this component E this unit is this component is more prone to faults. So, what we need? We need to have some back up identical such units which should be connected in parallel so that the overall composite reliability becomes high ok.

That is what exactly done here. So, what we did? We use B, C, D as they are and we use multiple components multiple E components which are connected in parallel like this so that equivalent reliability level of this you know will improve. In fact, if we use 4 you know same units of this particular component E in parallel then the composite reliability level is improved to be 0.985 which is pretty high ok.

So, that if one unit fails so other unit can be used in service so that the overall functionality of the system will be intact ok. Similarly, this since this reliability level of this component is very poor. So, use 4 identical units, you can increase the number and that will improve this equivalent reliability level of the system ok. So, here we have seen that by increasing it to 4 numbers, we got at least this desired reliability level.

So, this is a kind of trial and error function you keep in increasing this particular component multiple times in parallel so that this you will get this composite reliability level to the desired value. And here we got that 4 E components are 4 E subsystems if you connect in series, and 4 A subsystems, if you connect in parallel. Then their individual reliability levels will improve to 0.96 and 0.988 985 which makes the overall reliability of the composite system to the desired level ok, to the desired level which is 80 percent ok.

So, how many number of this component A or component E will be using? That will be trial and error ok, you keep on increasing this component E so that you get a moderate value of, not moderate you get very high value of the individual reliability level. And similarly you can keep on increasing instead of these two components of A if you add another component.

Of course, you will get the composite reliability level will be higher than 0.96 ok and that will make this overall system reliability higher than 80 percent ok. So, how many component you will be using for A and how many components you will be using for E that can be determined in a trial and error method ok alright. So, the basic functionality should be understood that because of this you know very poor reliability level of this component E, we use several units are in parallel.

And this at a time it may so happen one particular unit will be in operational and other units would be kept as a backup; would be kept as backup so that if there is any fault of that unit which is in operational immediately, we can run the other component ok. Or it may so happen that you can keep two units operational both will perform 50 percent of their actual required work and other two can be kept as a backup.

Same thing you can see if you visit any power station, particularly there are some motors where which have some specific work and we need only two motors to have a full generation, but we keep the total number at least 4. So, that if anyone trips another immediately come another will be brought to the service immediately ok.

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Example...

- Assume that a three-phase transformer bank consists of three single-phase transformers identified as A, B, and C for the sake of convenience. Assume that (1) transformer A is an old unit and therefore has a reliability of 0.90, (2) transformer B has been in operation for the last 20 years and therefore has been estimated to have a reliability of 0.95, and (3) transformer C is a brand new one with a reliability of 0.99. Based on the given information and assumption of independence, determine the following:

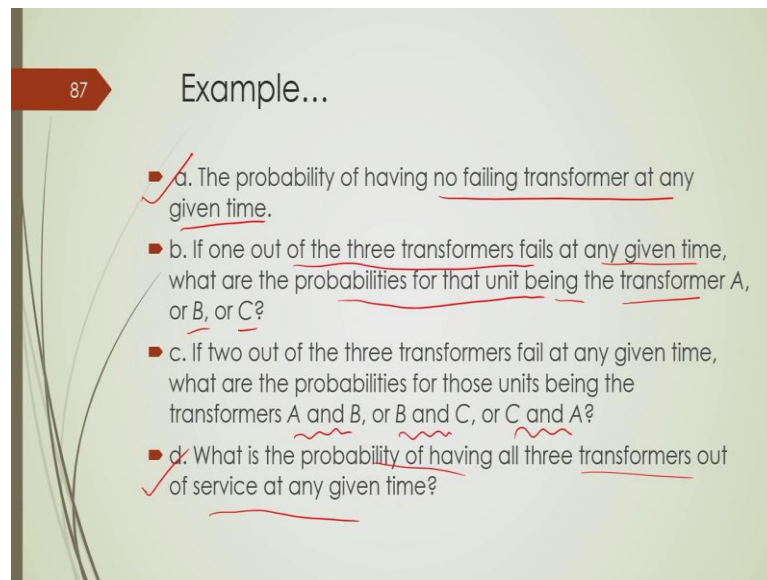
$R_A = 0.9$
 $R_B = 0.95$
 $R_C = 0.99$

Ref.: T. Gonen. *Electric Power Distribution System Engineering*

Now, we have another example, this example is of a three-phase transformer, this three-phase transformer bank consists of three single-phase transformers identity as A, B and C ok. And transformer we often know that one three-phase transformer is made of three single phase transformer bank same thing the problem is of similar concern ok.

So, transformer A is an old unit and therefore, reliability is 0.9 ok, transformer B has been operational for last 20 years and therefore, its reliability is 0.95 and transformer C is a brand new and its reliability is 0.99. Now, you can see this ageing effect is visible here with time this reliability level of individual component will reduce which I discuss at the very beginning of this mathematical description of failure function and the reliability function ok.

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Example...

- a. The probability of having no failing transformer at any given time.
- b. If one out of the three transformers fails at any given time, what are the probabilities for that unit being the transformer A, or B, or C?
- c. If two out of the three transformers fail at any given time, what are the probabilities for those units being the transformers A and B, or B and C, or C and A?
- d. What is the probability of having all three transformers out of service at any given time?

So, based on this information determine the probability of having no failing transformer at any given time, that is basically the reliability level of the overall system having three transformers. Second is if one out of these three transformers fails at any given of time what would be the probabilities of that unit being the transformer A, transformer B and transformer C; that means, what is the probability of the failure of individual transformers A, B and C ok.

And if there is a simultaneous failure of two transformers, what would be the probability of the transformers A and B to simultaneously fail and B and C to simultaneously fail and C and A to simultaneously fail? And what would be the probability of having all these three transformer out of service on a given time ok.

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88 Solution...

■ (a) The probability of having all transformers operational without fail at any given time is:

$$\begin{aligned}P[A \cap B \cap C] &= P(A)P(B)P(C) \\&= (0.90)(0.95)(0.99) \\&= 0.84645 \quad \underline{84.645\%}\end{aligned}$$

So, we know that individual reliability level transformer A, R A is equal to 0.9, transformer B, R B is equal to 0.95 and transformer C, R C is equal to 0.99 ok, this is something which is given to us ok. Now, what would be that probability of having all the transformer operational without any fail? So, that will be the reliability level of the system. So, you multiply all these three you get that is 0.84645 which is similar to 84.645 percentage ok. So, that much time, that percentage of time there will be no faults at all ok or system will be operational in healthy condition.

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89 Solution...

■ (b) If one out of the three transformers fails at any given time, the probabilities for that unit being the transformer A, or B, or C are:

$$\begin{aligned}P[\bar{A} \cap B \cap C] &= P(\bar{A})P(B)P(C) \\&= (0.10)(0.95)(0.99) \\&= 0.09405 \quad \checkmark \\P[A \cap \bar{B} \cap C] &= P(A)P(\bar{B})P(C) \\&= (0.90)(0.05)(0.99) \\&= 0.04455 \quad \checkmark \\P[A \cap B \cap \bar{C}] &= P(A)P(B)P(\bar{C}) \\&= (0.90)(0.95)(0.01) \\&= 0.00855 \quad \checkmark\end{aligned}$$

P(A) → probability for transformer A NOT to fail
This is a scenario of one transformer fault at any time

Now, what would be the failure if we have one fault of individual transformer at a time. So, if we have you know one fault at a time. So, this event is translated as A prime. So, probability of A is basically representing this reliability of the system that is probability for transformer A not to fail not to fail ok.

Now, what would be that probability of the transformer or failure probability of the transformer? Which is 1 minus this P A, which is equal to P A prime which is 0.1 and for that particular event that probably is this. Similarly, this you know this basically represents that A and C are healthy and B is faulty. This particular event is representing B and C healthy, A is faulty and this particular event representing A and B healthy, C is faulty and their individual probability values you can determine ok.

So, this is an example of one transformer fault at a time. So, this is a scenario of one transformer faults fault at any time ok. So, individual probably you know, you can determine.

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90 Solution...

■ If two out of the three transformers fail at any given time, the probabilities for those units being the transformers A and B, or B and C, or C and A are

$$P[\bar{A} \cap \bar{B} \cap C] = P(\bar{A})P(\bar{B})P(C) = (0.10)(0.05)(0.99) = 0.00495$$

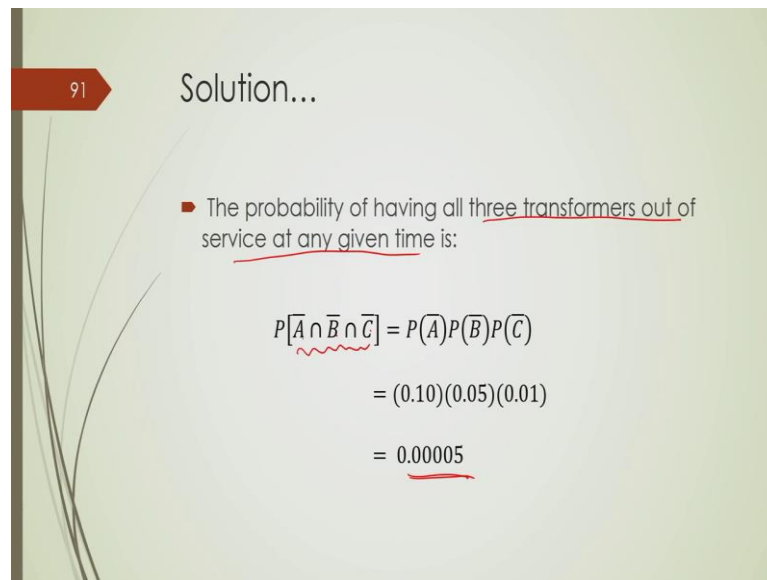
$$P[\bar{A} \cap \bar{B} \cap \bar{C}] = P(\bar{A})P(\bar{B})P(\bar{C}) = (0.90)(0.05)(0.01) = 0.00045$$

$$P[\bar{A} \cap B \cap \bar{C}] = P(\bar{A})P(B)P(\bar{C}) = (0.10)(0.95)(0.01) = 0.00095$$

Simultaneous failure of two transformers

Similarly, if we have simultaneous failures of the transformer, so here this is the example of simultaneous failure of two transformers ok. So, we have two transformer failed at a time that is A and B, C is healthy ok, here we have two transformer B and C are having this failure where A is healthy, and this is the case where this A and C are will encounter faults simultaneously when B is healthy ok. So, individual probability you can find out alright.

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Solution...

- The probability of having all three transformers out of service at any given time is:

$$\begin{aligned}P[\bar{A} \cap \bar{B} \cap \bar{C}] &= P(\bar{A})P(\bar{B})P(\bar{C}) \\&= (0.10)(0.05)(0.01) \\&= \underline{0.00005}\end{aligned}$$

Now what would be the probability of having all the three transformers out of service at any given time which is equal to this probability or this as you know this intersection of these three events where A prime B prime and C bar or A bars B bar and C bar they represent the failure event of individual transformer. When this happens simultaneously so this probability is equal to this ok.

So, here since we have three units there are you know four different scenarios. On the scenario 1 is all the units will be in healthy, scenario 2 is there would be one fault, one transformer failure at any time. Scenario 3 is there would be two simultaneous failures or two or two failures; that means, two units of transformer will fail simultaneously. And the fourth scenario is all this transformer will fail simultaneously ok.

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Summary of the solution...

Summary of the Computations

Number of failed Transformers	System Modes	Probability
0	$A \cap B \cap C$	0.84645
1	$\bar{A} \cap B \cap C$ $A \cap \bar{B} \cap C$ $A \cap B \cap \bar{C}$	0.09405 0.04455 0.00855
2	$\bar{A} \cap \bar{B} \cap C$ $A \cap \bar{B} \cap \bar{C}$ $\bar{A} \cap B \cap \bar{C}$	0.00495 0.00045 0.00095
3	$\bar{A} \cap \bar{B} \cap \bar{C}$	0.00005

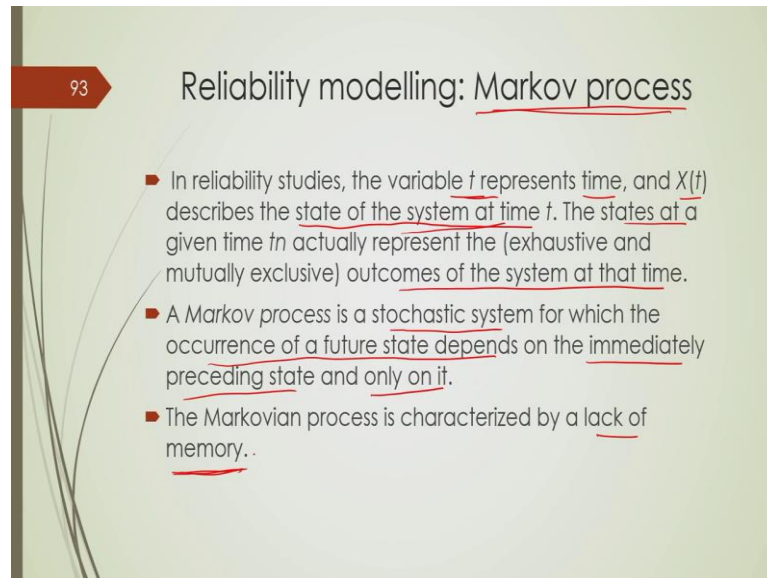
Handwritten notes:
no failure → 0
One transformer fault → 1
Two transformers faults → 2
All failure → 3

$$= \sum 1.00000$$

And if you add up all this probability value you will get this total value will be equal to 1, which is the sum total of all these 4 events. So, 0 stands for no failure, 3 stands for all failure, 1 stands for one transformer fault. So, this happens for 3 times; that means, when A failed, when B failed and when C failed and this is another scenario where two transformers fault, two transformers faults will take place simultaneously.

And there are three possible cases, one is when A and B here this would be B prime, A and B will simultaneously fail when B and C are simultaneously fail and when A and C are simultaneously fail ok. So, if you add all these you will get the overall event probability is equal to 1.

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Reliability modelling: Markov process

- In reliability studies, the variable t represents time, and $X(t)$ describes the state of the system at time t . The states at a given time t_n actually represent the (exhaustive and mutually exclusive) outcomes of the system at that time.
- A Markov process is a stochastic system for which the occurrence of a future state depends on the immediately preceding state and only on it.
- The Markovian process is characterized by a lack of memory.

Now at the end of this particular lecture I will talk about something called stochastic modelling of this reliability and I will discuss this Markov process ok; Markovian chain ok. So, in reliability studies the variable t represents time that we know and $X(t)$ represents state of the system at time t . Now, what do you mean by state of the system? So, the state at any given time actually represents the outcome of the system at that time. Alternatively, you can consider that a state is defined as the property of a system which adequately represents the system behaviour ok.

So, in Markov process which is a stochastic process it says that occurrence of any future state depends on the immediately preceding state and only on it. That means for any particular state occurrence or probability of the occurrence of that state is to only depend upon its previous state, not all the preceding states ok. And that is why this Markov process called by lack of memory, it is a kind of memory less stochastic process ok.

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Reliability modelling: Markov process

- A discrete parameter stochastic process, $\{X(t); t = 0, 1, 2, \dots\}$, or a continuous parameter stochastic process, $\{X(t); t \geq 0\}$, is a Markov process if it has the following markovian property:

$$\begin{aligned} P\{X(t_n) = x_n | X(t_1) = x_1, X(t_2) = x_2, \dots, X(t_{n-1}) = x_{n-1}\} \\ = P\{X(t_n) = x_n | X(t_{n-1}) = x_{n-1}\} \end{aligned}$$

And we have some mathematical description to describe this. So, this $P\{X(t_n) = x_n | \dots\}$ basically representing this probability of this state X at time t_n which is also represented by small x_n . Such that that $X(t_1)$ is represented x_1 ; that means, this state X at this time t_1 is represented by small x_1 ; $X(t_2)$ to represent by small x_2 and so on ok. So, this is basically similar to this probability of $X(t_n)$ is equal to x_n and its preceding state that is at t_{n-1} is equal to x_{n-1} ok so this is called Markovian property.

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Reliability modelling: Markov process

- For any set of n time points, $t_1 < t_2 < \dots < t_n$ in the index set of the process, and any real numbers x_1, x_2, \dots, x_n . The probability of is called the transition probability and represents the conditional probability of the system being in x_n at t_n , given that it was x_{n-1} at t_{n-1} . It is also called the one-step transition probability due to the fact that it represents the system between t_{n-1} , and t_n .

$$P_{x_{n-1}x_n} = P\{x(t_n) = x_n | x(t_{n-1}) = x_{n-1}\}$$

Now, we have a concept of transition probability from one state to another state. For example, this basically gives you the transition probability of state x_n from the state x_{n-1} . So, for any state of n time points where t is gradually increasing, t_1 less than t_2 less than to t_n , t is gradually increasing and there are real number x_1, x_2 to x_n the probability of or rather transition probability is the conditional probability of the system for being at x_n at t_n , given that it was at x_{n-1} at t_{n-1} ok.

So, it means that it gives a probability that any system will be at x_n at t_n is equal to t_{n-1} provided that it was at x_{n-1} that is this at t_{n-1} ok. So, this basically gives this one step transition probability ok.

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96 Reliability modelling: Markov process

■ For k-step transition probability,

$$P_{x_n, x_{n+k}} = P\{X(t_{n+k}) = x_{n+k} | X(t_n) = x_n\}$$

So, we can also find out k step transition probability; that means, any system state will be at x_{n+k} at t_{n+k} provided that it was at x_n at t_n is equal to t_n ok.

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Reliability modelling: Markov process

- A Markov chain is defined by a sequence of discrete-valued random variables, $\{X(t_n)\}$, where t_n is discrete-valued or continuous. Therefore, one can also define the Markov chain as the Markov process with a discrete state space.

$$p_{ij} = P\{X(t_n) = j | X(t_{n-1}) = i\}$$

- As the one-step transition probability of going from state i at t_{n-1} to state j at t_n and assume that these probabilities do not change over time. The term used to describe this assumption is stationarity.
- If the transition probability depends only on the time difference, then the Markov chain is defined to be stationary in time.

And also, we have a concept of Markov chain, Markov is a Russian mathematician. So, he defined this concept called as Markov chain which is defined as a sequence of discrete value random variable which is represented by $X(t_n)$ where, t_n is the discrete value or continuous. Therefore, one can also define the Markov chain as Markov process with discrete state space where this p_{ij} is equal to this probability for having the system at j at t is equal to t n provided that it was at i, state i at t is equal to t minus n ok.

So, at one step transition probability of going from state i at t minus 1 to state j at t n and assume that these probabilities do not change over time ok. So, this probability is constant over time and is that is why it is also called as stationary property ok.

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98 Reliability modelling: Markov process

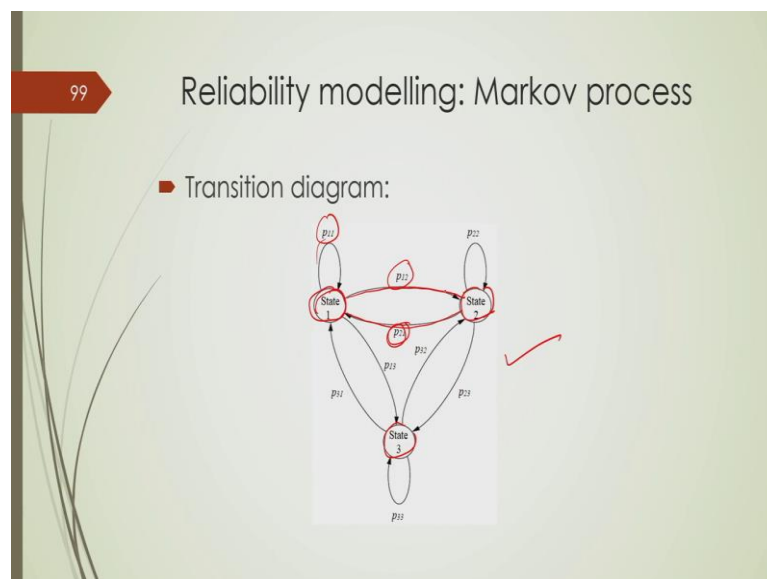
■ A Markov chain is completely defined by its transition probabilities, of going from state i to state j , given in a matrix form:

$$P = \begin{bmatrix} p_{00} & p_{01} & p_{02} & p_{03} & \dots & p_{0n} \\ p_{10} & p_{11} & p_{12} & p_{13} & \dots & p_{1n} \\ p_{20} & p_{21} & p_{22} & p_{23} & \dots & p_{2n} \\ p_{30} & p_{31} & p_{32} & p_{33} & \dots & p_{3n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ p_{n0} & p_{n1} & p_{n2} & p_{n3} & \dots & p_{nn} \end{bmatrix}$$
$$\sum_{j=0}^n p_{ij} = 1 \text{ for all } i$$
$$p_{ij} \geq 0 \text{ for all } ij$$

The matrix P is called a one-step transition matrix (or stochastic matrix), since all the transition probabilities p_{ij} 's are fixed and independent of time.

Now, in Markov chain we determine a probability matrix, it is represented by a probability matrix which is called one step transition matrix ok. Where all these individual elements are representing this transition probability; that means, p_{00} represent the transition probability of any particular system to stay at 0 state if it was at 0 state before ok. Similarly, p_{01} represents the transition probability to state 1 provided that it was at state 0 before and so on ok. And, sum of these, p_{ij} over j is equal to 1 that is basically equal to 1 for all i ok.

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Now, this is called a transition diagram where we have three different states of a system and this arrow basically represents this transition probability. So, p_{11} represents the transition probability to stay at state 1, provided that it was at state 1 before ok. And p_{12} is basically transition probability to go at state 2 provided that it was at state A before. Similarly, p_{21} is transition probability to go to state 1 provided that it was at state 2 before and so on ok.

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Reliability modelling: Markov process

- Example: The records of distribution utility indicate that only 2% of the transformers that are presently down and therefore being repaired now will be down and therefore will need repair next time. The records also show that 5% of those transformers that are currently up and therefore in service now will be down and therefore will need repair next time. Assuming that the process is discrete, markovian, and has stationary transition probabilities, determine the transition diagram.

UP
down.

Ref.: T. Gonen. Electric Power Distribution System Engineering

So, I have a small example which would be useful to understand the concept that a distribution utility having a 2 percent of transformers that are presently down and therefore, being repaired now will be down also and therefore, we will need to repair next time. So, there are 2 percent transformers which are presently down and after repairing there is a probability that they will be down ok.

And records show that the 5 percent of those transformers that are currently up and therefore, in service now will be down. And therefore, we will need repair next time ok assuming that the process is discrete Markovian and has a stationary transition probability, stationary transition probability means this transition probabilities are independent or constant does not vary with time. So, determine the transition diagram ok. So, here we have two states, one is state up another is state down. So, we have two states one is up another is down, so it is a two state model.

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101 Reliability modelling: Markov process

- Let t and $t + 1$ represent the present time (i.e., now) and the next time, respectively. Therefore, the associated conditional probabilities are:

$$\begin{aligned} P\{X_{t+1} = \text{down} | X_t = \text{down}\} &= 0.02 \\ P\{X_{t+1} = \text{up} | X_t = \text{down}\} &= 0.98 \\ P\{X_{t+1} = \text{down} | X_t = \text{up}\} &= 0.05 \\ P\{X_{t+1} = \text{up} | X_t = \text{up}\} &= 0.95 \end{aligned}$$

So, we represent that individual probability that those transformer which are down presently they will be down in after repair also, that probability is 0.02. Now, it is also given that some transformer which are down will be up that probability is equal to 0.5, 0.05 that is 5 percent transformer which are presently up and therefore, in service will be down ok. So, they are now up, in future they will be down ok, and so who are up will be up will be also up that will be 1 minus 0.5 that is 0.95 and presently who are down, who will be up will be equal to 1 minus 0.2 that is equal to 0.98 ok.

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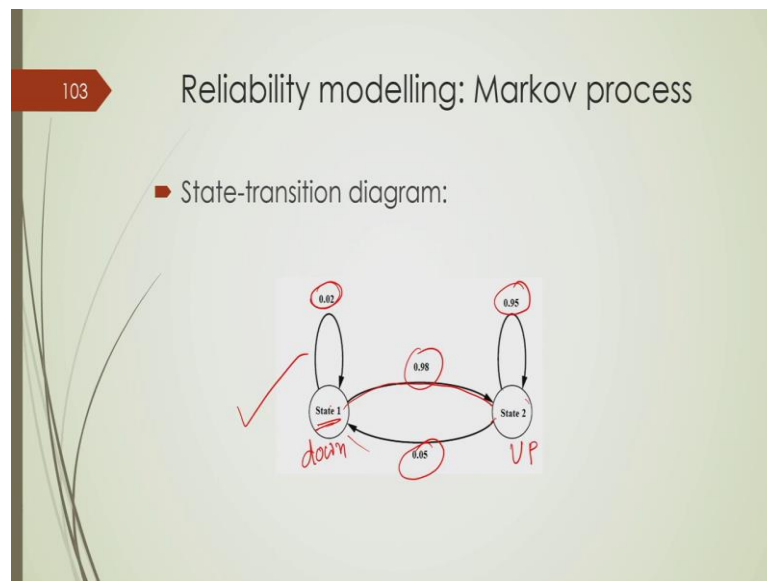
102 Reliability modelling: Markov process

- Let numbers 1 and 2 represent the states of down and up, respectively.

$$\begin{aligned} p_{11} &= 0.02 & p_{12} &= 0.98 \\ p_{21} &= 0.05 & p_{22} &= 0.95 \end{aligned}$$
$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \begin{bmatrix} 0.02 & 0.98 \\ 0.05 & 0.95 \end{bmatrix}$$

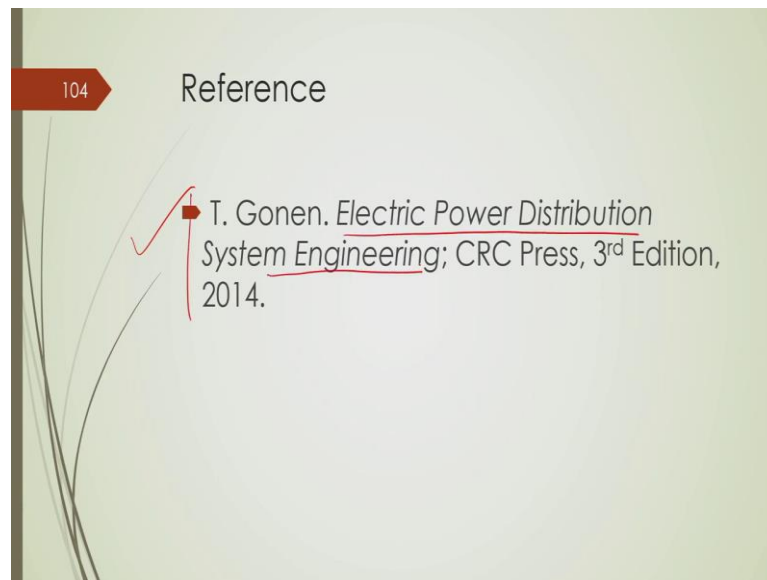
So, these are the elements a transition matrix and this transition matrix will look like this or at this represent this transition probability to stay at down for those whoever was down before. But this is the probability to switch from down to up ok. Similarly, this is the probability who are in up will be down and these are the probability who are up which will stay at up ok.

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Now, this is basically this transition diagram here you can see the state 1 is basically represented at down, state 2 is basically represented as up. So, those who are down will stay at down their transition probability is 2 percent. So, next 98 percent will go to up ok and those you know transformer who are up they will stay down that probability will be 5 percent and rest 95 percent will stay up ok. So, this is how we represent these fault events using this Markovian process.

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And all these materials I took from this book Gonen's book and this is the main reference for this particular module and this is of course, this part 1 this module 3. And another module, I will discuss next lecture, that will be part 2 of that module where we will talk about some power quality problems for typical distribution system ok.

So, thank you very much for attending this lecture.