Communication Engineering
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Lecture - 33
Behavior of Communication Systems in the Presence of Noise

I have been having to do a reasonable background now, about Random Process, in Random Processes. How do we characterize random processes, random signals? We are now ready to take out, the return to our main thing and where as Communication Systems Communication Theory. We have to study several modulation schemes by now, linear as well as non-linear, I am to choose frequency phase modulations.

And, we are seen the pros and cons in terms of system complexity in terms of bandwidth etcetera, we seen also their important properties in the presents of science squirrel interference, in some limited sense. However, what are the main problems in communications is the presence of noise, as I mentioned several time in this course earlier, noise is one of the dense of a communication engineer one of milli ampere.

So, whatever you do you have to keep that in mind because, noise is present rights from the point and which you have deleting your signal for transmission, through the procession that we do at a transmitter, then it keeps on moving while it is the signal is getting transmit to the channel. And, most importantly because the signal is already very equal it see, when it is see that the receiver at the receiver whatever noise gets edit, that is one of the most important components of noise.

Because, the signal and noise now more or less at about the same level, very similar levels whereas the transmitter whatever noise gets any much smaller in the signal power that you are finally getting, that you are working with. Therefore, the noise that is anywhere the transmitters not the crucial not the important, the voice that is doing the transcriptions to the channel, and more important to the one that gets edit at the receiver itself, is the one which really flex you very significantly in terms of system performance.

So, what you like to now studies how do this various do this various modulation, since that we discussed compare with each other in the presence of noise is something, any of them some of them better etcetera, and things like that will like to answer these questions. Now, we are ready with the tools of the required for carrying out this analysis will use the tools, we know how to model noise must be know how to model noise. We can analysis the performance of the system through, which noise as well as signal or getting process, so that is our objective for the next few lectures.

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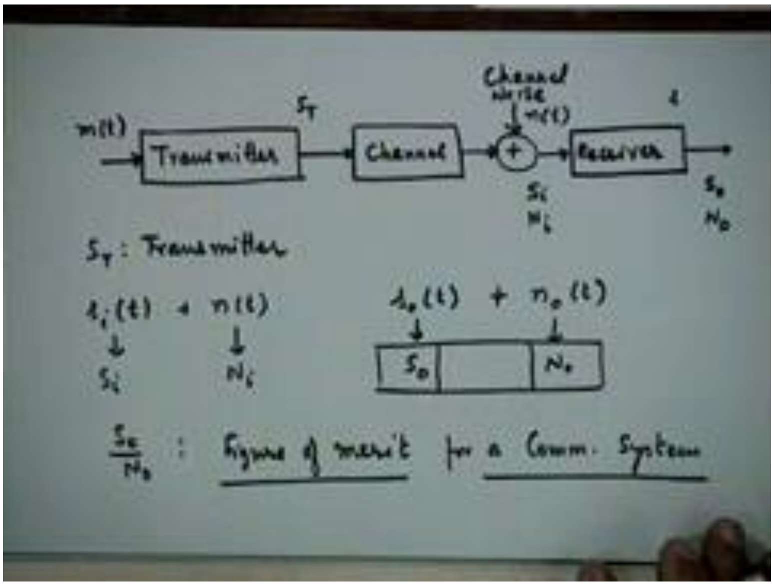

S_T : Transmitter

$s_i(t) + n(t)$

$\downarrow \quad \downarrow$

$S_i \quad N_i$

$s_o(t) + n_o(t)$

$\frac{S_o}{N_o}$: Figure of merit for a Comm. System

Now, start with let us look at the model that we used for the communication system for our discussion, you start with if you remember the message signal t, and then you have a transmitter and the transmitters in tools everything that you could have. They could be some filters wave, maybe like you have in FM some PF physics filters, and they could be a modulator they could be a power amplifier, before it is transmitted into your channel.

So, the transistorize everything here, I am not go into the big of the transmitter, that will depend in the specific scheme that you are working. And we are looking at the very basic level diagram now, and then the transmitter output is work with transmitters to some physical channel, to some physical media which could be a pair of wires. It could be a wireless transmission, it could be a coaxial cable, and it could be an optical fiber the physical channel could take any of these six steps.

Now, we also model the channel, channels so far as was an entity which could modify the spectrum of the signal, in terms of some transfer function, you can assume that the channel has a kind of a linear filter which could modify this spectrum of the transmitter signal. And, we had established conditions and which these modifications could be acceptable. We may establish conditions for discussion this transmission. For example, that is the amplitude responsibly flat, across the bandwidth of the signal at the phase responsibly linear, across the parameter signal.

So far, we are model the channel like that, now we also try to model the noise that gets edit, as more or less it is a channel which is to calculate. So, usually the noise that gets edit is show along with the channel, in the form of this block here. So, we called it the channel noise, and it is denoted by a symbol n of t, now one point that I like to make here is, not all the noise is getting edit in the passage to the transmission of the channel.

Are you said a lot of noise gets edit at the receiver itself; however, for the purpose of our modeling. We show analyze as getting edit to certain input and receiver, as input channel is a culprit and you will call it channel noise. So, whether the noise keep the transmitter, whether it is getting that is the receiver or whether it is getting edit or channel, all of them are getting club together. But the purpose of modeling the effect and we are saying that the total amount of noise, that is present and that is edit to be an incoming signal and that what you are seeing.

So, you are modeling this addition right of the input to the receiver, so the next mark is the receiver, and your receiver again is a very generic receiver, it includes everything that you may have to the particular system. For this discussion, example it will contain some front and filtering that you are want to do, to reject other signals there many signal coming in and you just like to except, tune to the signal of interest.

It could includes some, let us say demodulation it could do include some deemphasize filtering etcetera, so the receiver contains one of the main function to the receiver allow with the other function because, it has to give the demodulation. But, one of the main function of the receiver, is to limit the mode of noise that find the gets true to the output, where the signal should go through nicely without any destruction, the noise should get extended to that much as possible.

So, that is typically done through suitable process or suitable filters in the receiver. So that is the overall block diagram in which the field now concentrate, to introduce some parameters the transmitted output means say as some powers which will denote by S sub t. So, S sub t is a transmitted power, because our performance will be in terms of what you call signal to noise ratio, finally at the output.

So, we like to talk in terms of signal powers and noise powers, so this is about the power recipient transmitter, let us say at the most important parameterize which are filters or other input to the receiver, not the entire mod of power that is transmitted received at the receiver; obviously, it is a lot of continuation taking place in this channel. But finally, what you receive is some power S sub i, let us say the noise power of the input to the receiver is N sub i.

So, the subscript I here stands for input to the receiver, now let us say the noise the signal power at the output of the receiver is S sub o and the signal power at the output, the noise power of the receiver is N sub i. So, basically S sub o that is you know, you will have the signal, what you are saying is the input to the receiver consist of a signal component, plus a noise component.

The signal component has a power S sub i and the noise component has a power N sub i, the receiver at the output also will have the signal component which are divided by S sub o. The desire signal component plus a noise component which are designated as N sub o, if the receiver was perfect the noise could be completely eliminated, then only the signal will come through.

But, you can never design such a perfect receiver, for especially it should be more or less obvious, but they will become obvious if they are not right. Now, let us say this signal component is associated with their output power S sub o, as a noise for N sub o and the ((Refer Time: 10:56)) which is a filters to us is the relative values of these two powers. So, the output signal to noise power ratio then, I say output I mean the output of the receiver, the signal that is fined going to be heard or seen by the actual receiver, the actual user.

The power ratio of these two entities at that point is a figure of merit for any communication system, so that is our desired figure of merit for any communication system. So, that is our desired figure of merit this is, what we are interested to find out the value of the output signal to power ratio.

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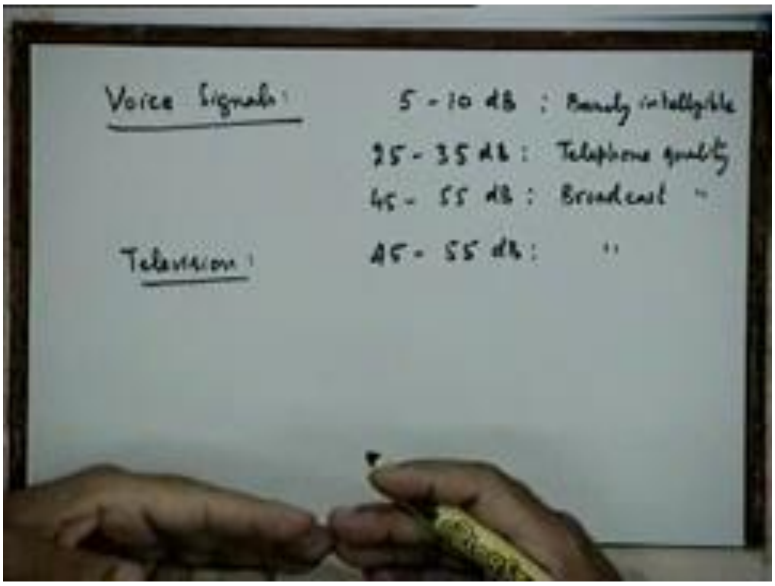

Voice Signals:

5 - 10 dB : Barely intelligible

25 - 35 dB : Telephone quality

45 - 55 dB : Broadcast "

Television:

45 - 55 dB : "

Just to give a field for what kind of signal to power ratios are useful or typically required values in various applications, for voice signal that is why if we are talking about voice signals, if we talk about voice signals or various situations that is why your voice signal or a telephone line. If the signal to noise ratio is in the range of 5 to 10 decibels, it typically all signal to power ratios are measure in the single unit which is, if you have the power units look at the 10 log of the power units.

So, whatever the actual value of the ratio the 10 log of that, log to the base 10 and multiplied by 10, so if the signal to noise ratio raise to 5 to 10 degree, we will find that the voice signal is barely intelligible. You can list about make out the something is being properly, typical telephone quality speech there is that you work, that you is associated with the signal to noise ratio of 25 to 35 degree at the output.

So, telephone quality you just give you some field for, what kind of values are desirable finally. When you come to broadcast quality for example, the music that you hear in radio receiver, the good broadcast quality signal to noise ratio is in the range of 45 to 55 degree. That is noise has to be really very small, finally that comes out along with the signal.

So, there is broadcast quality, similarly for the picture signals let us say in television, you have again the broadcast quality resonance, whether it is a picture as well as the voice resonance have been talking about is this order, again in broadcast quality. So, I have to understand what this, for example if a signal to noise ratio can d b, what are we talking about, we talking about the signal view about 10 time 10 times stronger than the noise, in terms of power.

Because, 10 log of 10 will be 10, whereas if it is 20 degree it will be 100 times stronger, 30 degree is 1000 time stronger, it is a scale of it is a log scale and follow so forth. So, 50 degree is 10000 times stronger 10000 or more, 1 lakh times stronger 100000 times stronger.

Student: ((Refer Time: 14:54))

No, when you talk about image is as it is to be see on a television, you required very good qualities more or less same as your voice. So, this figure gives you a feel for the kind of less noise which will be acceptable in various applications. Now keeping that in mind, just in the pattern in your backup your mind, let us start with some discussion on how do we calculate this output resonant, input in terms of input resonant value.

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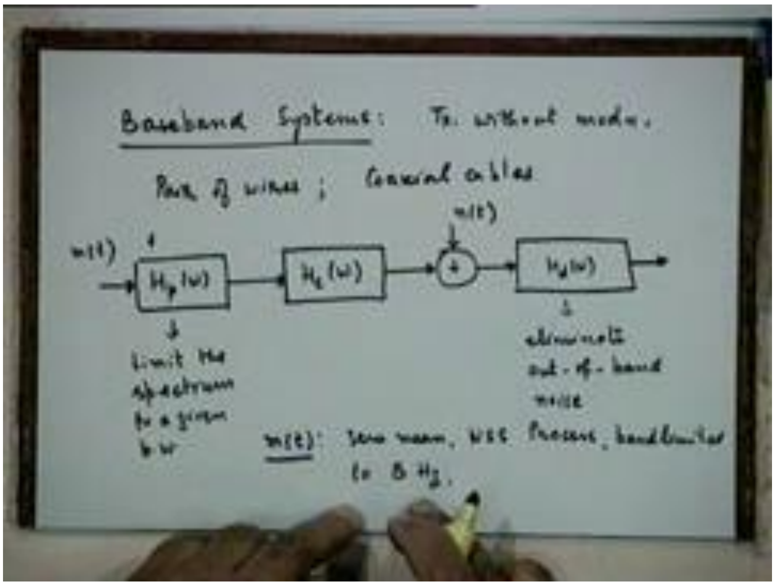

So, let us talk about we start with very simple communication systems, this the way we started earlier in our discussion, we will go through the same sequence, remember we start with discussion of baseband systems, then we went to AM system and then we went to FMN phase modulation systems. So, we will repeat that econology, except that we already know what the system is all about, we just want to understand how they have to be modeled and analyzed, in terms of ((Refer Time: 16:06)) performance that is our objective here.

So, let us talk about basic systems then you know, what are baseband systems baseband communication systems, system of the base which, in which you do not carry out a modulation process to carry out frequency translation. Basically, we take the signal itself and transmit as such without any modification, example in a telephone on a pair of wires connective your home to takes stage.

So, it could be a pair of wires the transmission could be pair of wires, so basically we are saying transmission without modulation that is, what I mean by baseband system. So, it could be a pair of wires a transmission it could be coaxial cables, but mainly these will be short or links. Typically if you have to transmit our along the distances, typically multiple signals are have to be transparent, and modulation will have to be carried out.

But, your short ((Refer Time: 17:15)) or between one user and the exchange, this is the kind of situation we will face, also example your digital communication that we take that we do on an Ethernet, it is time to a coaxial cable and a transmission is this. So, mainly short or links although we will do this analysis for this specific situation, we will find that whatever tools its really very simple analysis, and the arguments will be more or less directly valid for the modulation system.

So, let us go to the look carefully, the other reason why we want to do baseband system is, because in the other than the simplicity that is associated with that is, it will serve as a bench mark. Whatever performance you get in this case, inside the bench mark for all other systems. Because, if you have modulation and if you do not have modulation, how do the compare to each other.

So, this is the relevant question we will try to address that question, now to modern the baseband system how is of course, you are saying that it is without modulation by does not mean that you do not do any processing whatever. You could be example in the most general situation, have some kind of filtering develop the transfer itself, some at the transmitter.

The transmitter block no designated as a filter, for example it could be a pre-emphasize filter. It could emphasize and deemphasize you know in baseband systems, sub frequency components may have to be enhanced, sub frequency components may have to be deemphasize of the corresponding frequency component. We have to be Deemphasize at the receiver, so it could be or you may not have.

And then, it is a in this particular case the transmitter either contains its filter or does not have this filter that is about it, it is nothing else in the transmitter you know baseband system. You just amplify the signal, has sufficient power unit and so that it can drive the line through which it gets transmitted. That is the only processing you are doing amplified power amplified, have sufficient power to derive the pair of wires through which has to be finally transmitted.

So, that you get a reasonable strength of the signal at the other end of the line, that is the only process in we are doing, nothing else or at this you know between some filtering or some, then comes to channel the channel was said. In this case, both a transfer function which are denoting H e omega and the noise addition process and before, at the receiver you could have some kind of a filtering, for two resource one because you, if you use a pre-emphasize filter you must then use a deemphasize filter.

In any case you must have a filter which will limit the passage of amount of noise, that will finding goes go through. So, that is the main function of the receiver it has to eliminate, so let us say it could say that the purpose of this filter could be to limit the transmission spectrum. Limit the spectrum to some given bandwidth, what is the main purpose and the main purpose of the receiver filter is to eliminate, what we call out of band noise that is, the thing you can expect from any receiver filter in terms of what we can do to the noise.

If you try to eliminate noise which lies in the same bandwidth and the signal, will also try to eliminate the signal, so basically the anything we can really do is eliminate out of band noise, and that is what that is we will do. And, also this can also serve the addition purpose of deemphasize of the emphasize. So, with this as a model, now let us also talk about if you remember or input is the message signal empty.

Say to work, it also have some kind of a module for empty, because we talking about we can talk about power of the signal very stressly. So, let us say you will model empty also as some kind of a random symptom, actually signals most actual signals of interest which carries some information. Again this is a discussion we have the beginning of this course, they must necessarily the random, you know what you transmit something very precisely know, they no need to transmit such a thing.

So, empty also can we module as a random signal, so therefore, we module this as a zero mean wide sense stationary process. Now, we already start with talking the language of random processes, will assume that the signal of interest also is some kind of a random process excuse me, which is band limited to ((Refer Time: 22:42)) basically has a bandwidth of B H, so it is a its band limited to B H.

So, your desire signal empty which you want to it applicator the receiver output, is the model as a zero mean, wide sense stationary process of bandwidth B H. So, this characterization makes us where I say bandwidth. Bandwidth is to be B H what does it mean, that your power spectrum of this wide sense stationary process, it will not talk in terms of spectrum. Now, it will talk in terms of power spectrum is limited between minus P and plus P, that is for it means.

Let us assume, that we do not have any P L position deemphasize filters, but these are replace by ideal in pass filters, so both the transmitter filter as well as the receiver filter for this, in the first filter we will replace the work ideal depositor. The purpose of this ideal depositor filter is we make sure that the only signal that base three is the signal only this empty passes through. ((Refer Time: 23:53)) There is some other noise or something that is being generated here, that is an ((Refer Time: 23:58)) at least some portion of random.

Suppose, if I limit the spectrum to B hertz, similarly we will assume that the I this filter is as soon as ideal low pass filter of bandwidth P. So, whatever the channel we will say some kind of the channel also. In the first instance in fact the most of the calculation that we will do, will assume that, the channel for the purpose of symbol to noise ratio analysis that we are going to do. For this purpose we will assume that the channel is otherwise ideal.

That is, that is the distraction less channel, so if it is a distraction less channel what will be the characteristics of H c omega, will again is an ideal low pass filter. The ideal low pass filter will either zero phase characteristics or linear phase characteristics. So, there is H c omega H p omega and H d omega are all being modulus pass filter in this situation because, we are saying that the only effected channel, only effected channel that we are interested in studies of this ((Refer Time: 24:59)) the effect of noise.

So, we idealize everything else, we idealize the transmitter filter the receiver filter at ((Refer Time: 25:07)) channel we do not want to get confused, we do not want to mix of the effect of this as well as that. So, let us look at only the effect of noise that assuming that the channel is otherwise ideal.

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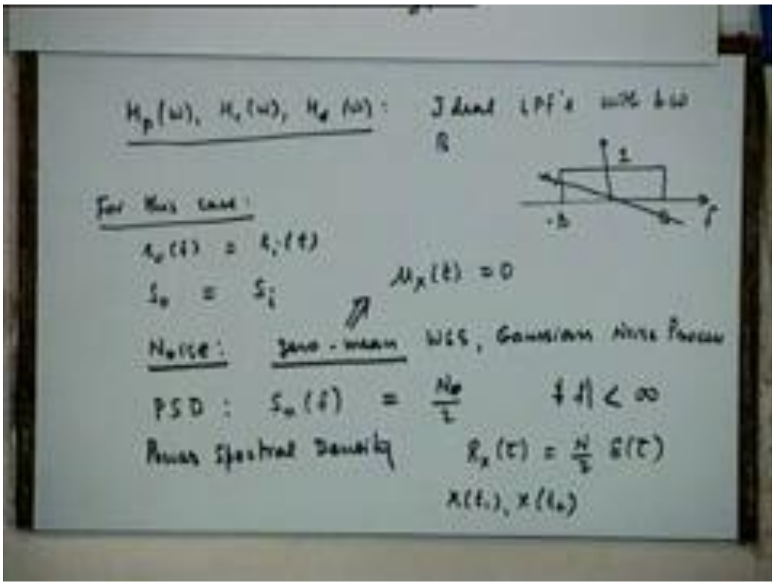

$H_p(\omega)$, $H_c(\omega)$, $H_d(\omega)$: Ideal LPF's with bw B

For this case:

$s_o(t) = s_i(t)$

$S_o = S_i$ $\mu_x(t) = 0$

Noise: zero-mean WSS, Gaussian Noise Process

PSD : $S_n(f) = \frac{N_0}{2}$ $\forall |f| < \infty$

Power Spectral Density $R_x(\tau) = \frac{N}{2}\delta(\tau)$

$x(t_1), x(t_2)$

So, we will say here that the H p omega H c omega and H d omega, will all these model as ideal low pass filters with bandwidth B, when I say what I am in the ideal depositor; that means, we have this kind of transfer function. Constant it will minus B to plus B and 0 outside, this is the amplitude characteristics, the phase response to be either 0 for frequencies or linear, over the again it has to be linear only over the bandwidth of filters.

Now, in this situation that this diagram here, you have a certain the signal input power here and the signal has a bandwidth of B, what will be the signal output power here, list being the data i data p of filter of bandwidth B, will you have difference in the two. The output signal will be the same as the input signal, because it has the input signal as a spectrum, power spectrum which extends up to B hertz, this is an ideal filter which does not do anything to the spectrum as long as utilize within B hertz.

So, basically you see the same thing at the output looks its perfect thing, so output symbol S not could be the same as the input signal power S sub i, so in a scale that for this case. If you have to calculate, the output signal power in the noise power since S o t could be equal to S i t the two will be the same. This implies that the output signal power will be the same as an input signal process, input to the receiver, is it.

What can you say about the noise power that will be come out, to give that you are to make some assumption regarding the noise power spectrum, noise again we will model as a, now we will have to first model noise, before we talk about output noise power we talk about modeling of the noise. What kind of model is appropriate for noise, first of all it is resizable resource that will be a zero mean WSS process just like a signal, in addition, we can usually assume that it is Gaussian.

Although, Gaussianity is not going to be central to our discussion because, as long as it is wide sense stationary we will anyway work in only for first elements, but it helps us to keep in mind that if it is a Gaussian process the first two groups are complete description anyway. So, it will be to assume to be zero mean WSS Gaussian noise process, in general it could have some arbitrary power spectrum.

So, let us say as a power spectrum or power spectrum density function, which is designated denoted by S sub n f or S sub n omega. Usually we will most of the time we will work with that model. Usually if you remember discussion in this context I said that the kind of noise is generally deal with in communication system can we thermal noise short noise etcetera.

Can we model as a very wide spectrum noise and therefore, all typical purpose is can be consider to be wide noise process, so it will usually be equal to constant pass density function, existing equal to let us a some value and 0 by 2 for all frequency with all values of f less than infinitive of all f. There is a slight confusion for in notation is in it, I will call it N sub o I simply call it N, because N sub o I already used for some other output noise power.

This is the respective, the units of this could be different from the units of the output noise power, tell me the units of this, it will be suppose the units of power of watts. This is a density function this will be watts or hertz, so here this is watts per hertz or it will be measuring watts.

Student: ((Refer Time: 30:27))

Parse spectral density if I got a

Student: ((Refer Time: 30:33))

Let us, this is a important point, you must understand this point, what is our understanding the second order characterization of a random process, Gaussian is not an important for that purpose. What is the important is, you must know the mean value function, of the process. If it is wide sense stationary the two parameters which will characterize, the second order description is the zero, the mean value function. What should be the mean value process and what should be the auto correlation function.

Or equivalently the parse spectral density function, because the auto correlation functions and parse spectral density function is a very transparent ampere. So, the second order description characterization of a wide sense stationary process is through the zero mean. Or is through the mean value function and to the auto correlation function, or equivalently to the mean value function and the parse spectral density function that is precisely, what I am specifying.

I am saying that the process is zero mean that is, the mean value function is constant is equal to 0 for all time, mew x t is equal to 0 can I say zero mean. It means the process mean value function is 0 where is then, what is specify its auto correlation function or equivalent I will specify the parse spectral density function, therefore it as a phase. When I say that it is white, it gives the parse spectral density function is constant, and the cross auto correlation function will be the retain functions, do you remember that.

Student: ((Refer Time: 32:18))

Gaussian is Gaussianity is not even required in this description

Student: ((Refer Time: 32:26))

So, whatever you has to do with the density function, we are in the first order second order descriptions are very cross characterization of a density function itself, density function at various times or joint density functions. However, if I know this also Gaussian process, then I know that given this knowledge I can generate the density function of the process and first order density function second order density function third order density function, through the general expression that I discussed last time for a Gaussian process.

If I may be auto correlation function, I can find out the joint density function of X t 1 and X t 2. I can find out the third order density function also, fourth order density function also for a Gaussian process not otherwise.

Student: ((Refer Time: 33:23))

I am coming to that, so let me if you assume it is white we also say it is white Gaussian, if you assume it is white, if you do not assume it is white it is simply a Gaussian process. So, all of us on the same view of point now, we are list on what we are talking about in terms of in the model, noise model is any point of discussion at this stage. So, given that the noise is white sense stationary Gaussian, and has a parse spectral density function of S sub n of f.

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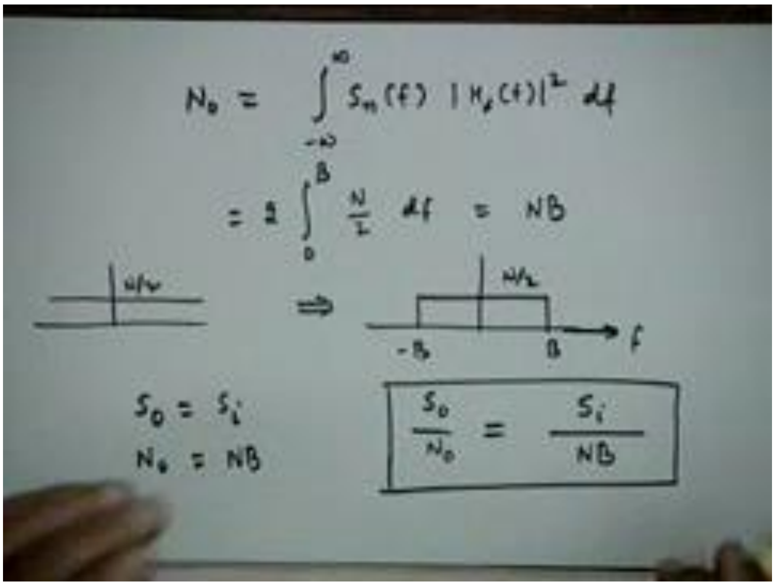

$$N_o = \int_{-\infty}^{\infty} S_n(f)\,|H_r(f)|^2\,df$$

$$= 2\int_0^B \frac{N}{2}\,df = NB$$

$$S_o = S_i$$

$$N_o = NB$$

$$\boxed{\frac{S_o}{N_o} = \frac{S_i}{NB}}$$

What can I say about the noise power of the output of the receiver, what we are saying is at the input to the receiver, at the input to the receiver here the noise function n t noise process m t, its parse spectral density function is S n f. What will be the input noise power, suppose I ask you what is the value of M sub i, how is the power noise power related to the spectral density, the area under its spectral density is the total noise power or the average noise power.

If you assume it is a white noise process, what is the amount of noise present to the input infinite, so what is the signal to noise ratio of the input zero, zero in terms of absolute value and minus infinitive in terms of d v decibels; is it clear? So, we can it is a white noise process; however, is the output noise power how do we calculate the output noise power to do that, first let us find out the output parse spectral density function.

What is the output parse spectral density function, it will be S n f into substitute that, so this is the output parse spectral density function, integrate this density function all frequencies and that gives you the various of the output noise process or the noise output power. In our case this will be, let us say minus b to plus b that is means it is a how is a even function, parse spectral density function has to be even before as a magnitude square, so you can implicit a twice of 0 to B.

So, let us assume it is white what is the parse spectral density function of N by 2, and this is Gaussian to equal to 1 between 0 to B, so d f what is that equal to, the value of this integral equal to N B and you can entity relative is wide is N B input, input noise as a plat parse density function, you are passing through a filter like this. This height is N by 2, so what is the total area of the output noise parse density, the output noise spectral have the same shape and the area under that is N by 2 into 2 B which is N B, it is tentatively to be obvious.

So, the output noise power is N B and that is something that you will have to work with tolerate, you cannot do anything about this noise, this is the noise that will come to if you want to a signal to come through, a signal of this for bandwidth. Therefore, you have to have filter with this for bandwidth, you have a filter width large with smaller bandwidth and this.

You can, if you have a filter with smaller bandwidth and this your signal will get distorted; however, what you can do is not no filter with the bandwidth larger than B. You should not change the filter larger than the bandwidth of the signal, because if you do that the noise that you will get in will be larger. So, this is the amount of noise minimum amount of noise if you will have to tolerate at the output effluence 1.

So, a signal power at the output is, say as an input signal power the noise power of the output is N B therefore, your signal to noise ratio at the output which is the figure of light of interest to us is given by S sub i upon N B, this your result for a baseband communication system. So, what is the result that the output signal noise ratio of the baseband communication system, is the ratio of the signal power this is your input signal power to the noise parse spectral density times, the bandwidth N by 2 times to B.

This N B is nothing, but N by 2 into 2 B we can consider 2 B bandwidth in the sense of at least in distance all actually, conventional sense is to say the bandwidth is B, but in the mathematical sense it is bandwidth 2 B. And, this is a benchmark we cannot have in any system is signal to noise ratio which is better than this, you cannot have any system simplification noise is better, is it clear.

Student: ((Refer Time: 39:15))

If noise is white, assuming that the noise with white, so this is therefore, that sense it is benchmark will call this benchmark ((Refer Time: 39:26)) now falling more or less logical notation here. So, finally easy to read this portion from that book, so we will call this parameter as gamma and we say that this is equal to gamma, so anything else that we do will be in terms of this benchmark will try to compare in terms of this benchmark.

Is I agreed one last thing that we can briefly talk about this, how can we express this S sub o in terms of m t, what will be the S sub i or S sub o in terms of this m t, also here in this discussion we assuming everything is ideal I have ignore the attributions. So, if introduce attribution there will be attribution factor which will come because, finally what is of important is what is the input power, that already takes here of the attribution.

But, suppose this attribution how in this S i t related to m t, it is not n f you do not talk in terms of n f anymore if you modeling n t to be a random process, again we are going to talk about the parse spectral density function m t SMF.

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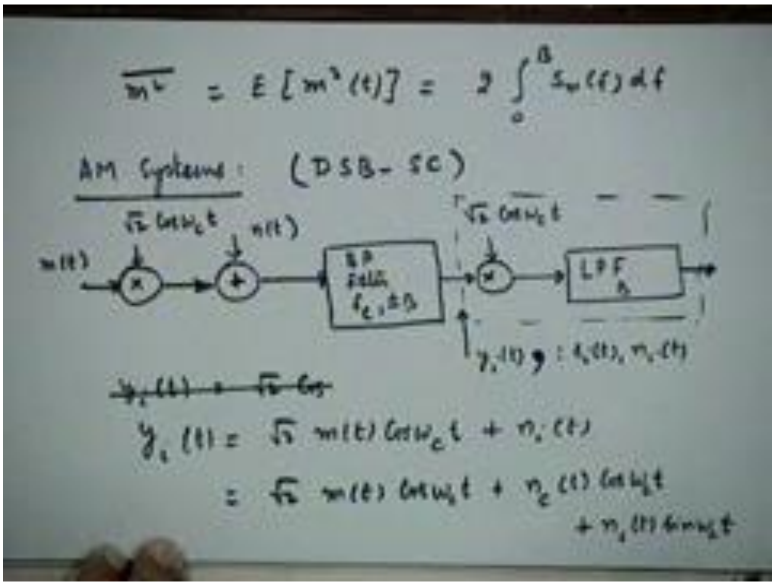

So, again we are m square that is the average value of m t m square which is expected value of m square t, that will be a signal power also that is given by twice 0 to B of S m f d f S m f is a parse spectral density function of noise. So, that is the main results there is output is S sub i in a baseband system is equal to S sub i upon N B, that is the best thing that we can do, now very AM systems stand in this context.

I was modify this statement, where is a this is the best thing that we can do, developing into linear systems you will find later, that you will lead modulations schemes like f m in any AM can do that better thing this, just how to do better is something that will become clearly later. So, I should need a statement we will say this is the simply a benchmark, that this is what the baseband system can be we will compare everything as with respect to this figure, I think that is the better way of putting.

So, let us come to AM systems and the first system AM system that we will discuss is the DSB-SC system, and we will find that the method of analysis is more or less as same. We will draw a picture of the system, look at the signal power input to the receiver noise power the input to the receiver noise parse spectral density and signal power of the output of and so on, so forth.

So, let us draw a diagram what you do, first multiply this at the carrier I denote the carriers here by root 2 cosine omega C t. Then multiply this root 2 has been included to normalize the power then the same power to the same value or before. We will discuss that is all transmitting that is all we are doing modulating the signal multiplying with the carrier.

Channel is again we will assume to be ideal, but it is not going to be ideal low pass now, it will be ideal than parse, there is it will have a perfect of flat transmit function between minus B to plus B, it have to have to a bandwidth of 2 B now not B center of C. So, it is a band pass filter, showing it here but keep that in mind, the only effective the channel that we will consider here is the.

Student: ((Refer Time: 43:55))

No, it is a minus B to plus B is F c minus B to F c plus B and similarly we will have, in the negative side between minus F c minus B 2 minus F c plus B that is implicit, I am not stating that. But, the bandwidth is 2 B the mathematical bandwidth if you like to use such a term will be 4 B, for the purpose of calculating the noise power, it is it will be N by 2 into 4 B.

So, that only effect the channel with therefore we will consider this again noise, we will assume that the channel is ideal band pass channel which is ideal and what will be the receiver consists of now. The receiver will have a band pass filter central alone F c, that is the starting point, so we have a band pass filter with frequency F C and bandwidth plus minus B.

And then, what will you have you have the demodulator, several noise we already have we have elementary part of the channel noise to this filter, and the rest of it is that demodulator. So, demodulator is essentially once again multiply, we need to cosine omega C t followed by the baseband low parse filter, bandwidth is B hertz and for convenience I look at, you know look at the input from let us say if I go to white Y i t here, what will I get here.

This is the input to the demodulator, this is your demodulator that is the input to the demodulator Y sub i t. So, the subscript I now denotes the input to the demodulated, what will be that equal to this is equal to root 2 what will you get here, what are you transmitting root 2 m t cosine omega c t plus n t, but after band pass filtering the signal power will be the same as before, is in it that n t which was white Gaussian.

So, for could become narrowband, narrow band Gaussian the parse spectral will get modify to have bandwidth of 2 B, so how do we denote this noise will be, let me call this Y i t which contains S i t and n i t. And, this is equal to root 2 cosine I am sorry Y i t is equal to root 2 m t cosine omega c t plus n i t where n i t is narrowband Gaussian noise.

And, before can be denoted by remember that the quadrature representation, that is call it N this is a standard notation n sub c t cosine omega c t and slightly modify earlier c o that stands that are cosine. So, in phase sometimes it is also this is n c and similarly n sub s t finally c t, so you can write the noise process here n sub i t with your normal Gaussian process, in terms of its quadrature components which are now been represented by in-phase component is things n sub c t, the quadrature component by n sub s t.

Student: ((Refer Time: 48:29))

You can call it plus or minus it does not make any big difference in, as for the properties of the resulting process of the result you could do that. What is the signal power here, the signal power that you will have here is this power, and the power in the signal power in this part of the thing, is it not? At the input to a demodulator this is your signal component, so the power in this component is your signal power of the input.

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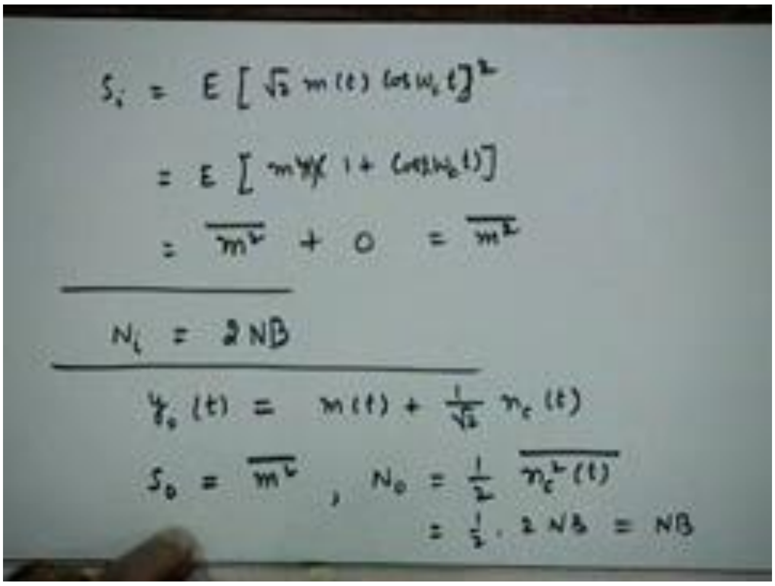

$$S_i = E\left[\sqrt{2}\, m(t) \cos \omega_c t\right]^2$$

$$= E\left[m^2(t)(1 + \cos 2\omega_c t)\right]$$

$$= \overline{m^2} + 0 = \overline{m^2}$$

$$N_i = 2NB$$

$$y_0(t) = m(t) + \frac{1}{\sqrt{2}} n_c(t)$$

$$S_0 = \overline{m^2}, \quad N_0 = \frac{1}{2} \overline{n_c^2(t)}$$

$$= \frac{1}{2} \cdot 2NB = NB$$

So, what is S sub i that is expected vale of root 2 m t cosine omega c t whole square, expected value of this square which we can write. Take the cosine for inside take the square inside its expected value of m square t into 2 cosine omega square sine square omega c t is again, we can write us m square t into 1 plus cosine 2 omega c t m square t here.

So, what is equal to the first part is nothing but your message power, in the second power could be what can if you a second part, it is 0 is it clear. So, this is equal to m square bar, so now you see why the factor 2 as included because. This will make sure that S sub i is going for m equal to m square bar the same value that you had earlier, what can you say about the noise power very quickly, where you demodulate this is a signal at the input of demodulated, what will be the output of the demodulated. I think, before that you will calculate n sub i, n sub i is how much it does not required much calculation for that.

Student: ((Refer Time: 50:51))

No, just even looking without doing any calculation can you tell me what is n sub i here, here what is n sub i infinity, same as before. What is capital N sub i at this point, it is N by 2 which is the parse spectral density input into 4 B, so this 2 N B. So, N sub i is 2 N B and what I am saying at the output, this is as for as the input is concern input is modulator.

What will be the output of the demodulator, let me call its y o t, this is the input to the demodulator what will be the output of the demodulator, you are multiplying with this into cosine omega c t and see what will you get you get m t plus 1 by root 2 of n c t. So, the output we will be m t plus 1 by root 2 n c t, so what is your output signal power same as the input signal power, what is your output noise power, it is half of average value of n c square n.

What is the important property there a gave this as a exercise in the quadrature representation of noise, you have to show that each of the two quadrature components as we same power of the noise process itself, as a major noise process itself. But, the narrowband noise process say you was n i t, n c t is an in-phase component of that. So, this power is same as 2 N B same as this, so this is equal to half of 2 N B in same as N B. So, once again if we call this as S i S o equal to S i, what you find, the output is r once again S i upon N B, it is same as gamma.

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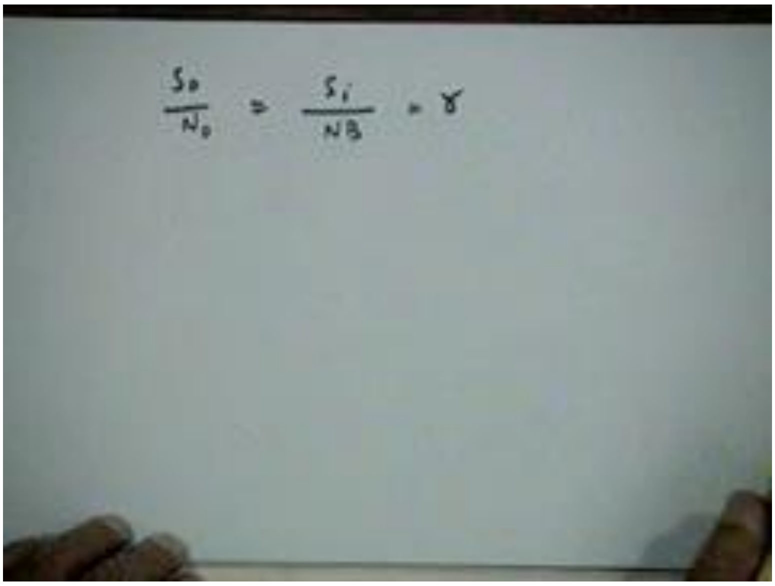

$$\frac{S_o}{N_o} = \frac{S_i}{NB} = \gamma$$

Using the fact, that n c square t average value of this, is equal to average value of this, is equal to average value of the noise power itself. This was an exercise as we do, a very simple to provide please provide that and that is the result I am using here.

So, we find that we just conclude that, in as for as signal to noise ratio figure as we concerned there is no difference between the baseband system and the modulation system, which was something that we can expect into two ways. And therefore, it is not a very surprising ((Refer Time: 54:05)) we will discuss it further next time.

Thank you.