

Communication Engineering
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Lecture - 35
Noise in AM and Angle Modulation Systems

I hope you recollect what we were doing in this topic. We quickly remind you some of the things we have learnt, so that you can at least in the same way that I am going to talk on. We discussed the noise performance of the AM systems, first for the case of synchronous demodulation and we found that in this particular case the signal to noise ratio was given by the expression.

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The image shows a whiteboard with handwritten mathematical expressions and text. At the top, the signal-to-noise ratio is given as $\frac{S_o}{N_o} = \frac{m^2}{A^2 + m^2}$. Below this, the text reads "Demod. of AM Signal via ED" and "provided that $A + m(t) \gg \gamma_1(t), \gamma_2(t)$ ". At the bottom, it states " $A + m(t) \ll \gamma_1(t), \gamma_2(t)$ ".

Output SNR was given by the, this directed expression, where this average message power, this is carrier amplitude. And this of course, is worse than what we get in DSB sc when you use DSBSC this is DSB AM with some percentage modulation with some modulation index or modulation index. And we found that there is a reduction SNR by this factor because of the inefficient use of carrier, inefficient use of the transmission power.

But other than that the fact was the output signal power was clearly dependent on the input signal to noise ratio. Then we perform the if we consider the case of demodulation of AM signal via envelope detector, we consider the noise in this situation again and we

found that the same expression is still valid. The same expression for the SNR is still valid provided that what is the condition when it is valid, provided that noise is comparatively small compared to the carrier that is $A + m(t)$ is much greater than $n_c(t)$ or the quadrature components $N_C(t)$ on $N_S(t)$ for all t that is if for all time or most of the time the noise remains much smaller than the signal amplitude.

The signal here we do not need here signal, the carrier plus signal then you get the same expression for the output SNR. So, there is no difference in the performance of the synchronous demodulator or an envelope detector under the small noise conditions, the performance is identical. Then consider the other extreme, when noise takes over that is when opposite occurs when $A + m(t)$ is much less than these two things that is signal is much weaker than the noise. And under this condition we have obtained the expression for the envelope of the output signal.

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$$\begin{aligned}
 E_o(t) &= \sqrt{(A+m(t)+n_c(t))^2 + n_s^2(t)} \\
 &= \sqrt{\underbrace{n_c^2(t) + n_s^2(t)}_{E_n^2(t)} + 2n_c(t)(A+m(t))} \\
 &= \sqrt{E_n^2(t) + 2n_c(t)(A+m(t))} \\
 &= E_n(t) \sqrt{1 + \frac{2(A+m(t))n_c(t)}{E_n^2(t)}}
 \end{aligned}$$

The whiteboard also shows a small diagram of a right-angled triangle with sides $E_n(t)$, $n_c(t)$, and $n_s(t)$, and a hypotenuse labeled $E_o(t)$. Below the triangle, the ratio $\frac{n_c(t)}{E_n(t)}$ is written.

Which was given by, let me just recap how did you obtain the expression the exact expression for the envelope was the in phase component that is $A + m(t) + n_c(t)$ whole square plus $n_s^2(t)$. This is the exact precise expression for the envelope with no approximations, evaluate this assumption of $A + m(t)$ much less than this basically what it means is when you expand this term here the square of $A + m(t)$ can be ignored that is going to be much smaller than either n_c or n_s .

So, we can write this as if I ignore the square terms this will be $n c \text{ square } t$ that will be the term coming from here plus $n s \text{ square } t$ that is the term coming from here and there will be also the cross product term. I want to ignore the $A \text{ plus } m \text{ t}$ the whole square because that we are assuming much smaller than $n c \text{ t}$, so the cross term will be $2 n c \text{ t}$ into $A \text{ plus } m \text{ t}$. Now, denoting by $E_m \text{ square } t$ the quantity $n s \text{ square } t$ and $n c \text{ square } t$ which gives you the instantaneous envelope of the noise alone.

Then you can write this as square root of $E_m \text{ square } t$ plus $2 n c \text{ t}$ into $A \text{ plus } m \text{ t}$ again if I take $E_m \text{ t}$ outside this I will be left with $1 \text{ plus } 2 \text{ into } A \text{ plus } m \text{ t}$ into $n c \text{ t}$ upon $E_m \text{ square } t$. Now, let us look at the last quantity, particularly let us look at the quantity $n c \text{ t}$ upon $E_m \text{ t}$ consider this ratio. Let me draw the normal trigonometric thing, this represents here $n s$ this represents your $n c$ then this will represent your $E_m \text{ t}$ and this will represent your $\theta_m \text{ t}$ $\theta_m \text{ t}$ is the instantaneous phase of the noise part. So, can I express this ratio in terms of $\theta_m \text{ t}$, cosine $\theta_m \text{ t}$.

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The whiteboard shows the following derivation:

$$E_i(t) \doteq E_n(t) \sqrt{1 + \frac{2(A+m(t)) \cos \theta_n(t)}{E_n(t)}}$$

$$\doteq E_n(t) \left[1 + \frac{A+m(t)}{E_n(t)} \cos \theta_n(t) \right]$$

$$E_i(t) = \boxed{E_n(t)} + \boxed{(A+m(t)) \cos \theta_n(t)}$$

\Rightarrow Totally Noisy, with no component of $m(t)$

So, this will be we can write this part as your instantaneous envelope becomes $E_m \text{ t}$ into $1 \text{ plus square root of } 2 A \text{ plus } m \text{ t}$ upon $E_n \text{ t}$, one of the $E_n \text{ t}$ will stay into cosine $\theta_m \text{ t}$ that is the expression we have for the instantaneous angle. You once again use the approximation that $A \text{ plus } m \text{ t}$ is going to be much less than $E_n \text{ t}$, $E_n \text{ t}$ is the noise amplitude, $A \text{ plus } m \text{ t}$ is carrier plus signal amplitude which is going to be much smaller than the noise amplitude.

So, once again if you use the approximation you can write this you can binomial expression of this square root and you can only get the plus term and you will get $E_m t$ this is approximately now equal to. Of course, this is also approximate because we have made small signal approximations. So, this will become $E_m t$ into $1 + A$ plus $m t$ upon $E_n t$ into $\cosine \theta t$, which you can write as $E_m t$ plus A plus $m t$ into $\cosine \theta t$.

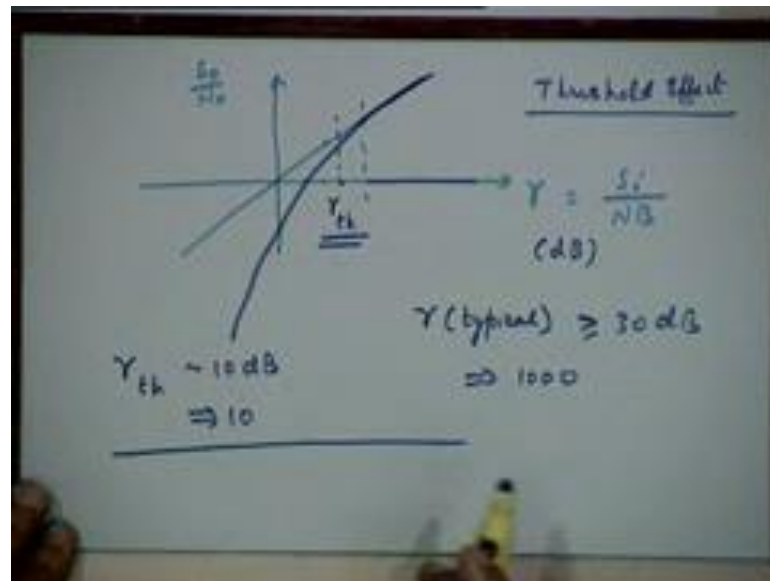
So, what we see from here that is the final expression we are looking for you, instantaneous angle out is this which is here envelope detector will produce this kind of an output. What will you have been like to see you will like to see the something proportional to $m t$, unfortunately in this term you do not find any term which is costing time $m t$.

It is not even that we have costing time $m t$ plus something, the plus something you could have considered as noise and you would have signal term you would have noise term, noise term while calculating the signal to noise ratio, what you get the output is the almost entirely noise, because this is noise envelope. So, this is noise nothing to do with the signal this is also noise this of course, is getting multiplied with $m t$, so the does not help that is still noise.

The signal $m t$ which is multiplied by the noisy wave form will be the noisy waveform, so this is not even signal plus noise term of expression. So, essentially the envelope is almost totally noisy with no component which you can consider as proportional to the message $m t$, which is the desired signal. So, what is it means; that means, that under large noise conditions the envelope detector fails to deliver the signal of interest, it completely breaks down.

Now, how does that compare with what does the synchronous detector would have done under this condition at low SNR at under large noise conditions. The synchronous detector when we derive this expression, when we derived this expression for the case of synchronous detector we did not make any assumption that the signal is small or large. This expression is valid for all condition of signal and noise, do remember that which means that.

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If I want to plot, for the case of synchronous detector a plot of output SNR versus left side gamma. Where the gamma if you remember is s_i upon NB the signal to noise ratio at the message, in the message bandwidth then what would be the for the case of synchronous demodulator it is a straight line, it has a slope given by this expression, it is a constant slope and it will pass through the origin. So, it is straight line curve like that do you agree.

So, what does it show that even here when the input SNR falls down that is when the noise becomes large the output SNR will of course falls down that is bound to happen nothing we can do about that, but the falling down is very grace full it is linear falling down. But what do we find, now our finding now is as the noise becomes in the case of envelope detector as the average SNR is high that is well this kind of region, let us give it out some value.

If your input SNR, if your input signal is much stronger than the noise we are more or less on the same curve because the two expressions are same. However, the moment you fall below certain input signal to noise ratio, when the input signal becomes weaker than some value or input signal to noise ratio falls below some values you start deviating from the straight line curve, you start moving down. Eventually at lower SNR's you get a signal which does not resemble, because the output does not resemble the input at all in terms of message, so you will get a curve like that.

Did you see early, you should have asked me what does the negative axis here denote, I am plotting this in decibels. So you can have negative dB value or positive dB value because signal to noise ratio in absolute terms of values will be positive, because it is the ratio of powers. But, when we take it in decibels you can have negative value of signal to noise ratio because it is $10 \log$ of the actual value, so this what it happens this is the behaviour.

So, this is, what you are going to see in the envelope detector and this kind of phenomenon that is exhibited by the envelope detector is called the threshold effect. Basically what the threshold effect says is that the output SNR does not degrade gracefully as the input SNR is reduced. Beyond certain value of the threshold, let us say below this value you find that the output SNR degrades much more than the input SNR degrades.

If it degraded by the same output it would have been the straight line, it degrades much more. And therefore, there is a concept of some kind of a threshold value of input SNR and you must operate always above the threshold value, because if you operate, if the receiver operates below the threshold values less than threshold then you will get it very bad highly deteriorated output, which will not resemble the signal at all.

Fortunately, this particular effect is not that important in typical broadcast applications in amplitude modulation. The reason being that the value of the threshold SNR at which it will start to happen unless is in the order of 10 dB that means, unless my input signal to noise ratio falls below 10 dB, I will continue to be in this more or less in the linear pair. And typical acceptable values of the signal to noise ratio in the broadcast applications is any way much larger typically.

So, gamma typical remember I discussed this earlier with you will be greater than 30 dB or like 40 dB, so which is much more than this because this absolute terms about 1000 and this will absolute terms about 10 is in it. So, since the typical values are going to be much larger than the threshold value, we never see that threshold effect in typical radio receivers. Because they are typically operating at input signal to noise ratios which are much larger than the threshold values at which this phenomenon will start to be seen.

So, this point of comfort that unfortunately for at least broadcast kind of applications it does not, we can still use what are motivations to use the envelope detector, the

motivation was to simplify the receiver, but at the same time you do not want to lose on performance this says that you will lose on performance. But fortunately the loss of performance comes in those regions where it is not important, you are not going to use that regions at all in typical applications.

So, it does not matters, so the simplicity can still be captured without much loss that is the lesson you have learnt from this, any questions one of the issues that arises here is, how will you calculate the value of 10 dB or whatever it is, so called the threshold SNR. So, I just give the brief indication, we will not go in detail discussion of this, but I think you should have some feel for how to calculate this SNR at which this starts to deviate significantly from the straight line curve.

Now, this point will of course is arbitrarily define some error, you could define it here, you could define it here or you could define it here. Let me some definitions and do the calculation accordingly, so what will you do is we will define the point of threshold in the particular manner and then calculate at what SNR input SNR the point of threshold will occur.

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Calculation of $\gamma_{\text{threshold}}$

pdf: $E_n(t) = \sqrt{n_1^2(t) + n_2^2(t)}$

$n_1(t) = n_2(t) \cos \omega_c t + n_2(t) \sin \omega_c t$

Given $p_{n_1}(x) = p_{n_2}(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-x^2/2\sigma^2}$

Then $p_{E_n}(x) = \frac{x}{\sigma^2} e^{-x^2/2\sigma^2}, x > 0$

So, let us look at the very typical calculation of the threshold SNR, you understand the meaning of gamma threshold, gamma threshold is that value of s_i by NB such that below this you will start seeing significant deviation of output SNR curve compare to the linear curve that is the definition. Now, we can do this calculation by looking, at by

talking in terms of probabilities. So, we talk about let me first give some important results which hopefully you should know by now.

If you read the probability theory properly, the probability distribution function let us consider the probability distribution function actually I should talk about small p d f the probability density function of the noise envelope $E_m(t)$ is the noise envelope if you remember and how is the noise envelope expressed in terms of the quadrature components, it is this what will be Gaussian noise, where $n_i(t)$ is given by $n_c(t) \cos(\omega_c t)$ plus or minus $n_s(t) \sin(\omega_c t)$.

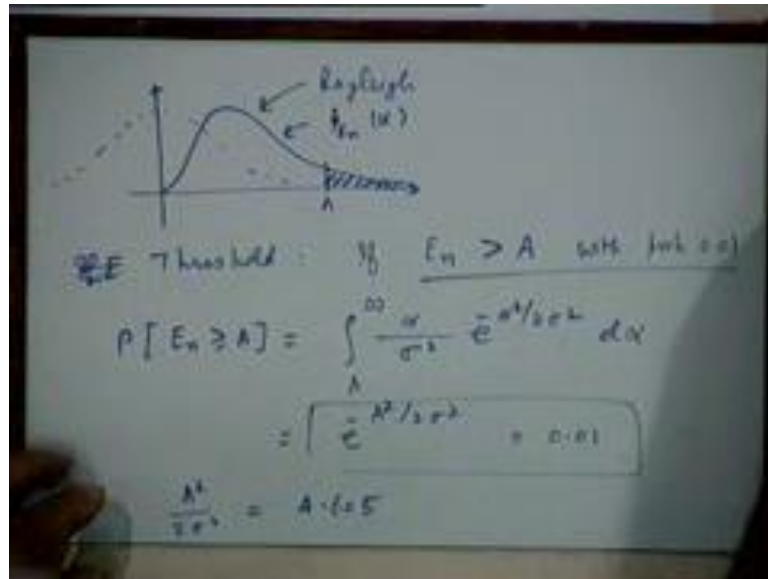
So, this is Gaussian and what else do you know, this is also Gaussian these two quadrature components are also Gaussian. Therefore then if you review properly from the probability theory we would know that if we take the sum of the squares of the two Gaussian normal variables and take the square root of that then the resultant normal variable is not the Gaussian density function it is Rayleigh, it is a Rayleigh density function.

If you not reviewed this please review it, so given that density function of n_c and density function of n_s lets us say all Gaussian g_n is the Gaussian variables, assuming both n_c and n_s has variably has variable signal square then it is very easy to say that, so if this is given then the density function of E_n the density function of instantaneous envelope will be given by this expression for α greater than 0. Almost this has to be positive values always, so this density function is defined for the positive values of the argument, $E_m(t)$ should always be positive and that is the Rayleigh density function.

Incidentally, what will be sigma square in this case, sigma square is the noise variance of n_c and n_s that will be the same noise variance of n_i and what is each of them is equal to $2NB$ if you remember, is in it, this is the noise variance of the variable signal which is coming out of the band pass filter before the envelope detector. What is the bandwidth of that filter $2B$, what is the noise pass frequency height n_0 by 2, n by 2, so n by 2 into $4d$ $2d$ on the positive frequency side and $2d$ on the negative frequency side.

So, signal square is the noise variance, which is the power of the noise signal, which is the area in the power spectral density function, which is going to be equal to n by 2 into $4NB$, so sigma square is going to be $2NB$ in our case. Is this calculation understood, good.

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So, this density function looks something like this, If you would have known it, I am sure that you know it, the Rayleigh density function, the Gaussian look like that, the Gaussian. The Rayleigh density function since it is going to exist for the positive values of alpha it looks something like that, so that is the Rayleigh density function. So, that is the little bit of background that is needed, now let us come back to our threshold calculation this was to review one of the important results that were required for the threshold calculation.

So, you have to define the threshold, you can say that if say threshold occurs if E_n noise envelope becomes greater than the carrier amplitude A with the probability of 0.01 or more. It is basically a if the probability becomes 0.01 normally we will like the noise to be much smaller, when the noise is much smaller the probability if exceeding this will be very small. What we are saying is if the probability of the noise amplitude exceeding the carrier amplitude becomes 0.01 that is the point we say that the threshold has started, this will be the arbitrary definition you can change this you can take 0.1 if you wish or 0.05.

So, depending on the what probability value you fix here, the resultant value of gamma threshold will be different, so this is just something that arbitrarily you have to fix and then calculate the probability of that. So, basically what you are saying is that if the noise amplitude is greater than the signal and the carrier amplitude with this much probability

the probability of this happening is 0.01 or more then the threshold phenomenon all in the threshold region.

So, this is the point of threshold, at which that is what you need to understand, so what is the corresponding signal to noise ratio which will result in this kind of a situation that is what you need to calculate. So, basically what you are saying is if this is your probability density function curve, this is basically the quantity of, this is the density function $P E m$ alpha, what you are saying is, this is the area under the tail of this curve what is this probability this point is A.

So, when this area is the 0.01 that is the point of threshold, so what you are saying is if you look at the probability that $E n$ is greater than or equal to A that will be this area that will be given by the integral from A to infinity of this Rayleigh density function, alpha by sigma square divided by e power minus alpha square by 2 sigma square d alpha. This is very simple integrated to evaluate this value will be e power minus A square upon 2 sigma square.

And this is going to be equal to this probability you are saying, should be equal to 0.01 at the point of threshold. We are defining that the threshold point to be the point at which the probability of this event becomes equal to 0.01. Now, from this you can calculate the corresponding value of A square by 2 sigma square, so that means, that A square by 2 sigma square should be the natural log of this and that turn out to be 4.605.

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$$\sigma^2 = 2NB \Rightarrow \frac{A^2}{4NB} = 4.605$$

$$S_i = \frac{A^2 + m^2(f)}{2} \quad m(f): \text{ tone } 100\% \text{ modulation.}$$

$$= \frac{A^2 + 0.5A^2}{2} = \frac{3A^2}{4}$$

$$Y_{\text{threshold}} = \frac{S_i}{NB} = \frac{3A^2}{4NB} = 13.8$$

$$\Rightarrow 11.4 \text{ dB}$$

That means your sigma square is $2NB$ this means you are A^2 by $4NB$ is 4.605 , now let us talk in terms of input what is that we want to talk about, we want to talk about s_i , s_i by m what is the s_i in our case s_i if you remember is A^2 plus m^2 average value of that upon 2 that is the message signal power that is the carrier power plus message power. Let us assume that for the sake of simplicity of calculation that m is a tone, a pure tone, pure sinusoidal and let us assume that modulation is 100 percent.

If the modulation is 100 percent the amplitude of pure tone, what is amplitude of the pure tone, what is the power is maximum power in m^2 , A^2 by 2 , so this will be A^2 plus $0.4 A^2$ upon 2 , which is $3 A^2$ by 4 . So, what is the value of gamma threshold, now that is s_i by NB this is $3 A^2$, s_i is $3 A^2$ by 4 and the A^2 by 4 is 4.605 . So, this becomes 13.8 or in dB's this is 11.4 dB which is consistent with the figure which I gave you minutes ago I said approximately 10 dB.

So, this is typically how you do the threshold SNR calculation is, this understood any questions.

Student: ((Refer Time 30:23)) Sir in what basis the value of probability was given to be 0.01 .

Arbitrarily I told you, It is the matter of defining, you agree on some definition because otherwise at which point it starts it is really difficult to say in that sense, this is more or less the((Refer Time: 30:43)) you agree on some definitions that is all. Basically what I say is the point is, the idea is to say at this point there is a significant probability that you get away from that linear curve, you get away from the linear curve by a significant amount So, I think that is brings us to the end of the discussion of noise in amplitude modulation systems.

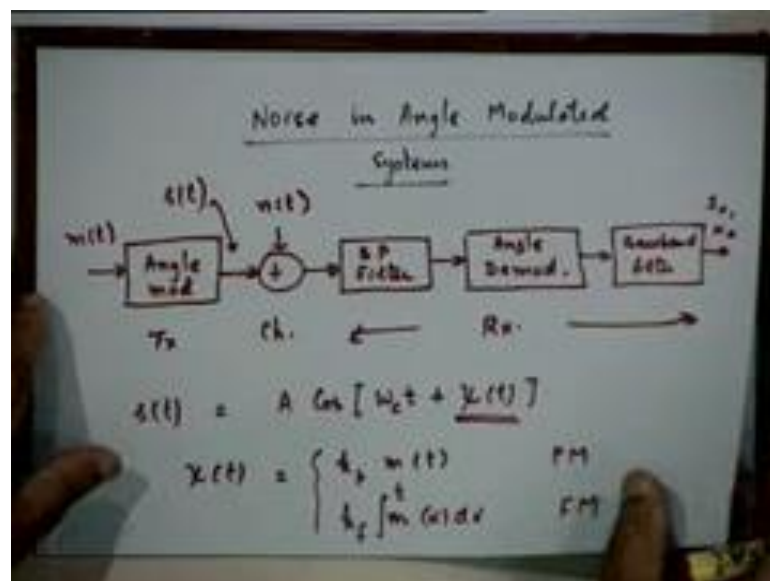
So, we discussed all kinds of amplitude modulations DSBSC, SSB, VSB, synchronous modulation then DSB AM with carrier both synchronous and non-synchronous demodulation. So, more or less that exhaust in the same picture. What are the various messages that you can summarise at the end of this discussion, message number one the SNR performance of typically all synchronous modulation schemes by the one with the carrier are same, the output SNR is same as the input SNR in the message bandwidth.

There will be no difference, they all are same in terms of SNR performance they are close in terms of bandwidth and other issues which we have already discussed in terms of noise performance they all are equivalent that is message number one. Message number two, when you have non-synchronous demodulation using an envelope detector you get a performance loss corresponding to the power loss that is embedded or implicit in the use of this modulation schemes, nothing surprising there also because not all the power is been used to convey the message signal.

Somehow just the transmitting carrier which contains no intelligence of the component and therefore, you get the SNR reduction by that factor. Third message there is again no difference between the performance of synchronous detector and an envelope detector as long as the noise is sufficiently low; however, the noise is higher than the certain value you can see the threshold phenomenon in the envelope detector, not in the synchronous detector.

Synchronous detector continues to perform with the same expression, same expression for the output SNR of course, the output SNR degrades as the input SNR degrades, but it degrades linearly, you say it degrades gracefully whereas, now it is not degrading not so gracefully.

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Next thing obviously, what we have to do is noise in angle modulated systems, what is the model now the model, now is as follows at the modulator, at the transmitter we have

the message followed by an angle modulator then we have the channel which introduces the noise, at the receiver at the front end you select your signal of interest using a band pass filter this is followed by an angle demodulator.

And finally, you have the base band filter whose bandwidth is same as that of the message signal and you are interested in calculating the output SNR by calculating the output signal power and the output noise power. So, that is your transmitter that is your channel, this is your receiver and that is your model which you have to use for doing your SNR calculation before you proceed on to the calculation just couple of remarks to keep in mind.

We cannot exactly proceed the way as proceeded for the amplitude modulation for two reasons, One is the band pass filter here has a very different band width as compared to the band pass filter that we had at the input to the amplitude demodulator, you have to say that because your angle modulated signal has the much wider bandwidth of course, we are considering the wide band FM is in it, it has much wider bandwidth.

So, it has band pass filter, if you want to ensure that the angle demodulator receives an distorted angle modulated signal, it must ensure that most of the significant frequency components of the modulated signal pass through. So, whatever is the bandwidth of this signal that is coming out must be allowed to pass through this band pass filter. So, bandwidth of this message, band pass filter is not dB, if the these are the bandwidth of $m(t)$ this is much more than it will depend on the peak frequency deviation of the angle modulator.

That is the one difference as compared to the AM signals it will have some implications which have to understood. The second complication is the amplitude modulation systems were linear at least when we were discussing synchronous demodulator, it became non-linear when we were discussing envelope detectors, because there is square root operations and things like that happening, otherwise they were linear systems.

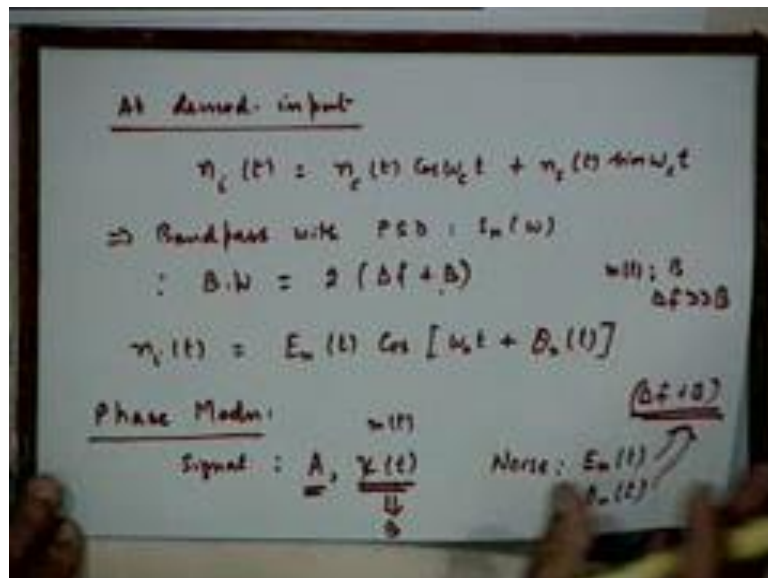
That means, I could and we have actually used the principle of super position that is we assumed that if we calculate the effect of signal in the output and the effect of noise in the output, I can do that separately. Independent of each other I can calculate this the input signal power and that is the output signal power, this is the input noise power what

is the output power noise power and then take the ratio, linearity imply that the ratio will be just the same.

I cannot do that anymore this is the linear system, so these are the two components applications that we have to take care of analysis of angle modulation systems so let us proceed. Let us start with the modulation signal s of t which is here, s of t is here this has a form $A \cos(\omega_c t + \psi(t))$ plus some waveform ψ of t which contains the intelligence where the waveform ψ of t is equal to $K_p \int m(t) dt$ if you are working with phase modulation systems and $k_f \int m(t) dt$ when you work with the frequency modulation systems $m(t)$ is the message.

This is the model in which you have to work now, so let us start from the very first point you have wide Gaussian noise been added on the channel, what kind of noise you see here, once again narrow band noise for this narrow band noise has a bandwidth much wider.

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So, at the demodulator input always look at the noise at the demodulator input that is a key thing, the noise n sub of i t when I say i it refers to the input what point in the demodulator what is the point in the receiver after the band pass filter input to the demodulator remember that. So, this again can be written as $n_c(t) \cos \omega_c t + n_s(t) \sin \omega_c t$. So, there is no difference here as far as the representation goes representation is still valid, it is the narrow band noise process it will have this representation. So, this is

band pass process with power spectral density function, which will be denoted by $s_n(\omega)$.

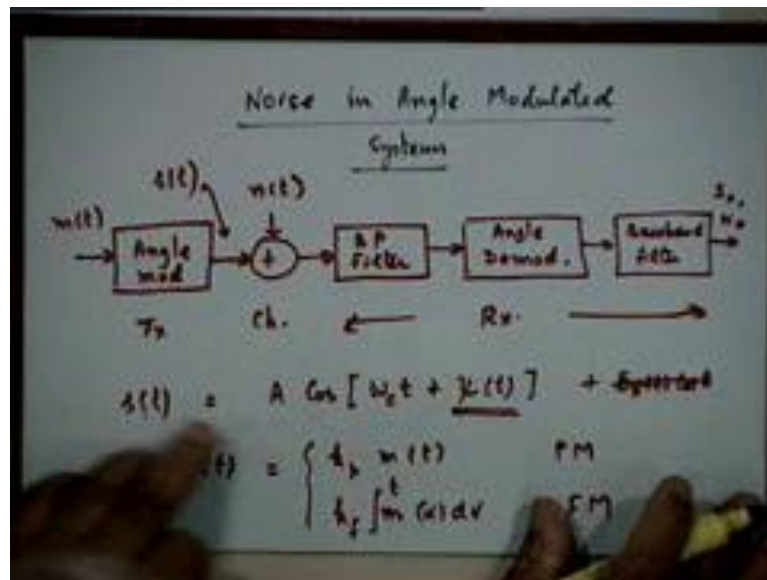
The important thing is that this power spectral density function will have the band width which is equal to $2\Delta f + B$ in it ((Refer Time: 39:58)), where Δf is the peak frequency deviation of the modulator whether it is FM modulator or PM modulator. Alternatively we can also write $n(t)$ in terms of its envelope $E_m(t) \cos(\omega_c t + \theta_m(t))$, where $E_m(t)$ is square root of $n_c^2 + n_s^2$ and θ_m is the tan inverse of n_s upon n_c or you can write other similar equations.

Next consider phase modulation and before you proceed one point we will like to you to understand what is the implication of the fact that the noise has this band width whereas, your signal what is the signal by the signal, I should say message actually the message has the bandwidth of B . So, $m(t)$ has the bandwidth of B , but the noise that you are considering at the demodulator input has the much larger band width typically Δf is much much larger than B .

Student: ((Refer Time: 41:31)) sir you are passing through the filter right the band pass filter

Yes that is the ((Refer Time: 41:46)) I am saying this is the band width of the filter that is going to I mentioned this earlier you have to ensure that the bandwidth of this filter is equal to the bandwidth of the FM or PM signal, if you do not ensure that your angle demodulator is not getting the right kind of input for the demodulation. So, the band width of this filter has to be equal to $2\Delta f + B$ that precisely that point whereas, the bandwidth of the message is B that is all I am saying message signal has the bandwidth of B , $m(t)$ has the band width of B the signal which is going the noise and the signal which are going into the angle demodulator both have the bandwidth of $2\Delta f + B$ that is all.

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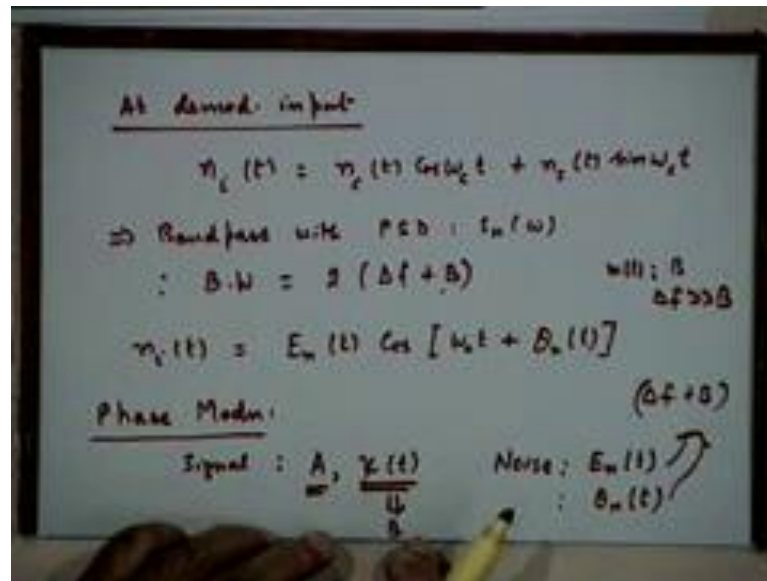


I am saying, so I am not saying different from what else, what is the implication of this, the implication of this is called as look at the signal, this signal also have the same value the signal which is reaching here also has the same value, noise also has that band width, but the message, the message is not embedded in the s t message is not s t message is in s i t and that has the bandwidth of ((Refer Time: 42:52))

What it means is to this variable noise, what you have seen here is not only this plus noise. So, that is the important point to note now to understand that if I look at the sum of these two things this signal plus this noise, this noise has the very large bandwidth this also has the very large band width, but this has the very small bandwidth.

Now, therefore, this will fluctuate very quickly $E m t$ plus cosine theta $m t$ whatever that.

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Let me write it separately, so at the input to the system the signal has the instantaneous amplitude which is constant A and instantaneous phase which is psi of t. The noise has the instantaneous amplitude which is E m t and has the instantaneous phase which is theta m t this is the time to compare these two, this is what we have to understand. A is angular constant, A is the amplitude of the input signal the modulating signal psi of t is proportional to m t for the phase modulation psi of t is K p times m t.

Now, what we are saying is E m t which is the instantaneous amplitude of noise and theta m t they will be having bandwidth proportional to delta f plus B. So, they will vary much more rapidly than psi of t this is the important point since psi of t has the bandwidth of B and each of these have the bandwidth delta f plus B is in it, these are low pass signals, the band pass signals has this bandwidth the corresponding two low pass signals will have this bandwidth.

n c t is E m t cosine theta m t and n s t is sin theta m t, so these two signals envelope and phase of the noise will fluctuate much more quickly, much more rapidly than the phase of the signal of intelligence than psi of t; that means, you can think of the phase of the carrier, which contains the signal which contains the message more or less ((Refer Time: 45:40)) On several constants whereas, the corresponding phase of the noise will go under the very wide fluctuations that instant because you know for the small period of time

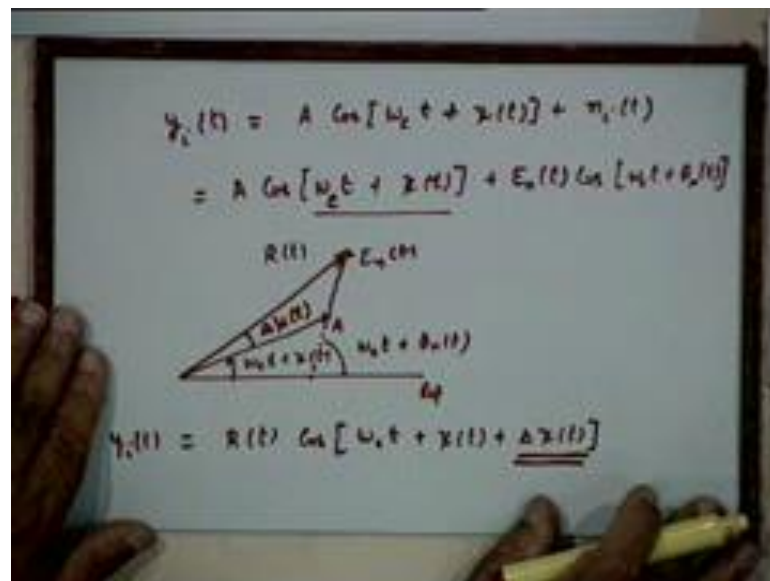
value of $\psi(t)$ is nearly constant the carrier goes through many cycles, you have many cycles in the carrier the value of $\psi(t)$ is nearly constant equal to some K_p times $m(t)$.

Student: ((Refer Time: 46:07))

Low pass has the much wider bandwidth this is also low pass with the bandwidth of B , I am comparing two low pass signals you see low pass signals. This is $m(t)$ which is the bandwidth of B and these are the noise, these are the bandwidth are very different, what is the difference that is the point I am trying to say. These will fluctuate more rapidly these will fluctuate, this is the point keep in mind as you proceed.

Now, how do we calculate the output SNR, we cannot use the linearity principle, I cannot consider the signal alone noise alone.

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I will use this fact, which I just discussed and the best thing to do is to start with $y_i(t)$ with the sum of these two this is what you have at the demodulator input and the demodulator input, what is going into the system into the demodulator is cosine $\omega_c t + \psi(t) + n_i(t)$, which you can write as a cosine $\omega_c t + \psi(t) + E_n(t) \cos[\omega_c t + \theta_m(t)]$. Now, to proceed further it is best to understand the nature of this signal, this added signal this sum to the phasor diagram.

Now, the phasor diagrams that we discussed earlier that are going to be very important now, very useful to carry out this analysis. Let us remember the basic points this is the

reference phasor at the time instant t what is the instantaneous carrier phasor $\omega_c t$ plus ψt . So, let me denote that like this $\omega_c t$ plus ψt , if you are assuming that the whole system is rotating at ω_c radius per second then you simply write ψ of t otherwise write $\omega_c t$ plus ψ of t this has the amplitude of A , to that you are adding this noise which has the instantaneous amplitude of E_m and has an envelope θ_n with the reference to the reference phase.

So, we will denote that by this has an angle amplitude of $E_m t$ and this angle is $\omega_c t$ plus $\theta_n t$ agreed. So, what we are seeing is the vector sum of these two and the vector sum of these two is this let me call this vector sum as $R t$ or actually $y i t$ this is the same thing. So, the amplitude of the vector sum is $R t$. So, to write $y i t$ as how it has the amplitude of $R t$, because this plus this sum of the these two vectors into cosine of $\omega_c t$ it is a phasor, because the instantaneous angle is $\omega_c t$ plus ψt and let me call me this angle as $\delta \psi$ of t .

So, this is ψt plus $\delta \psi t$ it is clear that effect of noise is, where is the effect of noise going, it is going to $R t$ to some extent and in $\delta \psi t$ if these noise was not there the $\delta \psi t$ would have been 0 is that clear. This whole, the time variation of this would have been better if you remember, because the demodulator will also contain a limiter. So, this will be converted into constant amplitude this is of no consequence really speaking the angle demodulator will produce an output proportional to the instantaneous angle.

So, the effect of noise will be obtained by considering the effect of noise on $\delta \psi t$ because that is what you should not have got which are getting, because of noise as you can see it is not a linear analysis. Once again to proceed further we have to consider two situation separately the small noise case and the large noise case, so let us quickly consider the small noise case, and then we will look at the large noise case, if the time permits.

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Small - Noise Case: $E_n(t) \ll A \quad \forall t$
 $\Rightarrow \Delta \psi(t) \ll \frac{\pi}{2} \text{ rad} \quad \forall t$

$$\Delta \psi(t) \doteq \frac{E_n(t)}{A} \sin[\theta_n(t) - \psi(t)]$$

We will not be able to complete it, but we will write few expressions, so let us first consider the small noise case, defined as the situation where noise amplitudes for all times much smaller than the carrier amplitude, that is the definition of the small noise case, now in the internal of phasor diagram what does it mean you are saying that this amplitude is much larger than this. So, basically implies that this angle $\Delta \psi t$ will be a very small angle.

So, you can use all kind of interesting approximations under that situation, so therefore, this says this will imply that $\Delta \psi t$ will be much less than $\pi/2$ radians for all time. And if you want to just imagine let us extend this I can write an expression for $\Delta \psi t$ where this angle is very small I can approximate by let us say this upon this and this amplitude the length of this line, if it is really very large, this length is very small basically this noise is very small, this angle is approximately equal to this angle.

Assuming this length is very, very small, so these two lines become parallel at this point more or less they occur at the extreme situations, so in at that case what can you say about the length of this perpendicular, it is $E_m t$ into \sin of this angle. So, therefore, you can write approximately under this situation $\Delta \psi$ of t as equal to $E_m t \sin$ of and what is this angle incidentally it is this angle minus this angle, so it is $\theta_m t$ minus ψ of t upon A .

So, I think this is a good point to stop, we will start from this point onwards and try to understand the output SNR in terms of this expression.

Thank you very much.