Communication Engineering

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Lecture - 30
Random Processes (Contd.)

So, if you recollects we have looked at the definition of random processes. These are the 2 ways of looking at random process 1 is to look up on it as a collection of waveforms which we are randomly available right. Any particular waveform is certain probability depending on the probability distribution of the underlying probability space right. There is a mapping from omega ((Refer Time: 01:33)) to the set of functions which are functions of time. How a ways to think of it have a sequence in this sequence of ((Refer Time: 01:40)) we are there is some kind of index which gives out the functional dependency eventually. It could be a functional dependency on t index could be t the index could be some special variable or any other kind of variable right.

So, you have a sequence one after another or random variables occurring the certain probability distributions in that constitute of random process. The later approach is convenient from the point of view of characterization of the random process in terms of a probability distribution functions. Only thing is the complete characterization is a very elaborate affair and we discuss that it is very very difficult for a complete characterization of an object random process. Because you when need to characterize the process at every time instant individually at every pair of time instant jointly and so on so forth for every ((Refer Time: 02:35)) the points every quadrate the points that you select. And therefore, 1 ((Refer Time: 02:40)) and infinitive number of going to distribution functions ((Refer Time: 02:44)) distribution function as well as joined distribution function of various orders you completely correct raise the process.

To simplify the process to simplify the characterization you made some assumptions above the process. We can make some assumptions of course, they must be valid if you are going to use this typically many of this functions are valid why assumption that we made is the stationary assumption. Then we defined ((Refer Time: 03:09)) processes as 1 whose turbidity distribution functions or whose characterization whose statistical characterization is independent of timology right time shifts. So, these are the things are we cover last time, we also looked at the definition of certain limits of a elements process mainly the new value function and the auto correlation function. So, we will start from ((Refer Time: 03:36)).

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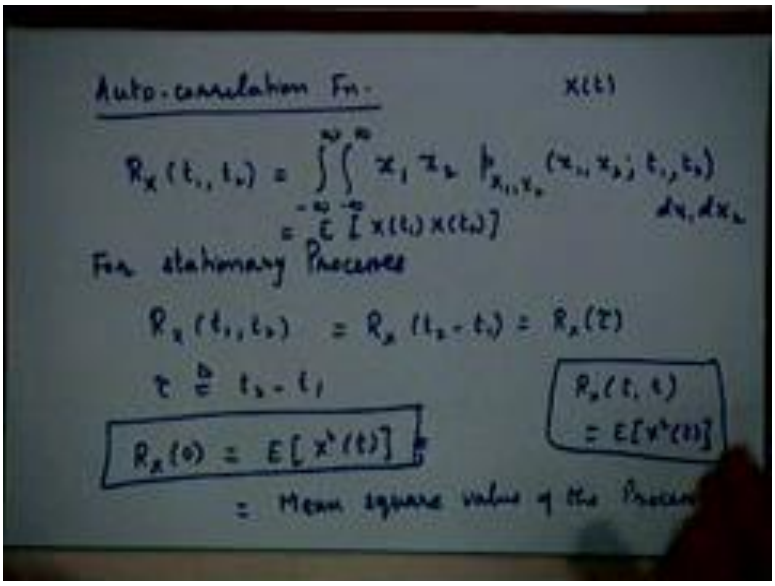

So, look at the auto correlation function is second order characterization why do we say ((Refer Time: 03:52)) size of characterization? Because required the joint density function of the 2 in the ((Refer Time: 04:00)) Xt way and Xt. So, next call it Rx t 1 t 2 Then integral ((Refer Time: 04:12)). So, take the random variables values random process values are time instance t 1 that will be denoted by the value x 2. Of course, this could occur anywhere from this value could be anywhere lying between the value of this minus infinitive plus infinitive x 1 into x 2 multiply these two. Multiply to the join density function of these 2 random variables. So, this is 1 notation by which we can denote the join density function of random variables t 1 by samply the random process x t at the time instance t 1 and t 2 right.

So, this is the notation this t 1 and t 2. This ((Refer Time: 05:01)) in general there will be a time dependency. Yes there will be a time dependency of the processes not stationary. If the processes is stationary the time dependency only being in terms of P 1 minus 2. This ((Refer Time: 05:13)) t 1 and t 2 separately right. But yes in general this is evaluate as this joint density function depends on this dimensions t 1 and t 2. Therefore, the auto correlation function that dependent the various of symptoms of random process right. So, this is the general definition for strictly stationary processors. For stationary processors this will be a function only or you could say this you could right as Rx t 2 minus t 1 if the be if you are Rx tau the tau is defined to the variable t 2 minus t. This is I thing we are we stop last. So, now let us look at a few things.

Let us look at the values of Rx tau for tau equal to 0 that is it mean that we are looking at what is this equal to? You can think of this portion in terms of expectation of portion are equal to expected value of x t 1 into x t 2 right. This integral is nothing for the expected value or average value of the product of the p random variables ((Refer Time: 06:40)) sampling the process of time instance t 1 and t 2 right. So, where is Rx ((Refer Time: 06:55)) if you choose t 1 equal to t 2 That will be that will give you tau equal to 0 right. So, it could say this is equal to expected value of x square t any arbitrary time instance t. So, this the value of the auto correlative function for a stationary process at all could 0 it going to be equal to the mu square value of the process it is a constant. Suppose the process was not stationary then with this be a constant? No. In fact, we will not write Rx ca in that case. We will write Rx t t or t 1 t 1 right ((Refer Time: 07:48)) dependent the value of t.

So, this equivalently for a non stationary process the corresponding relation will be Rx t t it represent t 1 equal to t 2 equal to t. And this will be equal to expected value of x square t that is correct. But this is going to be a function of time it is are to be constant. In this case it is not going to be a function of time it is going to be constant right is it clear? So, for a time for a non stationary process this is the relation for a stationary process ((Refer Time: 08:25)). It is clear from the definition of the auto correlation function at least for real value processors that if I interchange t 1 and t 2 if I interchange write x t 2 first x t 1 later it would make any difference. We just multiplying the same 2 numbers right the average value will be the same right what did it mean? The Rx Rx tau will be equal to Rx minus tau right. So, this is a important property of the auto correlation function for real value processors right.

Student: ((Refer Time: 09:05)) what is the difference between Rx ((Refer Time: 09:10))

They are same for stationary processors. This will become this for stationary process; for stationary process this will be equal to t minus ((Refer Time: 09:22)) t minus t. But in general it is not a stationary process this we have ((Refer Time: 09:28)) have to work with. If it is a stationary process work with this place.

Student: ((Refer Time: 09:32)).

That is right because the value of this auto correlative function has been discussed we depend only on the separation of the t time instance have a on ((Refer Time: 09:45)) value in the time instance. Because if it is a ((Refer Time: 09:47)) the basic function ((Refer Time: 09:49)).

Student: So for Rx 0 should not be used ((Refer Time: 09:54)).

No no no please understand if it is what is the Rx t 1 and t 2? t 2 value x t 1 and x t 2. So, ordinary ((Refer Time: 10:08)) choosing t 1 and t 2 will be the same time instance. The tau 0 any kind of term ((Refer Time: 10:15)) look nearly these value that is R 0. Rx stop is a functional stop then tau equal to 0 if it is it becomes it could be a least four validity function that is the point that you are trying make sure. Look at the basic definition can place of the ((Refer Time: 10:36)).

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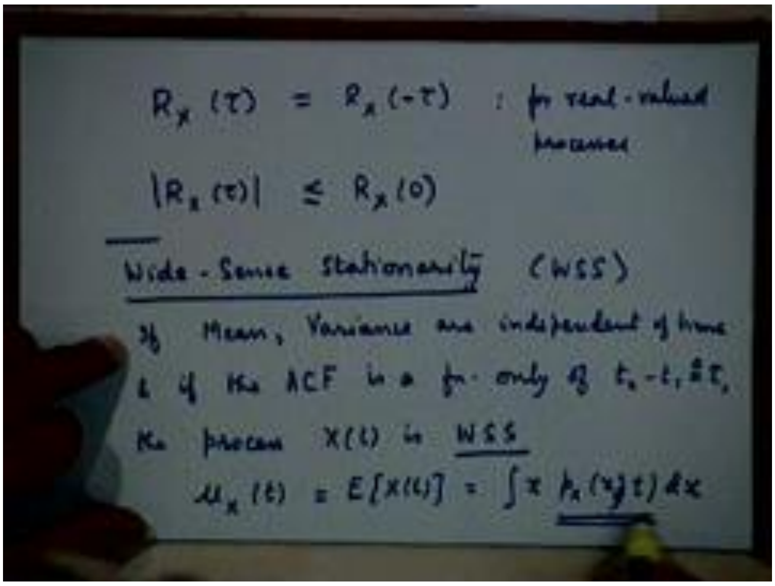

So, this is true for real value processors. So, sometimes I simply use a word process when I want to say random process. In general you could also ((Refer Time: 10:56)) field that the ((Refer Time: 11:01)) Rx stop. Of course, we can prove it more formally like it is with that yourself the magnificent of RX stop cannot be more than the value of tau equal to 0 is not it? Again for the same reason we can expect next line correlation to occur when the p random variable of the same right. The way you example of a different time extend the additional random variables it can have correlation right. But; obviously, that will be less than the correlation that you will have of random variable with itself that will be perfectly ((Refer Time: 11:40)) right. So, of course, one can ((Refer Time: 11:42)) with mathematically which are likely to be yourself. It is very easy to do easy to check that the magnitude of auto correlation function for a laptop the variable size as sometimes sometimes called the long variable, because long between 2. Time instance t 1 and t 2 are the time difference t 1 and t 2.

It is always less than or equal to the value of the auto correlation function the tau equal to 0. Please understand that Rx tau is not a random quantity in case this there is any kind of confusion in your mind. Rx tau is the highly deterministic function right it is a deterministic function, because a basically look here this interview right it is a function of I like x t which is a random process Rx tau Rx t t or Rx t 1 t 2 They are deterministic functions which are properties which are which describe the second order of the random process x t right. It tells you the behavior of the random variables with respect to each other on an average by looking at the average value of the product. By sampling a random process the different kinds of things the multiplying this random variables and looking at the average value of this product it is some kind of a characterization of the process some kind of a characterization of the process. It is a value it is a fixed value for again process even though the random process x t random in nature.

It is the value that you will see random in nature just nothing random in a Rx t 1 t 2 or Rx t t or Rx 0 or Rx stop right. Lot of correlation function is a deterministic function. So, these are some of the properties of a stationary process. You say it is just I will mention possibly here and we leave it if ((Refer Time: 13:40)). If the process x is a complex value process. For example, if you are working with in a naroboam process naroboam process when we looking are its complex envelop then it will be a complex value process right. In that case the definition are slightly modify that correlation function is defined as x t 1 into x t 2 right we do a ((Refer Time: 14:00)) correlation that you will use that one in the ((Refer Time: 14:08)). Suppose in a same instinct ((Refer Time: 14:15)) process could briefly simplify our characterization of the process right we discussed earlier. In many

many instrances your real life when you are working with random signals or random processors.

We do not really need to worry about distribution beyond second order right. Typically we are interested in collectors in the process in terms of individual value that you might see are various time instance or in terms of join behavior of a sphere of values right. Typically very early need to work work a situation very need to look at more than 2 values at a time right, which essentially means that we do not need to worry about characterization beyond certain order of ((Refer Time: 15:00)) in many many application in practice right. So, if that with the case wherever the process executes stationality in the stripes or not is another key all of them really then want us that the process be second order stationary right. The second order stationary means that the first order density function E x t 1 x should be independent of t. And second order joint density function E x t 1 and x t 2 should be independent of should over dependent on t 1 minus t 2 right.

So, if the process set as files this time origin you various only the respective these 2 distribution function we say the process is second order stationary ratherstic sensationally right. In reality you do not to worry about second order sationarity as engineers. It is sufficient for us if the first 2 elements of the process execute these properties right. So if that happens we are working with that process which is set to be stationary in the widest sense or wide sense. So, we define a nation of wide sense stationarity which is a special case of stripces rather stripes sentationallity imply of course, this. So, wide sense stationarity is defined as follows. If the mean value function the various function which of course, is redundant, because if the auto correlation. And the third thing auto correlation; if the auto correlation is a lead to various as a special case or if these 2 are independent of time. And if the automobiles function is a function only of t 2 minus t 1 t 2 minus t 1 is a function only of the difference variable tau.

Then the process x t is set to be void sensation this, your usual notation WSS. So, what I am saying? We not asking even in the density functions to be invariants even the first and second order density function you only say that the new value function. You know what is new value function how will you new value function define? Mu x t will be simply equal to expected value of x t. Now, it is a that is the definition of mew looking at the random variable of time t look in survey value what is this going to be equal to x E x x it will also dependent t in general dx right. So if it is a first our stationary process Px t could not dependent t right therefore, it is obvious that mew x with constant right. But other definition of wide sense stationary process does not even require this to be independent of t all as various if this is independent t that for enough for us right. We look all the other first of the movement associate with this x t function ((Refer Time: 18:53)) right that is one.

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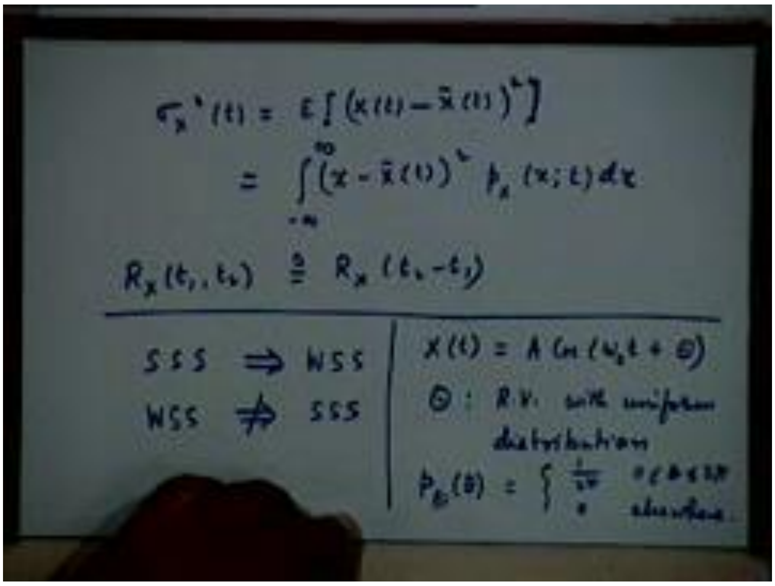

What is this various? Various is sigma x square t it will be expected value of x t minus x bar t whole square right. Again this will depend only on the density function p x t right. So, this will be equal to x minus x bar that the definition of various is in it that expected value of x minus x bar whole square that is integral of x minus x bar and the density function associated with x right. Again this is this is independent of t therefore, this should be in dependent of t, but again in WSS whereas, in the, this should be independent of t are you say yes this should be independent of t right. The second order movement also independent of t. And finally, what we are saying is that auto correlation function Rx t 1 t 2 as defined earlier should be a function only is tau right.

Of course, the various function is related to this function this will display this property actually this will be automatically implied this is something that we can check right. So, if these three conditions satisfy actually 2 conditions then we the mew value function is independent of time. And the auto correlation function is dependent only of on the time difference between the 2 dimensions t 1 and t 2 These are the processors stationary wide sense stationary. And many times we are quite happy the processors wide sense stationary be many times you do not you are need to worry about the distribution functions right. Do not need to worry about the first and second order movement functions then we the mean value function and the auto correlation function.

The mean value function is a first order ((Refer Time: 21:15)) function, because the race power race power of x that we racing 2 is 1 right when we taking the movement is a first order movement the auto correlation function the various function in a second order function, second order movement function. And many time it is sufficient to work with the first movements right they contain most to the physical information's we are generally interested in in looking other processes right. So, therefore, introduce the concept of strict sense stationary process a process which is stationary up to second order or third order could be a special case of there. But wide sensational process is something which is all together much more tolerate of non stationary, process maybe non stationary ((Refer Time: 22:10)) stationary in the wide sense right.

So, what your ((Refer Time: 22:17)) is that if a process is strict sense stationary this stands for strict sense stationary it could; obviously, implied that it is wide sense stationary is not it? That obvious should be obvious I think this could imply WSS property the ((Refer Time: 22:36)) will happen only very very special cases. WSS could not in general imply strict sense stationary right because this is independence only in terms of movements there also if the first thing first 2 orders right. And therefore, we cannot say that all density functions or the complete statistical characterization could be independent of time right. But in very very special cases this could imply this for example, for a class of association which we call casion processors right that is very very special.

To clear example of a process which could be which is not strict sensationary and get it is wide sense stationary right I will just give you one simple example. I leave a ((Refer Time: 23:33)) file to work out yourself ((Refer Time: 23:35)) we can do that quickly where itself suppose, I generate a Raymond process in a following bit. The many reasons of ((Refer Time: 23:41)) n m process right that we define a random process which essentially we can random by virtual of its dependents on it is a finite function of time right. It is a finite sense function of time, but A that 3 parameters in the cosine functions the amplitude A is if you ((Refer Time: 24:09)) and phase theta right. Let me look to one of them that it the phase random way that is say the value of theta is something that of course, randomly from 1 function to another function.

If you look upon this as an example of functions right all every function in this collection should be cosine function. But with different phase and the value of phase is govern by some distribution right. So, as much as you have a collection of function it is a random process is it clear and that every function occurs with a certain probability right. So, becomes random if any one move of its parameters random variable. So, for this becomes random because we assuming the theta is random variable let us say with uniform distribution. That is why if I say its uniformly distributed between 0 and 2 pi there could be the density function of theta with B equal to 1 by 2 pi 0 0 and 2 pi and 0 elsewhere. This will imply this is the density function of the random variable theta we have could be.

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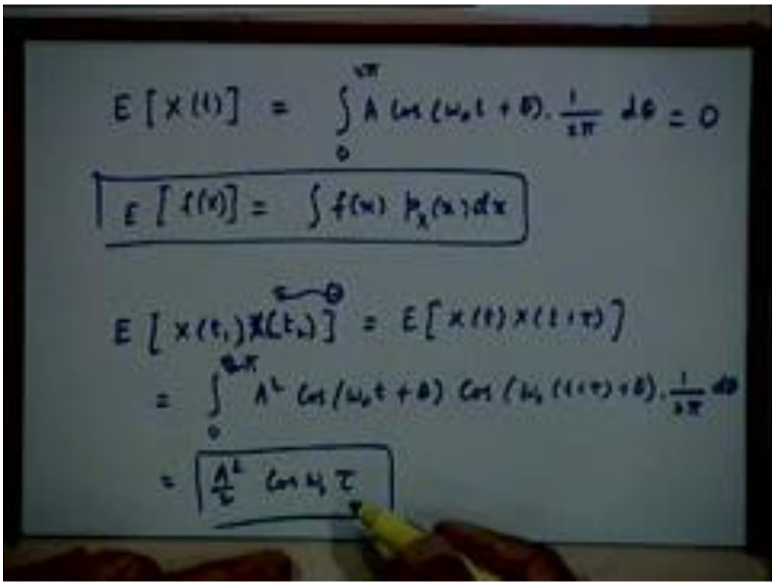

Next look at the mean value of this next we look at the mean value function we want to see whether this function is a function of time or not right. Mind you we are not looking at density function of x t here right now. But sometimes we can work around that even the doubt completely the base density function of x t you can compute. So, this is the important point we can compute this quantity even without move density function of x of t right, because we know how to compute the expected value of or function of a random variable. So, you can think of you can think of this constant as a function of random variable theta and complete this every value on that process. So, will be at the it will be A cosine omega a t plus theta you multiply with the density function of theta right think of this as a function of theta is entire thing at any given time right. Multiplied to the density function of theta which is 1 by 2 pi or 0 to 2 pi d theta is there. Your function of theta multiplying the density function of theta integrating over the range of theta or the d 1 of theta right that is

That is one way of computing the expecting this ((Refer Time: 27:20)) and using the ((Refer Time: 27:24)) let me recollect file the discussed. Expected value of any function of x requires simply do this a portion is not it? I am using this solution here. X here is our theta variable theta; f of x is this function right. We are multiplying the density function of x or density function of theta the integration of theta. Now, look upon what will be the value of this integral? 0 right. So, it is independent of time right. So, value of this is equal to 0. Just look at the auto correlation function just look at x t 1 into x t 2 or lets say more conveniently. Just write this as x t into x t plus tau right p 1 to be some arbitrary time is complete and t 2 T plus tau for the difference between ls equal to tau right. So, this will be equal to 0 to 2 pi again now the function is different that is how? The function is now this kind of function everything else with the same. So, become A square cosine omega t plus theta into cosine omega 0 t plus tau plus theta multiply this with the joint if the basic function of theta right integrate theta right. And a simple evolution of this intergral we will show you that this is equal to A square by 2 into cosine omega 0 tau.

Student: ((Refer Time: 29:39)).

No no no no ((Refer Time: 29:42)) joint density function I am just looking at this product as a function of theta I am again using the same definition here, because I am not going through the path of first joint density function of x t 1 and x t 2 That could be 1 way. That is very complicated in this phase it is not necessary. In this case mush easier to look upon this simply as a function of random variable theta this product function right multiply the basic function of theta and into our theta. So, you can see that after this integration it is not going to depend on t it depends only tau right.

Student: ((Refer Time: 30:35))

Please repeat your question. No no it has do dependent time theta is a parameter of this function right. Depending on the value of theta you have a different time function this different time function are all cosine a same frequency in same ((Refer Time: 30:52)) but ((Refer Time: 30:54)) different phase right.

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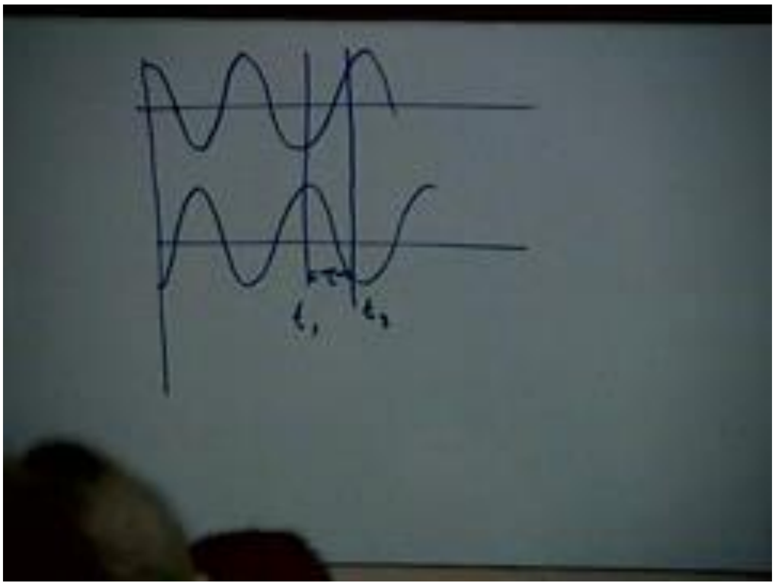

So, therefore, basically this collection if you are looking at is a collection like this and so on and so forth. It is a infinite collection you can define a fairly different value of theta right. So, ((Refer Time: 31:24)) this shape of article this shape of article this shape of right. The way document the mew value function yeah say if I pick up some times is empty what is the average value that I am likely to see here across the example it also is 0. If I pick up 2 Time instance t 1 and t 2 which are separated by time instance separate by some interval at all. What is the average cos ((Refer Time: 31:49)). It is these 2 random variables answer is cosine variable ((Refer Time: 31:53)) top it does not depend on the class of t 1 and t 2 This is what you have demonstrate right is it clear? So, here is an example of a process which is; obviously, not strict sense stationary is it obvious it is not strict sense stationary. It is not obvious I am assuming that it obvious you think about it. It is not obvious, but it is possible to argue very easily to see that.

Let us looks at let us say 1 time instance like this. I think it look at little bit of thinking I think like linear of the timing, because we are going to ((Refer Time: 32:45)) a square a bit. It is possible to argue the density function is not constant with I right. You will a first add density function is not constant this specific time actually it is one of the problems in the book. Please look at the problem with very carefully and you will arrive this argument. But even though the density function itself is a function of time the first elements then will the mean value function expected value of x of t and the auto correlation function. Set if pi the required properties of a wide sense stationary process there is not a strict stationary process and we are taken a wide sense stationary process. It is. So, only you need to wide pi rates not a strict sense stationary process. So, please do the as an exercise. Now, what are let us take talk about what we are discussed so far.

We say that if we are working with random form random process we can characterize it for all technical processes as ((Refer Time: 33:50)) engineers most of the time it is sufficient for is to characterize it with 2 kinds of functions. The mean value function that gives us an idea of what is the average behavior of the time functions. What is the average behavior the various time functions which constitute the process right that is that is 1 property. The second is the auto correlation function which tells us if I look at 2 random variables which are ((Refer Time: 34:22)) which other bian interface tau seconds what will be the obvious value of the cross correlation of the 2 random variables right. So, rather than trying to specify the density function and the joint density function which has much more information many times as engineers. We are sufficient you are sufficiently happy with these 2 information's namely the mean value function property and the auto correlation function property right. In some cases we need to going to more detail, but many times this is good enough.

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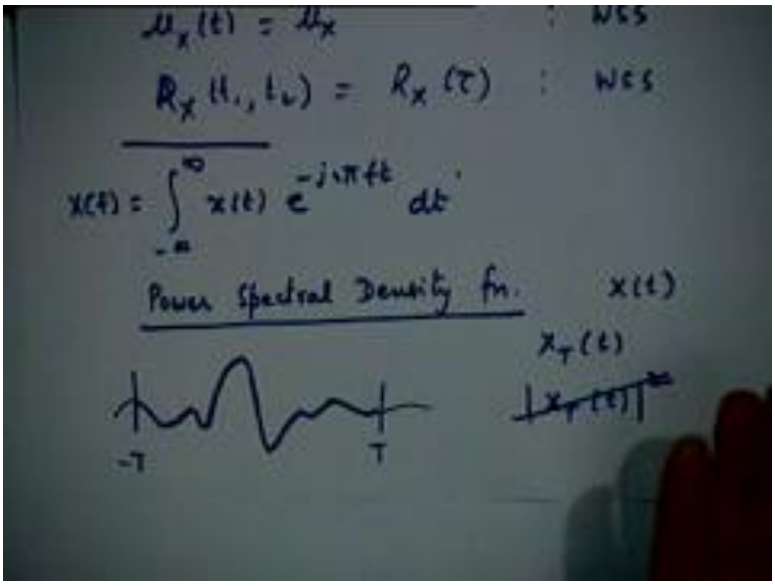

So, all purposes the function may mew x t which is going to be a constant function for a wide sensation process. And the function R x t 1 t 2 which is going to be the function only of tau for wide sensation processors is good enough us. You do not going to worry about the density functions in many many cases right. Now, as electrical engineers we are used to describing things in the time delay as well as in the ((Refer Time: 35:33)) right. When we talked about the time delay function immediately ask ourselves what its specters domain description? What is it spectrum live is not it? I you have worked out the elaborate theory for bring that in the ((Refer Time: 35:47)) transcription. You could have similar interest here you have occur whether the signal is deterministic signal; however, is a random signal. You can ask some properties post in the time domain as well as in the frequency domain right. So, we like to also see whether it is possible to characterize a random process in a frequency domain.

Now, let us see look me just discuss a few basic concepts or conceptual difficulties associated with this and then just give you the important results in this connection. Let us look at the difficulty just say you are defining a random process as a collection of waveforms right that is from you are looking at. Now, when we take a tele transform, what is the tele transform? Tele transform is ((Refer Time: 36:37)) let us say you have function x of t. And you multiplied e to power minus j 2 pi ft and take the intervals from minus infinitive to infinitive that this is the ((Refer Time: 36:54)) that is the initial 1 thing that you work with what are the assumptions in this? The assumption x of t is a energy signal right it is a its absolutely integral right. Remember the base shape condition specify with the definition of a whether the system ((Refer Time: 37:16)) transform associated with a system approach transform.

So, what are the requirements are that x t should be absolutely integrable that way we should be a ((Refer Time: 37:23)) power signal hen you are working a random processors. We do not know where we are be working a energy signal of process that is one issue one difficulty with need to work with right. A functions maybe power signals your function maybe energy signals. But that is the problem that we are also direct with the in the context of deterministic signals and we typically better around that by introducing in ((Refer Time: 37:47)) function in the frequency domain right if you recollect that is one problem. The second problem is more difficult more conceptually more difficult. Except you have a very well defined waveform very well defined mathematical function of time right. It is very transform you are taking for example, e to the power minus alpha t or cosine omega 0 t right. But the collection when we talk about x of t be a random process you need not know what waveform you are working with actually.

It is one of infinite number waveforms an every one of them ((Refer Time: 38:19)) transform what we exist for that waveform you have a different transform and therefore, you could have a different kind of spectrum. So, what is we talk about does it make sense to talk about ((Refer Time: 38:36)) transform or the spectrum in the normal sense. Are you ((Refer Time: 38:38))? No it does not make sense right. However, what that make senses on an average where is a energy distribution as a function of frequency? What is the power distribution as a function of frequency? Depending on whether we are working a energy signals of approximate right. So, important frequency domain concept for random processors is not the usual spectral which is just the, which is just a ((Refer Time: 39:02)) transform of a function, but a average kind of function which is known that the layer of power spectral density function. First just define this process density function conceptually right let us say we have random process x of t.

So, what we will do is to convert this into an energy ((Refer Time: 39:43)) alpha can takes this process. So, lets ((Refer Time: 39:49)) random waveform just look at this waveform between that is a minus 1 to plus t right. This ((Refer Time: 40:00)) waveform ((Refer Time: 40:03)) one same sample function in the process ((Refer Time: 40:05)) with here one sample function the process. As you know random process is a infinite collection of such sample functions right its some arbitrary selected sample function from that collection. So, you pick up 1 and truncated between minus t to plus t you note the result in process x sub T t. So, x sub T t has been generated from x t by looking at its interval between minus t to plus t making it 0 outside this interval. This artificial construction ensures that high converted even a power signal into an energy signal right is it clear?

Because now clarifying that energy and for the peri transformer this could be define again sense again this is your random quantity I do not want to take the full transform of this right where I am tested in what is first of all I scored this function. Because of I do not interested in the power energy I am not interested in the individual functions values by themselves right. Squaring is measure of the energy at various time instance right is of course, not literally, but some approximations. ((Refer Time: 41:20)) is no that thing in beginning long ((Refer Time: 41:28)). So, 1 let me define a free transform of X T all here using the capital letter please remember I have pick a 1 sample function right. I pick a one sample function and that sample function is low energy signal I am taking pay transfer right. This pay transfer what exist now whatever the function maybe whichever ((Refer Time: 41:57)) it does not matter. Because I have converted this into energy signal the pay transform will exist all.

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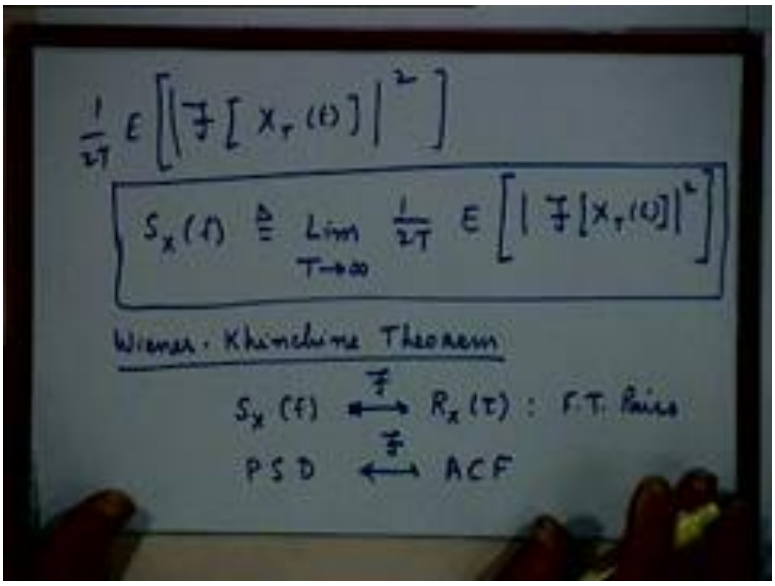

So, I take the pay transfer. This will now become a function of frequency right take the magnesium square of that right. Now, this will be the magnesium square or the energy as a function of frequency of one sample function ((Refer Time: 42:27)). If I want to look at the average value average properties what shall I do here is the average value of this right. And if I want to convert this energy function this a energy function is not it?. Into a part function what shall I do? Do varied the 2 T, because of my function duration is 2 T and if I want look at the original function take the limit us theta theta as infinitive. So, this motivates my definition of the parse with density function of a random process S x f. I denote it by S sub x f as limit as T tends to infinity of 1 by 2 T expected value of magnitudes square of the free transform of X T t. Look at this carefully I have gone through the argument reading to this that if you have a doubt please ask your questions.

That is a formal definition of the ((Refer Time: 43:46)) density function. It is a measure of the average distribution of power for given random process X t in the ((Refer Time: 43:58)) right. How is the power distributed among difference frequency components in the frequency in the net? Are you agree, you have any questions? So, that is you can take that as a definition. Now, without going through the details of a result I just like to give a result which is should no off which is a very important result in the characterization of NM processors. It relates the density function the ((Refer Time: 44:42)) density function when we talk about density function as a process you must ask the where you talking primitive density function we are talking about ((Refer Time: 44:52)) density function. ((Refer Time: 44:55)) density function is a it is what so, say it is a ((Refer Time: 45:00)) once again. Do you agree with that?

Because you are taking a random process in this I just not enough scoring in that in the time domain ((Refer Time: 45:10)) in the ((Refer Time: 45:14)) limit. Now, we are any way squaring it up right and you taking the away value of the x squared in a function of frequency right. So, it is a second order movement just like the auto correlation function was the second order movement right. So, if you look at this way it must be natural to expect the least 2 second order movements must be some more related is not it? We are saying in the auto correlation function is a second order movement description of the random process x t. And now we are saying similarly that the less function at the just define the ((Refer Time: 45:52)) function is also a second order movement characterization in some cases only thing this is the frequency ((Refer Time: 46:00)) characterization that was ((Refer Time: 46:03)) characterization.

Therefore, it sounds logic error the, these 2 second order movement description should be related to each other right. And that is the important result that I am talking about there is a very simple way to not exactly very simple. But it possible to show that these 2 Things the second order element description in time delay which is auto correlation function. And the second order movement remaining in the frequency domain which is the ((Refer Time: 46:25)) density function or essentially free transform phase right. So, which is your result to very similar to what you are used to doing for deterministic signals what you thing is where reject the free transform of the signal directly where we taking the free transform of the auto correlation function right.

This result is note the name Wiener Khinchin theorem which essentially states that the ((Refer Time: 47:08)) density function of a process x t and the auto correlation function of a process x t or the auto correlation function of a process x t or previous ((Refer Time: 47:22)) one determines the other. So, p s t we put here like that in auto correlation function of free transfers. And therefore, in as much as the mean value function and the auto correlation function are complete second order characterization of a random process. Similarly, the mean value function that the ((Refer Time: 47:55)) density function of complete second order characterization of the random process. Let me now finally, define one more concept and then next time we will now will essentially concentrate on concept that we are specifically go to need in our treatment of the communication systems.

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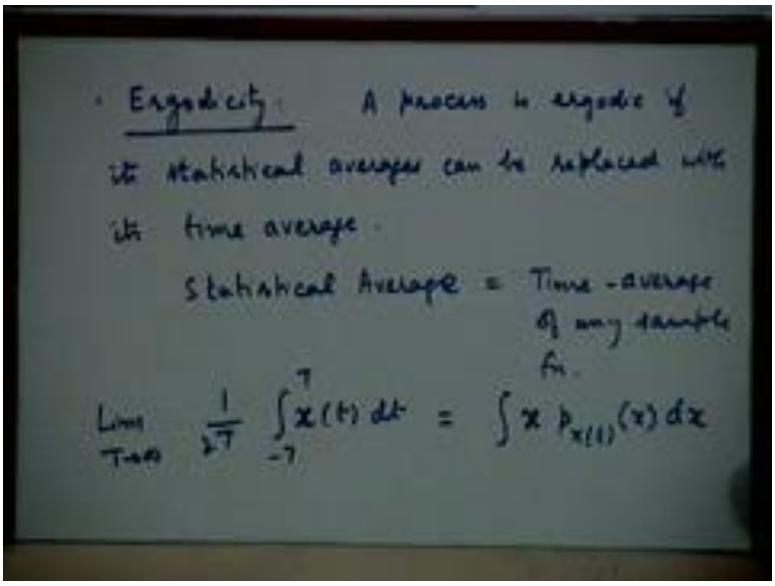

The final concept is the concept of ergodicity. I will just mention it here will have to ((Refer Time: 48:30)) discussion of this concept, but we need to know it right a twice. The concept is as follows we say that the process that I need a random process is ((Refer Time: 48:45)) if it is statistical average is or equal to or can be replaced with its time averages. That is statistical averaging of any ((Refer Time: 49:30)) is equal to the corresponding time average of a given any any sample function. Now, this is the very peculiar concept that the very useful concept because without this concept you know really we particulate we have to work with random process. Particularly when comes to measurement of what properties of random process just look at the very quickly a modification for this property before talk about the property itself.

Motivation is as follows I was said our concept of random process is limit of how you recovery basically it is a collection of infinite number of waveforms. And you not know which waveform we are going to see suppose ((Refer Time: 50:19)) perform the experiment are you look at the random processes as it as it occurs and display waveform that you see ((Refer Time: 50:27)) some arbitrary waveform which we cannot predict right. Now, anytime in typical situations I will see just one such waveform from this infinite collection. So, for what ((Refer Time: 50:38)) measures properties average properties let us us say if you want to measure the mean value. So, ((Refer Time: 50:47)) measure the auto correlation function right suppose I do not know a ((Refer Time: 50:25)) density functions.

How will I have find out this qualities? right very difficult, because I have only 1 sample function available in front of t right. I have just 1 sample function which I have observed I do not know what are the other infinite what are the other members of the this infinite family. In the other hand this property if it is valid for a given random process helps to with find this properties just from a single sample function that we might have observed. So, what we are saying is even if I do not know if I even if I ((Refer Time: 51:20)) average they cross the whole family because I done have the whole family with we. If I just look at the average across the time of one sample function of the family it is good enough I get the same value right. So, processes with exhibit such properties are called algolic process. Fortunately for us many physical processes which we work with I got it in nature right.

And therefore, many times we can obtain this the auto correlation function by looking at the time mean auto correlation function of a given sample function or the mean value function by just looking at the mean value of a given time to mean function. Just like you do for any time ((Refer Time: 52:00)). So, what is a time mean average are you simply this right take the limit as the T junction infinite that the time mean average. This does not involved the density function of x is taking a sample function right and integrating minus T to t that is the sum of all the values that you see between minus infinitive the vary for the time interval that is the time mean average. What is the correspondent statistical average? It will be x Px t x dx that is the correspondent statistical average. So, what you are saying is this is equal to this similarly for the auto correlation function for any other kind of average. If this kind of relation hold for all time averages corresponding statistical averages the process is continue step by step I ((Refer Time: 53:11)).

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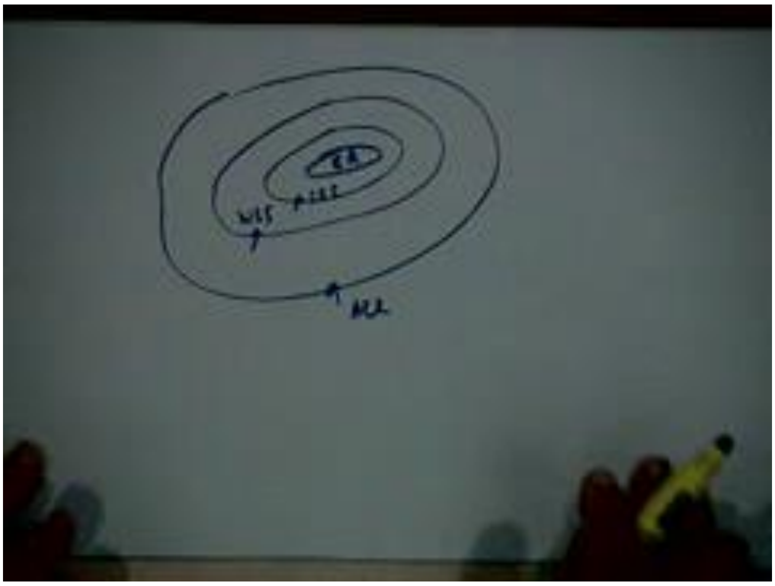

Final statement in this connection I know time this one final statement, because overall this continuous. If I denote by this space as a clause of all stationary clause of all random processes right clause of white sensationary processes. So, this is all processes clause of white sentationary process is a sub clause its satisfy those 2 conditions which I mentioned linearly function is independent of time auto correlation function is dependent only on time difference. Strict sense stationary is a further sub set of that right and ((Refer Time: 54:00)) it is a smallest sub set that just I want to it make and complete this discussion. So, if a process I go to it will also be strict sentationary it will also be wide sentationary etcetera.

Thank you.