Communication Engineering
Prof. Surendra Prasad
Department of Electrical Engineering
Indian Institute of Technology, Delhi

Lecture - 36
Noise in Phase and Frequency Modulation Systems

We continue with our discussion on Noise in Angle Modulation Systems, it has been quite some time, few days now.

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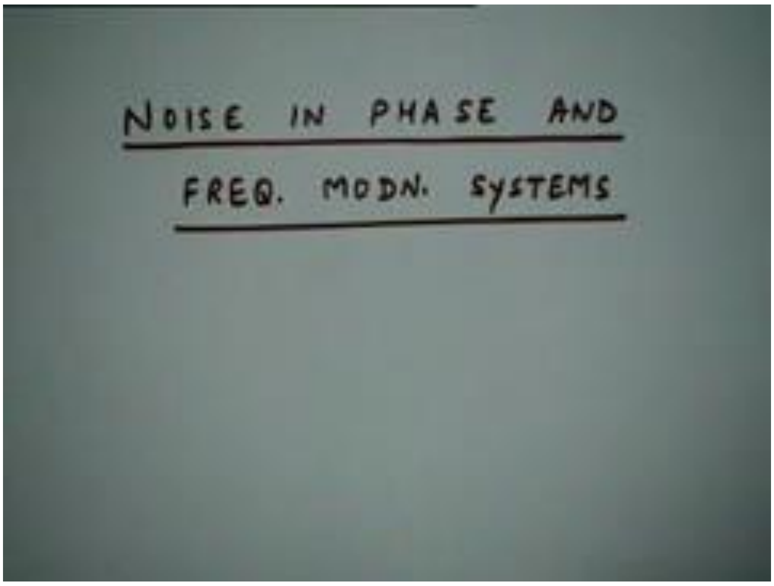

Since we discussed in class, now let us recap the basic principle on which we are going to discuss the analysis.

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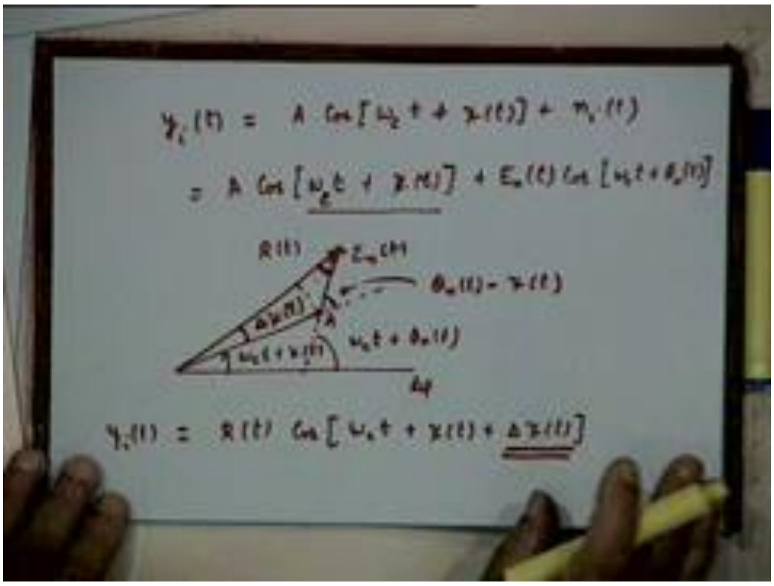

If you recollect, we have decided to draw the phasor diagram of the signal plus noise. In this case, you have received signal at the output of the bypass filter at the receiver is the sum of the desired PM signal plus the narrow band noise or band pass noise and n sub i t which you can write like this, E sub n t denotes the envelope of the noise and theta sub n t denotes the instantaneous phase of the noise. So, look at the sum of the these two signals and that comes in the form of phasor diagram.

This phasor represents the signal, the modulating signal then the modulating signal is equal to omega c t plus psi of t, this phasor represents the noise with the amplitude of E m t and angle reference to the reference phasor of omega c t plus theta n t. And therefore, the resultant phasor that you see y i t has this resultant envelope, this is the envelope of the resultant phasor, horizontal signal and the total angle that you will see will be omega c t plus psi t plus delta psi t.

And therefore, we can write this signal also equal to this signal where R t is this phasor and delta psi t is this angle. I think this are required much explanation and if you have any questions you can discuss it now. Is there anything, shall we start from here, then if we proceed a little further, try to write the expression for the delta psi t in terms of quantities indicated here. You notice that this angle is theta n t minus psi t that is this angle minus this angle, from this triangle.

Having figured out that this angle is theta n t minus psi t and making the assumption of small noise, which is the place we are considering first. The small noise plays with the assumption first corresponding to the situation where the length of this phasor is much smaller than the length of this phasor. So, that we can proceed actually the scales are not appropriate in the figure that I am show.

If it is really true, you can assume this two lines are going to be parallel because these are very far away from the origin and therefore, this two angles are nearly equal. And therefore, you can write an expression for this length, this perpendicular from this R we can say from the point A to the resultant phasor as sin of this phasor times E m t, which is what we had discussed last time.

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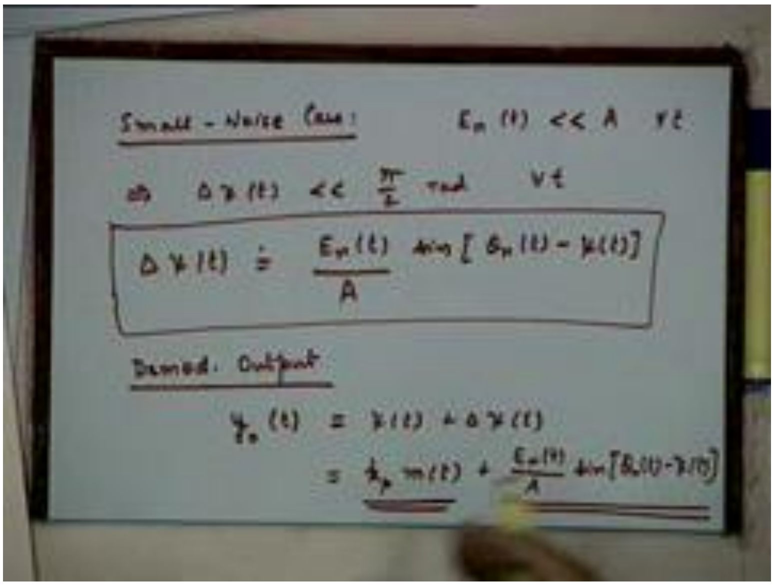

Assuming delta psi t is very small, which should be the case when E m t is much, much less than carrier amplitude. You can write the expression for delta psi t as equal to E m t sin theta n t minus psi t which is associated to the length of this arc E m t into sin of this angle upon the radius, you can approximate the angle by arc length upon radius which is note down here. So, this is the arc length that is numerator is arc length and A is radius, so you can approximate delta psi t by this angle.

So, this now becomes the basis for the further discussion, so what can we say for the demodulated output. Now, what will be our demodulated output like is y naught t, remember what is the input to the demodulated input, what is the input here to the demodulated it is this signal. An ideal phase demodulator what will it do, It will produce the output which is proportional to the instantaneous phase deviation, so it will produce output which is psi t plus delta psi t.

So, assuming that you have a ideal phase demodulator, the output of this demodulator will be psi t plus delta psi t, desired output would have been psi t if you remember psi t is k sub p times message form of m t and delta psi t is E m t upon A sin theta t minus psi t this is the desired output and this is the contribution of noise. Now, the analysis is, on the phasor are still complicated because the noise term also confusing the signal term that is why it is a non-linear system its exact analysis is very, very difficult.

However, now we make approximations or simplifications that I taught about last time, look at the comparison between theta m t and psi t both are phase terms, but theta m t is the phase component of the noise cross, psi t is the phase component of the modulating signal crosses. Modulating signal is bandwidth of t. So, psi t is bandwidth of t, but theta t is a bandwidth of very large of 2 delta plus 2 D, theta m t is coming from the noise, these are low pass components envelope and phase of the band pass noise passes. And this band pass noise passes is coming out of band pass filter, whose bandwidth must be equal to 2 delta plus 2 D.

So, the band width of the baseband signals E m t and theta m t are much larger as we discussed last time, what does it mean, in this point can we concede this point, these two signals this envelope and phase are much larger than the psi t. And therefore, the consequence of that is that the rate of ratio of theta m t is much larger than the rate of position of psi, the fluctuations in theta m t will happen at much faster rate than it will happen in the phase psi t.

Which means that if I have to do the analysis of this output related to short period of time you can consider over that period of time psi t will be nearly constant whereas, theta n t goes through lot of fluctuations. Let us assume that let us do the analysis as if you are doing the analysis for short period of time, comparatively short period of time not infinite time comparatively short period of time and of course, you can extend the arguments over period to period.

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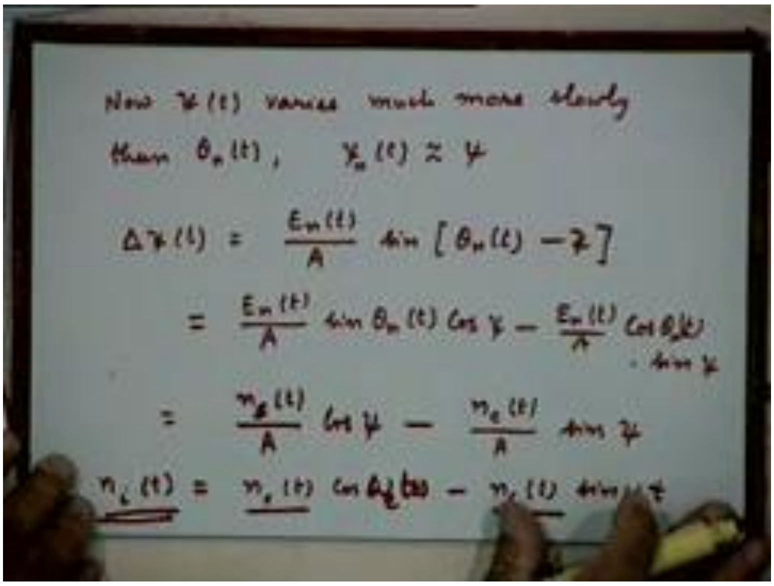

So, if you if you make this assumption, or rather which is really true this psi of t varies much more slowly than theta n t theta sub nt. We can assume that psi of m t is nearly constant over the period of consideration is equal to some value, constant value equal to psi, is it. So, what can be say about delta psi t in this case, we can write this as E sub on m t upon A into of sin of theta n t minus psi.

How does it help, how does doing this approximation help, basically I am trying to get rid of dependence on the signal, this noise part, this is a output noise delta psi t is a kind of output noise that you could see. I want to expose it purely in terms of noise properties if possible, so simplifications I make use of this is where as this fluctuates much faster whereas, this remains constant over certain period of time, are you comfortable with the point that I am making here.

If there is a case, we will expand this using trigonometric identities, we can write this as sin theta n t into cosine psi minus E m t upon A cos theta n t into sin psi.

Student: ((Refer Time: 11:30))

Delta s i t is a wide band process because theta m t is a wide band process.

Student: ((Refer Time: 11:40))

This I have discussed last time may be you have forgotten, let us bring out this issue the signal has this phase psi of t. Noise is this, this is the noise process what I am saying is this noise process which we have seen in the output of the band pass filter at the receiver, that band pass filter must have the bandwidth equal to the bandwidth of the FM signal, is in it. If that band pass filter does not have the bandwidth equal to the bandwidth of the FM signal you will not allow the PM signal to come through.

So, the band pass filter receding demodulator is a filter of this bandwidth and therefore, the noise process which was wide at the input to this filter will now be the band pass passes. Whose bandwidth is this that means both E m t and theta n t are low pass processes whose bandwidth is delta f plus B. This is the point I explained in detail last time.

So, I thought of not repeating it today, since you have forgotten this is the result because our demodulator is preceded by a band pass filter, which must have sufficient bandwidth to allow the desired signal to come through which has the bandwidth of 2 delta f plus B, is that clear. Therefore this fluctuates must faster than this also means that delta psi t fluctuates must faster than psi because delta psi t depends on theta m t is that, so let us now go for this.

What is E m t into sin theta m t that is nothing, but n c t that is in phase component of noise with band pass representation, so this is n c t upon A into cosine psi and similarly what is the E m t this is n s and this will be n c into sin psi. Now, if you remember the characterization of n s t and n c t in terms of n i t, what are those n s and n i. Let us recap the band pass process, noise process of the input of the demodulator is n c t cosine omega ct minus n s t sin omega c t.

This is the representation of band pass noise process, which can also be represented as E m t into cosine of omega c t plus theta n t. Now, what is the representation of these two quadrature components in terms of this, let us say in terms of power spectral density, the power spectral, this is the band pass process whose height will be N by 2, let me plot it.

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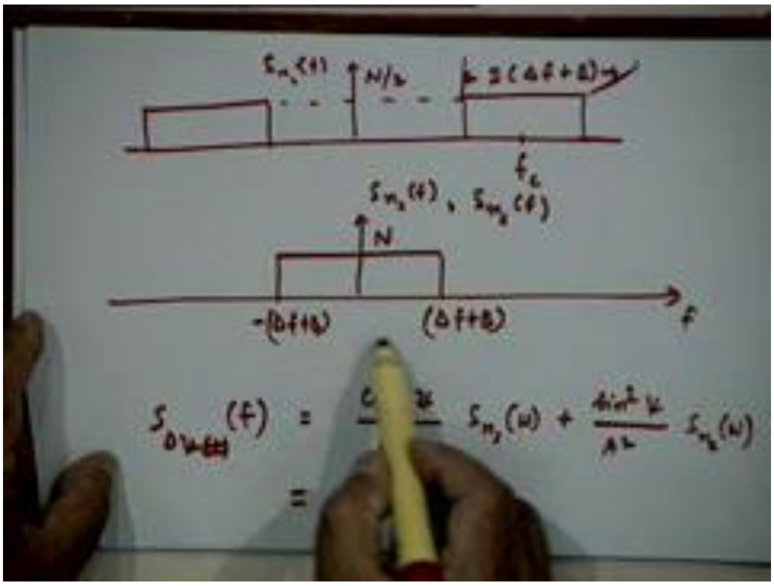

N i t will have this kind of power spectrum, do you agree with this, around f c which will have band a width of 2 delta f plus B, this is going out of this region and the height is N by 2, what can you say, this is the spectrum of n i what can you say the spectrum of n c t and n s t they are low pass processes whose spectrum will be N from minus delta f plus B to delta f plus B, height is N because the two components must have the same variance as this, so the height ((Refer Time: 16:24)) is it clear.

So, this is the power spectrum n c t and also the power spectrum of n s t, so this is the power spectrum density functions of the two quadrature components in terms of the power spectrum density function of the band pass noise process. So, if we want to write this now, suppose I want to look at this expression think of delta s i t is the output noise processes, a random processes output noise process it is a linear sum of two noise processes, what can I say about the power spectrum density of this noise process assuming that n c and n s are unquadratural ((Refer Time: 17:11)).

The quadrature component of noise band pass noise are unquadratural, so what is the power spectrum density of this, the autocorrelation of this will be the autocorrelation function of this, plus the autocorrelation function of this, because the cross correlation is 0 and therefore the power spectrum density will be the power spectrum density of this part plus the power spectrum density of this part. So, this should be essentially, this coefficient square times the power spectrum density function of this plus this coefficient square times the power spectrum density of this.

So, you can write s delta psi t f and this simply say s delta psi f will be cosine square psi upon A square s n omega plus sin square psi upon A square into s n c omega and that is nothing, but this two are equal this two are same. We can defected out, so we will left with essentially any one of them s n c omega upon A square and cleverly like this we have dependence on psi. Because there is some assumption involved while doing this analysis for a fixed period of time on which psi remains constant.

But the noise processes fluctuates, Is it okay, so that is what we say that the power spectrum density function of the output noise in terms of the power spectrum density function of the input. So, if I have to expand this further.

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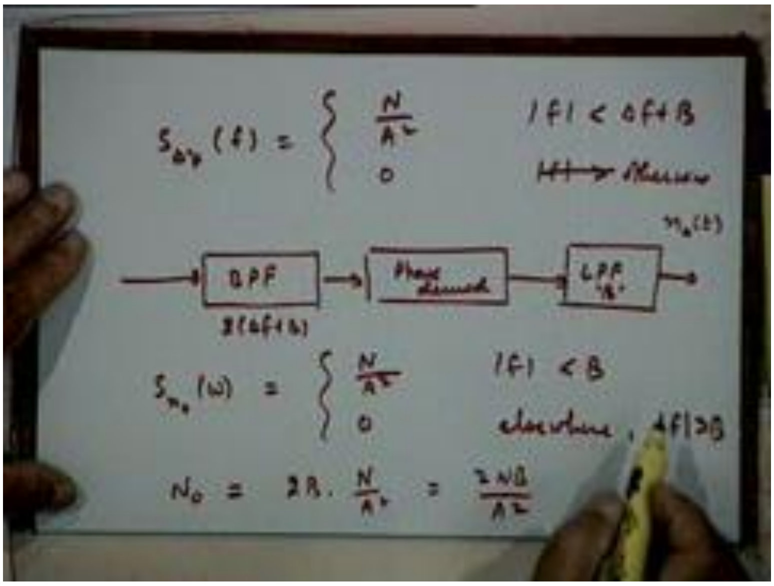

This reduces to S delta psi f equal to N by A square for mod of f less than delta f plus B and is equal to 0 otherwise and I am just substituting for the fact that s n c or s n s f is equal to N from minus delta f plus B to delta f plus B which is written here. So, interestingly the output noise processes has the same power spectrum shape, but it is height is much lower, its lower value is equal to A square, the amplitude of the carrier and that is really a very interesting phenomena

Let us come back to this, if you go back to the calculation of noise power, now what do we need to consider for that, this is the power spectral density function as seen in the demodulated output, demodulated output contains two parts the signal part and the noise part. But when we are going to look at this is not the total output noise, can you see that see it is because, so far the demodulator was working with signal whose bandwidth was delta f plus B or 2 delta f plus B, but my signal of interest has the bandwidth only of B, so at the demodulated output I can have a filter whose bandwidth is only B.

I do not have to look at the entire noise process, this is the power of power spectral density of delta psi t, but you know my senior would have this, if you go back to the block diagram and do that we have this band pass filter whose bandwidth is 2 delta f plus B, this will be followed by the phase demodulator and relating delta psi t at this point and this delta psi t has the bandwidth of delta f plus B. So, as far as the signal part of the demodulator output is concerned that has the bandwidth only of B.

So, I do not have look at the entire noise process, I must go for the pass this to a low pass filter whose bandwidth is B. So, the final output noise process which we see here will have bandwidth only of B, so the noise power that will contributed by the noise processes is delta psi t will be the one which will lie in the bandwidth from minus B to plus B, so if I consider this noise processes as n o t then the power spectrum density of this processes will be same as this N by A square, but not over delta f plus B, but simply B.

So, what is the total noise output power, it will be 2 B into N by A square, which is 2 NB by A square, what is the output signal power here, what is the output message that you will get here K t times n t, if the phase denoted is perfect.

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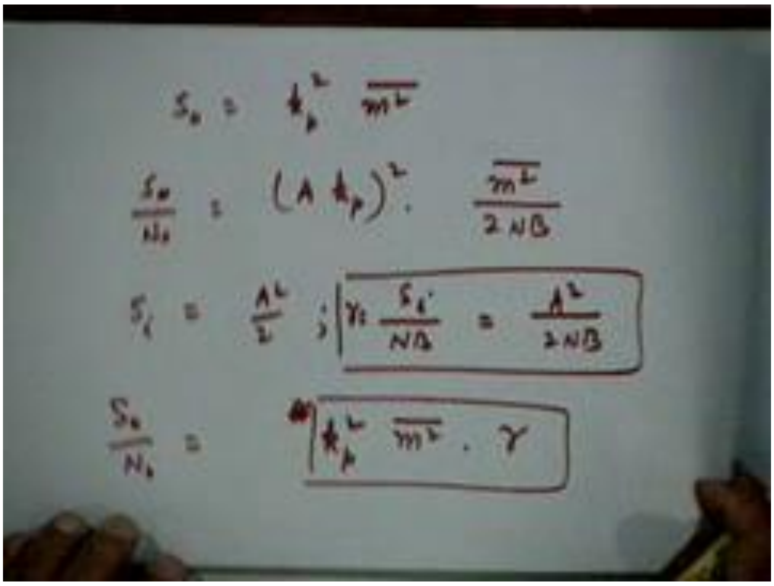

So, the signal power will be k sub p square m square bar therefore, your output S N of S naught by N naught will be A k sub p hole square into m square bar upon 2 NB. Now, let us do the final job of expressing this in terms of input signal to the noise ratio in the message signal, so let us recap what is the input signal power in this case, what is the nature of input signal, just the input signal component it is A cosine omega c t plus psi t what is it is power A square by 2.

So, s i is simply A square by 2, which implies that s sub i upon NB which is the value of gamma if you remember gamma is defined as the s sub i upon NB the input signal to noise ratio only NB, that is going to be equal to A square upon 2 NB.

Student: ((Refer Time: 24:35))

No, this has nothing to do with n i, gamma if you remember the definition of gamma is it is a benchmark parameter, it is defined as the input signal power upon input noise power in a message bandwidth, message bandwidth not in incoming signal bandwidth. Because this is bench mark for comparison, how much you see, why this benchmark, this benchmark because the nosie that is present in message bandwidth which you cannot get rid of ever.

That is why for all AM systems, the output is same as equal to the gamma itself, so this is going to be the universal benchmark for comparison, we would like to always express the output SNR in terms of gamma. So, gamma is what you could have got in any way by a simple modulation scheme like E m or just by not using any modulation scheme, for example, if you are transmitting the signal at the base band itself this is the output SNR you would have got.

I am comparing with that always the benchmark is the figure which you are going to compare is this particular, so in this case s i upon NB is equal to A square upon 2 NB, do you agree with this, any questions on this. So, let us now expose S naught by N naught in terms of gamma. So, if I combine these two expressions what can I write S naught upon N naught is equal to A square will cancel off, we will left with k p square m square bar into gamma, so this is the key expression if you write this separately this makes an impact on this.

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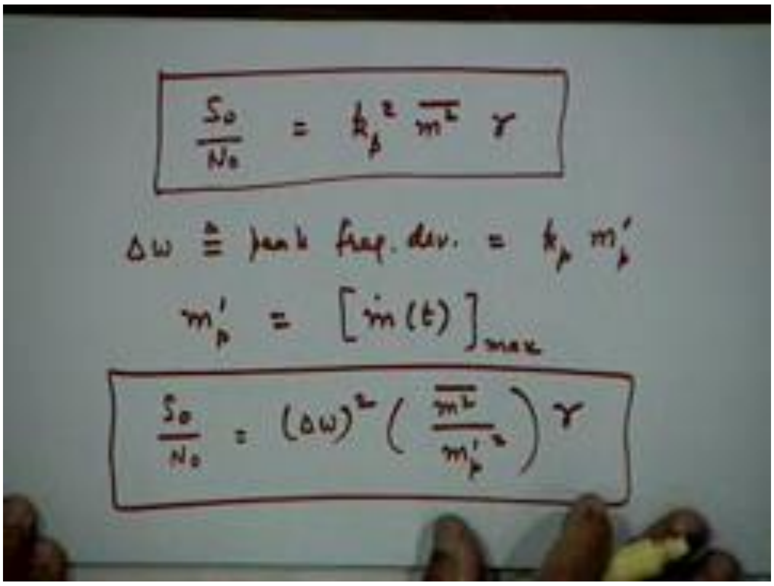

To see the impact of this expression and how compares what we have discussed earlier let us do some more calculations just try to express this m square bar in terms of standard parameters which we describe angle modulating signals. What are the parameters in terms of which we describe angle modulations systems in terms of peak frequency deviation for example, so let us light in terms of peak frequency deviation rather m square bar which is slightly unknown entity here, so in the case of phase modulation, what can I say about the peak frequency deviation, delta omega or delta f.

The instantaneous value of phase is equal to the k p times m p, what is the instantaneous value frequency or frequency deviation it will be k p times derivative of m p. So, what is peak frequency deviation going to depend on, they will depend on the peak value of derivative of the signal. So, I can write delta omega of the peak frequency deviation, in this case envelope frequency deviation this is going to be equal to k sub p times, let me denote by m p prime as the peak value of the derivative.

So, let me make it very clear m p prime here denotes, if you look at m dot t the message waveform, looking at its maximum value is it ok, that I am defining as m p times. So, let us substitute for what is delta omega substitute k p for here, so S naught by N naught that equal to delta omega whole square into m square bar upon m p prime square into gamma, so let us now look at this expression.

This expression is very informative because this in terms of parameters of modulating signal is the peak frequency deviation, this is parameter of the message signal, the ratio of the mean square of the message signal to the square of the peak derivative, maximum derivative some property of the message signal. It is going to be some constant for even kind of signals, so you can take this as some constant value, which depends only on the message signals, I mean the message signal we have.

For example in angle modulation, you can calculate what this value will be it will be some fixed value and this is the input SNR in the message band signal the benchmark when you are comparing against any modulation system. So, what we have find, we have find that the output SNR is proportional to if you forget about this constant delta omega square the maximum peak frequency deviation and larger the peak frequency deviation the better the output SNR.

That means that if I double the frequency deviation from the existing value my output SNR will increase by 4 times in terms of decibels 4 times is 6 decibels, 10 log of 4 is 6. So, every doubling of the peak frequency deviation is going to improve my output SNR with respect to the input SNR by an additional 6 dB's, so this is very, very important parameter to note.

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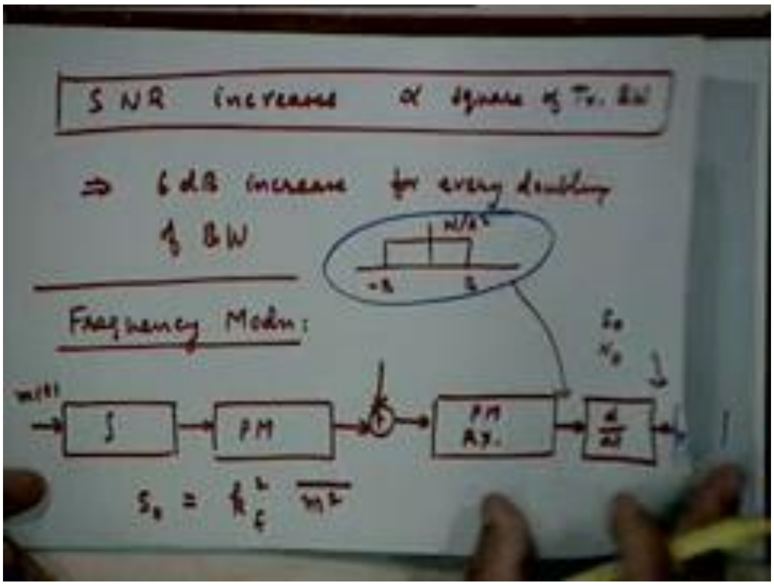

So, SNR increases and delta omega in some sense is the measure of the bandwidth of the message signal is in it because actually bandwidth is delta omega plus 2 by B, delta f plus B. So, if you assume that this much smaller than delta f, so delta f is the measure of bandwidth in some sense, so larger the bandwidth of the phase modulated signal the larger the improvement or better the output SNR, so that is the key result here.

So, SNR increases in proportion to the square of the transmission bandwidth, this is the key result there is a 6 db increase in the output SNR for every doubling of bandwidth, where does this benefit come from why does it happen. So, basically what we are saying is wideband phase modulation will be much superior to either base band transmission or AM transmission what does this improvement due can you see, that can you visualize that if you go through the steps of the analysis, which particular step leads to this great improvement of output SNR.

I suggest that you look at the steps that I had given in the correction with the phasor diagram, go back to this expression this phasor diagram, you see the phasor diagram of the signal plus noise losses and important point is delta psi t, this is the output noise and that is this expression is this deviation of A which causes the real improvement. The larger the amplitude of the carrier, irrespective of the noise that is when you are working with small noise condition.

Thus smaller the effect of noise on delta psi t is like this the noise is additive, let us go back to this expression this noise is additive, this noise will affect the amplitude significantly can affect the amplitude significantly. But effect of this noise as though the effect of this noise will not be large on phase of this signal is going to be much, much smaller and fortunately for you the phase demodulator will look at the phase not the amplitude.

In fact, we are going to have the band pass limiter not just the band pass filter, the band pass limiter at the input of the band pass limiter at the input of the demodulator. So, it will be totally insensible to the amplitude variations. It will be only sensible to the phase variations, the phase variations is came down by the effect of A and that is what causes the spectrum density to come back to be multiplied by 1 by A square and things like that N by A square and not N and that is the crucial factor.

The spectral density is not N over the bandwidth of interest it is N by A square over the bandwidth of interest that is what causes the reduction of noise. Basically the phase of the signal is much less sensitive to the noise process than the amplitude distance and you are pulling amplitude distance and phase demodulator is only sensitive to the phase variations and that is ((Refer Time: 34:58)), so that is the reason why the FM and PM signals are much superior in performances to AM signals.

Because this direct this is trade off of course, if you want more improvement in the SNR we use more bandwidth. In fact, this always this is conventional wisdom of communication engineers, if you want better SNR you must pay a price in terms of bandwidth, this picture has changed slightly over the recent years, but this conventional wisdom generally holds.

If you do some clever things you can make the status better, but generally this status always exists, improved SNR will come at the cost of bandwidth. We will discuss more examples as we go on, so that is as far as phase modulation is concerned, let us now time to frequency modulation, how that things change now.

Student: ((Refer Time: 35:59))

We will come to the large noise separately, let me complete the small noise analysis both for PM as well as FM then I will discuss the large noise case. So, the entire analysis that

I did for PM is more or less correctly valified FM if you recollect that you can think of the FM modulator in terms of the PM modulator and FM demodulator in terms of the PM demodulator, so we have to do only minor changes in the analysis to rather required conclusion.

So, if you recollect that I could have message m t first integrated and then pass through the phase modulator remember this, that is equal to my frequency modulator, then you have noise, then you have phase modulation, a phase demodulator or phase modulation receiver, which you have in SSB band pass filter or band pass limiter in front of it. And this will be followed by d by dt and that will give followed by low pass filter etc. So, you can think the low pass filter as the part of this signal.

So, just follow what the phase demodulator receiver like differentiator and preceding the phase modulator by the integrator and that is your FM and the PM system. So, I do not have to repeat the arguments and directly write the output signal power here and output noise power here, can you do that. What can you say about output signal power, it will be; obviously, equal to k f square m square bar I do not think i have to repeat that argument

This is the frequency demodulator this entire thing, this produce an output which is proportional to k of m t, so output signal power is k f square into m square bar, what about the output noise. So, to look at that, let us look at the power spectrum density remember what was the nature of the power spectrum density at this point, it was the power spectrum density at this point of noise was like this, between minus B to plus B and constant was equal to N by A square.

If you recollect this, that is the power spectrum density function of noise at the input of the low pass filter and therefore the output of the low pass filter had the total noise power was the area under this, which is N by A square into 2 B, but now you are passing this by d by dt. So, what will happen to this noise power spectrum density and this is followed by a low pass filter or low pass filter can be any where this is for the reference.

So, what will happen to this power spectrum density here, it will get multiplied by the transfer function of this differentiator, what will be the transfer function of the differentiator, j omega magnitude square of the transfer function, power spectrum density function is modified by the magnitude square. So, it will get multiplied by the mod of delta omega square or mod of delta f square, so what will it look like.

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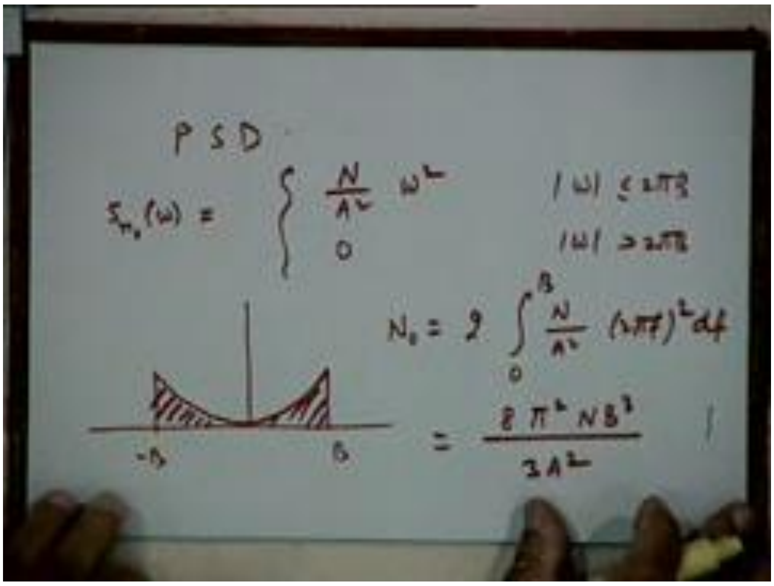

So, the power spectrum density function will be as follows, S sub naught omega will be equal to N by A square times omega square for omega lesser than equal to 2 pi B and 0 for omega greater than 2 pi B, same thing but is multiplied by omega square of f square depending upon the ((Refer Time: 40:26)). So, it will look like that this I am making it flat over minus B to plus B will look like a parabola.

What is the output, noise power you would see, it will be the area of this power spectrum density function, so output noise power N naught is going to be 2 of integral from 0 to B of N by A square into 2 pi f whole square d f. If you have to calculate this, term has to be 8 pi square into NB cube upon 3 A square, this integration is straight forward combine the expression for S sub and N sub, we can write the expression for S sub.

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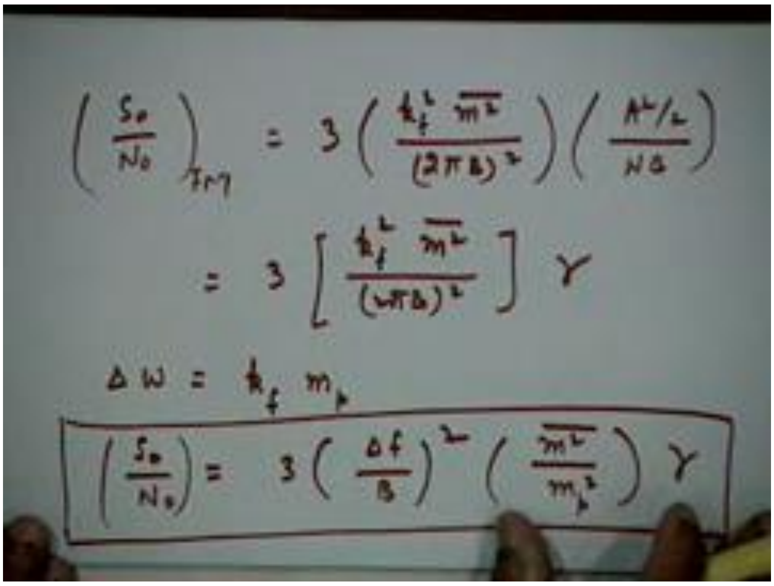

Signal to noise ratio at the output, in case of FM, can I just write it down rather looking at the details, it turns out to be k f square into m square upon 2 pi B whole square into I am skipping few steps, so that you can write directly, write in terms of gamma and that is 3 times k f square m square bar upon 2 pi B whole square into gamma. Again to write in the standard form I do not want to write it terms of k f like I did it in the case of PM I substituted k p in terms of delta omega the peak frequency deviation.

Similarly I substitute for k f in terms of the peak frequency deviation because that is the measure of the bandwidth, so directly writing the expression in terms of band width parameter it is more convenient. So, now what can you say about delta omega the peak frequency deviation it is k f times simply m p not m p prime, m p prime was the peak value of the derivative of the message signal, this case is peak frequency deviation which depend on the peak value of the signal itself is in it because in FM signal instantaneous frequency deviation is proportional to the amplitude of the message signal it is equal to the k f times m p.

Student: ((Refer Time: 43:21)) let it be like this, you need to adjust with this itself.

So, what we can say is S naught by N naught, if I make the substitution it turns to be the 3 times delta f by B whole square into m square bar upon m p square into gamma. By and large a very similar expression the details are slightly different instead of m square bar upon m p prime square you have m square upon m p square some other constant in the message signal.

But this factor depends only on the message signal, instead of depending on delta omega square it depends on delta f upon B whole square you see actually. If you think about it something very similar to the beta parameter, beta that is the expression we have, but by large the same confusion is valid that every doubling of the bandwidth will produce a 6 db increment in the output SNR.

The detailed expression is slightly different, actually the values are slightly different which means that there is a valuable question that you can ask here that for a given message signal, whether FM is superior or PM is superior that will depend upon the message signal you have, you cannot drive general conclusion, because that will depend upon the value of these constants. So, depending upon how these constants change either the FM or the PM of the signal. Next we will do a small comparison between these two.

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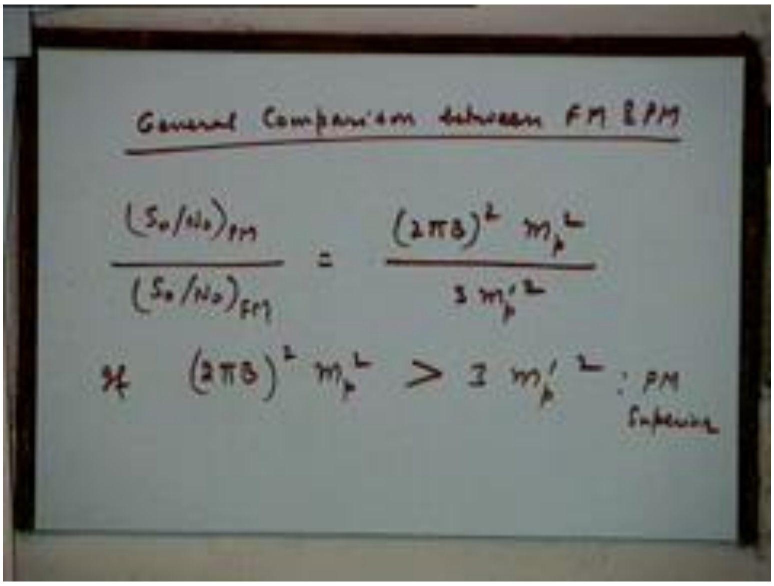

Any questions so far, let me write down the ratio of these two expression for the phase modulation phase and for frequency modulation phase, the output SNR’s for the two phases you have two expressions you just divide one by the other. And simplify the gammas square will cancel out and delta f square will cancel out and what we will be left will be the parameters of the message itself, anything else will go is in it delta f will go, gamma will go only message parameters will remain and that will give some idea of how it depends on, so this will become, now who want to say to find which one is superior you just look at this ratio.

If I give the message signal this is greater than 1 PM will be superior and if it is less than 1 FM will be superior. So, let us say 2 pi B square m p square is greater than 3 prime m p square then you say PM is superior, otherwise FM is superior. So, I think whether you can convert these things into some kind of physical picture very gross physical picture can be converted, if you understand what is the meaning of m p prime, m p prime if you remember that is the derivative, the peak derivative of the message signal.

So, if this value is small, then you can expect PM to be better what does it mean this equation indicates that if the most of the spectrum of the message signal is constructed in lower frequencies because in some sense the derivative, the derivative will be large if high frequencies components are large, just think about it some kind of physical picture I am giving. So, if the bandwidth is large you can expect large variations, bandwidth is large means large fluctuations, large fluctuations means larger derivative some way or the other.

So, this is the gross physical picture I am not trying to make it more precise, so generally if the most of the spectrum in the message signal is constructed at lower frequencies you can expect PM to be perform better, If in the other hand most of the spectrum is constructed in higher frequencies FM will be better, something like that it is just a gross picture.

And that is why in practice one does not either use pure FM or pure PM remember, I talked about some preemphasis or deemphasis earlier one tries to optimize the performances in that sense.