

Communication Engineering
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Lecture - 34
Performance of AM Systems in Noise

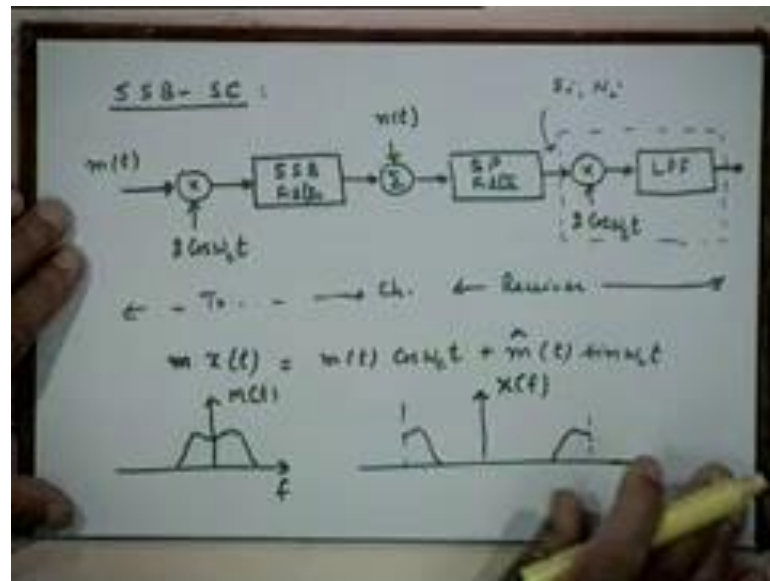
Let us continue our discussion that we started last time, about the Performance of Communication systems in the presence of noise, and we have discussed so far, the two situations one corresponding to transmission at baseband. And, you found that the signal to noise ratio that you can expect at the output of the baseband system is basically the input signal power divided by the noise power present in the message bandwidth.

So S_o by N_o is equal to S_i upon N_B and what is N_B , that is the total noise power that comes into the system at the output comes at the output of the system. Because, this much noise power is present in the message bandwidth. So, the output SNR is essentially what we can say, we say is an input SNR in a message bandwidth, whatever is the input SNR is the message bandwidth that is the output SNR.

The input SNR as seen at the receiver, which is at the noise power at the input to the receiver, is infinite. But if you consider the noise power in the message bandwidth that is equal to N_B and that is also the output process that is the main thing. And, when we went from the baseband system through the amplitude modulation systems DSBSC to find that still holds. That is the output SNR of an amplitude modulation system is equal to S_i by N_B , where S_i is the input SNR in both the cases S_i is equal to average expected, value of n^2 and N_B is the noise bandwidth in a noise power in the message bandwidth.

We will continue this discussion and try to see what happens in the case of SSB. We will briefly discuss against of USB and then go on to amplitude modulation with carrier, and then amplitude modulated amplitude modulation. Where is the demodulation, is not then single like, what we are discussing so far, but using envelope, so we will discuss early phases, before we go to the amplitude modulation cases.

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So, now today we take the case of single side band modulation, and for the movement we will consider synchronize system and plus carrier systems. So, model of interest could be once again you have a message, the first DSP modulated by multiplying with the carrier, we note it arrive n could different constant in different models.

Sometimes you put root to cosine omega c t is sometimes 2 cosine omega c t basically, the idea is I should be a constant in such a manner that the input power to the receiver $S_{sub i}$, in every cases respective number of m square. I just want to keep the same signal power as coming into the receiver input. So, as to keep the uniform it is a matter of scaling nothing else, it does not fact the results in many way even if did not do this you still this same of source.

((Refer Time: 04:45)) result should be the same and after this we will have the SSB filter, which will remove one of the two side bands that you will find the transmit or the channel. The channel noise again is going to be a assume to be ideal, and the only effect there is of interest to us is the present of noise n of t, at the receiver you will have band pass filter with center frequency of omega C.

But, it will remove one of the two side bands, of a side bands are already going to just you are selecting the required of frequency. So, selection filter and then the demodulated which is again synchronize system followed by low pass or baseband filter. And, that is your final output that is the wide system which we need to analyze, so again as in the

case of AM. I consider this is also really part of the receiver, but let us consider your S_i and N_i at this point.

At this point, the signal power could be... so this part is your transmitter; up to this your channel and actually these whole things are receiver. Because it's meaningless we talk about N_i here, it is more useful to talk about this signal and noise input signal input noise power at the output and the band pass filter front and filter of the receiver. Because, at this point the input noise power is infinity, so it is much more meaningful to talk about the input signal power and input noise power, immediately after the band pass filter, that is what we are doing.

So, now thing is you start with the signal our transmit signal is SSB signal, which we will call let us call it by x_t , and that we know can be represented in terms of the message signal m_t this, here \bar{m}_t will be transform. Whether it will be plus or minus m will dependent whether you are remove the lower side band or the upper side band, so that is the mater.

And, you remember this spectrum just to recap let us say this is your spectrum, just for take a discussion to mind you, your modulated signal will have a spectrum like that, that is if you are proceeding lower side band if you are proceeding upper side band, it will be the. So, corresponding to this message spectrum will have this, so what is the power of the message signal there is what is the power of $2 m_t \cos \omega_c t$ that you have a, will be the power here, could be $2 m^2$ bar is in it.

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Power of $2m(t)\cos\omega_c t = 2\overline{m^2(t)}$

$S_i =$ Power at input to demod. $= \overline{m^2(t)}$

$y_i(t) = m(t)\cos\omega_c t + \hat{m}\sin\omega_c t$
 $+ n_c(t)\cos\omega_c t + n_s(t)\sin\omega_c t$

$= (m(t) + n_c(t))\cos\omega_c t + (\hat{m}(t) + n_s(t))\sin\omega_c t$

$y_{fo}(t) = \underline{m(t)} + \underline{n_c(t)}$ $S_o = \overline{m^2} = S_i$
 $N_o = \overline{n_c^2} = \overline{n_s^2}$
 $= N_B$

If I want to compute this powers, so this powers, so let us look at power of $2m(t)\cos\omega_c t$, this will be basically integral m^2 of this, which will be equal to $2\overline{m^2}$ that is. When you what are the side bands, how was phi you illustrate half of this, so therefore, what is the input signal power that is coming in here, once again $\overline{m^2}$.

And, that is the reason why I am multiplying with 2 here, basically the idea is to normalize $S_{sub i}$ in every case to $\overline{m^2}$ if it is possible, just for safe of convenience. So, therefore, we can say that $S_{sub i}$ power the input to the receiver, input to density demodulate I should say not to receiver, is equal to $\overline{m^2}$, simply call it as $\overline{m^2}$.

So, at the demodulated input you have if I call this signal at the demodulated input as $y_i(t)$, can we write an expression for this signal $y_i(t)$, what will you what is the signal that is coming here, it is a transmit signal which is a SSB signal which as its representation in terms of the transform, plus the noise. And, what kind of noise is present here, narrowband noise which again one can write in quadrature form.

So, therefore the special form $y_i(t)$ like edit for the AM case, could become $m(t)\cos\omega_c t + \hat{m}(t)\sin\omega_c t$. This is the signal part plus using the same notation that we use last time $n_c(t)\cos\omega_c t + n_s(t)\sin\omega_c t$. This is the

noise power; this is the quadrature representation of the band pass noise that is coming out at the band pass filter at the receiver.

So, this signal on the whole will have in-phase component which is $m(t) + n_c(t)$ into $\cos(\omega_c t)$ plus a quadrature part which is $m(t) + n_s(t)$ into $\sin(\omega_c t)$. That is the input to the modulation, what will be the output of the demodulator after the low pass filtering, the multiplying in this with $2 \cos(\omega_c t)$. Obviously, this quadrature component will with this no outputs, the only output will be due to this and it will be simply $m(t) + n_c(t)$.

So, output of the demodulator if I call it $Y_o(t)$ some calling this signal $Y_o(t)$ then $y_o(t)$ essentially will be $m(t) + n_c(t)$. Because, when we multiply into $\cos(\omega_c t)$ we will get $2 \cos^2(\omega_c t)$ which is equal to $1 + \cos(2\omega_c t)$ and the only part that will be left after low pass filter will be $m(t) + n_c(t)$. This is the signal part of the output this is the noise part of the output.

So, what is output signal power what is S_o , once again $\overline{m^2}$ and which is equal to S_i . So, output signal power once again as in the earlier cases is equal to S_i and what is the noise power, is the expected value of n_c^2 which once again is equal to expected value of n^2 . If you remember and that is equal to once again how much what is the bandwidth of this band pass noise. It is not $2B$ it is simply B because, this band pass filter has a bandwidth only of B hertz not $2B$ hertz, in the case of AM it was $2B$. So, it is $N/2$ into D plus $N/2$ into B for the negative part negative frequency part, is it clear? All of you, so the total output noise power once again is equal to NB , so nothing I change.

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The image shows a whiteboard with handwritten mathematical expressions. At the top, the equation $\left(\frac{S_o}{N_o}\right) = \frac{\overline{m^2}}{NB} = \frac{S_i}{NB} = \gamma$ is written. Below this, the text "SSB \rightarrow VSB-SC" is written. At the bottom, the equation $\frac{S_o}{N_o} \approx \gamma$ is enclosed in a rectangular box. A hand holding a yellow marker is visible at the bottom of the frame.

So, output SNR once again is m^2 upon $N B$ which you go to write in terms of inputs message power, which is S_i and noise power. This is again the noise power of the message bandwidth. Noise power spectral density is $1/2$, the amount of noise power respective the bandwidth of the message into $N/2$ into $2B$ because, the base band signal is bandwidth of B or on frequency excess from minus B to plus B $N/2$ into $2B$, consider a noise power in the message bandwidth.

So, once again output SNR is the same as input is, so this is very interesting and of course, it is intuitively to be expected that as for as noise performance is concerned, these system could they are many reason from each other. So, essentially when you go from let us say, baseband to AM with double side band and AM with single side band, the consideration are different it is not because of better performance in noise.

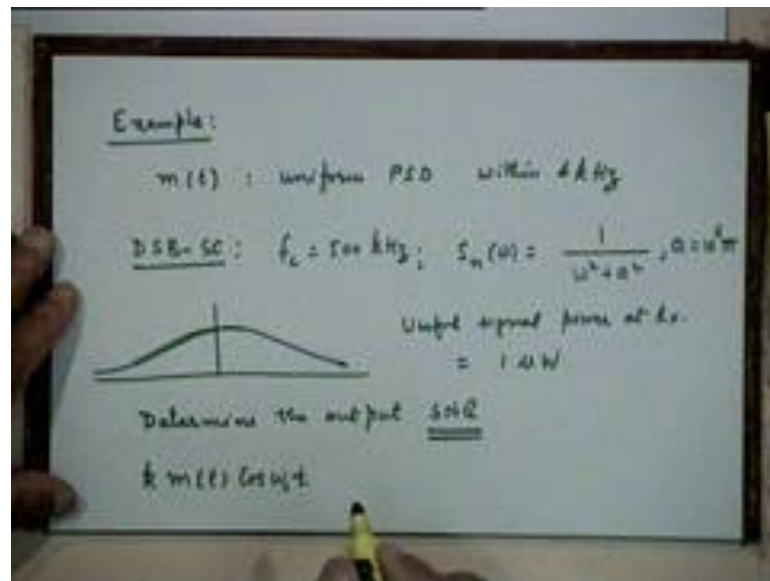
You go from double side band to single side band; it is not because you likely to superior noise performance, it is simply with say bandwidth. Incidentally as for as value is concerned, in some sense DSB SSB are same because, in DSB also the bandwidth twice you know that using quadrature modulation I can transmit two messages instead of one. So, really they are practically for all practical purposes is the equivalent if you can exploit that in a particular application. Otherwise of course, SSB, so once again this is equal to γ , so the signal of light, so for all the systems remain the same equal to

gamma. Now, when you go from SSB to VSB surplus by once again and carry out synchronize demodulation, the using the same origin set we have used here.

If you remember we have very similar quadrature representation for a VSB signal, such that this is transform it is a signal theoring by passing m t through an appropriate filter, what we call the VSB filter. However, the details are little complicated not too much I will leave that an exercise for you to do yourself. And, show that in this case also the output SNR is approximately equal to gamma, is knows to the same as gamma.

So, whether it is VSB SC whether it is SSB whether it is VSB, the performance of the system after synchronize demodulation is the same, same as that of the baseband system. So, this really you are not achieving anything in terms of SNR performance, when you are doing amplitude modulation that is the message that is the moral of the story that is, what we need to understand.

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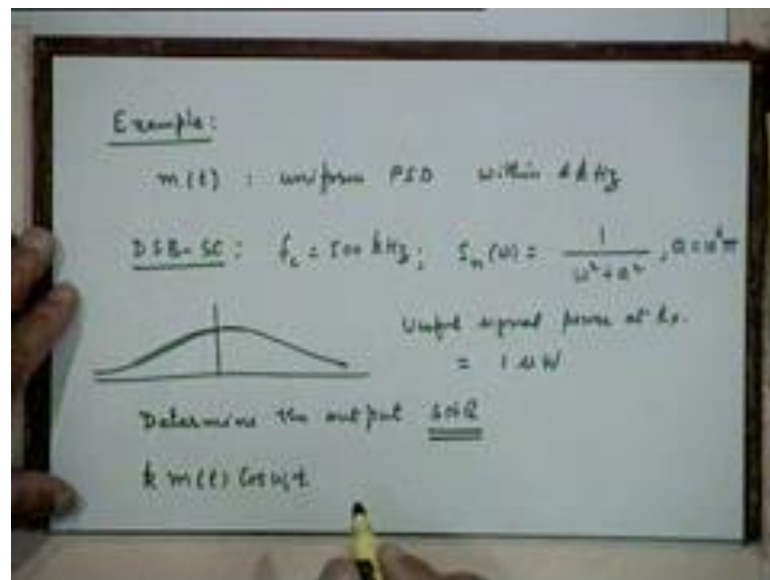
May be at this stage, I could take up a very simple example, this is an example from the book itself, but still its value in it, let us to see what kind of calculation to be actually have to do to calculate the output SNR. Let us say you have a message, which is which has a uniform parse spectral density will assume that as I said earlier, message is the modulus as a random waveform.

So, it is define in terms of this parse spectral density function in the frequency domain, and let say it has a uniform parse spectral density function over its bandwidth which is assume to be 4 kilo hertz. That is the parse spectral density function is flat selling from minus 4 kilo hertz to plus 4 kilo hertz that is the rate is, let us say we are working with DSB SC system, carrier frequency is 500 kilo hertz, just some figures.

Let us say noise is not white but colored, it means it has a non flat parse spectral density function and that is given by $1 / (\omega^2 + a^2)$. So, assuming the noise parse spectral density function to the central like that, which is what will happen if white noise what we first parse through a filter with this transfer function.

So, it is some kind of a filtered noise, it does not matter just for given and the value of a is 10 to the power 6 pie, this some numerical value to give some makes sense because, if your carrier frequency is 500 kilo hertz which is noise up to about that bandwidth which will easily going to play role if it all. So, the final data that is given to us is that the useful signal power, at the receiver is equal to 1 microwatt.

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There is either receiver input, the signal part signal power that you are receive is about 1 is 1 microwatt, and we have our objective is to determine the output SNR from this picture, of the receiver. So, how we do that, so you transmitting the DSP as a signal which is $m(t) \cos(\omega_c t)$, in get attributed by some spelling factor that is why the receiver signal is $k m(t) \cos(\omega_c t)$.

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Demod. input : $y_c(t)$

$$k m(t) \cos \omega_c t + n_s(t) \cos \omega_c t + n_n(t) \sin \omega_c t$$

$\times 2 \cos \omega_c t \rightarrow \text{LPF} \rightarrow$

Demod. output : $k m(t) + n_s(t)$
 $n_n(t)$

$$S_o = k^2 \overline{m^2} = 2 \times 10^{-6} \text{ W}$$

$$N_o = \overline{n_n^2} = \overline{n^2} = 2 \int_{t_0-B}^{t_0+B} s_n(t) dt$$

So, what will be the demodulated input now, it will be $k m(t) \cos \omega_c t + n_s(t) \cos \omega_c t + n_n(t) \sin \omega_c t$. I am talking about $y_c(t)$, $y_c(t)$ is a notation for the signal which is present at the input to the demodulated. It will have the modulated signal plus the noise at the input to the model. So, we multiply this with $2 \cos \omega_c t$ and the demodulated output could be if I multiply this with that is the demodulation operation, followed by low cost filter.

If you do that, the output will be, you could not remember anything, if you always treat from basics will be $k m(t) + n_s(t)$, this is S not t the signal part of the output this is the noise part of the output. So, your output power is $k^2 \overline{m^2}$, this phase and that is what is given to be this is given to be 1 microwatt, one has to be little bit careful there, what is given to be 1 microwatt is the input power at the receiver, that the signal power that is incident the receiver.

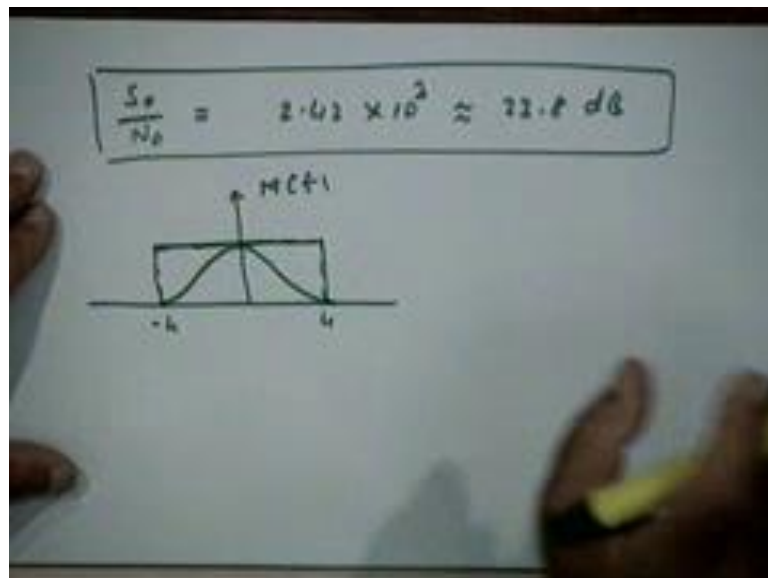
What is the signal which is incident the receiver, it is not $k m(t)$ it is $k m(t) \cos \omega_c t$, what is the power of this, what is the power of this $k^2 \overline{m^2}$ upon 2. Now, this is given to be 1 microwatt this is 10 to power minus 6 watts, which means $k^2 \overline{m^2}$ is 2 into 10 to the power minus 6 watts. So, S not is 2 into 10 to the power minus 6 watts, so output signal power is only thing left to these because, it is the output noise power.

So, what is the output noise power, it is equal to the power in n_c , this in-phase components of noise that is n_c^2 which is the same thing as n^2 , so basically is the noise if the power in the narrowband noise have bandwidth $2B$. Now, this is not a flat parse spectrum, so I cannot simply multiply n by 2 into $2B$, which is what I have done in the previous examples.

What will happen is, integrate the noise spectrum between $f_c - B$ to $f_c + B$ and take twice of that, this is for the positive frequency part then will be corresponding for the negative frequency parts. So, that is why I am making doubling with that surely calculate this integral, f_c varies 500 kilo hertz B is 4 kilo hertz, so this becomes 496 kilo hertz here and this becomes 504 kilo hertz here.

So, take this expression for the noise parse spectral density function and integrated what this terminal. And if you are to do that, the numerical also that you will get let me write it know itself to be 8.25×10^{-10} watts.

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And, if you what you now calculate the signal to noise power, the ratio that to be 2.42×10^3 , it is of the order of 33.8 decibels. So, you go through one sample calculation to get this idea, how we actually calculate the signal to noise ratio and what is the order of values that you can expect.

Yes please

Student: ((Refer Time: 25:26))

This is just simplify if I otherwise the noise power calculation, I do not think you are using with the exact for the filters will be assume to be ideal parse filters, if I fill the filters to be ideal parse filters. Otherwise, the filters regard the ideal parse filters if I if your... this is the let me spent a couple of movements in this question as well as you asked, just to give some field first some of the things that, some of the details which we are not discussed here.

By saying that the signal has this kind of a spectrum basically, what you are saying is we can use the final low cost filter after demodulation to be a filter of this kind of shape, suppose I want to tell you that the parse spectral density within this bandwidth is distributed like that, the message spectrum is not like this, but like this. Then, usual in ideal low pass filter as you final baseband filter is not the best thing one can do, can you appreciate why.

Because, I guessing that different frequency components have different importance for the signal, in the signal that different important to different frequency components, so why should I allow. So, much noise to pass through, in the same way to all frequencies between minus 4 to 4 because, all in the signal is going to be weak. I multiply by large amplitude unnecessarily, which will simply amplify the noise components without giving any benefit about this symptom, how this no signal present.

So, the best filter to gives them will not be an ideal low cost filter. But some kinds of a different transmit function which gives higher weightage stress frequency components, where the signal to noise ratio is good and low it is best frequency components where the signal noise ratio is poor.

Student: ((Refer Time: 27:28))

Not the B emphasize this is as the best filter to use at the final output, could have to deal with the kind of signal to noise ratio distribution you have, across the bandwidth. So, that is the only significance of that is that we can use this, as we can assume that the final baseband filter is an ideal low cost filter.

Otherwise, if it is not that the filter is not ideal low pass filter; however, we are not going with base issues because, these issues are really deal within area called this estimation theory. How do you estimate the signal with any kind of spectrum in the present of noise with any kind of spectrum is a much more difficult problem and requires some formal background which we do not have at this stage, thank you.

Good question, what is the next system you should consider, I am teaching modulation with carrier and see how to. Again we can intuitively expect how things should change, can you intuitively expect sometime, but we have two cases there.

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AM: Sgn (with carrier)
 Synchronous Demodulation

Received Signal $\sqrt{2} [A + m(t)] \cos \omega_c t$

$$S_r = (\sqrt{2})^2 \frac{[A + m(t)]^2}{2} = [A + m(t)]^2$$

$$= A^2 + \overline{m^2} + 2A \overline{m(t)}$$

$S_r = A^2 + \overline{m^2}$: if message is zero mean

One when we take amplitude modulation with carrier, and demodulate synchronously that is the case we will take first, and the second situation could be when we demodulate use the envelop vector. So, let us consider AM whereas, an AM here I am talking about AM with carrier, that is not surprised carrier I am let us talk about synchronous demodulation first.

Write the core in the synchronous demodulation, so what is your kind of signal that you are transferring now with, now draw the diagram is more or less same that is how that you also the carrier. So, what is the kind of your received signal that you directly come to the receive signal of, you can say root 2 there is a carrier component and there is a message modulated message component, excuse me.

This is a transmitted signal AM signal with carrier, the factor of root 2 of just to make sure that the final S_i , the message power is the same please know that requirement. So, what is the receive power in this case, that will be root 2 square A plus m t square upon 2. So, it makes it clear as A plus m t square writer, which you can expand as A square plus m square bar plus 2 A m t.

Now, just expanded that and if you assume the message is 0 mean, it is A square plus m square bar, that means message has no DC component, so this is your expression for the input column. This is different you along a m square bar because; you are also transporting some carrier.

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$$\begin{aligned}
 & \left[\sqrt{2} [A + m(t)] \cos \omega_c t \times \sqrt{2} \cos \omega_c t \right]_{LPF} \\
 & \Rightarrow m(t) \\
 & S_0 = \overline{m^2} \\
 & \frac{S_0}{W_n} = \frac{\overline{m^2}}{NB} = \frac{\overline{m^2}}{\overline{m^2} + A^2} \cdot \frac{S_i}{NB} \\
 & = \frac{\overline{m^2}}{\overline{m^2} + A^2} \cdot \gamma
 \end{aligned}$$

What is your demodulation process once again, this root 2 into A plus m t that your receiving, we are going to multiply this with that is a root 2 cosine omega c t and the result will be low pass filter. Do the operation we are going to do for demodulation, multiply the root 2 cosine omega c t followed by low cost filtering, what will result what will be the demodulated output.

Student: ((Refer Time: 32:05))

No, the A part is not message, so the message part is only m t, so really speaking m t as for the signal part is concerned. So, your output signal power will once again equal to m square bar and as for as the noise is concerned, there is no difference. So, what can I say

that about S not by N not that is N square bar upon N B and now, I must express it in terms of signal to noise ratio of the input because, that is our benchmark.

Our benchmark is in terms of S i by N B, suppose I would do that I can rewrite this as m square bar upon m square bar plus A square, which is your S i vary to S i. Basically, I am multiplying with S i these two will concerned of will be left this only upon N B. So, this is your result into γ , so what do you find, that the output S i is now smaller and there is expected because, when you are transmitting an AM signal with carrier, not the entire input power entire useful input power is useful in terms of the message.

Part of this input power is carrier and part of this input power is message, so that is therefore, you using in terms of performance, that is to give look at the same SNR as the other systems, you will have to have a and γ . Because, of the saturation factor that is coming in the process is this, so obviously, in terms of SNR performance AM could be imperial, AM with carrier could be imperial have you all understood this, good. Let us continue this discussion little further to appreciate how much more power will be required in typical situations, as compare to AM with AM with without carrier.

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Handwritten mathematical derivation on a whiteboard:

$$\begin{aligned} \text{If } |m(t)|_{\max} &= m_p, \quad A \geq m_p \\ \text{For maximum SNR, } &A = m_p \\ \left(\frac{S_o}{N_o}\right)_{\max} &= \frac{\overline{m^2}}{m_p^2 + \overline{m^2}} \gamma \\ &= \frac{1}{1 + m_p^2/\overline{m^2}} \gamma \\ \frac{m_p^2}{\overline{m^2}} &\geq 1 \quad \leq \frac{\gamma}{2} \end{aligned}$$

Let us consider the case, we are let us say the maximum value of your message is the peak values m sub b , why that is important because if you, if you remember your A plus m t for this to be well of course, the synchronize modulation that is not important. But typically you could keep you A plus m t to be positive. One of synchronization

modulation that the not a requirement, it does not have the show, but that the usual condition.

Because, we want that basically what does it mean, that A will then greater than or equal to m sob p , because, if you have a negative peak for example, A plus N B should still be positive, so the value of A should be greater than or equal to m sub p . So, for maximum SNR what should be do, we are change the maximum possible SNR you should choose A is equal to the largest possible value is it.

Because, then give you a 100 percent modulation A is equal to m p and S not by N not maximum could be m square bar upon m square bar plus m p square upon m square into γ . So, that is in terms of the peak value is it A must be greater than or equal to m p since A accounts their denominator, you must choose a smallest possible value of A and the smallest possible value of A is m p that is what I have done.

So, this is some kind of an expression for the output SNR in terms of only the message properties, nothing else and now if you want to take some typical example, this one way is which you can write this before I take, that if I write this as 1 by 1 plus m p square by m square bar. This actually this gives know bar here, this is just the peak value now, what do you by the typical value of this ratio m p square by m square bar.

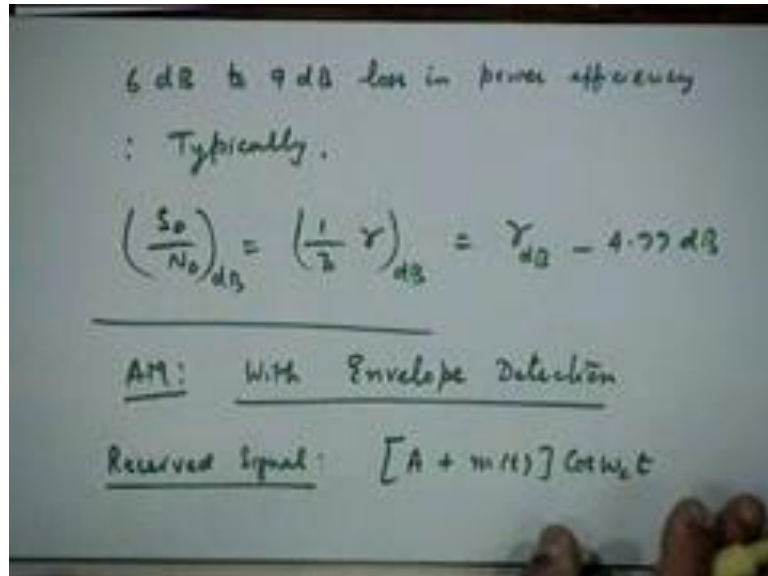
What will be the typical value of this could be less than 1 or more than 1 , more than 1 because, m p denotes the peak value m square bar is a new square value. It is a square of a peak value this is the square of the m p square value, so obviously, the peaks square to the average peak to average results typically going to be greater than or equal to 1 .

So, this obviously, means that is saturation factor, this factor by which the effective SNR gets reduce is going to be, so this is going to be less than or equal to γ by 2 . This of course, a very rough bound, but what this bound tells you is that there is at least a 3 dB disadvantage, the 3 dB cost associated with use of carrier then not using in the carrier. Then, at least 3 dB is in it that is I must require at least 3 dB, low power twice as much as power for transmission γ , should be at least twice as much as earlier.

So, as to get the same value of the SN, typical situations will be actually more than 3 dB because, this is very rarely we will have m p square by m square bar equal to 1 , that will be typically much better than 1 at least some more greater than 1 . So, depending on the

actual value of this ratio, actual SNR reduction or the SNR cost or the power cost associate with the use of AM with carrier, this more than 3 dB.

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Typical figures or more like 6 dB to 9 dB, there is a 6 dB to 9 dB loss in power efficiency, this is no typical 3 dB is this is absolutely minimum value for example. Suppose you are modulating message signal voice as sinusoidal. Just for the sake of discussion, suppose m t as a sinusoidal what is the p 2 i m s value for that root 2 p 2 i m s is root 2, so what is N B by N B square by A square bar, it is 2.

So, instead of 1 it is 2, so what will be the value of S by N not now, will be 1 by 3 gamma, so which is about 6 to be actually 4.77 dB less, so this is if you are to talk in terms of db, this will db could be gamma minus about 4.77 db. So, require 4.77 more db more SNR to get the same output SNR at the input, at the input you require this much more look at the same output SNR.

So, 3 dB is a minimum will actually range somewhere between 6 to 9 dB, any questions so for. So, what you are learned is that there is a cost associated with performance power efficiency etcetera. When you transmit with carrier, of course the reason why we do it is because, that is simpler to demodulate we do not actually have to do synchronize modulation.

And, that insist to the next question this power noise, this power efficiency that will calculated output SNR is equal to place it this for the situation, where the demodulation is synchronous. But, actually we transmit carrier, so as to get the simplicity of D modulation not to have previous synchronize modulation. So, the next question is could the envelop detector give you the same performance as the synchronize detector is, as core detector is, that is a next question that we need to look at.

So, let us look at AM with envelop detection, the process is very similar start with the receive signal, anywhere more or less the same structure involved. So, once again let us say it is $A \cos(\omega_c t)$ plus $m \cos(\omega_c t)$ into cosine $\omega_c t$, we want get confused by the fact that sometimes or use some factor and sometime I do not use some factor. It could be absolute no difference to the final result, please be clear about that.

So, what is the, what is how should I proceed before I actually proceed that we see whether you can give me an idea. How we should proceed now to calculate the output is, because you now done some sample calculations just discuss very briefly what will be the process now. Because, you are not at the receiver we are not going to multiply due to cosine $\omega_c t$ or cosine $2 \cos \omega_c t$.

Go to do I am not detection, so what kind of mathematics we should do now?

Student: ((Refer Time: 42:51))

Some kind of filter now what is actual operation we do in

Student: ((Refer Time: 43:00))

He saying that if you look at the, you know that is the way the other mathematically what are we doing noise produce an output which is propositional to envelop of the input. And, you know how to write the mathematical expression for envelop is in it, that is what we with the actual thing let us very. Because, that what envelop that is actually suppose to do produce an output which is proportional of this envelop.

So, if I write down the expression mathematical expression for envelop in this case that is what?

Student: ((Refer Time: 43:56))

I know that is not particular amount to receive, this receive signal is the part the actual receive signal $Y_{i,t}$. You have a signal part plus σ_x , it is envelop for the signal plus noise which will dictate the output, is it clear, so we need to look at that.

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Input to Demod.

$$y_i(t) = [A + m(t)] \cos \omega_c t + n_i(t)$$

$$= [A + m(t) + n_i(t)] \cos \omega_c t + n_i(t) \sin \omega_c t$$

$$S_i = \frac{A^2 + \overline{m^2}}{2}$$

$$= E_i(t) \cos [\omega_c t + \theta(t)]$$

$$E_i(t) \stackrel{\text{Envelope}}{=} = \sqrt{[A + m(t) + n_i(t)]^2 + n_i^2(t)}$$

So, input to the demodulated the input through demodulated is $Y_{i,t}$ which is $A + m(t)$ into $\cos \omega_c t$ plus the narrowband noise that is coming along. Remember when I say input to the demodulated, we are considering this is the output of the band pass filter which is the front end the first page of the receiver. The receiver will always associated with the band pass filter to pick up your result signal which you have bandwidth of B or $2B$, in this case it will have a bandwidth of $2B$.

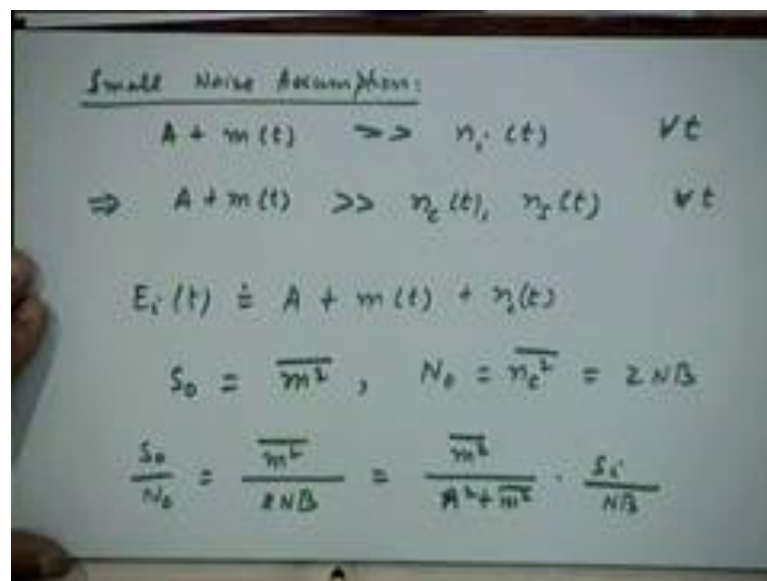
This is $A + m(t) + n_i(t)$, you can write in terms of quadrature representation in-phase component of that is $N_{i,c}(t)$, this entire thing into $\cos \omega_c t$ plus $n_i(t) \sin \omega_c t$. So, once again let us write down the expression for S_i , S_i is the same as before except that since I do not have a factor of $\sqrt{2}$, I will write $\frac{A^2 + m^2}{2}$.

If I have with factor of $\sqrt{2}$ then I did not have this involve you carrying different whether you start with the factor or do not start with the factor things will get modify accordingly. So, that is a special for the input signal power, this $Y_{i,t}$ now I can write as equal to the envelop of this signal that we call it $E_{i,t}$, what is the special value using now, so for E or R it does not matter let us call it $E_{i,t} \cos \omega_c t + \theta(t)$.

I can always write this metric expression in terms of this metric expression, where E sub i t which denotes envelop will be given by square root of A plus m t plus n c t whole square plus n s square t. And, they will be some θ t which you can write as \tan inverse of n s upon this entire thing, but that is interest to us, so I am not going to write in term.

Because finally, the envelop detector will produce an output which is proportional to the envelop, so this is the main thing of interest to us and this contains as you can see the signal part as well as some noise part, but there is a non-linear dependency now. And, that what makes the situation in general, in little more difficult and to simplify the situation one make some assumptions and to usual assumption that we make to do the initial analysis is the small noise assumption.

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Small Noise Assumption:

$$A + m(t) \gg n_1(t) \quad \forall t$$

$$\Rightarrow A + m(t) \gg n_2(t), n_3(t) \quad \forall t$$

$$E_1(t) \doteq A + m(t) + n_2(t)$$

$$S_o = \overline{m^2}, \quad N_o = \overline{n_2^2} = 2NB$$

$$\frac{S_o}{N_o} = \frac{\overline{m^2}}{2NB} = \frac{\overline{m^2}}{A^2 + m^2} \cdot \frac{S_c}{NB}$$

So, let us make a small noise assumption basically, what is the small noise assumption that is essential to say that the signal power at the input. This is much greater than the noise power in the input that is signal is much stronger than the noise.

Student: ((Refer Time: 48:24))

Typically yes, in if particularly depends on the kind of situation you are working with in broadcast application it is justified. Because, you have to maintain a good signal to noise ratio, in many other application it may not be justified in that case you have to make the

other simplified things, you may have to make the assumption that the signal is much stronger than noise, that also can be done otherwise you go to write a general expression which is very complicated.

So, just to simplify things and to give some kind of understanding, it is convenient to make these assumptions, so small assumption will imply here that $A + m(t)$ is much greater than $n(t)$ for all values of time, all the $n(t)$ is actually random. So, making this kind of a statement is that kind of new statement. But I think it is convenient to this takes to make this kind of a this will of course, also imply that $A + m(t)$ will be much greater than $n(t)$ for all time.

If that happens look at the envelope expression obviously, this term will be dominated by this square plus this square be dominated by the square of the first term and you can more with contribute the second term. So, what I will left with, so your demodulated output for the envelope $E(t)$ under this is could be approximately equal to $A + m(t) + n(t)$.

And of course, this will be blocked by you not interested in this you are only interested in $m(t)$ the useful signal power of the output will then become m^2 now use and corresponding noise power will be equal to n^2 which is once again in this case how much will it be. Remember this is an AM signal this bandwidth is $2B$, from $F_c - B$ to $F_c + B$.

So, $N \times 2B$ is $2NB$, corresponding negative part is $2NB$ total output noise power is $4NB$, so same picture as before therefore, S/N , so let us says things are absolutely the same as before, this is you can write it as m^2 bar upon. Because, your S/N is if you remember I have to your S/N is this, so if you take care of the factor of 2, this finally disappear you are left with the same expression and before is in it.

(Refer Slide Time: 51:48)

The image shows a whiteboard with handwritten mathematical equations. At the top right, there is a small note: $+ n_2(t) \sin \omega_c t$. The main derivation starts with the signal $s_c = \frac{A^2 + m^2}{2}$. An arrow points from this to the next line: $= E_c(t) \cos [\omega_c t + \theta(t)]$. Below this, the envelope is defined as $E_c(t) \triangleq \text{Envelope} = \sqrt{[A + m(t) + n_1(t)]^2 + n_2^2(t)}$. A horizontal line is drawn below this. Under the line, it is noted that $n_1(t), n_2(t) \gg [A + m(t)] \quad \forall t$. Finally, the envelope is approximated as $E_c(t) \triangleq \sqrt{n_1^2(t) + n_2^2(t) + 2n_1(t)[A + m(t)]}$.

So, your output SNR now, this is very important to appreciate on the way expression is same, but there is a very important message here, that is same as before. What I mean by same as before what I mean by this is that, the output SNR is exactly the same that we had for the of synchronized the modulation that is expression for the output SNR.

Now, what is the difference whereas, for the synchronized demodulation this expression is valid irrespective of an input SNR, for the case of envelop detector this expression is valid only one input as for the sufficient in large, is it clear.

Student: ((Refer Time: 52:35))

That we do not know we defined at least this all your saying is this assumption that we made in the analysis. It is not valid and therefore, this expression will not be valid we will not be able to calculate the output is SNR using this expression. That is all we can say whether it will be a better we do not know, we have to find out.

But in, so basically what we can say is highest SNR situations it highly matters we do not lose any further in terms of efficiency, if you go to use the envelop detector rather this, is it clear. Highest SNR situation there is no loss, so envelop you can just go out a new envelop without even thinking twice.

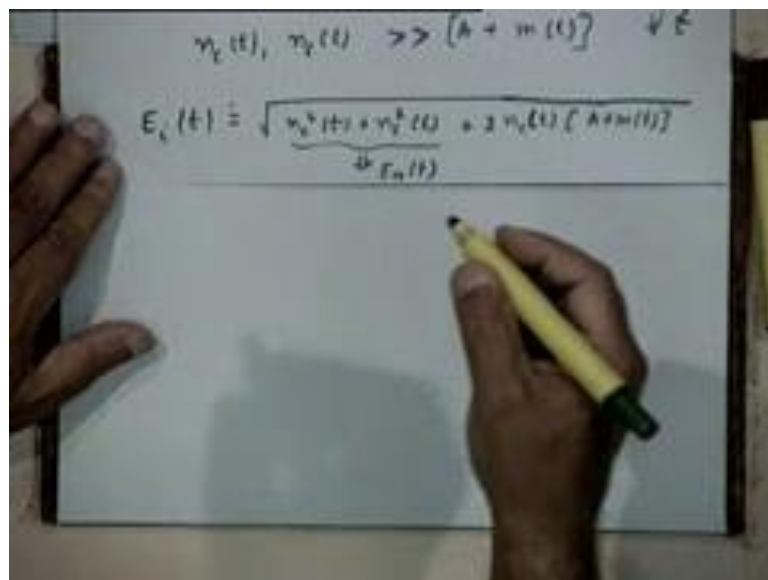
Because, you are lose any performance yes as your point you out for in the situation. The envelope actually we will work I am little more careful although it some obvious, but

that is the next thing I will take up. So, the situation we can analyze by to simplify things is when the, you see motivation is the same in some other want to simplify the expression for the envelope.

So, one why they are simplifying it is by assuming that, this is much larger times and this the other way is assume that noise is much larger than the signal and then again we can do things like differently. So, the large bias takes, so we will consider both the situations and what you mean by large bias case what you are saying is the opposite of what you have said earlier. That your noise amplitude most of the time is much greater than $A + m(t)$, the receive signal is much weak at the noise which essentially go to imply that both $n_c(t)$ $n_s(t)$ are much greater than $A + m(t)$.

Now what can we say that envelop. So, $E_i(t)$ could be approximately equal to next thing is to expand this expand this square term, we will get A square plus m square plus n_c square plus cross term. In things like that and the significance there could be n_c square t because, that is large plus n_i square t plus $2 n_c t 2 n_c t$ into $A + m(t)$. I just write this expression completely and then do a discussion next time.

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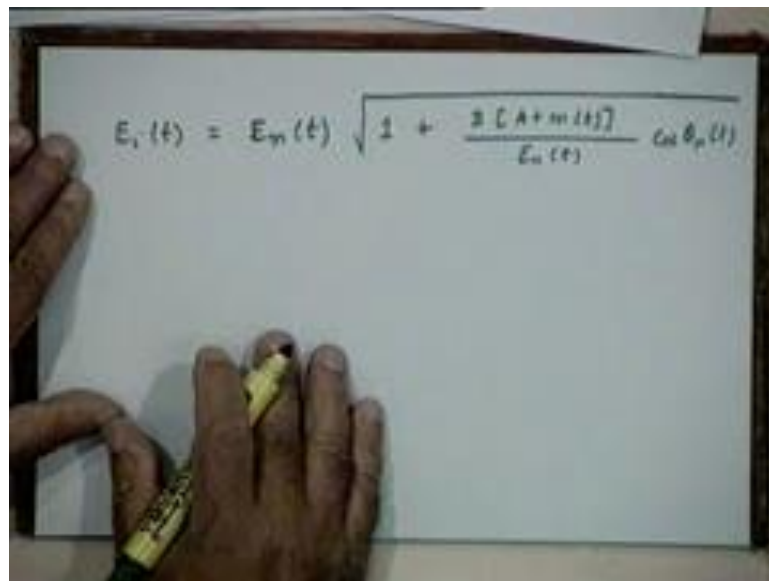


So, that we write this again we can write this as if I go to do white this and now this things about it, can I give some interpretation to this quantity this is the envelop of the noise process itself alone. Because, noise is $n_c(t) \cos(\omega_c t) + n_s(t) \sin$

$\omega c t$, so you can think of this as envelop of the noise process, so I think rewrite this as E_1 square t etcetera.

So, I can write this as if I take this $E_n t$ outside, this square root sign which you can 1 plus 2 into A plus m t upon $E_n t$ and cosine theta n t. We think about that then start from this existing one.

(Refer Slide Time 56:45)


$$E_1(t) = E_n(t) \sqrt{1 + \frac{2(A+mt)}{E_n(t)} \cos \theta_n t}$$

But, I am saying is become rewrite this, in this form theta n t is time inverse of n s by n t, we will start from this discussion next time.

Thank you very much.