

Communication Engineering
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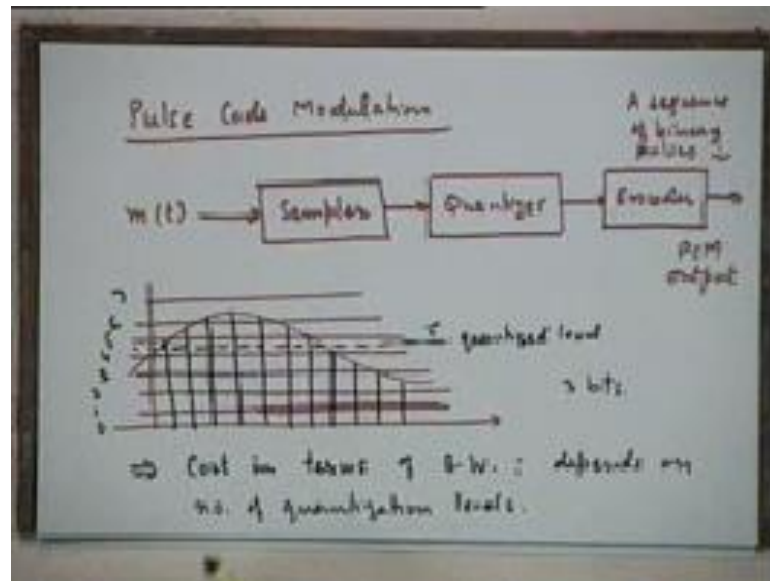
Lecture - 41
Pulse Code Modulation (PCM)

So, if you remember we have been talking about digital transmission, transmission of information in the form of pulses and our reason for doing that was we had discussed that couple of lectures earlier. We are able to trade of bandwidth with performance against noise, we can really get very good performance if you do this and our immediate concern was how to represent our analog information via digital representation of the same.

We discussed one simple method of doing that, in the previous class namely delta modulation and it is improved version adaptive delta modulation. The many variance of these delta modulations and but the purpose is the same to try to achieve a good representation of a basic analog information into a sequence of binary pulses, two level pulses. These two levels are typically opposite polarity, positive and negative polarity, now this is one way of doing things.

Another way of representing or converting to analog information into digital form is through what is called pulse code modulation, which is essentially a quantization process followed by an encoding process. So, we will discuss pulse code modulation in the context of communication theory into this class.

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So, if you look at this second method of doing things, basically we have this block diagram for a Pulse Code Modulator, you will see where the name comes from in a few minutes. You start with a lesser signal as before and the first thing you do is sample the message signal using sampling theorem, that is at a rate sufficient to of the assuming of course, the message signal will have some band limitation, it will be within some bandwidth.

And therefore, it should be possible to sample it adequately by sampling at a suitable rate; if it has a bandwidth of b you can sample it at, either $2b$ samples or more per second. And that will be a good representation or adequate representation of the continuous signal $m(t)$, because of the fact that we can recover the message signal, continuous message signal from a discrete sample signal.

This is same to a block or a device which we call it quantizer, I will describe the quantizer in a few minutes and the quantizer output essentially carries out a mapping, some kind of a manipulation, that if this particular sample value that has come out, it lies in a certain region in the amplitude range a certain interval. Basically, the amplitude of the signal is now mapped into certain intervals and then, depending on which interval it lies in that is the output of the quantizer in terms of which interval it lies in.

And then, we convert our that information or encode that information into a binary form, so that is encoder and that in fact, is a reason why we call it pulse code modulation, so it

is this encoding pulses they are typically binary in nature. This mapping of the level from the quantizer output to a binary code, which is the final output, is the job of the encoder and this is transmitted in the form of these sort of binary pulses. So, the final output is a sequence of binary pulses that is the final output of the pulse code modulator, this is PCM output.

Let me discuss the quantizer in a little more detail, so let us say this is a message waveform, what you are really doing is you are dividing the amplitude range that you could consider into intervals. And you are now saying, you are sampling the signal at a certain rate, you have the sample values, you look at each sample value, let us keep some numbers to these values 0, 1, 2, 3, 4, 5, 6, 7.

So, basically what you do is the following, look at each sample value let us say at this time instant you have this as a sample value and you find that this lies between the level 4 and level 5 which are drawn in this picture. And right in the middle of this level these two levels you have a corresponding quantized level, so any sample that falls between this the 4th and the 5th level will be represented by an amplitude corresponding to this quantized level.

So, I am going to effectively once again do some kind of an approximation, you are not really sending the true sample value to the signal, you are sending the sample value which is close to the true value, close in the sense that it lies in a small interval along the quantized level. So, basically the process of quantization creates an error, but we decide to live with this error, that is essential important point to note about PCM.

There is an array just like in delta modulation there was error in the presentation, after all ultimately you are reconstructing only a staircase approximation to the actual signal. Similarly, in PCM you are going to create an approximation, you are going to create an error at this issue is important; in both these cases we are actually introducing a kind of noise. Both in delta modulation as well as in PCM, in the process of converting this information into digital form a certain kind of noise is getting introduced, we call it quantization noise.

So, this is as distinct from the noise that is added by the channel, so you may well ask what is the use, you are replacing one kind of noise with another kind of noise, the answer to that is fortunately this kind of noise is under our control. We can reduce it to

the level required by choosing parameters of the modulators with you and channel noise is not really entirely in our control, so anyway this is a quantized level.

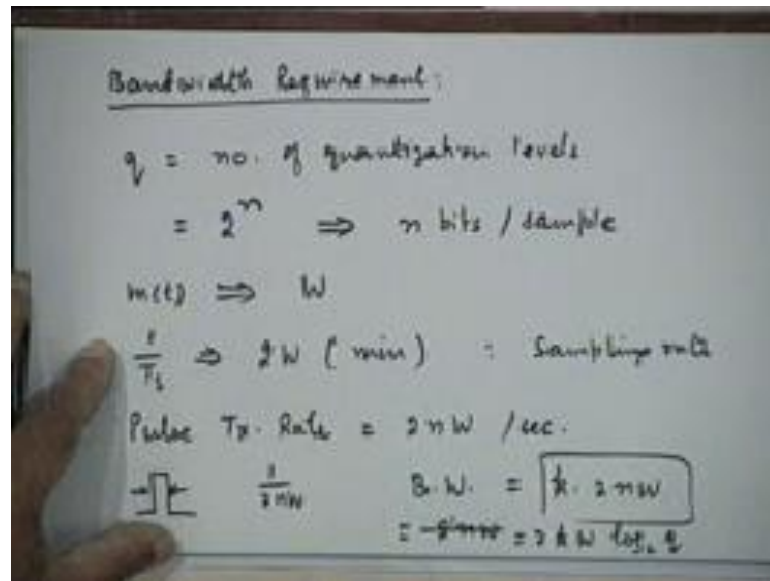
So, this let us say this will be level number 5, so the quantizer output would produce a mapping which says the level to which it is getting quantized is 5. And the encoder will produce binary representation of the number 5 and it is output and that is a one which will be actually transmitted, that is basic idea of... So, you will be transmitting number 5 in a binary form, you will be use an appropriate number of bits to be done, so now it is important to appreciate that between one sample and the next sample, you will have to transmit not just one pulse.

Suppose, you are transmitting number 5, suppose the total number of levels that you have as I have shown here is 8, you have 8 possible levels here, so you will require a minimum of 3 bits to represent this 8 levels. So, an m s ((Refer Time: 10:01)) 3 bits to represent just this sample value, so I need to send a sequence of 3 pulses with a particular 1 0 combination to represent the sample value.

So, between one sample and the next sample value I have to transmit 3 pulses, so therefore the pulse rate at which you will be making the transmission is much larger than the sampling rate. Therefore, the bandwidth requirement obviously, will be high, so it has a cost in terms of bandwidth which depends on number of quantization levels. Example if you have a larger number of quantization levels in a given interval you will obviously, have a better approximation of a signal by the quantization process which will be nice.

But, the price that you will have to pay for that is, because you have a larger number of quantization levels, you will require a larger number of bits to be transmitted between two successive samples. And the rate of transmission would definitely correspondingly much higher, so let is look at if this is understood can I proceed further with this.

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Let us look at the bandwidth issue. How much is the bandwidth requirement can we quantify it. To do that, to quantify this let us note by q , the number of quantization levels, I showed in quantization levels in the example that I drew in the figure a few minutes ago. In general, if this much larger, because you want a good approximation of the signal, approximation has to be rather small.

Typically, for convenience you will choose this q to the power of 2, so that you can represent it as an exact number of bits every level to be in terms of a finite number of bits. So, typically it will be chosen to the power of 2, so 4, 8, 16, and 32 etcetera and therefore, it would imply that you need to transmit n bits per sample, for every sample that you have you will have to transmit n pulses.

So, to signalize a bandwidth of W , so message bandwidth $m(t)$ let us say is W , then your sampling rate $1/T_s$ should be minimum of $2W$ and the pulse rate, so that is the sampling rate. So, the rate at which you will have to pulses or the pulse transmission rate would be equal to how much $2nW$, because for every sample you are transmitting and pulses, so that is the number of pulses that you will have transmit per second.

And it is more or less common sense to electrical engineers that the bandwidth of that is required to transmit pulses will depend on the duration of the pulses. So that means, if I transmit $2nW$ pulses per second, what is the maximum duration of the half range pulse.

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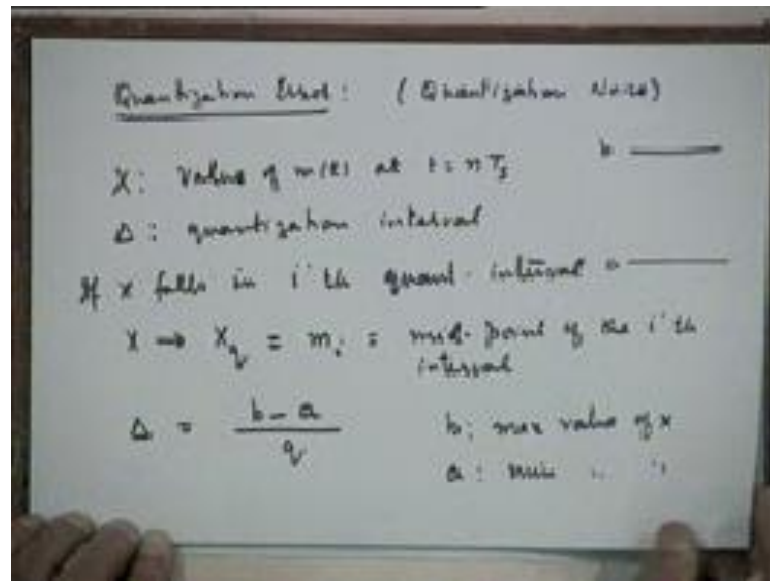
The maximum value that you can have is $1/2nW$, and the corresponding bandwidth will be proportional to the reciprocal of this duration, if the pulse duration is τ , the spectrum of the pulse is a sine spectrum. And what is the first process of the sine spectrum, proportional to $1/\tau$, minus $1/\tau$ to plus $1/\tau$, so roughly of course, the actual bandwidth is little larger. In fact, it is infinite to strictly speaking, but even if we assume that most of the energy lies in a certain finite band, that will be proportional to $1/\tau$.

So, the actual bandwidth will be therefore, some constant times $2nW$, where this constant is appropriately chosen to say that 99 percent of the energy of this signal lies in that band, so it will be proportional to $2nW$. So, that is the typically this cost in lie between 1 and 2, so that is the required bandwidth and as the increase in number of quantization levels, that increase the number of value of n and that will increase the required bandwidth.

So, the pros and cons are we must use large number of levels, if you want a good approximation and it is achieved at the use of more bandwidth higher bandwidth, any questions so far. To be we can just go one level further and write the solution as $2nW$, $2kW$, this n is you can also think of this n as $\log_2 q$, n is $\log_2 q$ to the base 2, so that is another way of writing the same result.

Next thing we try to understand about this process is we have said that there is an error involved in the quantization process. So, we need to appreciate what and therefore, this introduces a kind of noise right at the transmitter itself or otherwise if you have transmit an analog signal that kind of thing is not happening, you are introducing some noise deliberately here. Of course, you have no choice if you want to do quantization.

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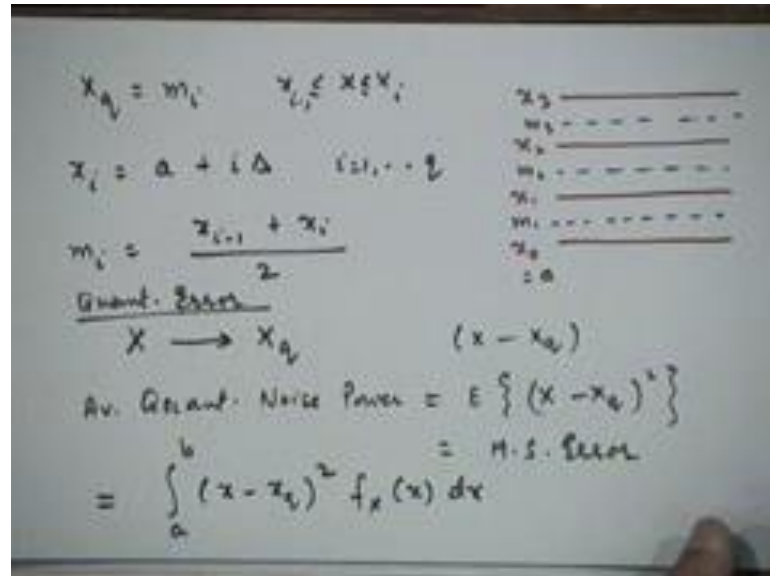
So, one of the important things to understand is what is the magnitude of this quantization error as a function of the number of quantization levels that you like to quantify quantization error, which in turn will allow us to quantify the quantization noise. Sometimes, it is also called quantization noise, let us calculate what is the signal to noise ratio that you can expect from a PCM system, which uses a certain number of quantization levels that is the analysis I like to be feed you.

What you do some notation for that, let me denote by X the value of the message at a given sampling instant, do not confuse this n with, n that I have used a few minutes ago, this is the n th sample instant. Denote Δ the quantization interval, we will assume that your signal amplitude varies between some fixed values, let us say from some value a to some value b and you divide it that interval of values into q intervals, q sub intervals.

So, the mapping is if X falls in the i th quantization interval, then what we do is we convert this X into a value $X_{sub\ q}$, go back to this picture, ((Refer Time: 18:29)) this is actual value converted to this level which I am calling $X_{sub\ q}$. This is what is this value, this value is the middle value between the two intervals typically $m_{sub\ i}$, I will denote it by $m_{sub\ i}$. So, $m_{sub\ i}$ is the midpoint of the i th interval and therefore, you can say Δ the quantization interval will be equal to b minus a where b is the largest value, a is the smallest value, you are dividing with q sub intervals. So, what is the value of the quantization interval, b minus a upon q , where b is the maximum value of X and a is a

minimum value of X , where q is the number of intervals, so this is some notation let me depict this in the form of a picture.

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Let us see how this level, it is 0, it is 1, it is 2 these are not the levels these are intervals, so call it x_0, x_1, x_2, x_3 and the midpoints are represented by m_1, m_2, m_3 and so on, and so forth. So, we said that $X_{sub\ q}$ is equal to $m_{sub\ i}$, when X is lying between X_{i-1} and X_i , you are mapping the value of X , to the value X_q which is equal to $m_{sub\ i}$, the value of X_q is equal to $m_{sub\ i}$, where if X lies between the i th interval, the i th interval is defined as between X_{i-1} to X_i , this is what I have been saying so far.

Let me write an expression for $X_{sub\ i}$, it will be equal to a plus the smallest value is a , we can denote it as X_0 here, a is nothing but, x_0 this is equal to a , a plus i delta, i delta 1 to q or if you make it 0 to q minus 1 it will be.

Student: ((Refer Time: 21:35))

I think I am defining from x_1 onwards, x_0 we define it equal to a , it does not really matter that is ok and $m_{sub\ i}$ is the middle level of these two, so it is x_{i-1} plus x_i upon 2, that is the $m_{sub\ i}$. So, with this notation we are able to compute the quantization error, how will you define the quantization error, you have a sample value X , so you have to define quantization error and you are converting this into a value $X_{sub\ q}$.

So, what is the error $X - X_q$ is the error and average quantization error or average quantization noise power I am denoting the error. I am thinking of the error as some kind of a noise, it is basically the expected value of, of course when you talk about error; error can be positive and negative.

When you talk about power we do not have to talk in terms of positive or negative values, we want to talk about the square value; essentially we are talking about the mean square value of the error. So, we get the expected value of $(X - X_q)^2$, that is the definition of average quantization noise or average least error which is there, so this is a kind of mean square error.

So, how will you compute this, X it is average because X can take any values, we are taking X as a random variable and as much as X is a random variable at different time instance for different values of X means different values of the i th depending on what you get. So, you have to take the average over the distribution function of X and X lies between a and b with some probability distribution function, the domain of X is a to b , lower value is a , largest value is b .

So, you have $(x - x_q)^2$ whole square into the density function of x which I am now denoting by $f(x) dx$, $f(x) dx$ and that is the value of the least per error, everyone agree so far. We proceed further please note that this interval from a to b is composed of a set of sub intervals, so I can carry out this integration with each of these sub intervals separately and add up the sum. So, directly carrying out the information from a to b , I can carry out the integration of this interval, then over this interval, then over this interval, so same thing.

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$$N_q = \sum_{i=1}^q \int_{x_{i-1}}^{x_i} (x - m_i)^2 f_x(x) dx \quad (1)$$

$$S_q = \text{Signal power at quantizer output}$$

$$= E \{ X_q^2 \}$$

$$= \int_a^b x_q^2 f_x(x) dx$$

$$= \sum_{i=1}^q m_i^2 \int_{x_{i-1}}^{x_i} f_x(x) dx \quad (2)$$

S_q / N_q : A measure of fidelity of Quantizer

So, you can also represent to it as sigma i going from 1 to q integral, so the i th integral is from x_{i-1} to x_i and when whether you are considering x to lie between this region what is the corresponding value of X_q , m_i , whole square $f_x dx$, any questions on this. So, let me denote this quantity by N_q , this is an average noise quantization noise that you (Refer Time: 25:50)), similarly I can define and have a signal power between a and b .

So, the signal power of the quantizer output would be expecting a value of the quantizer produces a value X_q , expected value of X_q^2 and what is this equal to, in the same way as we did for the case of noise. Basically we have to just take the integral of $x_q^2 f_x dx$, which again it was spitted into this is q integrals this is from a to b , in the i th interval what is the value of x_q^2 m_i^2 . So, m_i^2 and because this is a constant in that interval, I have taken it also the intervals and this would be x_{i-1} to x_i of $f_x dx$.

So, these are the two expressions we have 1 and 2, this gives you the average signal power, this gives you the average noise power and the ratio of these two will give you measure of the fidelity of the quantizer output to the original signal which you are quantizing. So, S_q upon N_q is a measure of fidelity or accuracy of a quantizer so far.

Student: Sir

Yes

Student: ((Refer Time: 28:05))

This one, this like in the same way you are taking average value of the quantizer output, X_q square then

Student: How did we separate ((Refer Time: 28:16))

So, ((Refer Time: 28:19)) a integral from a to b could be broken up into integral from x_0 to x_1 , then x_1 to x_2 , then x_2 to x_3 , that is all I am doing and summing them all up. So, this m_i^2 is actually part of this integral, but since it is independent of x in this integral, I have taken it also in the integral, just like here I cannot pick it out, because it depends on x .

Student: ((Refer Time: 28:46))

Please speak out the doubt, if you still have the doubt

Student: ((Refer Time: 28:51))

Sure, now to proceed further this unfortunately is not in a form which will make any sense to us, it is too analytical it is I mean simply does not give us a physical picture. To get a physical picture we have to assume a certain distribution for the signal samples, because unless we have a distribution I cannot simplify these expressions any further. And to keep the picture simple, let us keep this distribution, assume this distribution to be one of the simplest distributions we can have really for. Here, x could lie between a and b and let us say x can lie between a and b very uniform distribution, it is a simplest picture that you can have.

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Example: $f_x(x) = \frac{1}{2a}$

$m_i = -a + i\delta + \frac{\delta}{2}$

$$N_q = \sum_{i=1}^q \int_{x_{i-1}}^{x_i} (x - m_i)^2 \cdot \frac{1}{2a} dx$$

$$= \sum_{i=1}^q \int_{x_{i-1}}^{x_i} (x + a - i\delta + \frac{\delta}{2})^2 \cdot \frac{1}{2a} dx$$

So, let us take an example that will give us a better picture, so let us assume that, let us say a is minus a the lowest limit is minus a and the upper limit is plus a , so I am assuming the signal to lie between minus a and plus a and uniform distribution of 1 by $2a$. So, I have taken the lower limit as some value minus a and upper limit as plus a , this is again for simplicity, so then your N sub q , please note that in this case you can write here m sub i .

The i th quantization level to be minus a minus δ by 2 , let me rewrite it many mistake here, m sub i is minus a plus i δ , minus a plus i δ will take you to the next boundary and the middle level will be δ by 2 lower, so that is we can write this. So, m sub i can also be written like this, I will substitute for this m sub i in the expression for N sub q . So, let us now look at the expression for N sub q , which was summation i varying from 1 to 2 integral x sub i minus 1 to x sub i into x minus m sub i whole square and this f_x is 1 by $2a$ dx .

As usual for m sub i , ((Refer Time: 31:45)) this x plus a minus i δ plus δ by 2 whole square into 1 by $2a$ into dx , now this is a very easy, it looks cumbersome because of this constant. But, if you have to do this integration which is a very straightforward integration and simplify this integral and this evaluate this integral and skip that step if you permit me.

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The whiteboard shows the following derivations:

$$N_q = \sum_{i=1}^q \frac{1}{2a} \frac{\Delta^3}{12} = \frac{q \Delta^3}{12(2a)}$$

$$= \frac{\Delta^2}{12} \quad \left[\because q\Delta = 2a \right]$$

$$S_q = \sum_{i=1}^q m_i^2 \frac{\Delta}{2a} = \frac{q^3 - 1}{12} (\Delta^2)$$

$$\frac{S_q}{N_q} = \frac{(q^3 - 1)}{12} \cdot \frac{12(2a)}{q \Delta^3} = \frac{(q^3 - 1)}{q} \cdot \frac{2a}{\Delta^2}$$

When $q \gg 1$

You can see that $N_{sub\ q}$ becomes i is equal to 1 to q by $2a$ that basically comes out outside the integral and the rest of the integral it has to be simply delta cube upon 12, can I skip that, can you just evaluate this in this boundary, it is straight forward in that. So, this becomes delta cube by $2a$ and therefore, see this is independent of i , we can write this as q times delta cube upon 12 into $2a$, because basically we have two similar terms. So, you simply multiply by q , but what is the value of q delta, q times delta is the

Student: ((Refer Time: 33:20))

$2a$ the total interval in which the signal lies, the dynamic range of the signal in some sense, so this is $2a$, so you are left with delta square upon 12, because q delta is equal, so $2a$, so very simple neat expression you will get for the quantization noise. Now, you can see that smaller your quantization interval the smaller the power of the noise process, which was intuitively expected. Similarly, if you have to compute $S_{sub\ q}$ because you have got simpler of course, as for simply in this case order of this integral of ((Refer Time: 34:15)) I think it is obvious.

Let me just go through that expression this is 1 by $2a$, so what is the value of this integral between $x_i - 1$ to x_i with has a width of delta, delta by $2a$, that is the answer. In this case of course, there is supposed to be it seems to become m_i , it is again very simple to check that, if you substitute from this m_i and evaluate this summation you got to have a simple nice closed form. Q^2 minus 1 upon twelve into delta

square, essentially same delta square by 12 into q square minus 1 is essentially comes from the summation of these m_i square.

Substitute for m_i as given to you and you can check it out, again I am leaving that out for reasons of time and it is something which is straightforward to you, apart brings us to the value of S_{sq} upon N_{sq} to be given by $q^2 - 1$, very simple. This upon this all or you can say approximately equal to q^2 if q is large, when q is much greater than 1, so that is the real term which you should use for finding out what is the kind of signal to noise, quantization noise ratio that you can expect from you PCM quantizer.

Of course, remember that the they are certain assumptions that we have made in deriving this expression, the most important assumption we made is the signal samples I have a uniform distribution between minus a and plus a or a to b . So, with that assumption this is the answer, which is useful from a rule of thumb point of view, but very accurate for all kinds of signals.

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$$\left(\frac{S_v}{N_v}\right)_{dB} = 20 \log q$$

∴ 6 dB improvement for every additional bit introduced.
rms value of error = $\frac{\Delta}{\sqrt{12}}$
Uniform Quantization ⇒ OK for Uniform distribution

If you express it in the dBs the rule of thumb term becomes something like this, this will become $20 \log q$, so let us say double the value of q , what is the value of including the signal to noise ratio you will get how many dB's, 6 dB's. $6 \text{ dB's } 20 \log 2$. So, every doubling of q you will get a 6 dB increment in a signal and doubling is what, in terms of number of bits one more bit.

So, you get a 6 dB increment for every additional bit that you introduce in the representation, so that is the general rule of thumb, so if you want something at 40 or 50 dB's SNR, 40 dB SNR you must use at least 7 bits, rule of thumb. Typically use 7 to 8 bits in typical applications, 7 to 8 bits means number of quantization level is between 128 and 256, so either 128 or 256 are larger.

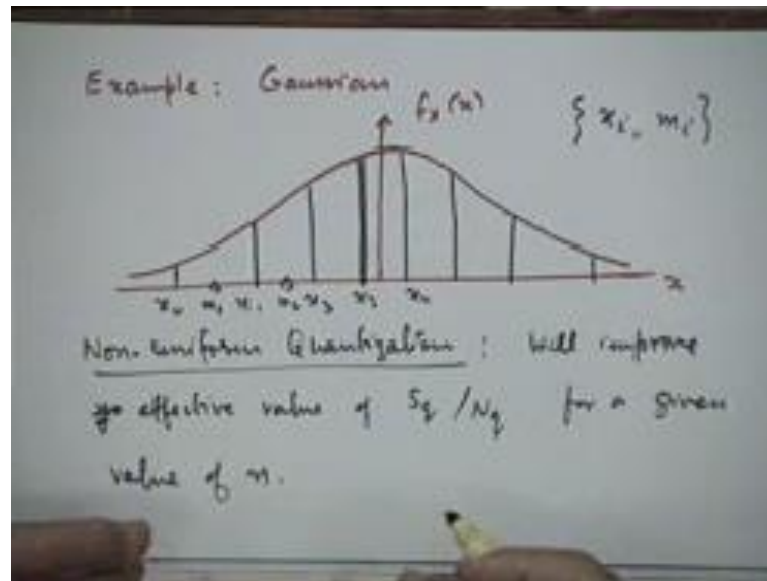
The rms value of the error if you are interested in more in terms of rms value of error rather than signal to noise ratio would be simply Δ upon root 12, because it is Δ the noise power is Δ^2 by 12, so rms value is Δ by root 12, any questions so far. Now, this picture is good as well as this the example that I have took was somehow matched to the kind of quantization that I did, let me elaborate what I mean by that.

You have to take uniform distribution that means, signal has equal quality of being there in any of the intervals and these intervals are uniformly spaced. But, suppose this the probability of the signal being in let us say one of these one interval is very different from the point of view of signal lying in some other interval, this can happen.

This the small values of signals let us say sometimes are much more likely than larger values in this signal, then this kind of quantization that we are discussed is not a very good idea to use. This kind of quantization that we discussed is called uniform quantization, so what we have discussed so far is a uniform quantization, uniform quantization is quite, if a signal samples are uniformly distributed. So, they are for the uniform distribution that we discussed, but more likely your samples are not going to be uniformly distributed.

You may have a Gaussian distribution actually speed signals have a very different kind of distribution, they are Laplacian kind of distribution, very different kind of distributions. So, in that case you can intuitively expect that is your uniform quantization process is not the best thing to do, it is probably better to...

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Let us take an example, suppose x is Gaussian that means samples are distributed according to this bell shaped distribution curve, probability density curve. Now, this picture it is clear that the smaller values of the x are much more likely than larger values of x , x lying in the interval here has this kind of probability. X lying in the interval here has a very large probability. So, if you are going to use your number of bits for representation efficiently I should crowd the quantization intervals in this neighborhood.

And space out them sparsely for larger values of x , rather than keeping all the quantization intervals of uniform size, because then although whenever I use a larger quantization interval the error is going to be large, but fortunately that error will occur with much lower probability. So, the continuation of that to the average value of noise power would be much smaller, so that is a reasonable thing to do.

So, whenever you have a thing like this you do not use uniform quantization, you use what is called non uniform quantization, and non uniform quantization will typically improve the effective value of S_{sq} by N_{sq} for a given number of bits by a, given value of n . And the drawing is basically to find out how you should, I have not a good picture. How you should distribute this interval boundaries and the interval and the quantization intervals all.

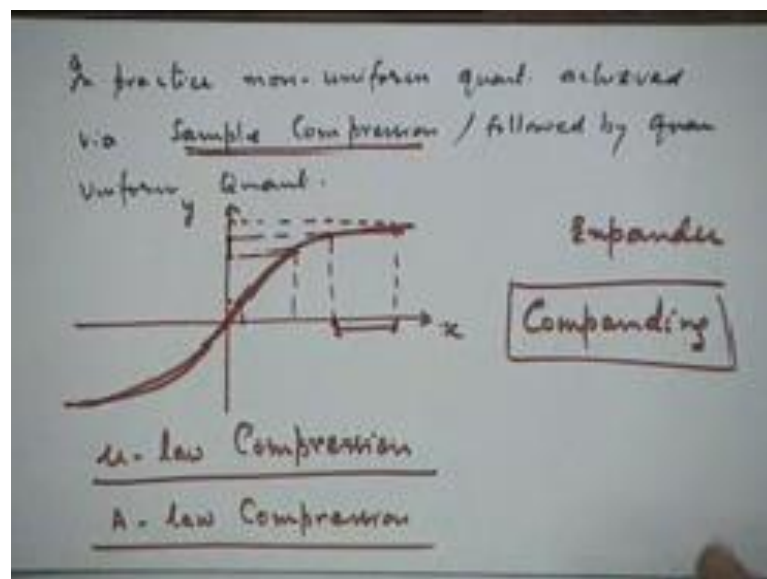
How you should space this value of x_0, x_1, x_2, x_3, x_4 etcetera and what should be the corresponding. Also it is clear that I should not now use the middle of these two for my

quantization level. So, at the end I will use something which is biased towards the region where the signal samples are more likely to be located, so instead of being on this side it is slightly to the right of the middle here.

So, this will be m_1, m_2 etcetera, so but this is a difficult problem, find out the set of x_i 's m sub i 's, so that you get the best possible signal to noise ratio, that is a that is the job of designing a non uniform quantizer, that is one way of designing a non uniform quantizer. Taking into account the distribution function that you have and using that non-linear distribution function find out the best pair of values of x sub i and m sub i for all the values of i , so as to produce the smallest value of error, least rate error.

So, that is a good interesting problem, it has been solved in the literature, but rather difficultly involved, hence because of the difficulty that is associated with this approach, there is a simpler approach to carrying out non uniform quantization. And the simple approach is a one which is actually widely used in practice which I like to discuss.

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So, in practice non uniform quantization is achieved via what is called sample compression followed by quantization, so followed by uniform quantization, let me explain what I mean, let us say you have this signal which has non uniform distribution. So, what I will do is I will pass the signal sample values through a non-linear amplifier, the signal values are denoted by x and non-linear amplifier outputs are being denoted by y , it is not a linear amplifier does not produce y equal to t tau x y .

It is a non-linear amplifier in the sense that it will produce response something like that, so the basic idea of using this non-linearity. That signals sigma sample values which are going to occur with lower probability, which are typically going to be the higher values of x , for higher values of x they will be compressed into regions which are smaller, so that is this region that is compressed into this region.

Whereas the corresponding region here for the same interval only this much, so if I go to the next value it is going to be still here, because x has much smaller probability of lying in this interval, I compress this entire interval into a much smaller interval. And in this domain I do uniform quantization, now I give equal importance to all the values of y that I get, you are effectively doing the same thing in a different way.

A larger region interval of x is composed into a smaller interval in terms of y , but having got this having done this compression I will treat y with uniform quantization, so this is what we mean by sample compression. This we mean by sample compression and followed by uniform quantization and at the receiver of course, your signal value will get stocked now, you are not transmitting the actual signal value, you are transmitting some alternative value.

Student: ((Refer Time: 47:50))

No, but this is what I am finally, transmitting I am not transmitting this value, I am transmitting some value which lies in this interval, so instead of transmitting this value I am transmitting this value and there is a non-linear relationship.

Student: ((Refer Time: 48:08))

So, I must convert them back into x values, so what should I do at the receiver, I should do an inverse operation to this, so I have a compressor of this kind, I have a compressor amplifier at the transmitter. At the receiver you have an inverse characteristics and an amplifier with an inverse characteristics which you call the expander, so we carry out a compression of the transmission and expansion at the receiver. And for various reasons this operation of compressing and expanding is typically talked about together is called companding.

It is a short form for compressor plus expander, so this companding is a very common practice used in PCM pulse code modulation to effectively utilize your number of bits you allocate in a much better way, that two commonly compression laws in the industry. One is called the mu law compression, but these laws bit complicated, but they are definitely easy to implement and they have been arrived at through a lot of empirical research.

The other is called A law compression, these are industry standards, you all know what is an industry standard, European simulation process; the other is based on the American standardization process.

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The image shows a whiteboard with handwritten mathematical formulas. The top formula is for mu-law companding: $|y| = \frac{\log(1 + \mu \left| \frac{x}{x_{max}} \right|)}{\log(1 + \mu)}$ with a note $\mu \approx 100$. Below it is the piecewise definition for A-law companding: $|y| = \begin{cases} \frac{A \left| \frac{x}{x_{max}} \right|}{1 + \log A} & 0 \leq \left| \frac{x}{x_{max}} \right| \leq \frac{1}{A} \\ \frac{1 + \log(A \left| \frac{x}{x_{max}} \right|)}{1 + \log A} & \frac{1}{A} \leq \left| \frac{x}{x_{max}} \right| \leq 1 \end{cases}$ with a note $A \approx 100$.

And this is what they look like, do not ask me too any questions on this, because one we do not have time for it that I am just giving the expressions just for the sake of completeness. These basically specify the non-linear characteristics of the amplifier that you need to have at the compressor, so this is a mu law compressor, basically a log compression the logarithmic relationship is some kind of a compression relationship, this is based on that.

And typical value of the parameter mu is taken to be about 100 and the A law is similarly given by more complicated expression. I am doing this just for the sake of completeness; let us not worry too much about where these expressions come from. Here A is typically taken to be in the order of 100, anyway both are some kind of logarithmic compressions,

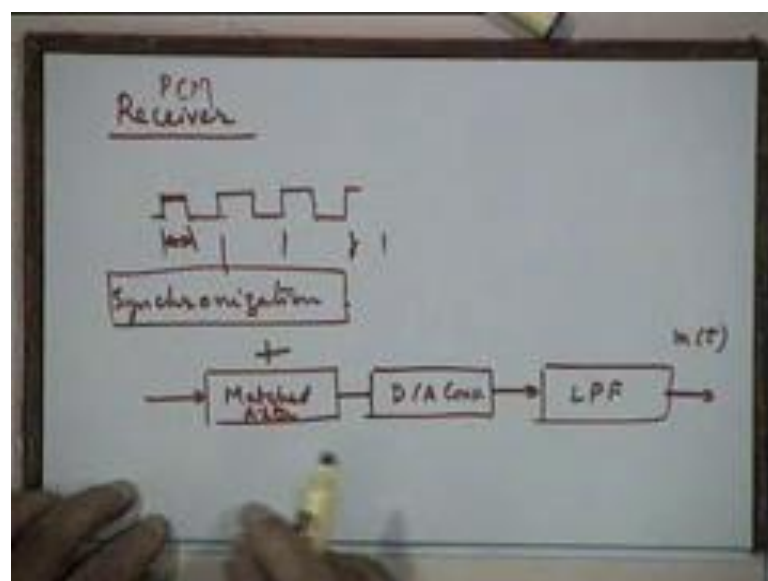
and I write that independently by different people. The important feature of this compression characteristic is these are largely independent of the distribution function that the signal might have, incidentally.

Suppose, I ask you the question that I have some distribution of around in terms of x , that I want to carry out a transmission y of y equal g of x such that, y has a uniform distribution. What kind of function will do the job, do you know that, y is equal to, if you choose this mapping function this will convert y into uniformly a distributed random variable.

So, ideally I should choose this compression for ((Refer Time: 52:15)) based on the distribution function of x , what these A law and mu law will do is they make it more or less independent of this. They are fairly robust to distribution function, they do not of course, there will be some variation it will not be really an optimum, but they come pretty close to the optimum.

From most of the kind of distribution that you see, in ways a signal, that you use speech and music and things like that. Now, one final thing before I finish with the PCM, I have discussed the transmitter, a few things that you should know about the receiver, I have not discussed anything about the receiver, what should you have at the receiver, and what does a receiver look like.

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Before you talk about anything else, please remember that there is already one complication that we have received for the receiver; we are transmitting groups of bits to represent one sample. And these groups of bits are following one element and now I have an additional problem which I never had in any analog system, even pulse amplitude system that I, pulse modulation system that I discussed, so far.

Namely, I must know which group of bits represents which sample, I must have synchronization, and I must know precisely, here is a continuous sort of bits coming along. Whether it is these 3 bits which represents the sample or these 3 bits which represent the sample, this has to be specified. So, you need to have, you need to introduce some way to tell the receiver how fairly has been done, so that introduces a very important problem of synchronizing at the bit level as well as the block level.

You must know precisely the time instance at which bit levels might change and also how the good bits are grouped, which 7 bits represent the sample, which group of 7 bits and of course, once you establish the once, then there is no problem. So, one additional computation is that of synchronization, now let us say we sorted that out, I need to go through discussion of that.

You will learn more about all these things in a course in digital communication if you desire to take it, but suppose we have taken this problem, this is an additional problem. This ((Refer Time: 54:57)) of what you are doing there will be noise coming in all the directions, you what you really get is not this clean samples that I have drawn here, but noisy versions of this and what are the jobs that you have to do at the receiver now.

Before you can do the decoding and things like that, you have to simply be able to carry out saying whether in a particular interval, the signal that you have seen is a pulse of amplitude some value 5 volts pulse of amplitude 0, it is a positive pulse or 0 pulse. It is not going to be 5 volts it would be 5 micro volts or 50 micro volts along with noise, does it suppose to be a 1 level or a 0 level, how do you decide that.

Student: ((Refer Time: 55:36))

No, you cannot do a straightforward quantization, because you can a noise ((Refer Time: 55:41)) easily make you take a wrong decision, so somehow you have to filter at the receiver which will average out the noise first. Remove the effect of noise within the

pulse and then, you take a decision whether it is 1 or 0, this is very important subject process.

The decision making process which looks so trivial actually is a fully fledged subject by itself, in communication theory; we call it detection theory, the subject of detection theory. And one of the simplest results is you use a filter at the front end which is called a match filter which will remove the noise, follow this match filter output by a D to A converter.

Follow this by low pass filter; because now the match filter output will give you these groups of bits, these groups of bits will convert, the D to A converter will convert this group of bits into a sample value. The sample values are represent the signal in a pulse amplitude modulated form, pass that through a low pass filter and you are able to recover a message signal $m(t)$, so you are required to all this plus synchronization, that is your PCM receiver.

So, in a very short form an over view of pulse code modulation which I normally do in 2 lecture, but I do not I have done it did only first, do not have time, but I think I have still told you most of the things. What we have missed out on for both delta modulation and to some extent in pulse code, not too much in pulse code modulation is the quantization noise performance. For PCM I have done the quantization noise performance, at least for a simple case of uniform quantization, but for delta modulation it is left as self-reading exercise.

Thank you very much.