Communication Engineering
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Lecture – 5
Analytic Representation of Band pass Signals – Hilbert Transform

If you remember, we were talking about the Hilbert transform yesterday and we looked at the properties in the Hilbert transform and we were on our way to using the Hilbert transform for representation of band pass signals and that is our topic for today. We will look at how the concept of Hilbert transform helps us to have a convenient representation of band pass signals, any band pass any general band pass signal, in terms of appropriate, what we call low pass signals and a sinusoidal signal. So, we will see that and that is a very convenient representation and has many applications, as we go along. So, let us review what we talked about yesterday.

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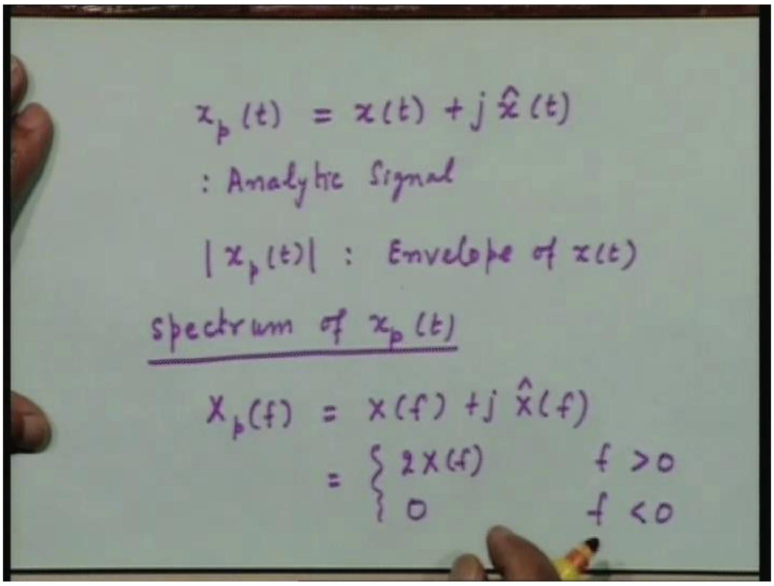

We have just refined the concept of an analytic signal x sub p t for a given real signal x of t as a complex valued signal, whose real part is the given signal x of t and whose imaginary part is the Hilbert transform of x of t. So, this is the analytic signal and one of the motivations that I gave you for defining the signal is of course, we have to appreciate that a little more, as we go along is that, while x sub p t we also call pre envelope of the signal x t.

The magnitude of x sub p t, the modulus of x sub p t would be the envelope of x t. Now the concept of envelope; I try to give you by taking a specific kind of signal. Particularly, for example, when a low pass signal m t, multiplies a carrier kind of signal cosine omega naught t and we found that the modulus of m t defines the envelope of this signal m t cosine omega naught t. However, not all signals may be of this kind; that is the low pass signal multiplied by the band pass signal.

Whereas, if you look at this definition of the envelope, as I have defined here, this is valid for all kinds of signals. So, in that sense, this concept of envelope is a generalization of the physical concept, that we discussed yesterday, that physical concept was something, that you could visualize by looking at the waveform. If you trace the peaks of that those oscillations the peaks define the envelope, the trace of these peaks define the envelope, we do not necessarily have to look at the envelope in that way.

You can look upon it mathematically, in this particular way and you can, we will see later that if you proceed like this, you will get the same answer, even for those kinds of signals, but then that say, that something that they we will look at later. Right now, let us proceed further and look at the spectrum of x p t, we are looking at the spectrum of x p t, now can we do that; let us look at take the Fourier transform of this. This will be x of f plus j times x hat f, what is x hat f, now this value for x, for f greater than 0, what will be the value of x hat f, j.

Student: ((Refer Time: 05:21))

minus j times x f, so what will happen to this, it will become minus j square minus j, ((Refer Time: 05:38)) so this it will plus, so it will become twice of x of f, for f positive. What happens for f negative, it will become 0, very interesting result, so what do you find.

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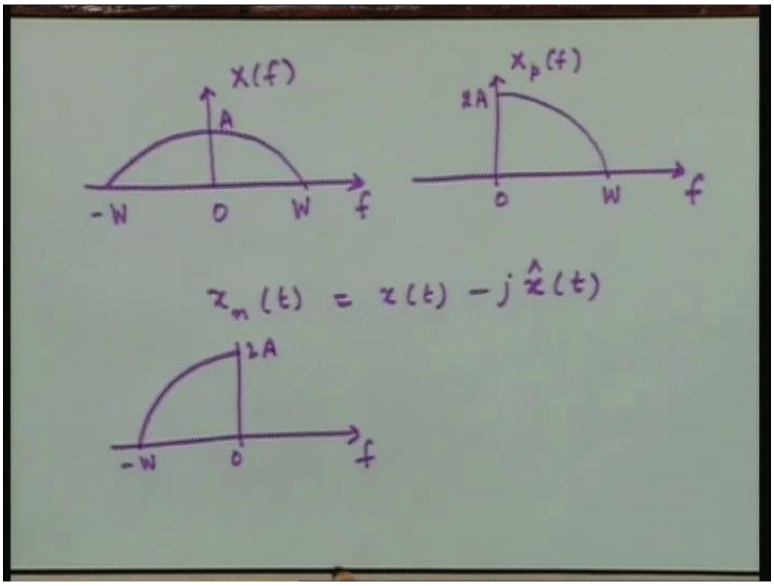

So, if you were to plot this spectrum, so suppose you had a spectrum like that, for x of t, a spectrum of x t, let us say x tends from some frequency minus sub w, minus w to plus w. Then, this spectrum of x p t of x p f will be extend, suppose this is A so this will be 2 A, going from 0 to w, so it becomes a one sided spectrum, from a two sided spectrum for the real signal, you get a one sided spectrum. Do you now see some, further analogy between what we are doing here and what we do when we go from a real sinusoid to a complex sinusoid.

The real sinusoid has a component at both f naught as well as minus f naught, the complex sinusoid has a component only at f naught, so we are doing the same kind of thing for any arbitrary signal. The real signal has both negative frequency components and positive frequency components. This spectrum would be amplitude even symmetric, amplitude spectrum of the even symmetric and phase spectrum of the odd symmetric.

The complex signal that we have constructed, the analytic signal that we have constructed, only has positive frequency components, just like the complex exponential. It has only a positive frequency component, because if you want you can also construct, the corresponding signal will only negative frequency components, what will you have to do for that. Suppose, I want to only negative frequency components, all you have to define is a new signal, x call it x sub n f which is equal to x sub sorry, x sub n t, which will be equal to x t minus j x hat t.

So, such a signal will have only negative frequency components, it will be also a one sided spectrum, so the spectrum of x n t will look something like that. So, you see a direct analogy, between what we are doing here and what we are already used to, when we try to represent a sinusoid with a complex exponential, as a real part of a complex exponential. So, again here also, the signal of interest which is a real signal is a real part of either x p t or x n p, the same relationship holds, very similar relationship.

So, the development is absolutely parallel to, what you already use very frequently in a context of sinusoids, one thing is now, you can use it in the context of an arbitrary signal. So, as you can see therefore the, concept of analytic signal that we have, now fixed up is an extension of the concept that you already know, for sinusoid signals. You agree with this and you feel as comfortable as, you do when you use complex exponentials for representing sinusoids. If that is the case, you can move forward and come to the main thing of interest to us here that is the complex envelope representation of band pass signals.

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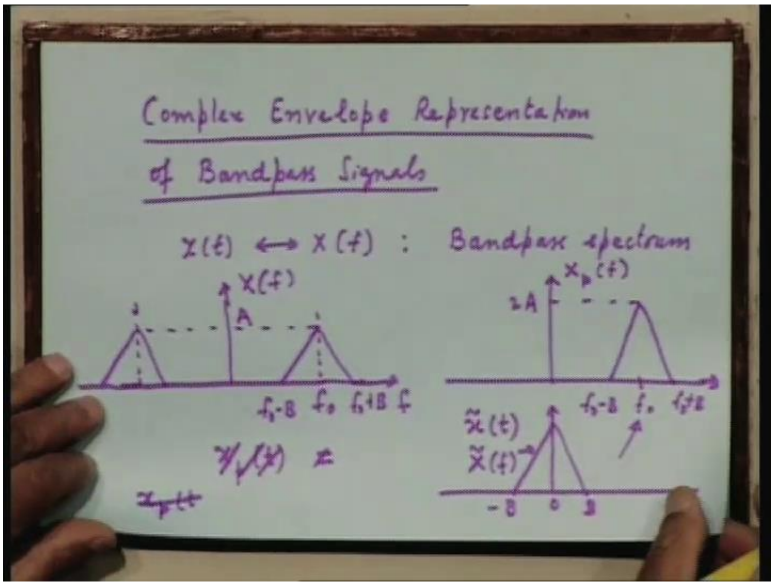

This particular representation or this particular theory is particularly useful, when you want to represent band pass signals. So, let us start with a signal x of t such that, it is Fourier transform x of f, corresponds to the band pass spectrum, so let us say, you start with a spectrum like that, band pass spectrum you understand what, that means, when I say band pass spectrum. That means, most of the energy of the signal will be centered, will be contained around some non-zero center frequency.

Not around d c, but around some finite non-zero center frequency, so there is a band of frequencies around which the signal contains most of it is energy and when a plot a signal like this, like a triangle or anything, it is just arbitrary shade that I have drawn, to indicate the extent of this spectrum. The actual shape is not important here, in our discussion, whether take a triangle here or some parabola or something else, is not important.

It is only and again we are looking at only the, plotting only the magnitude plot here, if there is a phase plot, it will be correspond it could be correspondingly plotted. So, the main purpose of drawing these figures when I draw the spectrum, is only to show the extent of spectral support, on the frequency axis, what is the frequency band or which nonzero or which, significant energy exists in the signal or some energy exists in the signal.

Example, for this signal, suppose this is f 0 minus B to f 0 plus B, here 2 B is the bandwidth, so we say roughly 2 B is the bandwidth of the signal. This signal has a bandwidth of 2 B and has a center frequency of f naught, again it is not necessary, that is spectrum is symmetric around f naught. I have drawn it, I mean just like that, there is no particular significance of this symmetry, ((Refer Time: 12:52)) it does not have to be symmetrical, what is required to be symmetrical is, the spectrum should be even symmetric around 0.

That is the required property of a real signal, the spectrum of the real signal, but you know symmetry requirement around the center frequency of, that you may want to define. I could as well have defined the center frequency anywhere in this band, instead of showing it here, I could have shown the center frequency anywhere in this band, it really does not matter. Only thing is, if I do it, if it is actually symmetric, then it is better to do it in the center of the spectrum.

Suppose, it is not symmetric, it has the value of f naught is not particularly important, as long as it lies within the band. You could choose it arbitrarily, but it has to be somewhere in the band that you are talking about, so this is the plot of x of f or magnitude of x of f, right here assume that the phase spectrum is 0, so we can assume that this is the plot of x of f itself. It can happen, you can have a phase spectrum which is 0, so for this band pass signal, there was a question here, you wanted to ask something, it is taking care of.

So, for this band pass signal, suppose I find the analytic representation of this signal, let us go from x of t to x p t, what will be the corresponding spectrum, what will be x p f. It will only contain positive frequency components with twice the height of the, suppose this is A, this peak value is A here, the peak value now will be 2 A. The center frequency will remain f naught, it will go from f naught minus p to f naught plus p and the peak value will be now 2 A.

And I know that, this x p t corresponding to this spectrum would be a complex valued signal, whose real part is x t and whose magnitude part is x, the Hilbert transform of x t. Can I see this signal this spectrum in a different way, can I see this spectrum as a translated version of some spectrum around 0. I can see that. I mean imagine a spectrum of this shape here and you have frequency translated it to f naught and how do you do frequency translation by f naught only. By multiplying it with the exponential, so therefore, it stands to reason, that I could represent this x p t as a frequency translated version of some low pass signal whose spectrum is this,

Student: need not be symmetrical. ((Refer Time: 16:07))

Need not be symmetrical, that will depend on whether this is symmetrical, need not be symmetrical. Therefore, that signal may not be real, it can be complex.

Student: ((Refer Time: 16:18))

No no here, suppose we had chosen a particular band pass center frequency f naught and I am shifting it, I mean I am considering this signal as a frequency translated version of a low pass signal. The way the frequency translation is by the amount of f naught, so same shape instead of centering it around f naught I will center it around 0. Let me first instead of first instead of writing the expression, what I am saying is, you can think of this as a translated spectrum, where the original spectrum is like this and the translation is by an amount f naught.

In this particular case, because I have taken this to be symmetrical, this looks symmetrical here, but if this was not symmetrical, this symmetry would not exist here.

So, all I am saying is, there is a frequency translation relationship between this signal that I am defining here and the signal x p f, the which is analytic representation of a original band pass signal, are you with me everyone. Now, how do I represent this mathematically?

You just told me, that if I want to if I call this signal x t dot t, suppose not the of course this is the spectrum of the signal. So, what I am plotting here is not x tilde t but, x tilde f here, this is a plot of x tilde f and the corresponding time domain signal I am calling x tilde t. This signal is a low pass signal, because it is spectrum centers around 0and in general it would be a complex valued signal, in general. In particular it could be real value, if the symmetry here exists.

In general it may complex valued signal and these two signals would be related by the frequency translation operation. That means, x p t would be x tilde t multiplied by e to the power j 2 pi f naught t plus because it is shifting to the right. So, you can think of x p t, maybe I should use the next page.

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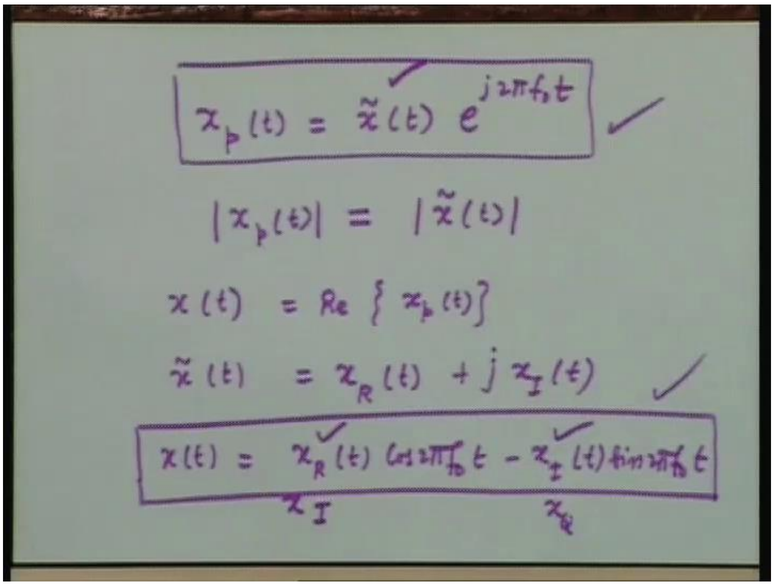

As the frequency translated version of a low pass signal, complex valued low pass signal x tilde t, so that x p t is equal to x tilde t e to the power j 2 pi f naught t. Now, that is a very interesting way of looking at it, you have already done half the job, what you have got is, that the analytic version of the band pass signal which was of interest to us, can be expressed as the product of, in complex valued low pass signal and e to the power j 2 pi f naught t, a complex exponential. Fine, any questions on this, also it is clear, that this envelope of this signal would be equal to what,

Student: ((Refer Time: 19:49))

Modulus of the signal is what, the modulus of this signal; now let us proceed a little further. My purpose is, final purpose is to express x of t, the original band pass signal in similar terms, because what I have got right now is a representation not for x of t, but for the analytic version of x of t. Suppose I want to go from here to x of t, what should I do, I should take the real part of the signal, x of t is real part of x p t, now since I said that x tilde t in the inner could be, complex value.

So, x tilde t, in general would have some real part and some imaginary part, let me call the real part as x sub R t and imaginary part as x sub I t, is it. I hope I am not using too much, introducing too much notation and confusing you, we have agreed that x tilde t is some low pass signal, we have also agreed that in general, this would be a complex valued signal. So, all I am doing is giving some notation to it is real part and some notation to it is imaginary part.

Nothing more than that and what I would like to, just like I have a representation of x p t in terms of x tilde t, what I would like to find out is, is there a corresponding representation of the original band pass signal in terms of these components, it is very simple. All you have to do is now, substitute for x p t as this into e to the power j 2 pi f naught t and take the real part of that, so what will you get. So, we will get x of t, what will be the real part of this multiplied with e to the power j 2 pi f naught t.

X R t cosine 2 pi f naught t minus x I t sin 2 pi f naught t and that is the result we are looking for. These two results, are what we are looking for, very interesting and very important results, so what do you find here, that the original band pass signal x of t, can be give a representation in this form, in this most general form, where it could be considered as sum of two product terms. One product term is of the kind x R t cosine 2 pi f naught t, the other product term is x I t sin 2 pi f naught t.

The band pass nature of the signal comes out from the fact that, there is a sinusoid present at frequency f naught, what is important is, it is a product of a low pass term and a single frequency term or a band pass term. So, the band pass signal can always be thought of as a product of a suitable low pass signal of an appropriate low pass signal with a carrier signal. That is ((Refer Time: 20:40)) what it really boils down to, a very significant result. For every band pass signal, there is a correspondingly corresponding low pass signal that is in the sense that, from that low pass signal I can generate the required band pass signal, by a simple frequency translation operation.

Student: ((Refer Time: 23:59)

Yes.

Student: what is the band pass ((Refer Time: 24:02))

What is the, which is low.

Student: what is the band pass signal is there any ((Refer Time: 24:06))

You tell me, you started with what was the assumption of x t, x t was band pass, you have forgotten. Our whole discussion on this point started with x t having this spectrum, you started with an x t will have a, which had a band pass spectrum. So our x t is band pass, we represented x t in terms of x p t, a real part of x p t, that is still band pass. We represented x p t in terms of x tilde t, which is low pass, x tilde t is low pass means, x sub R t and sub I t are both low pass signals, is it clear.

The real and imaginary parts of x tilde t are therefore, both low pass signals, sox R t and x I t are both low pass signals. So, this is a low pass, this is a low pass signal, this is a low pass signal, there is a complex valued low pass signal, these are both real valued low pass signals. So, I can represent x t, the original band pass signal, an arbitrary band pass signal in terms of two low pass signals multiplied with quadrature components of a carrier, we call this, sorry quadrature components of a carrier.

We call this cosine 2 pi f naught t as the in phase carrier and sin 2 pi f naught t as a quadrature phase carrier right and x R t therefore, is called the quadrature phase, the in phase component of x t and x of I t is called the quadrature phase component ofx t. So, sometimes in fact, you will find in the books, the notation here, so x sub I here and x sub Q here. Here, I used the notation real part and imaginary part of x tilde t, but in many books we also find the notation x sub I t here and x sub Qt here.

To indicate, that this represents the so called, in phase component of x t and in phase component is a low pass signal and this represents the quadrature phase component of x t. This, so I can basically what is also known as that the band pass signal can be resolved into two low pass signals, one of which is the in phase component, the other is a quadrature component, any questions.

Student: ((Refer Time: 27:01))

Let me explain again.

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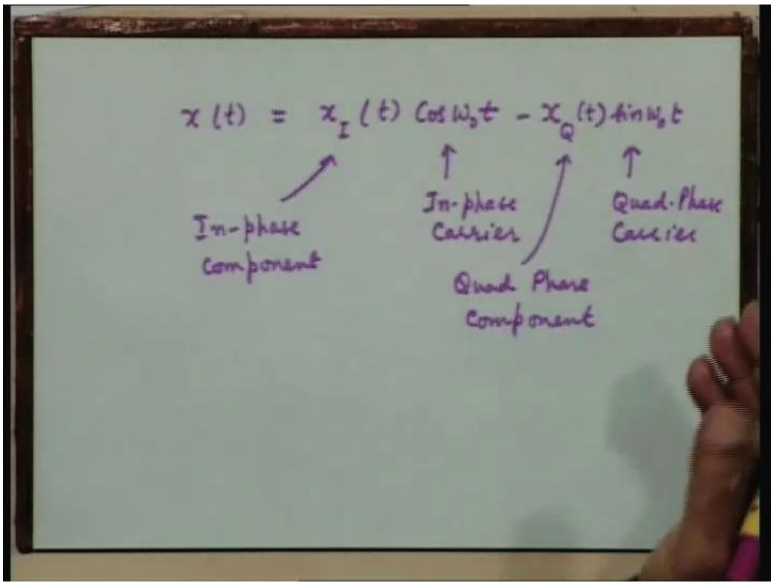

I am saying that, any band pass signal x of t we have found, the representation for it, if I slightly modify the notation myself, here itself. Let me call the real part now by x sub I t, although it is little confusing, but if I want to talk about in phase of quadrature phase, this is more appropriate, into cosine omega naught t minus, so I will call this x sub Q t, sine omega naught t. These two carriers are quadrature components of the carrier, e to the power j 2 pi f naught t, they are the real and imaginary.

So, you can just, it is a terminology, we call this as a inphase carrier, this as a quadrature phase carrier and these two are the in phase and quadrature phase components of x t, that is all, these are definitions, just to remember. So, again let me summarize, a even signal, a given band pass signal x t can be expressed or can be resolved into it is low pass components, one of which will be in phase with the carrier, the other will be in quadrature with the carrier.

Rather one of which, the in phase component of, it will have two components, one of which is called the in phase component, the other is called the quadrature phase component and I think that is more appropriate, rather say it is in phase with the carrier, that is actually meaningless, that does not convey anything. So, that is the band pass that is the representation of band pass signals.

Student: ((Refer Time: 29:12))

I do not know whether, that means anything, we have preserved whatever needs to be preserved, ((Refer Time: 29:35)) we have gone from here to here with twice the amplitude, this will remain twice the amplitude here.

Student: what are we going to get an amplitude here ((Refer Time: 29:43))

Sorry.

Student: what are we going to get an amplitude here ((Refer Time: 29:45))

Need to be what.

Student: ((Refer Time: 29:49))

Yes in this context, now how do these representations help us, this representation helps us in visualizing a band pass signal in terms of a low pass signal. So, that is that itself is a very satisfying result and you will find that, something that will appreciate more and more, as you deal more and more with band pass signals. You find that this representation makes a lot of sense and whenever you want to analyze the behavior of a communication system to band pass signals, you like to say, I like to feed the band pass signal to this.

And then you like to mathematically model that band pass signal and what is the mathematical model which is appropriate to use, this model. That is, assume that the input signal has this form and then proceed with the analysis, so very useful mathematical representation. Keeping in mind, that in this representation, you have two low pass signals and a carrier, it really helps to define things very precisely, equally useful is the other representation, from which we derive this one.

((Refer Time: 31:21)) So, this representation, let us see, in fact this is, in fact more powerful representation on this, in some sense, when we analyze. It has a same power that, the complex exponential has ((Refer Time: 31:38)) when say thus the real sinusoidal signals. For example, what I like to say is that the information dealing part of x p t is in fact where, you say x tilde t. This is just happens to be the center frequency and this is an exponential corresponding to the center frequency.

It has no particular information here we expecting, for example, it is a carrier, it is some it has I could as well change this to f 1 or f 2 or f 3, certainly x p t will change and x t will also change, but the information ((Refer Time: 32:17)) bearing part will be the still the same. So, this representation is, in that sense extremely powerful and therefore, it makes sense to think, that as far as my processing is concerned or as for my analysis is concerned, I could do most of the analysis without even worrying about this part.

In fact, that is the case, you could exclusively deal with x tilde t and get back to the result of processing or result of analysis by just multiplying whatever you have got with e to the power j 2 pi f naught t, that is the power of this. Let me illustrate this by taking a very specific example, have you understood what I am trying to tell you, but I will try to make it more clear by considering the situation.

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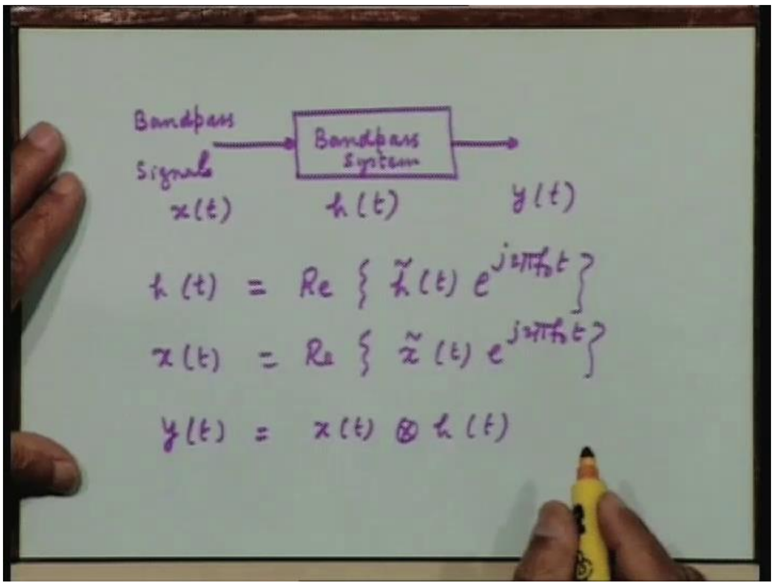

Where let us say, we are dealing with a band pass system, whose input is also a band pass signal, so at the input we have a band pass signal. I want to illustrate, how the query that you have developed, how the representation that you have developed, helps us, in dealing with band pass signals and systems. So far, you have looked at signals and systems purely in terms of, it could be any signal and it could be any system, now we are specifying that the, both the system as well as the signal are band pass in nature.

And in dealing with such systems, how do we, how is it easy to, how can we make it easy to deal with such systems, that is our concern here. A band pass signal is the input and the, so let us say this, this band pass system has an impulse response, h of t and you are feeding an input signal which is x of t, which is band pass in nature. For example, this could be a band pass filter; you are familiar with band pass filters.

That means, it will pass a certain set of frequency components in the neighborhood of some center frequency f naught and obviously, it will make sense to have a signal which has frequency components in that neighborhood. If it does not have any frequency components in that neighborhood, the output of the filter will be identically equal to. So, it makes sense that the input signal is also of at least some ((Refer Time: 35:02)) and let us say, it has it is a band pass signal in the same neighborhood.

And we want to shape the spectrum of the input signal by passing it through this filter and let us say the output is y of t. I know, how to deal with the systems, so you say what is the big deal, if I know h t, if I know x t I can find y t, by convolving x t and h t. But, what I am saying here is, I know that both h t and x t, they are essentially band pass in nature, sothey may have this quadrature representation that we just discussed, all the complex envelope representation that we just discussed.

Incidentally, this representation I probably forgot to mention that, excuse me, this representation ((Refer Time: 36:01)) is usually known by the name of complex envelope representation, soif I talk about a complex envelope representation, it is this, where x tilde t is called the complex envelope now. You see that the actually envelopes are both the, the real envelopes are both the, both of them are same and this is called the quadrature representation.

So, both cases representations have links, this one is called the quadrature representation, this is called the complex envelope representation. So, h of t itself has a complex envelope representation, is it not, because h of t is also band pass in nature. So I could think of this as real part of x sub p t and what is x sub p t, h tilde t e to the power j 2 pi f naught t, I am just repeating what I have already said for x of t. Similarly, x of t is the real part of x tilde t e to the power j 2 pi f naught t.

Are you with me?

Student: ((Refer Time: 37:38))

What did you say, yes we are keeping f naught same in both of them, because we assuming that, both the input signal and the band pass signal can be considered to have a same center frequency. Now, I want to find y of t, which is x of t convolve with h of t, let us go through this process and see, where do we lack, after doing that I want to make a few other remarks. Please remind me if I forget that and that is also very interesting remark.

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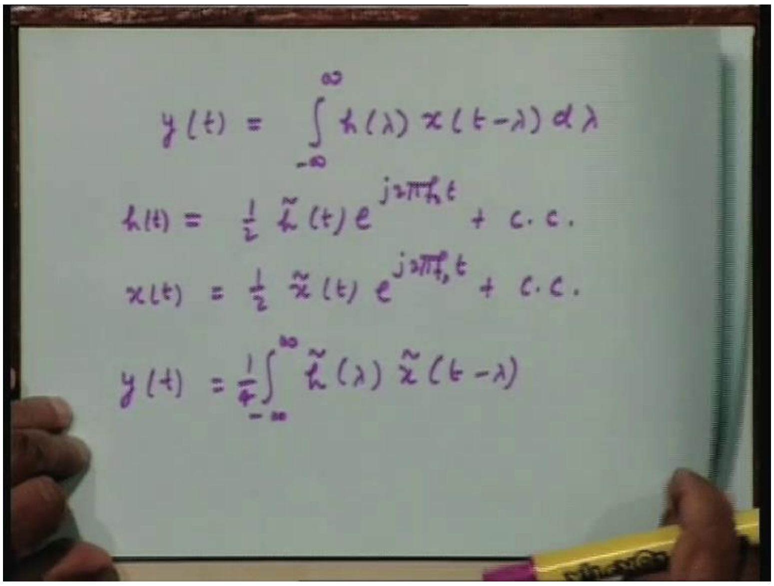

But let us first see, what is the result here, so let us proceed, let us use these two representations to find y of t which is a convolution of h t with x t, which you can have write like this. If I want to make use of ((Refer Time: 39:00)) these representation of h t and x t, in this integral how should I do, I should substitute for h t as, this quantity this term plus it is complex conjugate, half of that, the real part is, real part of any complex number is, the sum of the number and it is complex conjugate divided by 2.

So, I could represent h lambda here, lambda of course, this ((Refer Time: 39:34)) variable, which is minus infinity to infinity. I could write h of t as, half of h tilde t e to the power j 2 pi f naught t plus it is complex conjugate, so c here represents complex denotes complex conjugation. Similarly, x of t can be written as, half of x tilde t e to the power j 2 pi f naught t plus it is complex conjugate, therefore, y of t it is a fairly mechanical operation that I am carrying out.

So, what will you get, you will multiply this with x t minus lambda, this has two terms, this will also have two terms, so when you multiply this, we will get four terms. Let me write down these four terms, when you multiply this, the first term and the first term, you will get 1 by 4 h tilde lambda x tilde t minus lambda, what will happen to, maybe I should explicitly write the expression first, excuse me, let me explicitly write, y of t completely first, so that there is no confusion.

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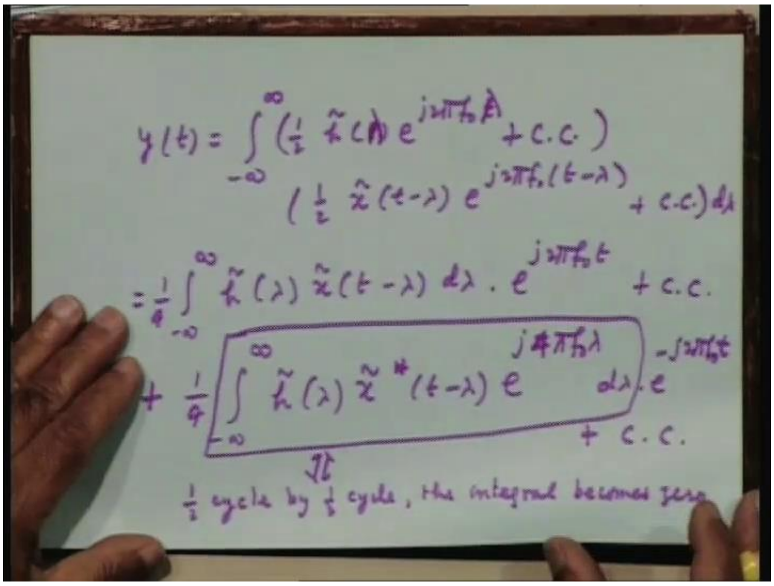


So, y of t is, I substitute for half h tilde t e to the power j 2 pi f naught t sorry lambda, this also is lambda plus it is complex conjugate into half x tilde t minus lambda e to the power j 2 pi f naught t minus lambda plus complex conjugate of this into d lambda. Well, complex conjugate also will contain half, so now, if I proceed term by term this is lambda there, what will be the product of the first two terms. This will be 1 by 4h tilde lambda, x tilde t minus lambda and what happens to the product of these two exponentials.

Student: ((Refer Time: 42:22))

e to the power j 2 pi f naught t, because the lambda term will cancel out and you can expect that the same thing will happen if I when I multiply the conjugate term with the conjugate term, except that it will be the conjugate of this whole term. So, I can say it will be the conjugate of this whole term, when I multiply this complex conjugate with this complex conjugate. Similarly, when I multiply across now, this with the complex conjugate what will you get.

You will get 1 by 4h lambda h tilde lambda into x complex conjugate t minus lambda and now what happens to the exponential product, it does not cancel off, you will get two terms, e to the power j 2 j 4 pi f naught lambda and also a term, e to the power j 2 pi f naught t. So, plus you will get a similar term which will be the conjugate of this term, I will keep this therefore a while, see if I made any mistake in writing this, is it. Now, what can you say about the third and the fourth term.

Just look at the third term, can you say something about the third term, that we claim that it is 0, do you agree with this. Can anyone justify my claim here, that this the third term and therefore the fourth term, would be both equal to 0, integral is around the center, what does that mean, I do not understand that.

Student: ((Refer Time: 44:30))

No we are just carrying out an integration with respect to the value of lambda, of course this term e to the power j 2 pi f naught t and e power minus j 2 pi f naught t, they come outside the integral, integration with respect to lambda. So, you do not even have to look at this term and this term, but look at the rest of the integral, you look at this part, concentrate on this part, I take this integral is zero. Could you agree with this and can you give me a reason for that, no.

Let me explain, what kind of a signal is this, what kind of a entity is this function, it is a low pass ((Refer Time: 45:21)) what about x, x tilde, that is also low pass, what is this, this is a carrier. Let us say x naught is sufficiently high, what will happen now, what it means is that the rate of variation, time variation in this function is small as compared to the rate of variation of the sinusoid, the rate of fluctuations here is much larger than.

So, if I consider let us say one cycle of this sine wave, which that cycle is completed very fast, over that cycle this hardly changes. This function hardly changes; it remains more or less constant.

Student: ((Refer Time: 46:11))

Assuming that this is varying much more slowly than this, this will be so, in which case, what will be the value of the integral over one cycle.

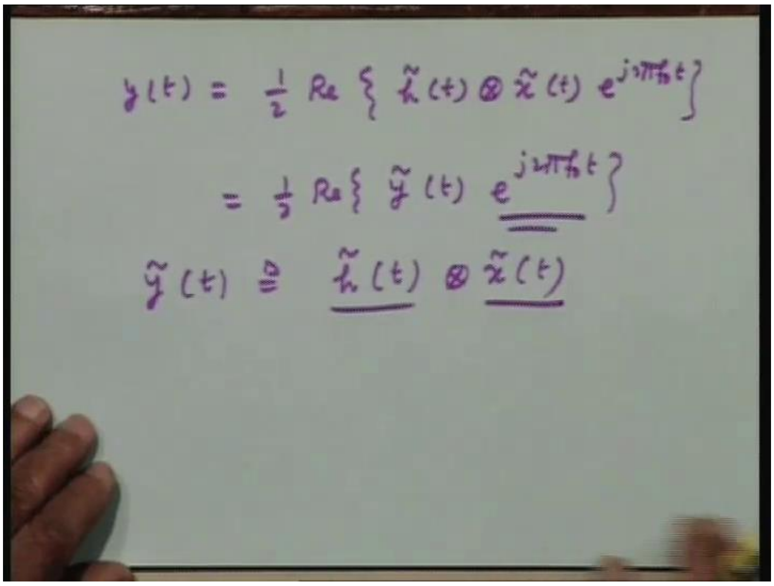
Student: 0

0 and I am integrating infinity, half cycle by half cycle, the same argument can be repeated and therefore, boils down to saying that this integral is 0. So, basically the argument that we give here is based on the fact, that this is a low pass signal and this is a high frequency band pass signal and therefore, half cycle by half cycle. This kind of argument is very useful in communication theory, because otherwise you get stuck with integrals which you cannot look at, which you cannot evaluate.

So, half cycle by half cycle integral become 0, the argument is simply this, that the integral is a product of two functions, this part of the function is a low pass function and varies very slowly, fluctuates very slowly. We assuming here slowly enough, so that over one cycle of this, this remains more or less constant and therefore, over one cycle the integral is 0, repeat the argument for every cycle and therefore, the integral becomes 0.

So, what happens to the result, this term is 0, this term is a continuation of this term that was also be 0. So, therefore, basically we left with the first two terms, then what happens the first two terms, if you look at this think of this e to the power j 2 pi f naught t separately from this integral and the corresponding term there will be e power minus j 2 pi f naught t. So, you have some real, so you have something e power j 2 pi f naught t plus the conjugate of this into e power minus j 2 pi f naught t.

(Refer Slide Time: 48:39)



I can write that as, soy of t, I can write that as half of the real part, half h tilde t convolved with x tilde t e to the power j 2 pi f naught t. Do you agree with this, let me put this note in front of it and hide the complicated fact, for I am saying is that, these two terms is equivalent to writing this. This is nothing but a convolution of h tilde to x tilde, which is what I have written, into e to the power j 2 pi f naught t and since you are taking adding the complex conjugate of that, what are you doing.

You are taking a real part of that twice the real part, so that is why I have to, becomes this one, 1 by 2, look at it carefully, if you have any question let me know. This is what I am writing here as like this.

Student: ((Refer Time: 49:49))

It is outside, could be outside does not matter.

Student: ((Refer Time: 49:59))

Yes, it has to be real, but it is also tending that period, because you are adding something, some complex quantity when it is conjugate, so it has to be real and it is turning out to be real. So, I have not found, I mean there is no inconsistency here, x of t was real, h of t was real, therefore y t has to be real, it is turning out to be so and you are writing it as the real part of this.

Student: ((Refer Time: 50:29))

If what.

Student: ((Refer Time: 50:33))

No no, we are starting from x t h t means real, because normally we deal with real signals. This complex thing that we have introduced is a representation, you remember that, I got a complex envelope representation, it is a representation of a real band pass signal, do not forget that. The complex exponential is a representation of a real sinusoid, similarly the complex envelope representation is a representation of the real band pass signal in the same sense. We have using the representation and see what result we get and whether it simplifies anything for us, we have not yet looked at the simplification aspect, so coming to that now.

Student: ((Refer Time: 51:18))

Let me just complete, just one minute, so that the argument is completed, so now let us look at this, this is the final result we have got, can I write like this, half of real part of say y tilde t into e power j 2 pi f naught t, where I define y tilde t to be, the convolution of h tilde t with x tilde t and that is, this is a result which has a lesson for us. The most important lesson which I want to talk about in this discussion; you could have avoided all these lengthy integrations that we have just gone through.

If we had recognized this fact and this is what I was trying to say in the beginning, Ido not have to carry out the convolution of the band pass signal with the band pass impulse response. I could as well do the convolution, directly of the low pass representation of the band pass signal, complex representation of the band pass signal with the complex representation of the complex low pass representation of the band pass system, band pass impulse response.

Find out what the output would be, which obviously low pass signal is, a complex low pass signal, then go and do this operation. So, I totally can avoid working with band pass signals, even though I am working, even though my basic interest is working in band pass signal and the advantages is that ((Refer Time: 53:04)) are. I do not have to write these lengthy trigonometric expressions which will always be there in the band pass signal representation.

Because it is x R t cosine 2 pi f naught t, x I t sine 2 pi f naught t etc or in the complex representation, I keep on carrying that e to the power j 2 pi f naught t, e to the power j 2 pi f naught lambda, t minus lambda. Those terms are not necessary to carry with us all, in all our manipulations, directly deal with the low pass representations, carry out the manipulations, carry out the mathematics and even the final result will be this. This is the point, because these carry the information, this is just the carrier term to be added, so we will stop here and we will continue at this point.

Thank you very much.