Communication Engineering
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Lecture - 3
Brief Review of Signals and Systems

My subject for today's discussion with you is a very brief review of signals and systems, because I believe there is some of you, who might have slightly lost touch with the subject, because there is a gap of about a semester or a year for them. So, I just want everybody to be, on equal terms here and this is important for all of you, that you have a very good solid background in signals and systems, as I mentioned earlier also in the discussions.

Now, before I do that, let me complete, the discussion we were having last time, about communication channels. If you remember, I discussed a variety of communication channels with you, particularly channels, which are relevant to us. Then we electrical communication channels, by particularly one of these channels, is some kind of an electromagnetic wave channel. It can be a free space propagation channel like we have in their atmosphere or in free space, or it could be in guided electromagnetic wave channel.

The examples of guided electromagnetic wave channels are the pair of wires, through which you may communicate, let us say as we do from a home to telephone exchange. Or it can be ((Refer Time: 02:30)) axial cable, which can carry much larger number of signals, than a pair of wires can and because of the larger bandwidth of it can support, or it could be an optical fiber, which can support an even larger bandwidth and here, there is a small comment. I would like to make and ask all of you to think about it.

Typical, as you go up in the carrier frequency or center frequency of operation, at a communication channel, if you are able to do so, it increases your ability to take up larger bandwidth, it supports larger bandwidth, why is it so, think about it and we will discuss it sometime all. For example, why did we use optical communications, why did we use fiber optic communications, because it has a capacity to offer, you much larger bandwidth left.

Let us say, microwave communications can do it or even larger than a coaxial cable, can support and so on and so forth, so it is an issue, I like you to think about, before I respond to this. Now, come back, let us come back to our subject of interest today, which is a very brief overview of signals and systems.

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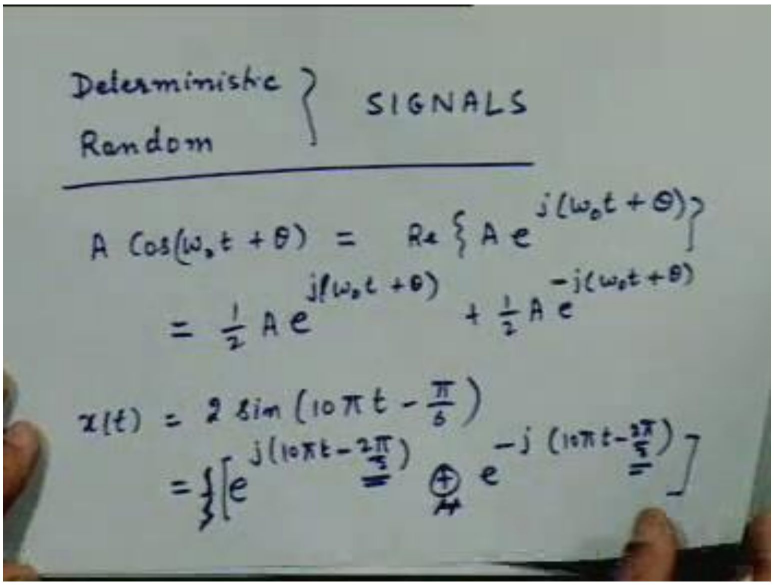

Now, to start with, let me start with the very beginning and if you remember, you must have dealt with a representation of signals, in terms of in a first starting with the most basic signals. Well to start with, if you remember we can classify signals into two kinds, namely deterministic and random signals, so you are talking about signals now. And in communications we deal with both kinds of signals, because deterministic signals are useful to us, as for example as careers.

We use a sine wave as a career, of a certain frequency and deterministic signals are also useful in, generating synthetic signals of various kinds, but random signals are absolutely, characteristic of communications. Why, because most of the time, then information that is of interest to you, is typically unpredictable in it is form, and therefore, it is random. You want the speech wave form, that is coming out of a person's speech voice is an unpredictable waveform to a large extent.

To some extent it is predictable, but to a large extent it is quite unpredictable and therefore, it is quite random. Similarly, in the noise that you encounter in a communication system will only be modeled as a random signal, now right now, I will not give you the random signals part, I will only look at the deterministic signals. The most basic kind of deterministic signals, which you come across, are complex explanations and sinusoidal signals, through a sinusoidal sequence.

So, if you take a real sinusoid, like a, very simple signals like this, we know some of it is features, but as you must have already learnt, that you can express a real sinusoidal sinusoid, in terms of complex exponentials. It is much easier to work with complex exponentials in many instances, than in the corresponding trigonometric function, so you take the complex exponential. As a ((Refer Time: 06:15)) phasor and the cosine signal can be thought of as a resultant of two ((Refer Time: 06:15)) phasors, which are moving in opposite directions, one in a clockwise direction and other in anti clockwise direction.

So, you can think of this as real part of, let us say this complex exponential, this is a very basic step, but it is useful to review this, because sometimes, people forget the basic step and they have difficulties later. So, this is the ((Refer Time: 06:54)) that, a phasor that is rotating in the clockwise direction, if it is in the clockwise then, this is the sum of these two phasors. The other one is rotating in the anti clockwise direction as indicated by this negative sign in the exponent.

Similarly, if you have, let us say a signal like this, this is just an example, you can express it, if you were to carry out some simple trigonometric manipulations, as e to the power j 10 pi t, minus 2pi by 3, plus e to the power minus j 10 pi t, minus 2 pi by 3. So, whether you are working in a cosine wave or sine wave, you can think of it, as sum of phasors and phasor is a complex exponential, rotating either clockwise or anti clockwise, and this term in the phasor indicates the phase, initial phase of the phasor.

You have any questions, have I made any mistake?

Student: ((Refer Time: 08:22))

It should have a minus, yes correct.

Student: ((Refer Time: 08:34))

That is right

Student: ((Refer Time: 08:37))

So, 1 by j

Student: ((Refer Time: 08:43))

This is 2pi by 3

Student: ((Refer Time: 08:50))

Currently this is

Student: ((Refer Time: 08:57))

Actually, I am sorry this is correct, there is no mistake, you please, check it up, please check it up.

Student: ((Refer Time: 09:06))

What I have written is absolutely correct, do not confuse me, you have to first, the way I have done is already starts ((Refer Time: 09:15)) in terms of the sine and then ((Refer Time: 09:17)) in both steps, please check it up, this is just an example. I do make mistakes at times and I like you to correct me, when I make mistakes, if I do not make mistakes, do not make me make mistakes, it is all right.

Now, having said, the sinusoids are some of the most basic kinds signals in terms of which you work, why they are, we consider them as basic. First, because they are very easy to generate for example, a typical oscillator that you might have used in your lab or you might have learnt about, in your analog electronic circuits or likely to learn soon, naturally generates a sine wave, oscillations are in the form of sinusoidal oscillations, that is one reason.

The other reason is many other deterministic signals, can be represented in terms of sine waves, for example, Fourier series is all about, which I will reveal in a few minutes. Now, just like sine waves, there is another set of basic signals, which are quite different in nature, sine waves are have infinite duration signals, extending from minus infinity to plus infinity, they are periodic signals.

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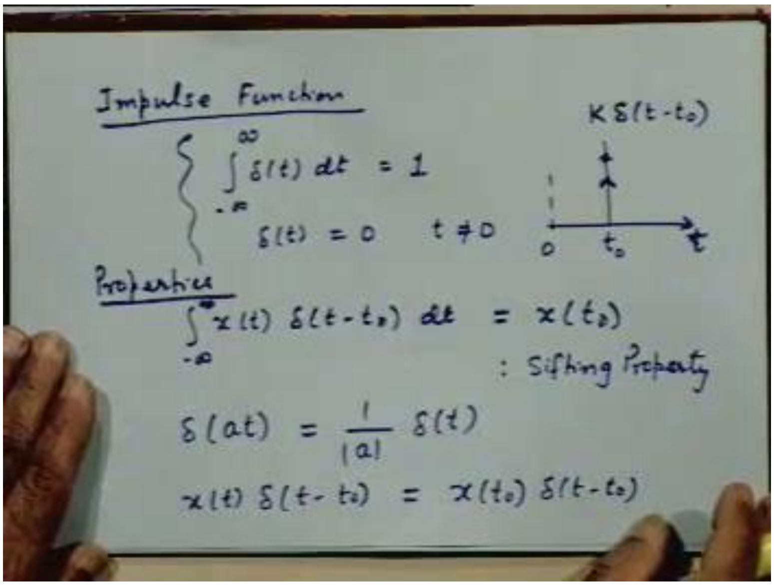

As against this, you can have a very different variety of basic signals, which are very useful in the concepts of signals and some communications signals and systems. And, one of those basic kinds of signals is the impulse function, I am sure all of you know, very well about it and let us quickly read, what is the definition. There are two ways of defining the impulse function, several ways, not just two ways, there are several ways, because this is a review. I will not go into a, very detailed description of all those various methods.

All we say is, that in an impulse ((Refer Time: 11:12)) t is equal to 0 is denoted by delta t and is defined to be a function, which encloses a unit area under it, and it is 0 ((Refer Time: 11:23)) other than t equal to 0, so this is a definition of an impulse function. This is one definition of a number of possible definitions of an impulse function, so some people sometimes, write by mistake delta t equal to 1 equal to 0, that is wrong, delta t at t equal to 0 is undefined, it is actually infinity.

But, the area under this is undefined and to indicate that effect, the usual notation for an impulse function is something like this. This is the time axis, suppose the impulse is located at t naught, then we will write this as. So this is your time ((Refer Time: 12:10)), this will be the impulse ((Refer Time: 12:12)) minus and this arrow here, indicates that the amplitude of the impulse is infinity or undefined, and the area under it is 1, so this is k time delta t minus t naught.

Everything will be the same, except that the area of the impulse is now k, or the strength of the impulse is k, that is the usual language of this ((Refer Time: 12:38)). Now, some of the very important properties are in impulse function are, if you have a time function ((Refer Time: 12:47)). I will just quickly review the basic concepts and discuss the basic properties. I will not discuss any proofs here, because the idea is to make you recollect these things, more than anything else.

If you do not understand something of this, I would suggest that is ((Refer Time: 13:07)), what should look for when you are reviewing your signals, and systems where you quickly go through the proofs and things like that. So, if you look at, this integral of x t multiplied with delta t minus t naught, minus infinity to infinity d t, what is the value,

Student: ((Refer Time: 09:15))

x t naught very important property also called the sifting property of a impulse function, all. Sometimes, this is also taken as a definition of impulse function, so it is a matter of starting from here and proving this or the other way round. So the other properties are, if you look at, if you scale the time axis by a, then delta a t would be, 1 by mod a of delta t and so on and so forth. One last property, that I would like to discuss is ((Refer Time: 14:19)) I consider this, this is the product of x t into delta t minus t naught, you can also write, this as x t naught into delta t minus t naught, all.

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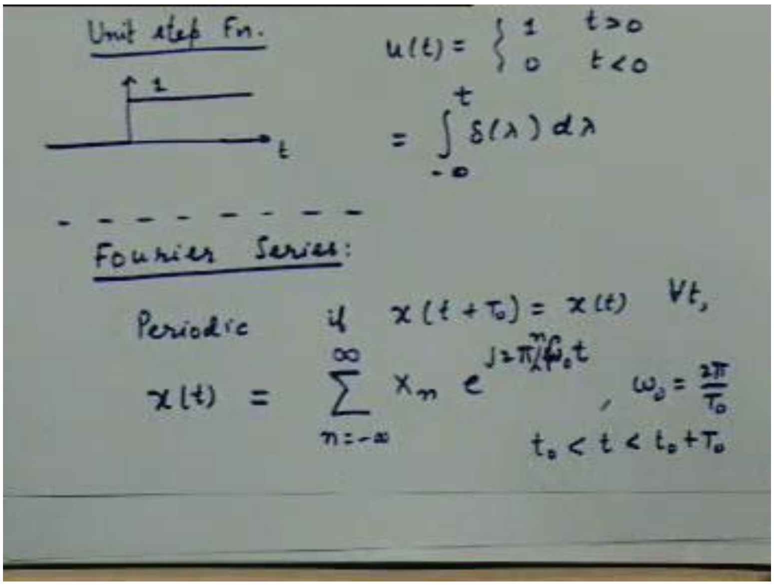

Now, I need a basic signal, which is very closely related to the impulse function signal is the unit step function. Pictorially, the unit step function is a function like this it is value is 1, for t greater than 0, so that is the definition, and 0 for t less than 0, or it can also be defined in terms of, an integral with respect to the impulse function, integration will be from minus infinity to t. As you can see, this would be equal to this, so that is as far as basic signals go.

One set of basic signals are your complex exponentials, trigonometric sinusoids of various frequencies, then we had the impulse function and the unit step function, and there are several functions, which one can derive from these basic functions. We quickly now look at Fourier series, because I am trying to, collect practically all important concepts of interest to us in this course, into this lecture, so we will quickly review all of these things.

Before, we talk about Fourier series you remember that Fourier series will be fine for periodic signals, so we are going to talk about periodic signals. You say that, a signal is periodic, with period t naught if x of t plus t naught is equal to x of t, for all values of t and for all values of t, and for given t naught. Now, if it is periodic, then we can represent the signal x of t, in terms of exponentials of period p pi by omega naught and it is, it is multiples, actually frequencies of omega naught and hence multiples.

So, we can represent it in terms of, some coefficients x of n e power j 2 pi omega 0 t, where omega 0 is 2 pi by, is there a mistake here, it should be 2 pi n omega 0 t, n going from,

Student: ((Refer Time: 17:58))

I am sorry, it should be either f 2 pi f naught t or ((Refer Time: 18:06)).

So, that will contain up to 2 pi n f naught t, where f naught is 1 by 2 pi of omega naught. And this representation is valid over a period of, so if you take any period, between t 0 to t 0 plus capital t sub 0, t sub 0 to t sub 0 to plus capital T sub 0, this representation is valid, it is valid for any period of function x of t.

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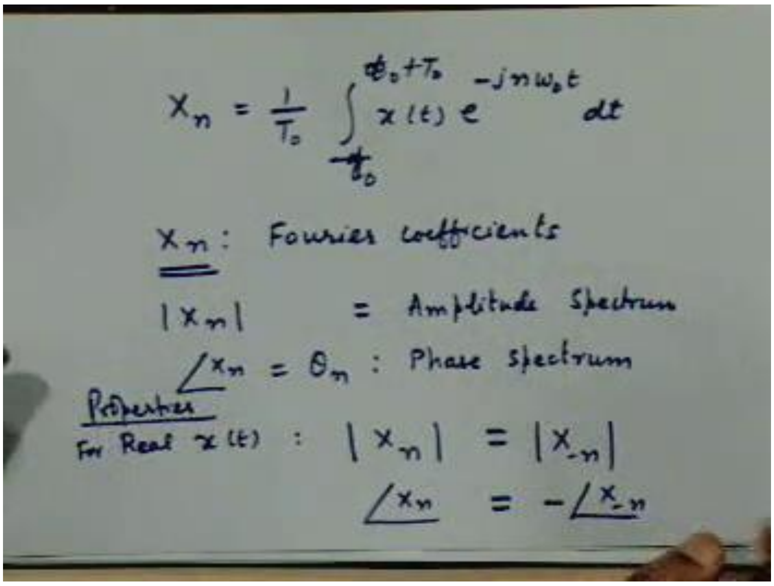

The coefficients x n in this Fourier series are given by, integral of x t e to the power minus j n omega naught t, n this is what you wanted to tell me to write. So, this should be, ((Refer Time: 19:08))t 0 to t 0 plus capital T 0, and these coefficients, we call the Fourier coefficients of the signal x of t. So, here we have a representation of periodic signal x of t, in terms of the set of, infinite an infinite set of coefficients, and going from minus infinity to plus infinity, this is what we call complex form of the Fourier series, we also have, real forms which use, trigonometric functions cosine n omega naught t and sine n omega naught t.

So, these Fourier coefficients represent the signal, in what we call the frequency domain, you can say x of n is a coefficient, which has an amplitude, which you can call mod of x of n, and which has some phase, which you can call has, an angle of x of n or call it theta n if you like. These amplitudes and this phase information, represents the amplitude and phase of the n the component in the Fourier series, the nth component being, e to the power jn omega naught t

Now, and therefore, we call this as the amplitude spectrum of the signal, when we plot x of n against n; n is an integer here, going from minus infinity to plus infinity. So, plot x of n against n, it will be discrete plot, the plot will be in the form of lines, it will be line spectrum in this case. So, if you plot mod of x of m against n that gives me the amplitude spectrum of the signal, if I plot theta of n against n that gives me the phase spectrum of the signal.

Now, for a real signal x of t, we are looking at some properties of the Fourier series now, for real x t, what can you say about x of n. In particular, if I ask you, the relationship between x of n mod, and x of minus n of mod, what will be the relationship, they are equal. Similarly we look at the angle of x of n, and the angle of x of minus n, what is their relationship.

Student: ((Refer Time: 29:09))

They are negative of each other, we see that the amplitude spectrum is an ((Refer Time: 22:16)) symmetric function and the phase spectrum is a non symmetric function, not for all kinds of signals, but for only for real signals real value signals.

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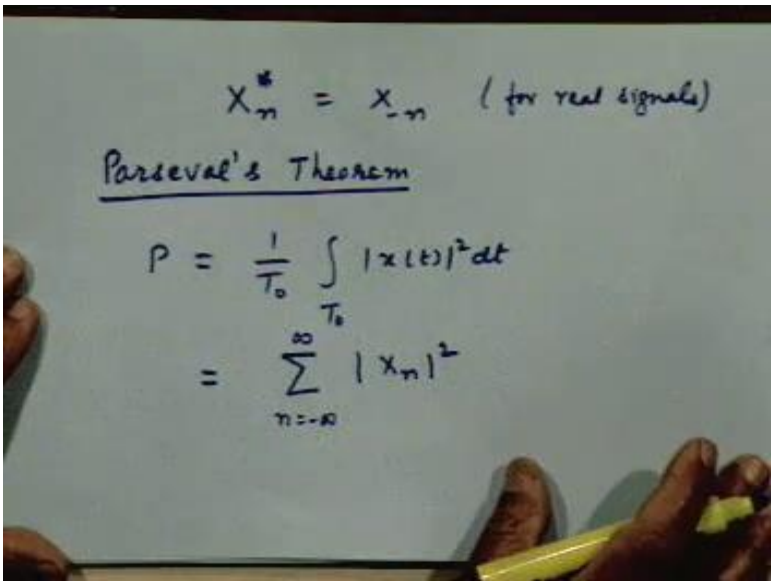

This can be neatly summarized in the form of a single equation saying that x of n conjugate is equal to x of minus n, for a real sequence. Well, the second most important there are many properties of Fourier series, but since they are very similar to the properties of Fourier transform, which I am coming to shortly, I will not discuss all of them, they are very they are parallel to each other. So, if you know, some of the properties of Fourier transform, you also know the corresponding properties of the Fourier series.

But, one property, which I like to discuss here, very briefly is the Parseval's theorem, you remember that, it is a very important result, because it relates the energy calculation of the signal in the time domain, energy at power. In this case, should you talk about power or energy it will depend on, what kind of signal we are dealing with, power signal or energy signal, a power signal is one, in which it makes more sense, to calculate power, because it has a finite power.

For example, a sinusoid has infinite duration, what is it is energy infinity, so it does not have a finite power, it does not make sense, it does not have finite energy, we do not call it an energy signal, we call it a power signal. On the other hand, a signal which is a, which has a finite energy, typically will have 0 power, because power is defined in terms of, energy divided by time, as time tends to infinity, time interval tends to infinity. So if the energy is finite, when time interval tends to infinity, the average power would be 0.

So, it makes more sense there, to talk about energy rather than power, so for sinusoids for periodic signals in general, there have to be necessarily of infinite duration. It is strictly periodic signal, has to be of infinite duration and therefore, it is better described in terms of power, and in terms of energy. So, here we talk about power therefore, which is the power calculation here would be, look at the integral of x t mod square d t ((Refer Time: 24:53)) t 0, that is the average for and the, for t 0.

This, the Pareval's theorem, can be obtained by simply summing the magnitude squares, of all the Fourier coefficients, very interesting result. The sum of the magnitude square of all the coefficients that, gives you the average power of the signal.

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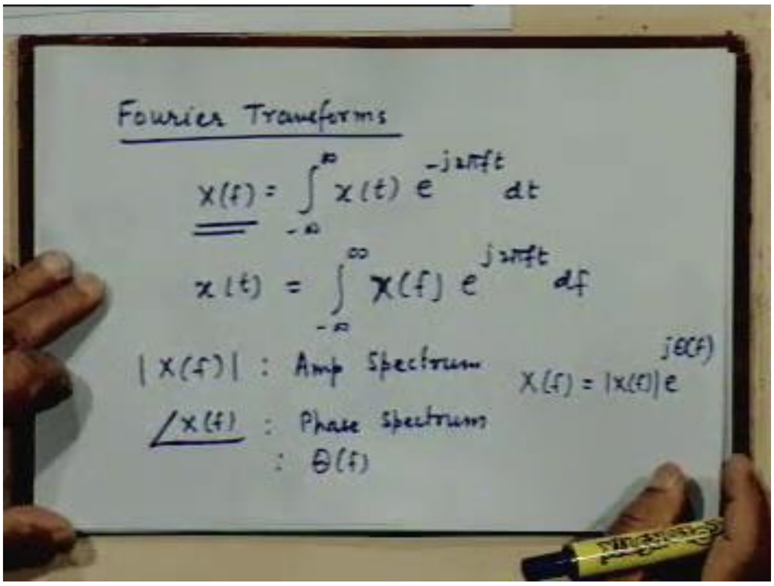

So, now we quickly come to Fourier transforms, all of the Fourier series, which is defined only for periodic signals, Fourier transforms are defined for any signal. Strictly speaking a periodic signals, but through some clever mechanism, it can also be made to represent periodic signals, particularly by the use of impulse functions in a frequency domain, you must you will be remembering that. So, if you have a signal x of t, which is no longer necessarily periodic, then I can define, it is Fourier transform x of f, as integral of x t, e power minus j 2 pi f t d t.

If you look at this, this will be it is very closely related to the expression we have for x of n. In fact, you can derive this expression, through a limiting procedure on the Fourier coefficients, where what is the limiting procedure like. You remember, you make the period, first start with the finite period in the periodic signal and when you meet the period infinity, and we have to look for a transfer, and the important thing is the line spectrum, of the original periodic signal, now becomes a continuous spectrum, continuous in the ((Refer Time: 27:01)).

So, x of f is a frequency domain description of x of t, and one can reconstruct x of t, from x of f, by taking what is called inverse Fourier transform, which will be x of t, x of f e to the power plus j 2 pi f td f. If you write it in terms of omega and d omega, then you have to put a 1 by 2 pi not otherwise, so once again, x of f is called the amplitude spectrum and the angle of x of f, as a function of frequency is the phase spectrum of the signal.

So, you could write x of f, if I call this theta of f, then x of f, can be thought of as mod of x of f, in e power j theta f.

Now, there are certain conditions, which the signal x of t must satisfy, in order that the Fourier transform exists, I simply name the conditions, these are called the ((Refer Time: 28:38)) conditions. There are three of them in number and most important that these three conditions is a requirement that the signal be of finite energy, that is the ((Refer Time: 28:50)) integrable, there should be a magnitude square integrable function of time, so please review these conditions yourself.

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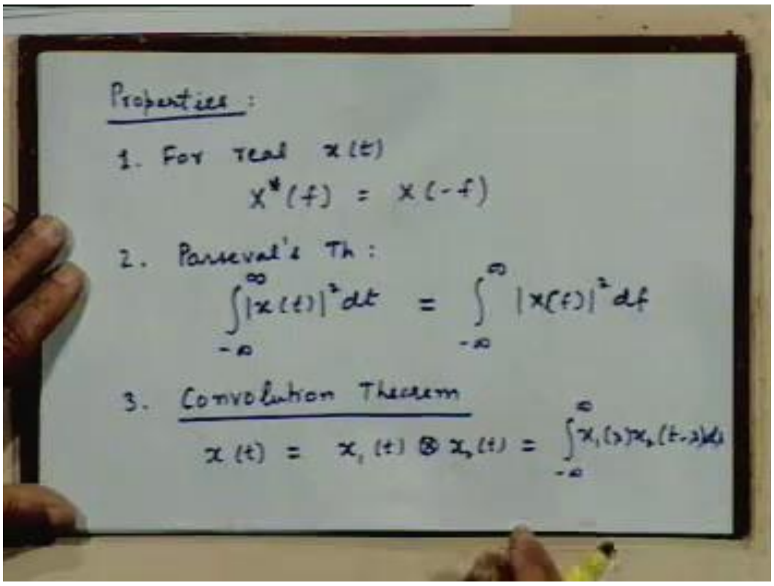

Now, a quick review of the properties, the most important properties of the Fourier transform. Some of the properties are the same that we already discussed, in the context of Fourier series, so for a real signal x of t, x conjugate f, would be equal to x of minus f, which is again equivalent to saying, that the magnitude spectrum is even symmetric and the phase spectrum is an odd symmetric, function of the frequency for real valued signals x of t. So, we need not spend too much more time on that.

Similarly, this so called Parseval's theorem, which incidentally sometimes is called a Rayleigh's energy theorem, is also valid here. So, the equivalent of the Parseval's theorem, here would be that if you look at, it makes more sense to talk about energy here. Because in fact, the Fourier transform, that exists in the free transform, is only for finite energy signals, although we can extend the definition to periodic signals, as a special case, but generally Fourier's transforms are defined only for energy signals, because that is the condition we discussed, they should be square integrable.

So, it makes sense to talk about energy here, so energy means, I simply integrate mod x t square between minus infinity and plus infinity, this is the same as, integral of, if you look at, the quasi function. That is a spectrum, sometimes x of f is simply called the spectrum of the signal, and spectrum has an amplitude path, and a phase path. So, if you take the amplitude part of that and square that, integrate that over an entire frequency range, these two appear the same, that is you can calculate the energy, either by a time domain operation like this or the frequency domain operation like this, whichever is convenient.

Now, let us look at a few other important results ((Refer Time: 31:23)), these are the same Fourier transforms, these are the same, as we discussed for the Fourier series, please stop me or if you have any questions, please point it out, if you find there is a mistake. The next important result, that I like to mention without ((Refer Time: 31:40)) like all other results, is so called convolution theorem. Now, I am not here to discuss the concept of convolution, which I will discuss separately, when I discuss linear time invariant system shortly.

But, basically assuming that all of you know what is convolution then, the result is like this. If you have, let me briefly, review what is convolution, if you have, two signals x 1 t and x 2 t, then I say that x 1 t star x 2 t, is a convolution of x t, where this operation is defined in terms of this integral, this integral represents the convolution of operation.

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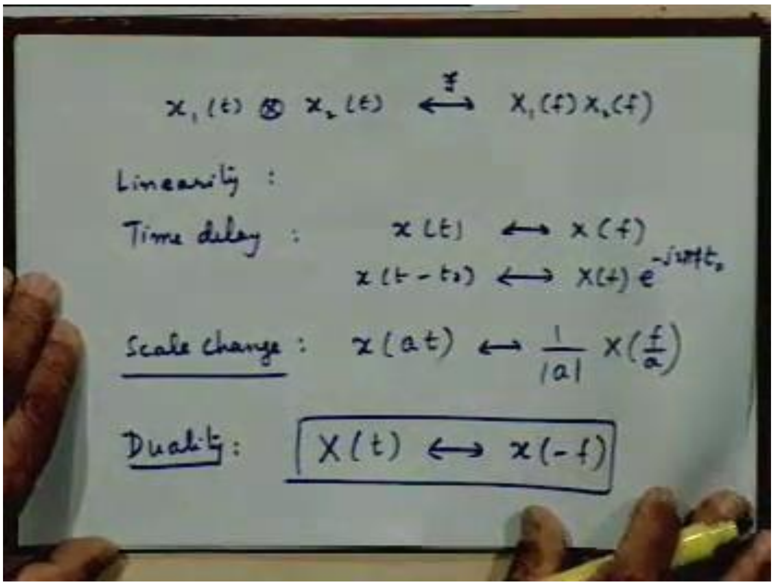

And then, convolution theorem says, that the convolution of x 1 t and x 2 t, if x 1 t has a Fourier transform of x 1 f, x 2 t has a Fourier transform of x 2 f, then x 1 t convolved with x 2 t has a Fourier transform, which is the product of these two ((Refer Time: 33:01)) Fourier transforms. This notation will be used to indicate that these, quantities on the left hand and the hand side are Fourier transform pairs, this is a very, very important result, as we will discuss shortly in the context of, linear time invariant systems.

Then in the standard linearity properties and the time delay properties, which are skipped, linearity means, the Fourier transform is a linear operation, that means, if I take the linear combination of two signals, the Fourier transform would be, linear combination of the corresponding Fourier transforms of the two cycles. And, the time delay property of course, ((Refer Time: 33:50))mention that, if x of t and x of f are Fourier transforms, then x of t minus t naught, will have Fourier transform which is, x of f into e power minus j two pi f t naught, that is the time delay property.

Scale change is another important property, which says that, if I modify the time scale time axis, by a scaling factor of a, it will modify the frequency axis in opposite direction. For example, if a is greater than one, then the frequency axis will be ((Refer Time: 34:39))by, one by a, which will be less than one, and so on and so forth. So ((Refer Time: 34:44))precise to the result is, this is an intuitively very interesting result, basically what it says is, that if you have actually it implies, that if you have a signal which is, small duration of the time, in the time domain, that you have, a large span in the frequency domain.

If you compress this time axis it expands the, frequency axis and vice versa, so it has a. It is a very interesting result and also leads to a very interesting concept, which I if time permits will discuss later, and that is the so called uncertainty relation, in communication theory. Have you heard of that, like you have, I am sure you heard of the ((Refer Time: 35:31))to the relation in quantum mechanics, there is a corresponding uncertainty to the measure of communication theory, which says that you cannot locate a signal, precisely in time, as well as in frequency together arbitrarily, there is a limit.

To either you can locate it very accurately in time or very accurately in frequency, but not both think about it and we will discuss it sometime. And, this is the consequence of this property of the Fourier transform appears another interesting property is that of duality. We will do a very quick review, because you have gone through course in signals and systems and this meant to brush you up on this matter. So duality is interesting, if you can take a time signal, which is, which has a shape of the Fourier transform in the signal, of course this will imply, that x of x value is really a complex valued signal.

If you consider a time signal, which has a shape of the Fourier transform of small x of t, then the Fourier transform of this, would be small x ((Refer Time: 36:45))signal, with I have to replace the, with t replaced by minus f, so this is the duality result.

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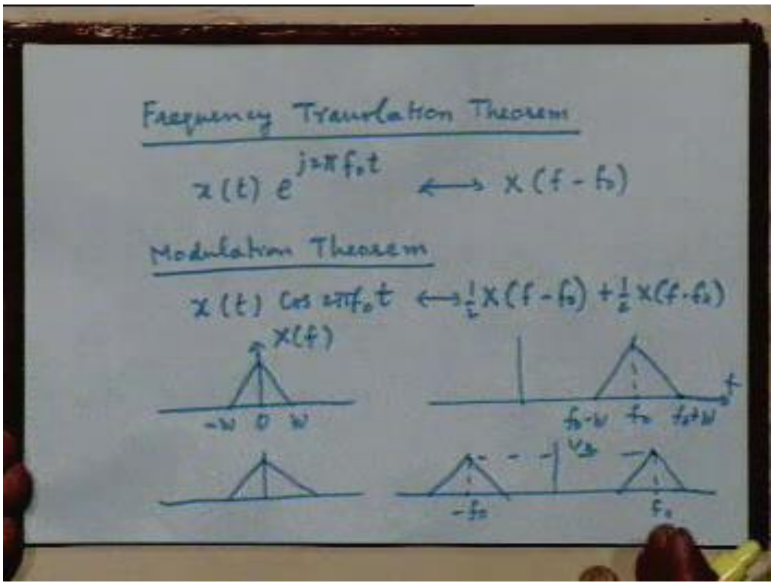

The next, we come to a very important term, result property of the Fourier transforms called the frequency translation theorem, and this theorem is particularly useful in our course, particularly when we discuss the modulation techniques. This theorem states, that if I multiply a ((Refer Time: 37:31)) signal x of t, with a complex exponential e to the power j 2 pi f naught t, then it is Fourier transform, get translated to the frequency f naught. That means, multiplication of x of t, with a complex exponential of frequency f naught, shifts the spectrum x of t or the Fourier transform of x of t, to this ((Refer Time: 38:00)), to f naught as a centre frequency.

So, it is suppose also a signal is centered the spectrum is centered to 0 frequency, after this multiplication it is spectrum gets centered to the frequency ((Refer Time: 38:15)) here. This is also similar to what is also called the modulation theorem, which we multiply x of t with, real cosine side like cosine 2 pi f naught t, and now since, this is equal to the sum of e power j 2 pi f naught t and e to the power minus j 2 pi f naught t. This gets shifted a spectrum of x, t gets shifted to both, f naught as well as to the frequency minus f naught.

In other words, pictorially speaking, if this denotes the spectrum of x t to the centre at 0, multiplication of x of t with this leads to a spectrum like this, whereas, multiplication of x of t, with sine wave or cosine wave, leads to a spectrum like this, that is, this gets shifted to both f naught as well as to minus f naught. This, these theorems play a very crucial role in modulation techniques, that we need to discuss in this course, in fact, it forms a basis for, the most important one of the most important functions of the transmitter, namely to modify the center frequency of a signal. This is also very useful in receivers, of them we want to bring a signal from a, RF frequency down to a base band frequency.

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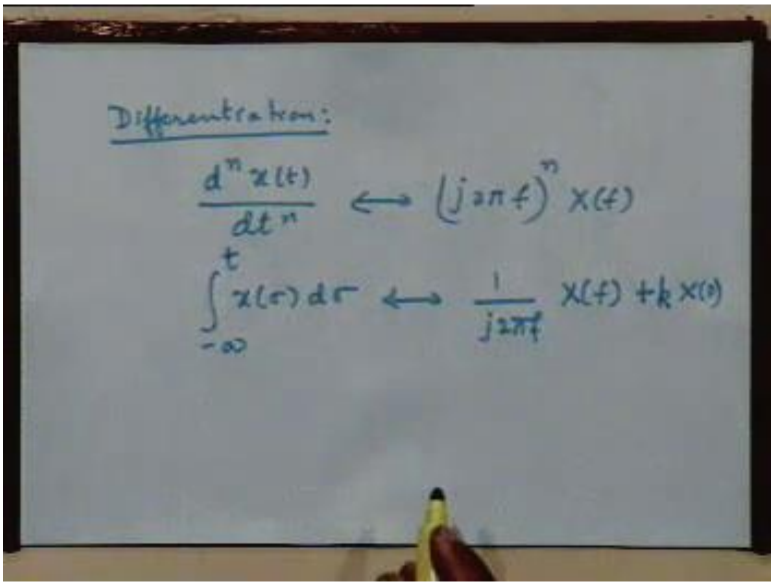

The next result which is important to some extent is the differentiation theorem and correspondingly the integration theorem. I will just state the result without proofs, if you take the nth derivative of this signal x of t, with respect to time and the corresponding Fourier transform is a product of the Fourier transform of x of t with a factor, which is equal to j two pi f to the power n, similarly, the reverse result holds, when we integrate a signal x of t from minus infinity to t.

The Fourier transform, now is given by this plus there is a component, which is proportional to the DC value of the signal or there is a scaling factor here, which you may check up from the book. This in a nutshell is the Fourier transform. We have omitted some of the most important properties of the Fourier transform, these, this treatment is not exhaustive, and it will very much appreciative, if you were to, review the entire ((Refer Time: 42:07)) properties of the Fourier transform ((Refer Time: 42:09)).

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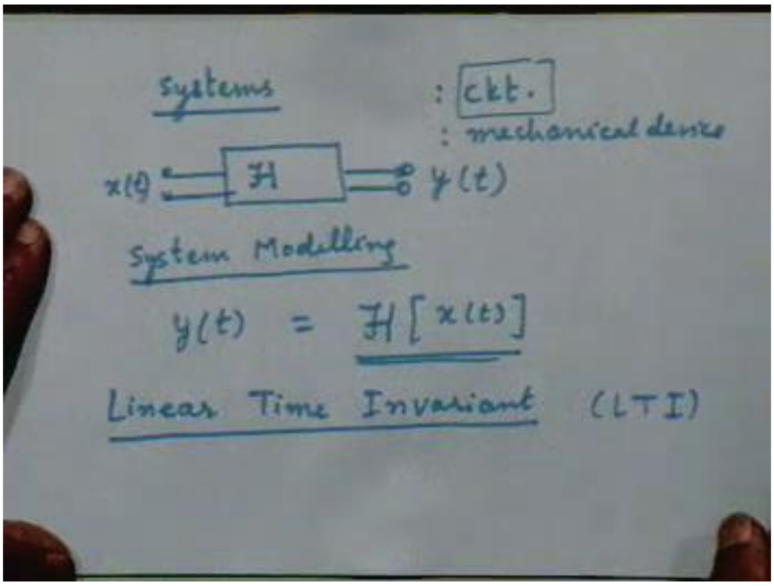

Our next we move on to systems, a system is a model, for some physical system or some process. So, when we do system modeling, the system for example, could be a circuit, could be an electrical circuit, it could be a mechanical device, it could be a ((Refer Time: 42:45)) system, it could be anything. Today, we will look at a system in this treatment is, that it has an input and it has an output, and when we talk about system modeling, primarily we are looking at, trying to understand, the relationship between the output and the input of the system.

So, when you study the input output relationship of a system, we can this system modeling. From the system modeling point of view, it is this relationship, which is important more than anything else, we look at the system. In fact, as a black box, it does not matter, what it is internally comprised of, it will be comprised of, electrical device, electrical components like resistors, inductors and capacitors. It could be, mechanical components like, springs and dampeners and whatever, things you may have in ((Refer Time: 43:45)) system etcetera.

What we like to understand is, how we understand, the response of a system, in relation to some excitations, that may be given to it. Now, from this point of view, it is convenient to think of the system as an operator, the operator h here, operates on x of t the input, signal x of t to produce an output y of t. Under certain conditions, the operator h is set to correspond to a linear time invariant system and these are the, kinds of systems, which will be of greatest interest to us.

It does not imply of course, that communication engineers do not deal with, systems which are not linear or which are not time invariant. However, a large part of our treatment will be devoted to handling systems, which are LTI systems, so to say, and therefore, it is useful to review, what LTI systems is all about. You say that, the system is linear, if superposition principle holds.

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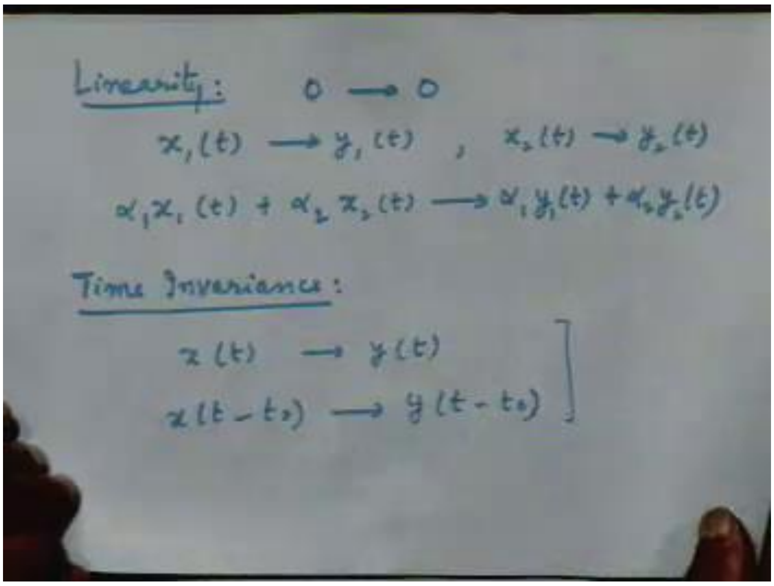

In other words, so linearity implies, that if the system responds to a signal x one t to produce an output y one t, responds to the signal x two t to produce an output y two t. Then if xi the system with a linear combination of these two signals, these two inputs in the alpha one x one t plus alpha 2 x 2 t. The system will respond, by producing an output, which is a corresponding linear combination of y 1 and y 2, and the time invariants. In addition the system should satisfy the homogeneity property that means, if I, 0, input to the system the output is 0.

And, the time invariance we mean, that the system response, is independent of the time of excitation of the input signal. So, if I, if x t produces an output, from the system which is y of t, and if I delay the input by t naught seconds, the response does not change in nature. All that happens is, that the response is correspondingly delayed, in other words, until the input comes, the system does not respond or the system responds, only by corresponding delay in the input or in the output. Now, from the point of view of describing input output relationships, it is important to understand or important to develop, some kind of a characterization of LTI system.

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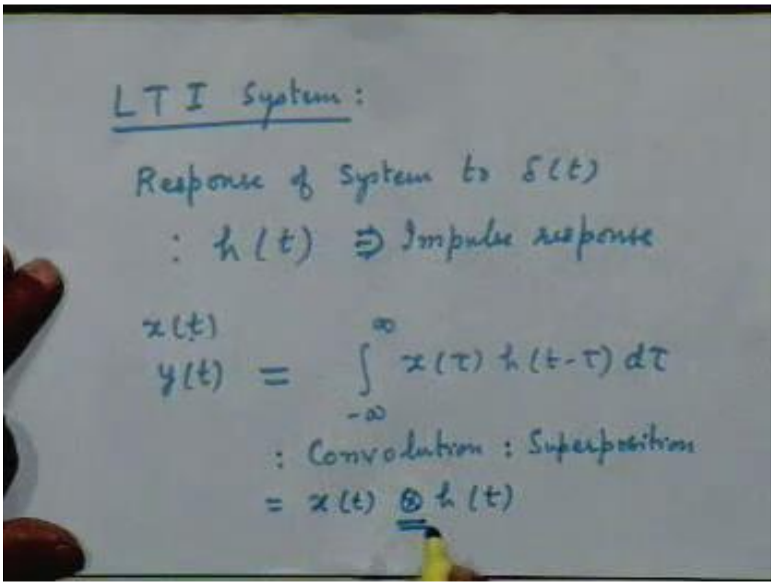

It so happens, it is very easy to check out, that for a LTI system, this characterization is very easy and can be done, by the response of the system, to an impulse. This response, usually denoted by h of tis called the impulse response of the system, the significance of the impulse response in the system is, that in view of the linearity of the LTI system. And in view of the fact, that any signal x of t, can be expressed as a superimposition of ((Refer Time: 48:12)) scale, and ((Refer Time: 48:14)) time displaced impulse functions.

It is possible to express the output y t of a signal, in terms of a linear combination of the, response to these impulses and this is best expressed in terms of the convolution relation, that exists between the input and the output, which is given by this. That is, given the input x of t, and the impulse response h t of a system, output can be expressed, thorough this superposition formula, which is also called the convolution integral, which is a manifestation of the superposition property, arising from the linearity of a system. And usually, expressed in terms of shorter notation, like x t convolved with h t, so this is a notation for convolution or let us say the convolution operator.

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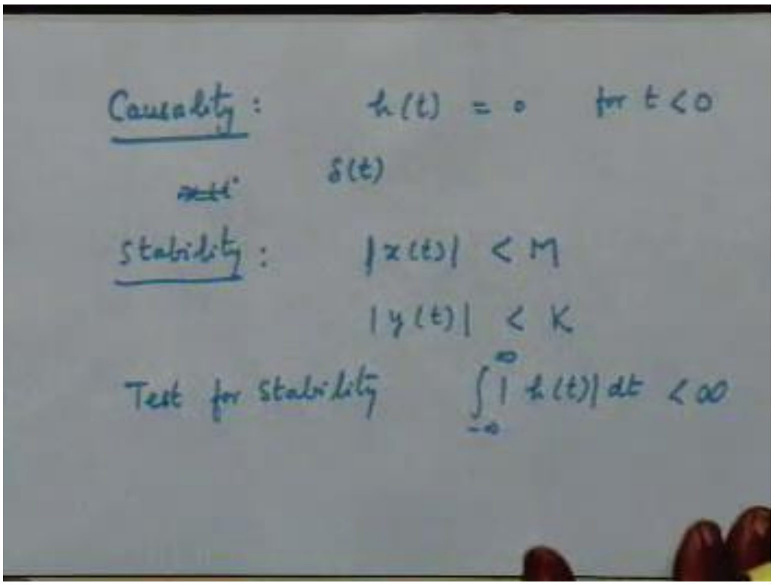

Finally, we just note two important, two other important attributes of ((Refer Time: 49:26)) system. One relates to causality, we say that the system is causal, if the impulse ((Refer Time: 49:40)) this is of course, not the definition of causality. This is a test for causality. If the system impulse response h t is equal to 0, for t less than 0, this is atest for causality, definition of causality is, the system responds toan input x of t, you can put it like this, until there is an input, there is no output.

So, since the impulse is response corresponds to an impulse ((Refer Time: 50:18)) of t is equal to 0, there can be no output from the system, no non-linear output of the system, before time t equal to 0, and hence the impulse response has to be 0 for, t less than 0. Another attribute of a system, which we shall assume throughout our course, to be valid for a section that we study is stability, and stability refers to, the bounded input bounded output property of the system.

That is, if your input x t, is always guaranteed to be less than certain finite value, then the output y of t, will also be guaranteed to be less than some positive value k and if this is so we say the system is, if every such input, the output satisfies this property, you say that the system is stable. And, a test for stability is, that the integral of the modulus of the impulse response should be bounded, should be less than infinity. So I think, with this I, will finish the review for signals and systems, and next time we will consider a very important, ((Refer Time: 51:38)) result for you.

And, introduce the concept of the ((Refer Time: 51:44)) transform, which we will find is extremely useful, for studying signals of certain types, which we have encountered in communication engineering.

Thank you.