Communication Engineering
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Lecture - 31
Random Processes (Contd.)

We will continue with our discussion on Random Processes and see whether we can find it up today, that is most of it and next quickly recollect what we did last time. We discuss the concept, basic concept of a random process and also the concepts of simplifying the characterization of a random process through assumption ((Refer Time: 01:26)) stationarity, wide sense stationarity and ((Refer Time: 01:30)). So, I think that is maybe work, let us continue the discussion now.

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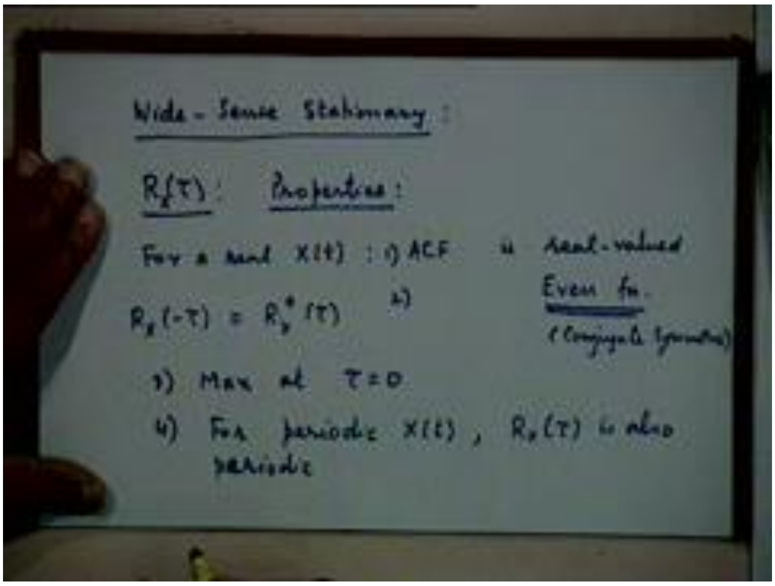

From now onwards, we will assume that, we are closely working with wide sense stationary process, processes which are at least wide sense stationary. They may or may not be strict sense stationary, but we can assume that they are at least wide sense stationary. And please recollect that, as far as wide sense stationarity is concerned, you are only required to worry about two properties, namely the mean value function and the auto correlation function.

The mean value function is, maybe the constant independent of time, the auto correlation function is going to be a function only of the log variable t 1 minus t 2, rather t 1 and t 2.

Now, therefore, if these two functions more or less are in complete certain order, characterization of a wide sense stationary process. That, they are not complete characterization, that complete second order characterization, now you just look at the properties of the auto correlation function in some more detail.

So, for now onwards, we will denote the auto correlation function by R tau, R sub X tau, R sub Y tau etcetera. So, we just looking at the properties, some of the, I am just going to mention some of the properties because they are very simple to prove, the more or less follow, directly from the definition of the auto correlation function or at does a little bit of manipulation. And I am sure you already know this, so I am not spend too much time on it, I am just mention them.

For a real process X t, X of t the auto correlation function could be real, if you also real valued, if it is a complex function, if your process X t is a complex valued function, then auto correlation function could be complex valued. It will be an even function, so even function, see already seen last time, because if I change the order of t 1 and t 2. You multiplying X t 1 and X t 2, it will not make any difference to the average value of the product.

So, whether you write t 1 minus t 2 or t 2 minus t 1, is the same thing, so it is an even function. So, if I replace tau with minus tau, the value will be the same, again for complex valued functions, the symmetry this even symmetry will change into conjugate symmetry. So, for complex valued functions, the auto correlation function will be conjugate symmetric. What is that, that means R x minus tau could be equal to, R x conjugate tau.

That is because, there is a communication operation involved in the multiplication, then you take the auto correlation function of to, a complex valued random process. Conjugate symmetry is also sometimes for called Hermitian symmetry and this third property, so this is property number 1, this is property number 2 and property number 3 is, that it is maximum, auto correlation function is maximum at tau equal to 0. Property number 4, if the random process x t happens to be periodic.

So, for periodic random processes, the corresponding auto correlation function also will be periodic. You have really state for properties to prove, just animating them for the sake of computers, so that you are familiar with these things.

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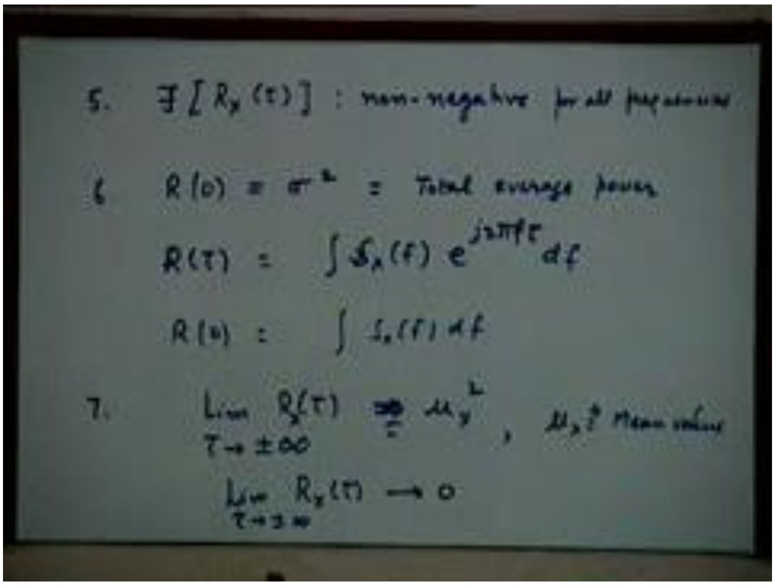

5, this is important, very important, the Fourier transform are the auto correlation function. So the Fourier transform of the auto correlation function, which we have seen, is nothing but the, so called power spectral density function and you know a how to define the power spectral density function, which have expected value of, Fourier transform of X t, t magnitude square. So, taking the expected value of some square quantity, so what can you say about such an expected value, it will always be positive.

And though for, the Fourier transform of the auto correlation function of any random process will always be positive. It will be a real valued positive quantity, is not, unlike the Fourier transform of an arbitrary function which can even be complex valued function, the Fourier transform of an auto correlation function will always be real valued and positive. So, is non-negative, actually note generally we should say it is non-negative for all frequencies.

And there is one of the important tasks, for let us say checking whether or not a given function, would be an auto correlation function of some process are there or not. We take it is Fourier transform, if there is a positive function for all frequencies, then it is likely to be an auto correlation function of some process are there. Then we how would you know, is this so that, this is understood by everyone, the motivation why this is, this property comes.

This comes from the definition of the power spectral density function, which we define to be the expected value of some square constant. And therefore must be positive and we already seen, that the power spectral density function and the auto correlation function are Fourier transform pairs, there is by Wiener Khinchin theorem. So, also the value of R is 0, which we also said is maximum, preservely ((Refer Time: 08:08)) for a periodic auto correlation function, people repeat itself, at periodic intervals with period t, whatever is a period.

But in general, R of 0 is maximum and it is value is equal to, if have a physical significance, is equal to sigma square. The variation of the process at time t, of all time, we are, since we are considering wide sense stationary processes, the various will be the same for all time instance. So, it is a take, which we sometimes also called the total average power, why because, as you can see if you remember R tau would be the inverse Fourier transform of, the power spectral density function.

So, if I put tau equal to 0, what do you get, R 0 equal to integral of S X f, that is the area under the density function. And therefore, it is a total average power and the last property there are like to mention here is, that in general for random processes, if you consider the limit limiting value of R tau, as tau tends to either plus or minus infinity. That is you are giving, here is the general nature of the auto correlation function, what kind of a auto, what kind of function would be the auto correlation function.

In general, as we approach ((Refer Time: 09:54)) infinity on either side, plus positive side or negative side, the value of the auto correlation function could tend to, this limit will be equal to mu x square. So, R X tau if you put mu x square, here mu X is a mean value and this is a process happens to be zero mean, then what will happen to this limiting value for either side, return to 0.

Now, this I am not proving, but very simple to prove, in ((Refer Time: 10:29)) even more simpler to appreciate, what we are say, suppose it is a zero mean process, just for the sake of ((Refer Time: 10:35)), what we are saying is, every auto correlation function for a zero mean process, would tend to 0 as tau tends to infinity. Does it may ((Refer Time: 10:47)) contiguity sense, what does R X, just remember try to ask yourself, what does the value of R tau represent, for a given value of tau.

It should presence the cross correlation between two random variables which are sampled, which have the team by sample the random process at two time instance which are separated by tau seconds, is it not? This is the correlation between two random variables which are separated by tau seconds and what this says is, the larger you make the separation, the smaller we correlation between them and it will become 0, as tau tends to infinity.

That is, if the separation between then becomes very large, there is every reason to expect, that the relative values will have absolutely no correlation with respect to each other. So, that is basically, that makes a lot of sense, if say truly random process it should happen that way, please close that parts, would have a larger correlation that is why R of 0 is a maximum and then it starts to decay and it decays to 0 for either side, so typical form,

Student: ((Refer Time: 11:55))

Yes please.

Student: Sir, what is this ((Refer Time: 11:59)) random particle are periodic function.

If it is a periodic function then this will not be true, this property is not valid for periodic auto correlation functions. This is only valid for a periodic auto correlation functions, so basically what we are saying is, you are right.

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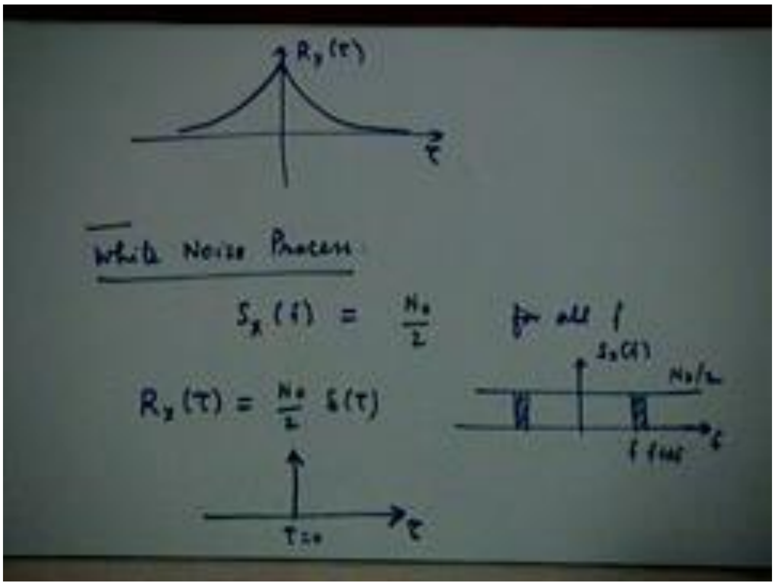

Auto correlation function typically will have, which was like that, in the case zero as tau tends to, this is the plot of, the typical auto correlation function as tau tends to infinity or minus infinity, it has to be even symmetric, if it is an even value for real value process and so and so further. At this point, having define the basic concepts which characterize random process, it is now possible for us to, look at a very important specify process which we shall using in modeling noise in communication systems.

And in fact, the process this particular random process is known by the name of, in white noise process. So, let me define a process, what you look kind of process which is called a white noise process, if I am take it up, I just like into like to reemphasize the fact, that the auto correlation function and the power spectral density function, they are equivalent second order descriptions of the random process, is it not. Because of free transform pairs, if you give one you know the other.

So, the more second order descriptions and secondly, these descriptions are, at that we are discuss or independent of what kind of density function the process has, because we are only looking at the first two movements of the density function. The first order movements, which is the mean function and the second order movement which is the, auto correlation function.

You could have any density function and you could have random processes with different density functions, different joint density functions having the same second order properties. So, get something that what you keep at the back of your mind, that when I am discussing, only second order properties, we are ignoring the density of a .function properties, because we are ignoring the detail, you only looking at these two cross properties.

The detail properties are not known to us or we are not talking about them at least, let we come back to the white noise process. The white noise process is one, is really the definition is very simple, for which the power spectral density function as this form, it is constant, in this you just the value, arbitrary value or arbitrary notation for the constant ((Refer Time: 15:03)) individually denoted by N sub 0 by 2, for all frequencies. For all values of the frequency f, the auto correlation the power spectral density function is a constant.

So, the power function power density function, plot as a function of f looks like this and like you see why, where the name comes from, the name comes from the fact that, all frequency components are present in equal measure. So, the analogy comes from white light we are, a large number of frequency components constitute the white light, so that is where the name comes from.

If the process is, if the power spectral density function is not flat, then if you want to call it we can call it a colored random process, the colored noise process, because it is not, so it is just as against wide, so it is not flat we call it is sometimes we just loose the quality colored noise process and they are various kinds of colors the one thing have, but usually the simply quality color tend to noise process. Now, what will be the auto correlation function of this process.

What is the R X tau, of a white noise process, what is R X tau, it will be delta function, it will be N 0 by 2 delta tau, because this is the Fourier transform pairs. The Fourier transform of this is equal to this, the inverse Fourier transform of this is this, so the auto correlation function therefore plot, looks like this. This is delta tau, sorry this is tau and this is N 0 by 2 delta tau, tau equal to 0, the delta function occurs at tau equal to 0, this N 0 by 2 is more or less a standard notation, using communication theory.

It essentially says, this is called two sided power spectral density value, the value of N 0 by 2 is called two sided power spectral density. In, as much as this negative frequencies are only abstract entities, the corresponding one sided spectral density sometimes called N 0, is denoted by Ns, the twice of this. There is a few reflect the negative access also the, or if you only talking in terms of positive frequencies, the total power of frequency in a ((Refer Time: 17:41)) frequency f, will be actually the some of these two areas.

Remember that, because every real signal, with frequency f will also have ((Refer Time: 17:52)) minus f, so really speaking, if you want to compute the power contained in, a region f to f plus delta, f equal to get a this area, with this area. So, N 0 by 2 delta f plus N 0 by 2 delta which will become N 0 delta, so N 0 is sometimes called single sided power spectral density for the white noise process and N 0 by 2 would become the double sided power spectral density.

These were just some general terms which are used in the free ((Refer Time: 18:23)). Now, this is a very convenient, modify, many physical random processes, that we come across in communication theory, but remember it is only a model, why, because such a process cannot physically exist, can you see that, why can you give why reason why cannot physically exist. That is, ((Refer Time: 18:49)) one we have to think at it, ((Refer Time: 18:51)) just look at it more slightly more slightly different way, what is a total power on the this process?

Student: ((Refer Time: 18:58))

Infinity, no physical process can have infinite problem, because which is, area under it is infinity, alternatively if you look in the auto correlation domain, what is it is say?

Student: ((Refer Time: 19:12))

That, other than the fact that, you know X t would be relative, X t will be correlative with X t, if you move even a slight, in the time domain x in the time domain, along with the time axis, if you move even slightly and if you look at two samples which are very very close to each other, but not the close this is not equal to 0, they will be uncorrelated. For such things to happen, for such wide ((Refer Time: 19:40)) sense to occur the process must have infinite problem, otherwise it will be correlated.

Otherwise, the process will have some degree of smoothness and there is even a small minute degree of smoothness, this kind may happen, in any time function that we are need. So, basically infinite power or such an auto correlation function or idealization, which will never occur in practice. However, inspite of the fact that this is, so this is convenient to model many physical processes in communication theory with this model, because the more or less satisfy, the broad characteristics that this model assumes.

For example, the kind of noise that if you deal within communications, the thermal noise, as a very large spectrum, very wide spectrum, more or less it is flat over that spectrum. So, it is, then it really go to infinity, infinity is anywhere in a spectrum, it goes very wide, much larger than the bandwidth of the signal which you are working with. So, for all practical purposes with a model that is white noise, so even though in practice this is not truly wide in the true size of the work.

Student: Sir, you say wide noise density ((Refer Time: 20:52)) most thermal noise, white noise.

Most thermal noise, short noise and various other kinds of noise, many of them can be modulus wide, but their situations maybe have to be deal with the non white model, their situations. So, it does not mean that, you will always have real product you always needs to work with only white noise, their situation when you have to look at the actual spectrum of the noise which is typically not wide, in some situations. At this point, I think it is important for us to understand one additional set of relations, which we typically need to work with.

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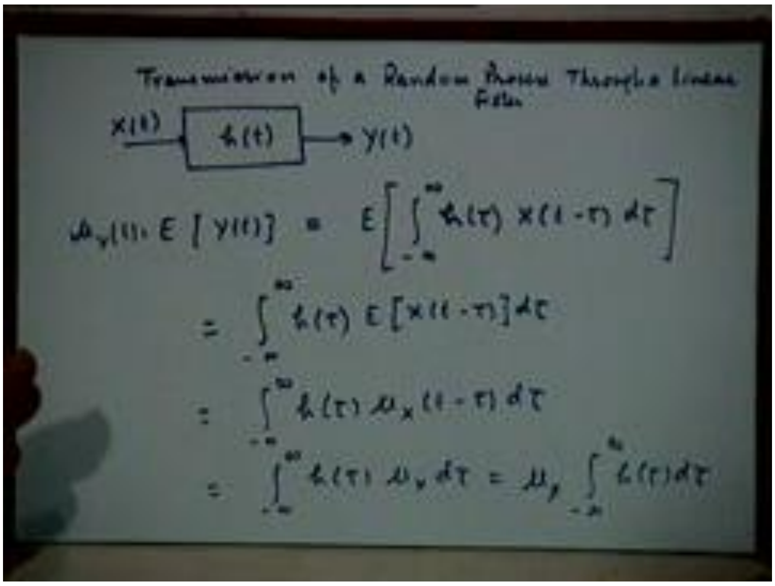

Many a times, you will be faced with, when an analyzing systems, analyzing the systems in the random inputs, you know after all when you work, when you our purpose of being this view right now is, to have the necessary tools to analyze the performance of communication systems in the process presence of noise and if you have, if you remember whatever we are discussed so for. Most communication systems will involve, one of the major components of any communication system, could be filters of various kinds.

You are doing a lot of filtering here and there, so like to know, if there is a certain kind of process that is, random process which is input your filter, what kind of random process will be the output of this filter. So, like to know this relationships, so let us look at these relationships that is, what happens when you transmit random process to a linear system, your linear system of a linear filter. So, we have a linear system with impulse cos h of t, input is random process X of t, output is some random process Y of t.

So, in as much as we are concentrating only on the wide sense stationary processes, that is we equal to assume that X is a wide sense stationary process. We will also try to characterize Y t, in terms of first two minutes of course, you will, it can be shown the pie t also will be wide sense stationary, maybe it could be obvious, even if it is not, it can be shown.

So, that means given the mean value function and the auto correlation function of the input process, you like to find out, what is the mean value function and the auto correlation function of the of ((Refer Time: 23:57)) output, there is a concern. So, let us look at the mean value function, how is Y t related to X t, through the convolution relation. So, basically use, that is the starting point, so if I look at expected value of Y t that is the mean value of function of ((Refer Time: 24:20)).

First let me express Y t, Y t in terms of h t is X tau X t minus tau d tau, since I think the expected value of this, basically where are you say, ((Refer Time: 24:40)) is a expected value of this integral for this convolution relation, X t is a deterministic filter X t is a deterministic function. An expectation is also linear operator, it is a linear operator with expected density function of the process and because it is a linear operator and a certain conditions, you can clarify the interchange these two integral operators.

The integral operators corresponding to the expectation operation and this integration, so I can carry the expectation of vector inside the integral, write the expected value of the product of these two, but X t is not random, so we can keep it like that x tau and really speaking the averaging will operate of the process X t. You are using the linearity property of the expectation operator, so what is this, this is by definition, the mean value function of the this is mu X t minus tau.

And since we assuming wide sense stationarity, what will be the value of mu sigma X star, will be constant. So, it will be minus infinity to infinity h tau into mu X, mu X is a constant it comes out of the integral, so just what you have. So, the mean value at the output is the mean value of the input, so that shows that, the input process is stationary up to first order, the output process also will be stationary up to first order.

Because, this is not a function of time, X t E X t was not a function of time, you find a ((Refer Time: 26:35)) new variety also lot a function of time. Because, this is going to be a fixed value, some number and what is that number, the multiplying E X with the area under the impulse response. Can you explicit it in terms of frequency domain valuation ((Refer Time: 26:51)).

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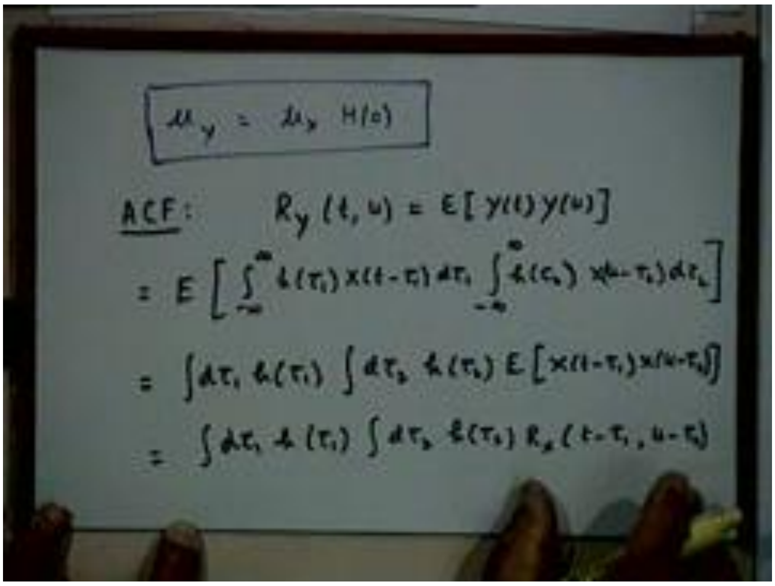

This is equal to?

Student: ((Refer Time: 26:53))

So, mu Y is equal to mu X into H 0, is it not, because this is nothing but H capital H of 0, H of f is H tau e to the power J 2 pi f tau, if put f equal to 0, you get this and that is ((Refer Time: 27:16)) very appealing because what is it, you can think of the mean value of the input as a DC component of input process. It is a zero frequency component, mean value is ((Refer Time: 27:29)) you can consider to be the zero frequency component.

And so what does it say, if the mean value of the output is the mean value of the input, multiply by the response of the system of DC. DC response of the system, so that is the first solution very simple basic relation, so if you know the frequency response, in particular if you know the frequency response or f equal to 0, that is how the mean values in the related, so this relationship is very simple. However, the next one is not that simple, but does not matter, it is still not very complicated.

So, let us look at the auto correlation function, so we want to find out R Y t u, which the definition is expected value of Y t into Y u. Now, you might have notice that are not ((Refer Time: 28:36)) done able to little t minus u, because I am not true yet that the output process will be stationary. I know that the input process is stationary, but we are ask to see whether the output process tends out to be stationary or not, so we start with writing like this.

And if Y of t to be a function of only t minus u, then it becomes the stationary process, so just look at this, this by definition is now I just, you just substitute the value of Y t and Y u, this may entirely mechanical process, use the dummy variable tau 1 x t minus tau 1 d tau 1, use a dummy variable tau 2, it is u minus tau 2 d tau 2, expected value of this, so fast, you substituting for Y t substituting from Y u, is defined dummy variables to differentiate between integrals.

Can write this like this, ((Refer Time: 29:59)) taking the linear operator on expectation of theta once again this side, combining it with these two random entities, these are the two random entities, so the expectation will now work on the product of this and this. So, X t minus tau 1, X of u minus tau 2, of course, certain conditions are required to be satisfied for this to happen to be valid, that we will assume those conditions are valid.

The basic conditions are that h of tau should correspondingly stable system and x of t should be a finite imaginary process and infinite power process. These are the conditions which are generally required, will not go into details, so this is theta 1, I think this is a, this purely mechanical manipulation, you should not have any difficulty and what is this quantity, this expectation, this is R x t minus tau 1 and then let me write the general expression u minus tau 2, if you assume with a stationary process, then it will be a function only of, this minus this.

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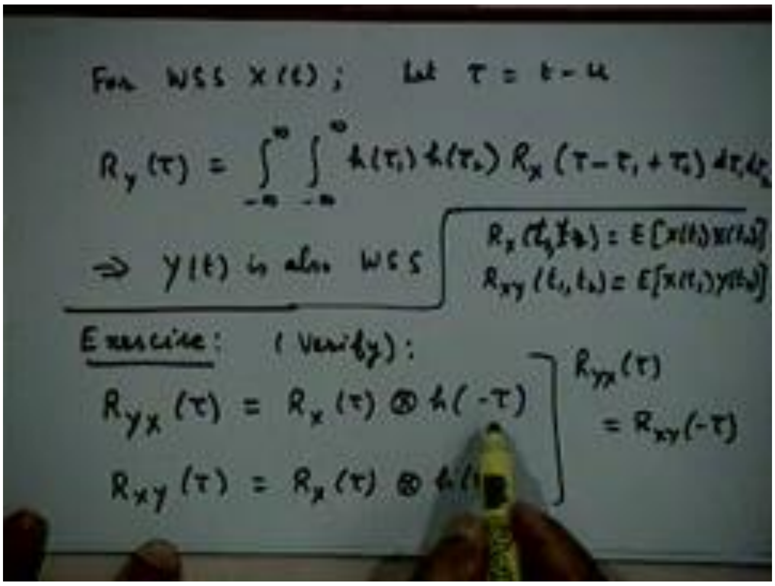

So, if for WSS of a wide sense stationary X t, let we just define tau equal to t minus u, so this becomes, can you see the region of function of t minus u, because it will be a function of this minus this, or this minus this. So, this because, I can write R Y tau instead of writing R Y t u, I can now write R y tau because it is a function only of t minus u would be equal to, just rewriting this, rewriting this relation.

You have h tau 1, I think you can look at a lower side check, you have extra to and what will be write here, R x take the difference, it will become tau minus tau 1 plus tau 2, d tau 1 d tau 2. So, as you can see, if a function this tau 1 and tau 2 will disappear after these two integral integration has been done. So, you will be left only with the function of tau, R Y is a function of tau, so which means which implies that Y t is also WSS, so if you pass wide sense stationary random process to a linear time invariant filter.

Because, X t was a linear time invariant filter, in fact, that is the reason the output process is wide sense stationary. For example, if the filter was a time variant filter, even though the input process was wide sense stationary, the output process then need not be wide sense stationary, because the fact your filter impulse response keeps varying the time. So, for the output process to be wide sense stationary, input the not only the input process should be wide sense stationary, the filter to which you are passing it, should be time invariant.

So, time invariants is the clue, which we have listed, so what you find, you find there is some kind of since like this relationship that you have, looks complicated, but actually it is a very simple relationship. I am I will not go through the ((Refer Time: 34:09)) I will give this as an exercise, I will give an exercise for you to complete, because these are, it is a repetition of the same steps arrive done so for and that we help you to see the relationship in a slightly better light.

Instead of directly looking at the auto correlation function of the output, there is a system where we are working with, ((Refer Time: 34:32)) what we have done so for is, you are related the output auto correlation function with the input auto correlation function, go through a two step process. First try to relate the cross correlation between the input output, in input and the output to the auto correlation function of the input. That is, consider expected value of X t into Y u, rather Y t into Y u.

Then we have to only dealing y integral, rather two integrals, is in it, I should have define, if the concept of cross correlation function which I have not done, but let me complete that. So, just like you have the auto correlation function of a process, which is R X t tau equal to R X t 1 t 2 equal to expected value of X t 1 into X t 2, I can define a cross correlation function between two processes, X and Y, any two processes X and Y in the same manner.

What we are do, I sample the random process X t at the time instance t 1, sample the random process Y t at a time instance t 2, I have two random variables now, X t 1 and Y t 2, take their cross correlation. That is the definition of cross correlation function of two random processes, so it will be X t 1 in to Y t 2, so basically what I am suggesting is, first find out for this ((Refer Time: 26:08)) see, for this picture, the cross correlation between these two processes, the input process and the output process..

And then, find the output auto correlation function in terms of this cross correlation, if you go through this two step process, what we just see the result please verify right, then this whole relationship becomes much more meaningful, so what have to verifies the following. Show that, R Y X tau is equal to R X tau convolved with h of minus tau, one simple relation and of course, unlike the auto correlation function or the cross correlation function does not have that even symmetric property.

Because, we have different processes, now if you change the order of X t and Y t, it will never change, it will make it will be different function. So, I mean, there will be some relationship, but it will not be in symmetric relationship, example if I replace if I change the order, a first take X and then take Y, then you find again the relationship is like this, R X tau convolve with h tau, so they are equivalent. The general relationship between R Y X tau and R X X tau, R x y tau could be this.

You see the difference, you are not saying R Y X tau equal to R Y X minus tau, it is R X Y minus tau, which is obvious, again just look at the definition of this things will more or less follow from that. So, this is a one relationship, we are able to express, what is that say, the cross correlation between the input and the output is the auto correlation function of the input, convolve with the impulse response either directly depending on whether you are, having this kind of cross correlation or within mirror image of the impulse response, around the origin.

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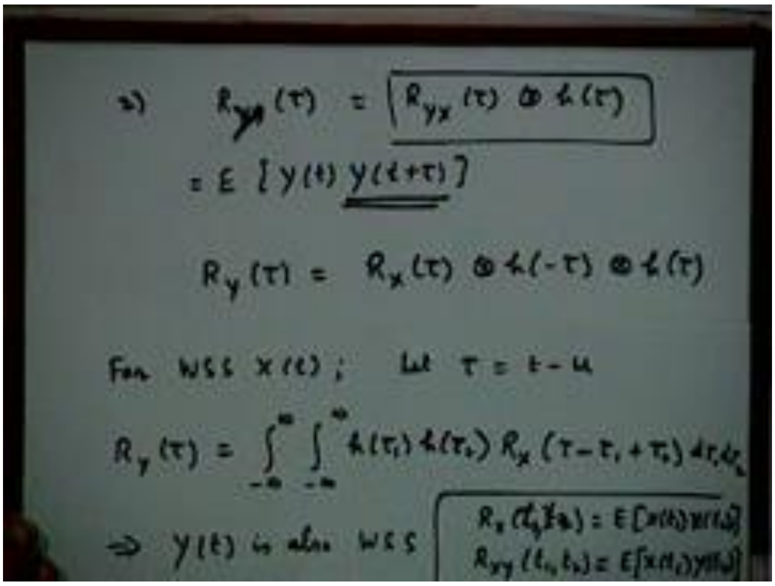


In the second step, we show that, we can express R Y Y tau or R Y tau, that is usually write simply R Y tau to simplify, can we written us R Y X tau convolve with h tau. How will you do this, to show this, you know you start with expected value of Y t into Y t plus tau, that is the definition of R Y tau, they substitute only for Y t plus tau. Keep this here, in terms of X t, I am go through that process, so you will ultimately end up here, so if I combine this solution with a previous solution which has this ((Refer Time: 39:43)).

What do you get, if I combine these two relations what will you get, R Y tau is equal to, I am substituting for R Y X tau as R X tau convolving X minus tau and this convolving h tau and let us precisely, what this relationship is, that we defined. So, now if you can look at this relationship in a much simpler manner, it is only a double convolution, double convolution of the auto correlation function of the input with h minus tau first and h tau later or the other variant.

Because, communication is the competitive operation, so this relationship which looks rather complicated, is actually this which is much more appearing. ((Refer Time: 40:45))

Student: ((Refer Time: 40:48))

No no, what I am saying is, very nice, when I substituting for X of minus t plus tau, it will be 1 Y and ((Refer Time: 41:04)) that is what we are need to R Y X, is it not. Now, this is, therefore the relationship between the input auto correlation function and the output auto correlation function. Incidentally, this relationships are very useful, let us look at these relationship for example, which we can prove very easily, which I am sure you will be able to prove in exactly one step.

I let us say, look at this relationship, this is extremely useful and this, I will discuss the application of this very briefly. This terms be there if a cross correlate, the input in the output random processes, what I am see is the, this convolution. Suppose, I choose the input auto correlation function to be a white noise process, input process to be a white noise process, what will be the corresponding auto correlation function, auto correlation function delta tau, some constant delta tau.

What will be the cross correlation of input and the output now?

Student: ((Refer Time: 42:16))

Will be simply h tau, so what is the result, if I feed a white noise process at the input to a linear time invariant filter, the cross correlation between the input white noise and the output noise which may not be white, which will not be white actually, would the cross correlation function would be proportional to the impulse response of the system. So, this gives me a method of finding the impulse response of the unknown system, experimentally.

I feed white noise at the input to the system, look at the output random process, cross correlate these two processes and the result of cross correlation will identify the impulse response ((Refer Time: 43:05)) which was otherwise not known to me, let us say it is not known to you. So, this is the very useful relationship, where can I use it in communication systems?

Student: Channel.

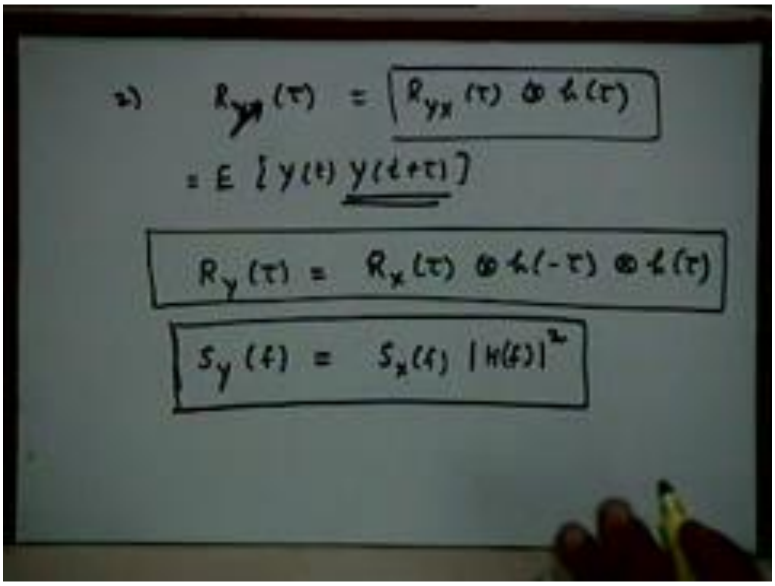
Channel, the typical unknown thing in a communication system is the channel system to which the signal is being passed. So, many, many physical communication system actually use this method, to identify the channel impulse response, for example your GSM standard which we are using in the mobile telephony, transmits voice in the form of packets, information in the form of packets.

Every packets has it is, as in the middle of it, a training sequence which is a socially some kind of a pseudo noise sequence, pseudo noise white sequence, which is power spectral ((Refer Time: 44:00)) whose you go for the out, if you when you look at the output of the corresponding to this training sequence of the receiver and see the most or can find out through cross correlation, between what was transmitting and what was received. The estimate of the channel impulse response and that is then used, to further process the signal, to get your signal nicely.

So, this relationship is extremely useful ((Refer Time: 44:27)) what I am saying, we will discuss it further if necessary. Now, finally this illustration implies a corresponding relationship in the frequency domain, let us look at that relationship, just take the Fourier transform of both the sides, so what will be the Fourier transform of this, the power spectral density function of R Y t. Fourier transform of R Y tau will be S Y f, this will be S X f, convolution in time domain will reduce to multiplication in the frequency domain.

So, what will be the product here, S X f into h of f and what will be the Fourier transform of this, h conjugate of f, so what you get, H f into H conjugate f, it is H f mod square. This is the corresponding very simple ((Refer Time: 45:25)) repeatedly appearing frequency domain relationship. That is the input the output power spectral density function is obtained by multiplying the input power spectral density function, by magnitude square of the transfer function.

(Refer Slide Time: 45:51)



In the sense, because power spectral density function is a positive function, you must multiplied by a positive number to get, make sure there using always a positive, that makes a lot of sense, any questions, any question so far? No.

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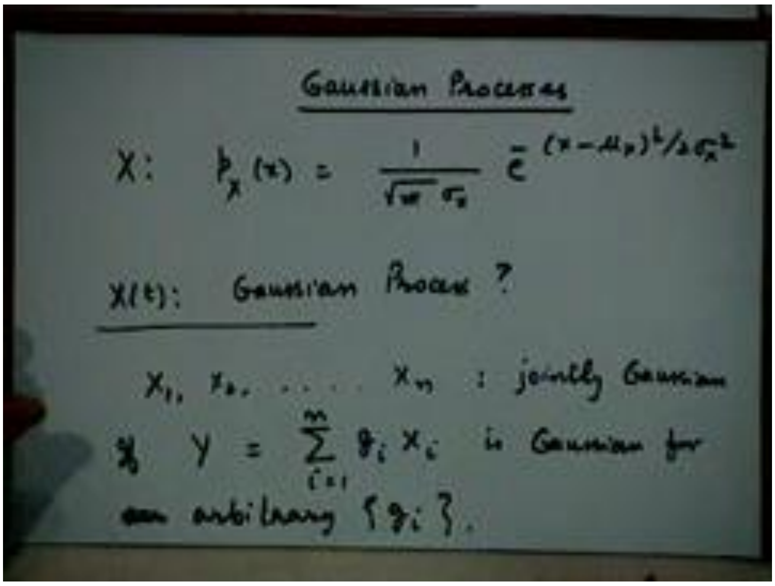

Finally, let we spend some time on, if returning to density functions so far, when I was discussing purely in terms of white sense stationarity and ignore the concept of density function, because I was looking at a second order characterization of the process. But, it is useful to know, which are mention before to you, that many physical processes the density function happens to be Gaussian. So, we need to also look at these functions or these processes in some more detail.

So, In fact, most of the time, the kind of noise process will deal with, will model the second order properties by saying that it is a white process, that is the power spectral density function is flat, but we are leave it at that, will also say we say something about the density function we say it is a white noise, it is a white Gaussian noise. So, now we not only talking about second order property, but we are giving some more detail information, that is which is the density function associate with, that is somehow Gaussian.

You understand the notion of the Gaussian random, the notion we like to understand is, that of a Gaussian process. When we say a process is Gaussian what is it mean, we know that when a random variable X is Gaussian, what it really means is, the density function is this. So, the question is, what do you mean by same, X t is a Gaussian process, what is this mean, therefore to like to understand a little bit, to start with, let we recap one very important property of a Gaussian of Gaussian random variables.

Suppose, X 1, X 2, X n all Gaussian random variables ((Refer Time: 48:53)) we will just take care of that. Suppose, these are Gaussian random variables, then if I construct a linear combination of these Gaussian random variables by saying that, by constructing Y in terms of X i's and g i is of some arbitrary coefficients of linear combination, i going from 1 to n. Then, Y will be Gaussian, we know this.

Now, I am say this is, a slightly different way, we will define a set of random variables X 1, X 2, X n to be jointly Gaussian, this is the definition now, you say that X 1, X 2 to X n are jointly Gaussian, such that if I take any linear combination of these, the output random variable, the the resulting random variable is Gaussian. So, if this is Gaussian for an arbitrary set of coefficients g i or arbitrary twice of g i, this will form the basis for definition of a Gaussian random process which are take up next.

Thank you very much.