Communication Engineering
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Lecture – 37
Noise in Angle Modulation (contd.)

We will continue our discussion on Noise in Angle Modulation. I think we have now, a reasonable idea of the fact that angle modulation is, indeed a good thing to do in as far as performance of the modulation scheme in the presence of noise is concerned. We find that there is a nice trade off that is available to us, for getting a good performance at the cost of increase manner that is of course here. Now, if you just recollect, you have derived the expression for both the case of FM as well as phase modulation.

Angle when is say angle modulation, it refers to both; it refers to any one of them, for any variant of. Now, if you look at, if you specialize these expressions, for the case of tone modulation, that is modulating the message signal is a pure sinusoid, you can deduce the expression. I will leave that as an exercise for you a very simple exercise to the following expressions.

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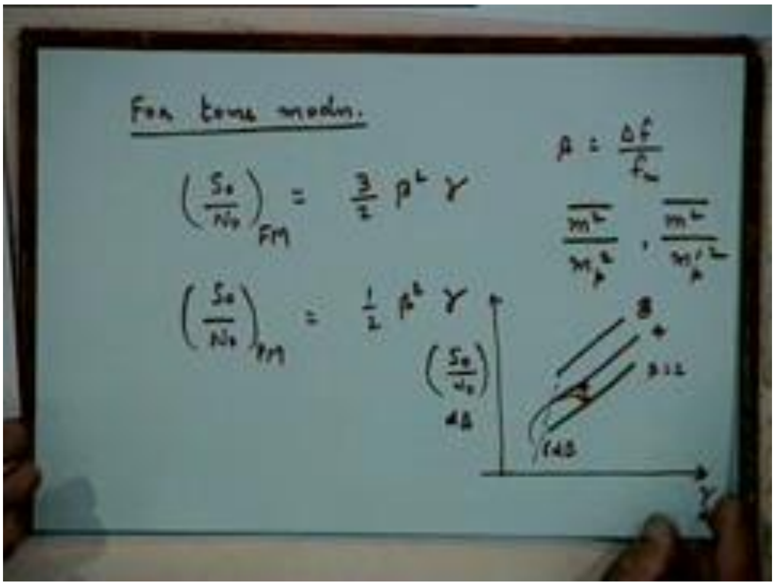

So, for tone modulation you can check that the expressions for the output SNR reduce to these very simple expressions, 3 by 2 beta square gamma, for the case of FM and half beta square gamma, for the case of phase modulation. Here, beta of course, is your modulation index, defined as peak frequency deviation ratio of peak frequency deviation to the highest to the frequency resonance side, modulating sinusoidal delta f by fm. You remember beta is delta f by fm and these expressions come, basically by understanding what is the nature of these values?

You remember this was the factor that appears in the expression for the FM case and the expression that appears for the PM case is m square by upon m p prime square. So, basically evaluating these two expressions for the case of tone modulation, you get this result. So, leaving the detail as an exercise, it is very for example, it is obvious that m square above upon m p square for the case of a sinusoidal signal would be equal to half. So, we use that fact and you will get this expression, use the definition of the beta ((Refer Time: 03:53)).

The reason why I wanted to cite this expression is because that, allows you to draw a set of very simple curves, for output SNR against gamma. So, that there are of course, before I go to that, please note that for tone modulation signals, which one is superior, FM is superior. In general we have discussed under what situation FM will be superior and PM will be superior, that was a separate thing we discussed, but specifically for the case of tone modulation, FM is superior by a factor of 3.

That means it produces three times output SNR, that is produced by a PM for the same value of gamma, three times means 4 dB. So, this superior by 4 dB 4 decibels, so that is one fact, the second is the interesting relationship between gamma and output SNR, it will go just like the case of ((Refer Time: 05:01)) straight line there is a straight line with a ((Refer Time: 05:04)) beta square.

So, of course, that is ((Refer Time: 05:11)) plot it on log scale ((Refer Time: 05:15)), if I plot it in decibels, this also be a log scale in decibels. Then, what how will they be different from beta square will come out, if different values of beta square suppose you take beta square as a parameter, then essentially will be log of beta square plus log of gamma. So, for different value of beta square, you will essentially get different straight lines, you will get a family of straight lines, which are parallel to each other. So, this is what you will get, a family of straight lines.

So, for example, this could for beta equal to 2, for beta equal to 4, for beta equal to 6 etcetera and as you know, for every doubling of the it should be 8 here, for every doubling of beta, it means doubling of bandwidth, the output ((Refer Time: 06:21)) 6 dB. So, this gap is how much 6 dB, as you go from beta equal to 2 to beat equal to 4, the curve goes up by 6 dB, ((Refer Time: 06:37)) when you go from 4 to 8, the curve again goes up by 6 dB.

So, that is something that you must know, you will also have notice ((Refer Time: 06:48)) thing that I have done here, I have not taken the straight lines like that, and I think you can guess why. Because, we discussed this analysis have been carried only for the small noise case, that in the signal to noise ratio the input, the value of gamma the input signal to noise ratio in the message bandwidth is sufficiently large. We need to make it smaller and smaller and analysis becomes invalid.

So, these results if you have plotted ((Refer Time: 07:19)) is simplified to be straight line result, we will continued and this side, actually it is straight line here and somewhere here, like this. And you will get a there is different situation, in the as the noise increases or the input signal power decreases. So, let us look at that issue now and that is the same issue, which is discussed in the case of AM earlier ((Refer Time: 07:49)).

That is as the signal to noise ratio decreases, unlike the straight line curves, which implies the corresponding ((Refer Time: 08:02)) deviation. Also the output SNR, beyond certain point the output SNR will decrease much more than the reduction in the input SNR. That is indicated by the reduction in the input SNR and that kind of factor occur much more pronounced now, than ((Refer Time: 08:22)) case of envelope detection in the case ((Refer Time: 08:25)).

So, this special effect phenomenon is the very important phenomenon in the context of angle modulation, It is much more complex in the context of amplitude modulation, if you remember, we discussed in the little earlier, for the case of amplitude modulation it is highly evaluated. So, theoretically it is there, but practically it is hardly insignificance, but for the case of FM, it is of great significance, so we will look at that issue, we will look at the threshold effect in the angle modulation.

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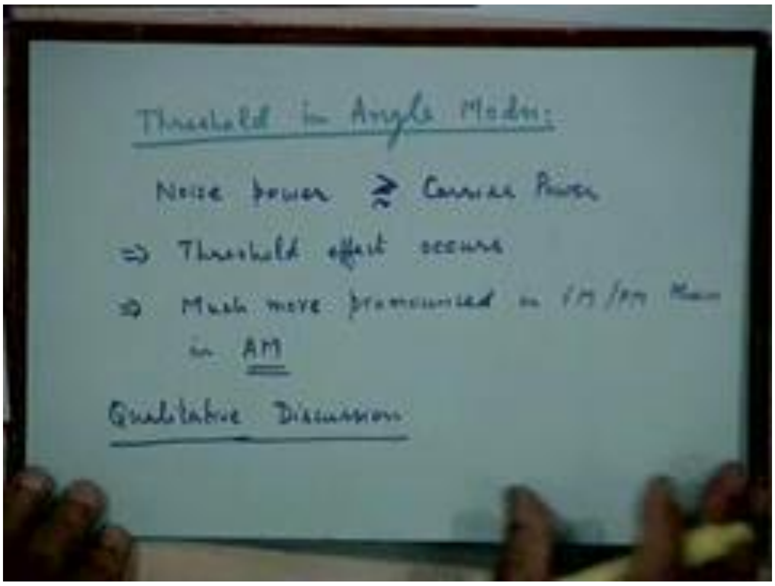

Unfortunately the analysis of this case is much more difficult and we will not be able to do the proper mechanical analysis, because that is quite involved and beyond the scope of this course, Then the case where the noise power is highly less than, which should either way round, the noise power should be greater than, in the order of the signal power or carrier power ((Refer Time: 09:55)), that is performance increase. So, this is when the threshold effect occurs, so all of you understand, what I mean by threshold effect, it is the output SNR performance becomes suddenly very poor.

They will go along the straight line anywhere, it goes much more worse than that, so you all understand, since the analysis will go much more difficult in this case, we will try to qualitatively understand the phenomena, I think there most of the effort relies. So, the second turning point, I want to make in connection which ((Refer Time: 10:42)) said that it is much more pronounced, the threshold effect is much more pronounced in FM or PM than in AM.

If you remember in AM it does not occur always, it occurs only in peak envelope detection, in invariably threshold effect is the phenomenon arising from the non linearity in the system. As long as you did not simplest demodulation, your entire system modulation and demodulation system was a linear system, the combined system as a linear system moving to put it an envelope detector, it is a non-linear system.

In case of angle modulation, modulation itself is a non linear, so the system is non linear and therefore, the threshold effect will occur. So, let us look at some kind of the qualitative discussion is to what is the threshold effect, why it occurs and what it does with the performance. So basically that what will limit ourselves to ((Refer Time: 11:50)).

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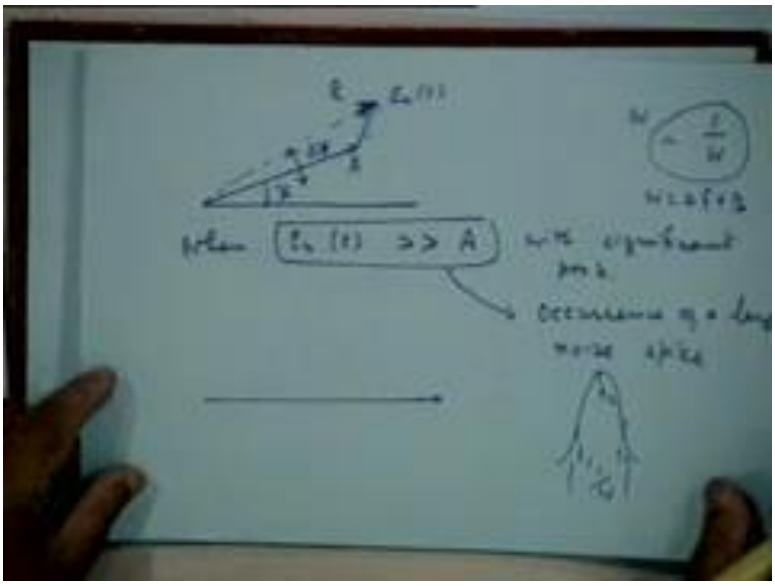

To do that let us return to the phasor diagram, that we have been study very well, this was the carrier amplitude A, for simplicity or call this angle psi, will define psi of t. This is a noise phasor and this is the resultant phasor, remember this. I am not calling it omega c t plus psi, because you can assume that whole reference is rotating at the rate of that omega ct, reference plane rotating at.

Now, notice that E m t is the fast fluctuating phasor and this phasor the carrier is the angle psi is the slow fluctuating phasor, because fluctuation in the phase of carrier phasor will be dictated by the related of psi of t and psi of t is kp times mp. So, the rate of variation of psi is much slower than, the rate of variation of angle an amplitude of an, because the bandwidth of E m t is remember, we used this argument earlier this is not the new argument.

So, bandwidth of this is much low than bandwidth of psi, which is the bandwidth of the message signal and itself, this is B and this twice delta f plus b or delta f plus b itself, at the base band level. So, this fluctuates much more rapidly, so that is why, you could make assumption last time, that psi is nearly constant, while this is fluctuating, we did the short term analysis, and this is the basis for the results obtained. So, basically what are we are saying is as psi remains fixed constant for some time E mt keeps on rotating keeps on fluctuating both in amplitude as well as angle.

But, as long as the amplitude of E m t is small, as a result of rotation of E m t or amplitude variation of rotation of E m t around the phasor, may give minor variations in delta psi. Delta psi will keep going up and down, around this mean phasor, as this carrier phasor keeps on rotating, why psi is fixed, psi is fixed for quite some time. The resultant of this arc will keep on fluctuating in this angle around the mean angle psi is that, that is the physical picture.

As long as E m t is small, now the important thing to study now is what happens when E m t becomes large, when you say E m t becomes large, we are not saying it becomes large all the time. What we are saying is there is sufficient probability, that there finite significant probability, that suddenly E m t becomes very large and remains large for some time.

Typically, if the signal has some bandwidth B, let us say the signal has some bandwidth W, then it can stay in particular amplitude in the level of some particular amplitude, for a time period, which is approximately in the order of 1 by W. This is just the matter of saying, that it is time constant is 1 by W, it changes its value at the rate of 1 by W, you can think of this being the highest frequency component present in the signal. And therefore, generally the rate of variation of amplitudes will occur ((Refer Time: 15:44)) by the highest frequency or it cannot be faster than this.

So, roughly you can say for this discussion that, the signal stays at the particular amplitude level ((Refer Time: 15:56)) for the particular amplitude, for the time which is proportional to the reciprocal of the other bandwidth. Just take that as a ((Refer Time: 16:04)) picture or ((Refer Time: 16:05)) meaning of bandwidth. So, we will just keep that in mind.

So, basically what are you are saying is when there is the finite probability that E m t suddenly becomes greater than A with some significant probability, that is probability of E m t becomes significantly large, when E m t much less than A when signal power is much less than noise power, the quality of this signal happening is going to be negligible.

The threshold effect occurs when quality, if this event becomes significant, now what can happen, so what we are saying is for some time duration, this happens remember true for all the time.

This happens for some time duration, typically the order of 1 by W, where W here is delta f plus b, it is a noise spike and the quality of the noise Spike occurring becomes significant. Noise spike or duration 1 by W, where W is delta f plus B here, such a noise spike, can occur in any case, can occur even in noise is small, but the probability of these occurrences is going to be very, very small, so we are not concerned about that.

But, the probability of the noise spike occurring becomes large, when the carrier power becomes much less than the noise power, so few statements, which I have made here are very significant. First this event has the significant probability and this event is the occurrences of large noise spike and duration of the spike whenever it occurs is going to be order of 1 by W, these are few statements, I have made. So, what happens during this duration, what happens to this phasor diagram, in this duration when this large noise spike occurs, that is what we need to understand.

When it is small, we know that the delta psi will rotate like this, it is basically the delta psi, which is going to vary around the mean value, when E m t becomes very larger the kind of situation, which you are going to have let me separate the picture. Let us consider this noise spike occurs ((Refer Time: 18:56)), this is your 1 by W, let me consider 2 or 3 instants during this spike, that is the t 1 t 2 t 3 etc whereas, the amplitude is significantly large. So, during this interval t 1 t 2 t 3, the amplitude of noise is significantly large as compared to A, much larger, so what happens to the phasor diagram.

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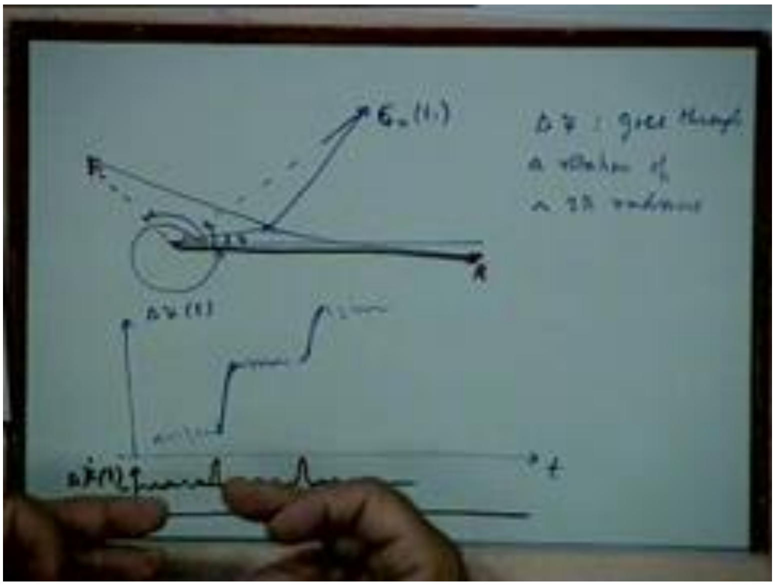

Let us say you have the reference phasor, at time t 1 your carrier phasor ((Refer Time: 19:30)) and suddenly you have very large spike at the time t 1. So, this is E m t 1, this carrier phasor remains of the same amplitude, because this is psi and within this noise spike duration, you have another value which is we are just talking about E m t fluctuating very fast. So, suddenly it goes to let us say another value like this, it s rotating around this remember and both angle is changing very fast and as well as amplitude is fluctuating.

During this entire duration the amplitude is very large, that is what we are saying and the angle is fluctuating, ((Refer Time: 20:21) be equal, but both are very large. So, what is the value of ((Refer Time: 20:27)) in this case the resultant phasor is here, so what is new resultant phasor, the new resultant phasor here. And it can happen at t 3 at your time t 3, you may have same carrier amplitude, let us say this phasor goes like that, this is fluctuating around it, what is the new resultant phasor.

So, now do you see, what is happening to delta psi, look at what is happening to delta psi, this is the value of psi, the value of delta psi is this much ((Refer Time: 21:13)) at time t 1, which this much and it almost becomes 2 pi at this point. So, noise phasor during that spike duration is going from this position to this position, I have just captured 3 positions. One can trace the whole path, I just captured 3 positions at time t 1 it was here, time t 2 it was here and at time t 3 it was here.

There is an almost a rotation of delta psi goes through, the rotations of 2 pi radians. So, instead of small value of delta psi around the mean value of actual value of the phase, what we all finding is that, the spike is suddenly becomes during the occurrence of noise spike very, very large. So, now what is the effect of this, so what you have is very large phase noise spike. So, if you want to plot this, as the function of, if you look at the output of the phase demodulator, which will be proportional to psi or psi plus delta psi, psi is constant. So, I am not worried about that, first look at delta psi as the function of time, because delta psi is the function of time.

So, what is happening is most of the time is noise is small and delta psi is fluctuating around some mean value of psi. Similarly, the spike occurs and almost the phase rotation is 2 pi radians, either 2 pi, see 2 pi is ((Refer Time: 23:14)) of saying, it could be 2 pi, it could be may be some other large value, that is not very important. So, what happens similarly the phasor jumps, that is the amount, the time interval is the order of 1 by W.

Student: ((Refer Time: 23:29))

Because it large becomes suddenly large the amplitude becomes very large and it goes through this phenomenon, being this time. So, output of the phase demodulator will show like this, you are getting some constant value of psi with some fluctuations and suddenly it jumps. And as long as we are here, it will be like this and once again after the second spike occurs, it has another jump.

So, this will be the behaviour of output of the phase demodulator now, instead of continuing to the ((Refer Time: 24:09)), now like this all the time, this indicating noise and this indicating the constant value of psi, it will go to this chance. The time interval here of this duration is going to be the order of 1 by W.

Student: ((Refer Time: 24:26))

No this is not modular 2 pi modular 2 pi value will be the same.

Student: ((Refer Time: 24:35))

Psi value is 2 pi, it is incident value, and it could be some other value, some other large value.

Student: ((Refer Time: 24:46))

For each 1 by W is the duration, for the each time spike duration, it is the typical average duration of the spike.

Student: ((Refer Time: 24:58))

This was the beginning of the spike; this is the end of the spike, during the end of the spike this phase rotation occurs. I am plotting delta psi t as the function of t, after that it starts behaving, it behaves as the same way here, after that it starts behaving in the same way, is that clear?

Student: ((Refer Time: 25:23))

Typically 2 pi, well theoretically it can be anything, because it depends on the kind of fluctuations, it goes through, but yes 2 pi will the typical value ((Refer Time: 25:41)). Now, what happens to this is the phase demodulator output, what happens to the amplitude demodulated output, no sorry FM demodulated output.

FM demodulated after all is obtained by passing the output of the phase demodulator by through differentiator. So, the behaviour that you see is more or less the same except that the differentiator ((Refer Time: 26:07)) of this. So, you will see some fluctuations and corresponding to this, you will pulse the derivative of this, the step function, similarly you will see something like that. So, this is delta psi dot t the derivative of this, this is what we look at, so FM demodulator will look like this. And phase demodulator output will look like this and now how does it makes difference to the noise power output.

Remember whether we are going to small noise case or large noise case, the demodulator is followed by a low pass filter of bandwidth B, corresponding to the message bandwidth. And our argument earlier was that most of the noise power outside this bandwidth will be removed by this ((Refer Time: 27:10)), so how does that, picture change when the behaviour of noise is like this.

The picture changes in the following way, although that picture is still valid, but the effect is this suddenly jumps in the output, this sudden jump in the output or this sudden this pulses in the case of FM. There are significant amount of low energy, if you look at the spectrum of these, spectrum of the unit step function or the spectrum of the pulse, most of the spectrum is constructed in the lower frequencies. So, there are the significant energies of the pulses or these jumps, which lies between the bandwidth of the message signal and you will not be able to eliminate that.

And this suddenly increases because of the occurrences of these pulses, as long as the noise amplitude is small compared to the average signal power, average carrier power, the occurrence of this pulses is very, very infrequent. So, does not matter, but when the reverse happens these pulses occur with the much more frequency and suddenly there is the large amount of energy that lies between the large amounts of noise, which lies in the bandwidth of the message signal, which you cannot eliminate that with finite filter that you have base band filter that you have.

And suddenly the noise output power becomes larger than, than otherwise, so that is the reason the suddenly the output noise power increases and the threshold effect occurs performance increase significantly ((Refer Time: 28:49)) this, this is the ((Refer Time: 28:52)) picture, this is what the qualitative picture.

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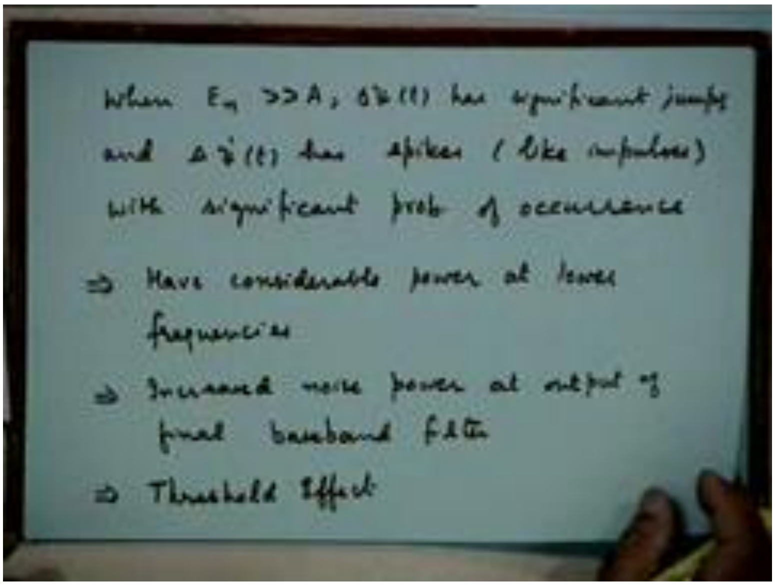

This is what it leads to, so let me write down the some of the points that, I just mentioned ((Refer Time: 28:58)). So, when based on this discussion, you can say that, when this event occurs, psi t has significant jumps and delta psi dot t, sorry delta psi t, not psi t. And delta psi dot t has spikes almost like in impulses with significant frequencies or significant probabilities of occurrences.

That is keep on occurring frequently this kind of event keep on occurring frequently and this in turn denote, that this kind of jumps or this kind of spikes, you have considerable power or energy at lower frequencies. And this leads to the increased noise power at the output of the final base band filter and that is the reason for, the disproportionate increase in the output power as compared to what you have expected. Because of the decrease in the SNR of the input and that is why, you have threshold effect.

See our analysis are did not taken in the account of spike, because this spike do not occur or occur in the very, very low frequency, very low probability, this mean analysis we have to take into account, this analysis for the case of high noise case. So, this leads to the threshold effect. So, more precise, but molecular analysis that one can do and one can work out the precise expression for the output SNR in the threshold region.

Since, I have not gone through the mathematics of it the expression. I like you to look at the book itself there is no point in writing the expression here, but let me draw the picture for you.

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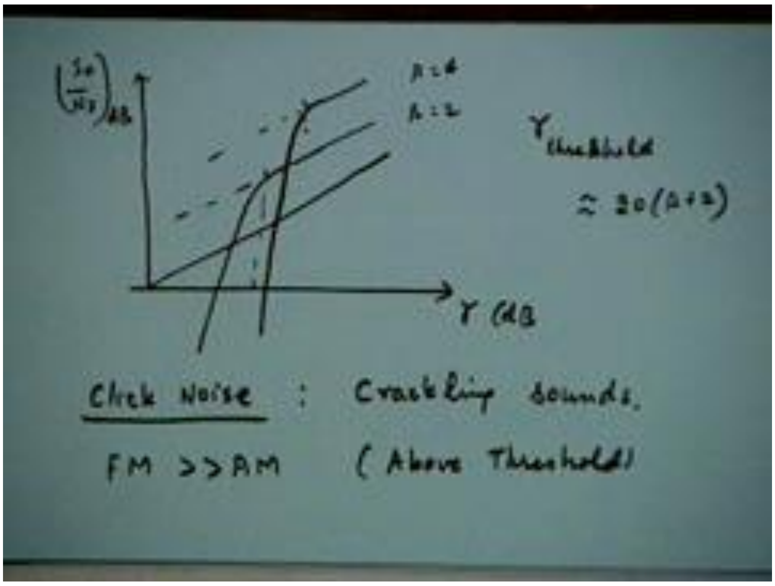

In terms of SNR curves, so if you look at the SNR curves, at the output in terms of gamma, I use the base band case as the benchmark, remember in the base band case the output SNR is equal to gamma, always this has been our bench mark. And what we learnt for angle modulation as we increase the modulation index, the output SNR increases. Now, taking into the accounts that at the threshold affect the actual curves instead of going like that, they go something like that that is the threshold effect.

So, at this point suddenly it starts deviating from straight line curve, very significantly and as we increase the value of beta, it is very interesting, I am not ((Refer Time: 33:06)) always as we increase the value of beta. This threshold becomes more and more significant; this will occur much earlier that it occurred in the narrow bandwidth signal. So, this can occur instead of going like this, instead of going like it will go like this.

So, ((Refer Time: 33:26)) in that case the threshold effect that is the first point, the second point is the point of occurrence of threshold, that is the value of gamma, at which the threshold becomes the threshold occurs is larger for the larger values of bandwidth, than for the smaller values of bandwidth. So, the point of occurrence for threshold is not same for all values of beta. It is different for different values of beta, if at one can prove that the point of threshold is at the same kind of arguments that we use for AM, but much more involved is approximately equal to 20 times beta plus 2.

So, larger the value of beta, the larger the value of gamma, that will be the threshold effect occurs and what will be like, it will be like to threshold to lie this side or side, on the nearer values of gamma. So, unfortunately that does not happen as we increase the value of beta the threshold becomes more and more significant as you go for the higher values of beta in order to get the better performance.

So, there is a kind of contradiction here, it goes for the larger values of beta, it get better output SNR and valued to stay above the threshold value, input SNR stays above the threshold value. At the same time we are buying the probability, that unless you significantly above the threshold value, you can come in to threshold value the threshold region, suddenly you get the large noise at the output.

If you ever listen to the FM radio carefully, sometimes the signal condition are not so good, I do not know whether that particular does not happen in a ((Refer Time: 35:21)) situation, where the power is significantly large. But, it were to happen what will you see is suddenly when that situation occurs cracking sound, very large cracking sound and that is because of the large noise. And that cracking sound is basically due to the noise spikes and this FM ((Refer Time: 35:42)) of clicks, you call this noise as the click noise.

This cracking noise is called click noise, I already explained how that click noise occurs, so it is heard in the form of crackling sounds, so you have to keep in mind, that while FM in general in much superior to AM above threshold, we must use, we must keep in mind that it can become very bad, that below the threshold region. So, you have to keep the input SNR at the significantly good level, so that you are above the threshold region.

Now, these contents are partially mentioned again, it is beyond the scope of our discussion here, when we discussed fm demodulators remember we discussed carious kinds of demodulators. Namely the simple slope detector and we discussed ((Refer Time: 36:58)) we discussed frequency compressive feedback.

At that stage, let us look ahead of the frequency compressive feedback, I do not know how much how many of you remember, what we do there what is the effect that we create there, in frequency compressive feedback system. If we recollect there is a discriminator inside the feedback loop and remember we discussed why should we go for that kind of system.

If you are going to ((Refer Time: 37:28)) know about discriminator, you can have the discriminator without the feedback loop, whether you learnt or if you do that the kind of FM signal that go into the feedback loop in to the discriminator is ((Refer Time: 37:41)) going to have much smaller value of beta. Remember that or compare that, beta that you have at the input. And now you see why does it important you have the large value of beta, because you want to work on large bandwidth at the same time, you do want large value SNR of threshold SNR.

So, you try to effectively reduce the value of beta, for the purpose of demodulation and frequency compressive feedback is the mechanism from which you can. And that is why it works, it is the preferred system when threshold effect is important, to worry about it. Similarly, the phase lock loop has the much better performance compared to the simple discriminator in terms of threshold performance. So, close this discussion here and connecting this what we had discussed earlier, you think about it, if you have some doubts you can discuss it.

Now, we have very limited time now, so I will try to optimise my time and to do that, one topic, which I intend to discuss in detail, I did not discuss personally now. So, I request all of you to read it up on your own for detail and that is I like to return to the subject of pre emphasis and de emphasis, remember we discussed this subject in some other context earlier.

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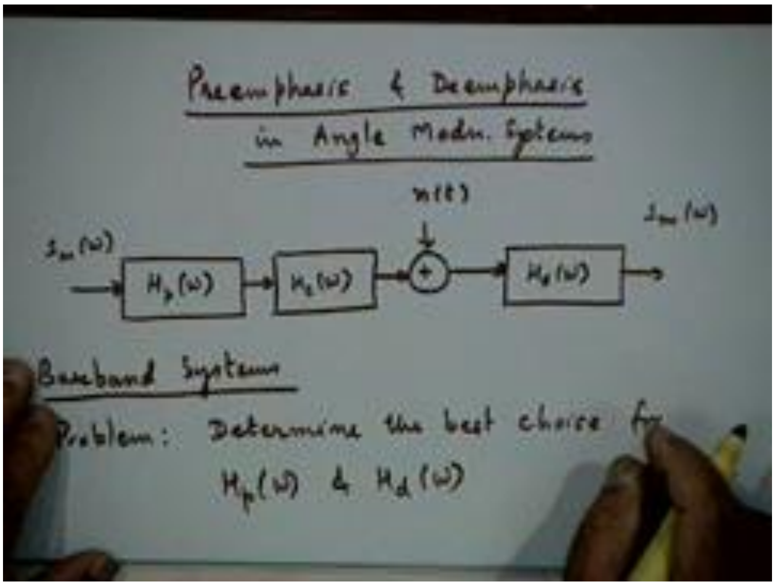


When we are saying angle modulation systems, this is what Ii will not able to do. I briefly discuss this the concept of pre emphasis and de emphasis in the context of noise, for the general base band system first, very briefly that also. And that then ask you to how to extend this concepts in the case of angle modulation.

When we discuss interference, we will always discuss this issue, but I want to know the discussion in the vase of noise and show that in general the concept or pre emphasis ((Refer Time: 40:18)) and de emphasis ((Refer Time: 40:19)) helps us to improve the performance slightly in the systems in all kind of systems present in noise in particularly. So, in the case of angle modulation systems. So, basically if you recollect the model, if you look at the base band system, in base band system, you have the message signal coming, in which is the power spectrum of omega.

Before it transmitted it passes through some ((Refer Time: 40:52)) occurs, this filtering basically modify the spectrum of the system, therefore in that sense it will discard that signal also. But, you want to do the filtering sometimes to take into account certain facts that we know about the communication system. For example the communication system may be better suited to work with let us say higher frequencies or lower frequencies for the message signal.

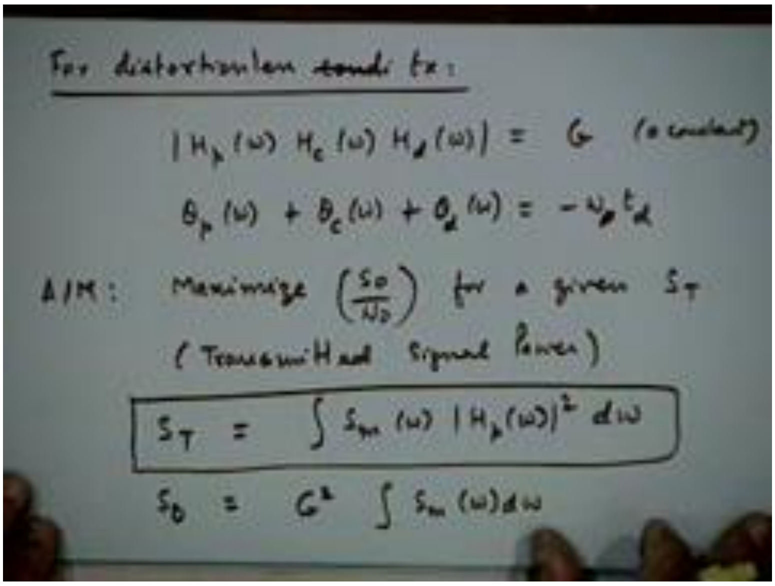
So, suppose it performs poorly for higher frequencies then what will you like to do, you will like to suppress the higher frequencies, if it performs sorry you like to enhance the higher frequencies. Because, you will like to increase the SNR of the higher frequencies, so pre emphasis will do that, but in the process you are going to distort your signal. So, what will do is you have to carry out the corresponding compensatory function of the receiver, which is called de emphasis filter.

So, complete communication system will at the transmitter have the pre emphasis filter at the receiver have the corresponding de emphasis filter. So, the combined product of these 2 is constant. So, keep the discussion more general, we will also have the channel with some transfer function H c omega and of course, you have noise, this the additional complication, you have now that is the complete picture.

Finally you want as far as the message is concerned, if it has the certain power spectrum shape to the input of the system, you must have the same power spectrum shape, at the output of the system. So, that the message signal remain undistorted, in the process the performance the SNR performance, at the output can be improved, if you choose the pre emphasis and de emphasis filters appropriately.

And I want to discuss very briefly, what kind of optimisation, you can do in the choice of the filters to improve the output SNR, I am not really doing this discussion in the context of angle modulation, which is what have written here. I do this in the context of base band systems, but the ideas can be extended very simply in the case of angle modulation, this the base band system modulation.

So, what is the problem the problem is to determine the best choice for H p and H d, what objective, how will you define this ((Refer Time: 43:57)) will be to maximise the output SNR that is the problem we did not consider this issue. So, far because we did we knew that our filter was ((Refer Time: 44:06)), so far with the out of band frequencies both H p and H d are out of band frequencies. But now we are allowing them to modify the spectrum also keeping the overall constraint that these two should be the same. (Refer Slide Time: 44:35)



So, if you want to do that, we know that the for distortion less transmission we must need certain condition and what about those conditions the magnitude of the product of these 3 transfer functions. H p omega into channel transfer function into h d omega should be constant throughout the frequency band of interest, let us call this as G.

Also the corresponding phases phase function of H p omega is called theta p omega plus phase function of H c omega, let us call it phase function will ((Refer Time: 45:30)), is not it because when you put three filters in cascade phase function will. Actual transfer function will get multiplied and see the phase function will occur in exponential of the complex function the exponential will get edit.

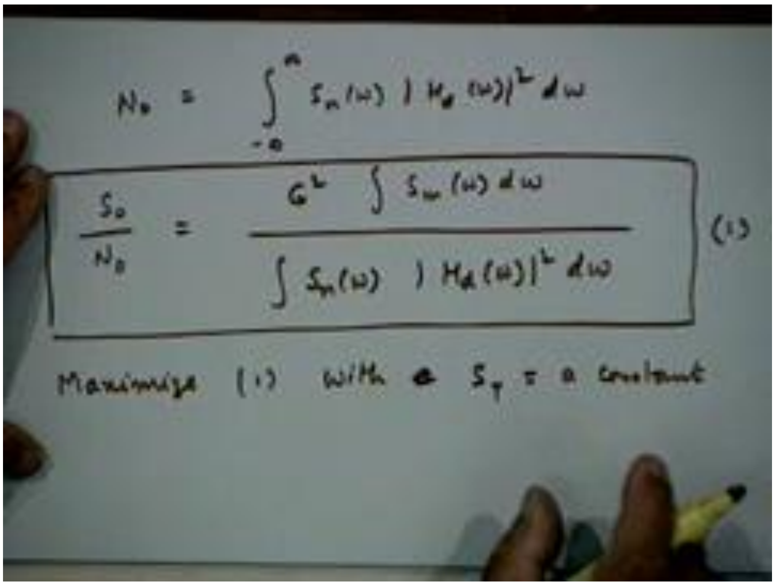
So, theta c omega theta p omega plus theta d omega will be equal to the straight line minus omega not into t sub d or sorry omega times t sub d, the function of omega should be the straight line, linear phase characteristics or constant corrected characteristics. So, if you want to formulate the problem mathematically, this is what we want to do, maximise output SNR S naught by N naught for a given transmitted signal power for a given S sub T.

S sub T is the transmitted signal power or received signal power can you express s sub t in terms of s of omega S of omega is the power spectrum of the message signal. It will be ((Refer Time: 46:53)) of the power spectrum of the message signal, so S sub T, the transmitted signal is not this, this is the transmitted signal. We are modifying the spectrum of the message, through the pre emphasis filter. So, power spectrum of the signal here, the area of that will be the transmitted power

So, S sub T is S N omega into mod H p omega square d omega, the area under that is the total power in the input of the channel, let us assume that the channel does not attenuate, in this case the attenuation factor is formed separately ((Refer Time: 47:50)). What can you say about, so your problem is to maximise the output signal to noise ratio for the given value ST, which can be written like, this which depends on S T.

So, now to do this you must write the expression for output power, output signal power output noise power and take the ratio and maximise, that is what you need to do, what is the output signal power output signal power final output is what S of omega. So, output signal power is simply, I have made this statement that assuming that, there is no attenuation that is not correct attenuation is continuing in the factor of G, attenuation or gain or whatever, you like to call it. So, output SNR Output signal power is G square times tell me integral to S of omega, this is the final signal, you are getting this is the attenuation factor, this is what W are getting, what is the expression for output noise power.

(Refer Slide Time: 49:05)



Let us say the noise, in general case, we are considering the general case you could consider the white noise, but less us consider the general noise, which has some power spectrum S N omega. When you consider white noise you can always put S naught is equal to N naught by 2 S of omega times tell me can you speak out H d omega, because noise will only go through this transfer function, the receiver transfer function d omega. So, therefore, the expression for s not and n not is G square integral of S N omega upon integral S sub omega into H d omega mod square.

So, now our problem becomes finite, this is the expression for the output SNR, we want to maximise this let me call this equation number 1 with a constraint, because 1 ((Refer Time: 50:24)) way of maximising. This will be to maximise the transmit power keep on increasing the S N omega keep on amplifying the signal, so you keep on varying and maximise will be the ((Refer Time: 50:34)) solution. Then the answer will be maximum or ((Refer Time: 50:37)) signal, but; obviously, there is limiter the amount of signal power to be transmitted various reason.

For various practical reasons, you will have to, so there is a constraint for which you have to maximise this. So, basically you have to depend only on full chain to do the optimization to increase the output SNR not full phase solution of increasing the transmit power where the constraint that S sub T should be equal to some fixed constant. Now, this is the problem, I do not know whether you solved these kind of problems in mathematics earlier, you then ((Refer Time: 51:27)) optimization problem.

In any case, you would not use mathematical formula tools to solve the problems, because the ((Refer Time: 51:35)) tools that are requires are calculus of variations, which I am sure you are not, I do not know whether you done it or not. But, we do not require fortunately the simple result in mathematics, which helps us do this optimization, almost in the trivial manner. But, since the time is up we will stop here and very quickly do the discussion next time and then move on to pulse modulation systems, this is the next topic.

Thank you.