Communication Engineering
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Lecture - 29
Random Processes

If you recollect we have been looking at some basic concepts in probability theory. And we had discussed the concepts of Random Variables, probability distribution functions, probability density functions and also concepts of how to deal with multiple random variables, two random variables, that is what we are discussing presently. So, we will quickly finish that review and then move on to random processes so, to continue with our discussion on, characterization of two random variables or more random variables.

Remember we had introduced the concepts of joint probability distribution functions and joint probability density functions. As with independent events or independent experiments that we discussed earlier, we could also have what are called independent statistically independent random variables.

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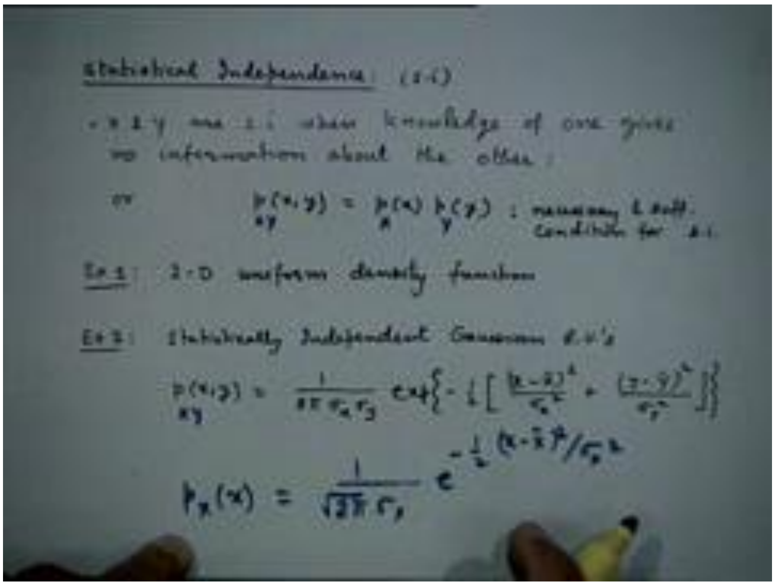

So, we say, that 2 random variable x and y are statistically independent, I have written x i in short, when the knowledge of occurrence of one, gives us no information about the other. And in terms of, the properties of the probability density function, 1 it implies is that the joint density function of x and y, will be product of the corresponding marginal density functions. P of x is the marginal density function corresponding to p x y, actually strictly speaking I should be writing sub x y here, sub x here and sub y here actually they are all different functions, you must appreciate that.

So, and p of y is the marginal density function, corresponding to the same joint density function. If the joint density function is related to the marginal density functions, through this product relation then, x and y are said to be, statistically independent random variables. Take for example, the 2 dimensional issue, from density function, that I discussed last time, remember this was a 2 dimensional uniform density function, with this probability density function.

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So, if you look at this, we can say that, this is really a product of the two independent p x x and p y y. Marginal density function, corresponding to this along x direction would be essentially 1 by x 2 minus x 1 and the marginal density function in terms of y, corresponding to this joint density function would be essentially 1 by y, y 2 minus y 1 and we can say that this is the product. So, therefore, in this particular case, this kind of a joint density function, would imply that x and y are statistically independent random variables.

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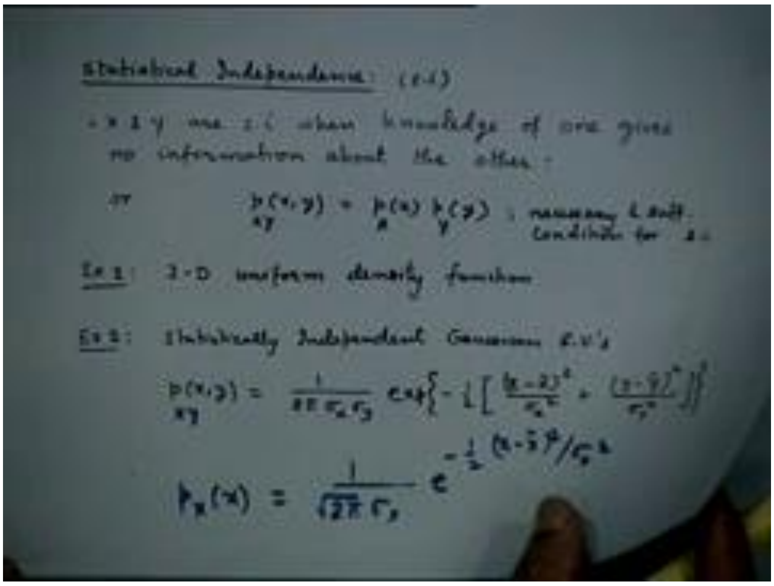

This is an example of statistically independent Gaussian random variables. So, I have if you recollect the expression for a Gaussian density function one dimensional Gaussian density function is something like this. So, if I multiply two such Gaussian density functions in terms of x and y, I get a joint density function which is like this. And since it is separated into, a product of two such density functions, the corresponding random variables are statistically independent once again.

However, one could have jointly Gaussian random variables, which are not statistically independent, but the expression for the joint density function would then be different, it will not be this. So, you could have jointly Gaussian density functions, which are not statistically independent. Let us we will see example for those later, is that now just like we defined the expected value of functions of one random variable, you could as well defined, very similarly expected value of function of two random variables, two or more random variables.

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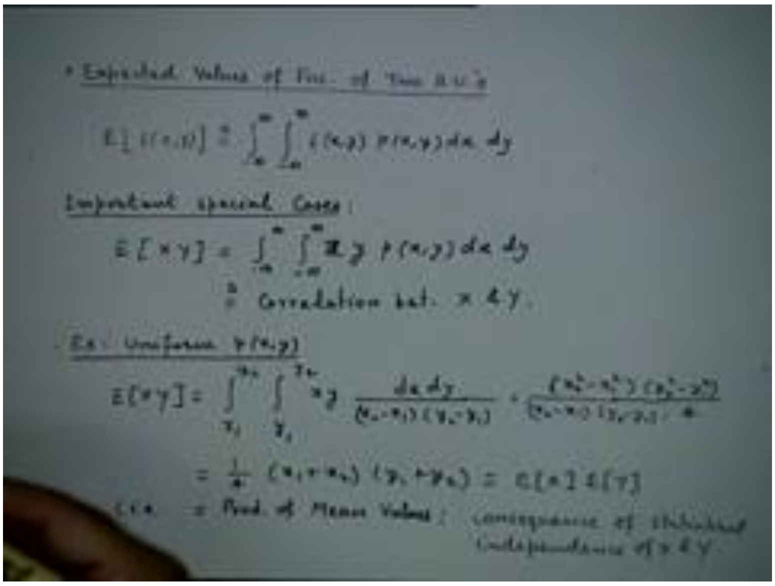

So, the general generalization is absolutely straight forward, if you have two random variables x and y and the function f, will acts this random variables x and y to some other variable. Let us say z, and then the expected value of this function is essentially, the same averaging operation that we defined for the case of a single random variable. So, you take the function, and now multiply it with the joint density function of x and y and integrate it about the range is, range of x and the range of y.

So, it is really a straight forward extension of this concept, so is nothing new here, conceptually there is nothing new, just you could easily generalized this notion to multiple random variables x y z, x 1, x 2, x 3. As a special case, when you take the function f x y, to be the product of x and y. Then you get this and that specific expectation, which is the expectation or the average value of the product of the two random variables is defined to be the correlation between x and y.

We say that, the value that you will get here, is a measure of the correlation between the two random variables x and y. The basic idea is the, if there is a strong correlation you will get a large positive or large negative value for this product, for this average value of the product. Example, if they are ((Refer Time: 07:09)) they if they are highly uncorrelated, what will tend to happen is, that this product will tend to have, a small value or a 0 value.

Actually the precise result is slightly different, so we will come to that a little later, so this is always define to be the correlation function between x and y. Take for example, the uniform density function that we discussed a few minutes ago, the joint density function of that is 1 by x 2 minus x 1 into 1 by y 2 minus y 1. So, I am, I am now computing the correlation between x and y, of those two random variables x and y, which were depicted in this figure.

So, there is, there is a joint density function and looking at these two random variables x and y and see what is the correlation between these two, given that the joint density function is a uniform density function, that is what we are looking at. So, I am multiplying x y and multiplying the product with, the joint density function integration over the range of x and y which is x 1 to x 2 for x and y 1 to y 2 for y, that is the domain of x y, for the particular case.

If you carry out this integration, I do not think we should spend time on that here, this will be result, which finally, simplifies to this. And interestingly, if you look at this result carefully, you can think of this as a product of x 1 plus x 2 upon 2 into y 1 plus y 2 upon 2. And, what is x 1 plus x 2 upon 2 that is the mean value of random variable x, because it has a uniform distribution between x 1 and x 2, similarly y has a mean value, which is y 1 plus y 2 by 2.

So, as you can see, in this particular case, the cross correlation or the correlation between x and y is nothing but, the product of the corresponding mean values. Actually, this is not surprising at all, this is not only true for this particular case, this will happen for every case, where p x y is a separable function. That is x and y are statistically independent random variables, which is very easy to see. In fact, I will take a general case in a moment, so basically this result, comes from the, it is a consequent of the fact that x and y are statistically independent.

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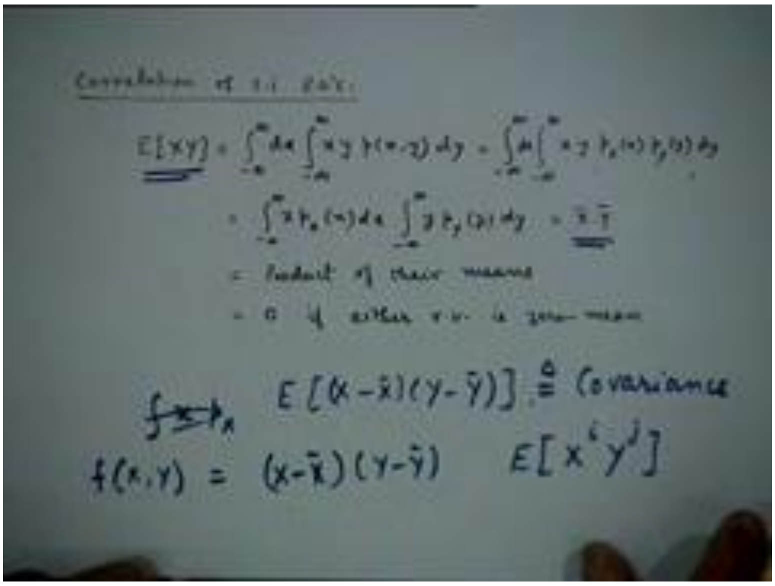

To show this, for the general case, let us look at the general case, you have the product x y multiply with the joint density function. If x and y are statistically independent, you can write this joint density function, in terms of the product of the corresponding marginal density functions, which is what I have written here. And, now you can separate out the two integrals, this is one step has been skipped here, but let us look at that step.

If I can separate this integral out into x p, I think there is no step you see, I have just separate it out the two integrals, one in terms of x. The other in terms of y and this is nothing but, p x x, the expected value of x, x bar this is nothing but the expected value of y. So, it is clear that for example, when two random variables are statistically independent, the correlation function the correlation value would be the product of the corresponding mean values.

If any of these mean values happen to be 0, the correlation would be 0, now closely related to the concept of correlation is a concept of covariance, which is defined like this. So basically, I am choosing the function f x y here, to get the product function not x into y, but x minus x bar into y minus y bar, so in that sense if correlation is considered as a second order movement, non central movement, then the covariance this is defined as a covariance, between x and y, can be consider to be a center element. In general, we say that expected value of x to the power i, into y to the power j, let me I will take a fresh page here.

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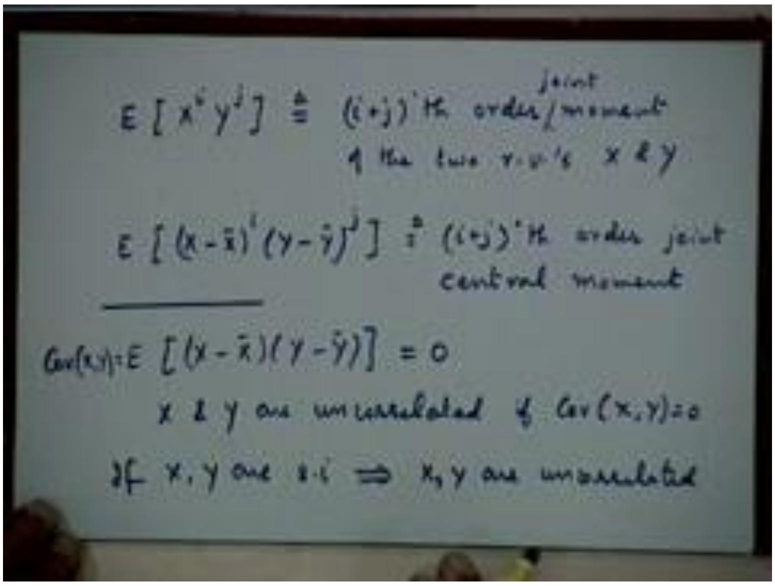

We will define expected value of x to the power i into y to the power j, I have chosen the function f x y here to be, the product function x to the power i into y to the power j. This is defined as the i th plus j th order moment or the joint i plus j th order moment of or let us say joint moment is, that is a better, because otherwise there will be a confusion, joint moment of the two random variables x and y. So, this like we could characterize a single density function, in terms of it is moments, and this moments for gross characterization of the density function of the random variable.

Similarly, the joint density function of two random variable x and y, are the two random variables x and y can be grossly characterized, in terms of joints moments of this kind. Correlation happens to be a special case of this, where i equal to j equal to 1, so correlation is the second order moment. Similarly, you can have the corresponding central moments, so this is the i plus j th order central moment, in a way basically, what I am saying is, that every concept that we discussed. For the case of a single random variable, can be extended for the case of multiple random, from the concept.

And of course, I hope you appreciate, why we call it the covariance, if you remember expected value of x minus x bar whole square was the variance, was the speared of x around it is mean value, the corresponding joint moment therefore, is called the covariance. So, covariance and correlation of course, related to each other, but when we say two functions are uncorrelated, this is a common terminology that ((Refer Time: 15:31)) uses.

The meaning of, that terms is, that we say that, this covariance is 0, so there is a slight possibility of a confusion here, which should take note of, we say that x and y are uncorrelated, if the covariance is 0, if covariance of x and y is 0. So, this is sometimes I will denote this expected value of x minus x bar into y minus y bar, simply a covariance of x and y, I just write this as covariance of x and y, not when the correlation is 0. It is clear that if x and y are uncorrelated, x and y are statistically independent, if x and y are what can you say about correlation, they will be uncorrelated.

If x and y are statistically independent, it will imply that they are also uncorrelated. However, if x and y are uncorrelated if that is not necessarily imply, if they are statistically independent, except in some very special cases, for example, then x and y jointly Gaussian. If they are jointly Gaussian and are uncorrelated, then it implies that they are also statistically independent not otherwise. So, good you seem to remember quiet a lot about the probability theory, from the feedback that I am getting, while we are discussing these things.

So, I think we can stop the discussion on the probability theory, review at this point we will look at things, if we need something else later on. There are few things, which I am omitting here, but ((Refer Time: 17:41)). I think, we will be able to take care of that, while solving problems or during discussion as the need arises, and therefore, we now move on to the discussion on random process.

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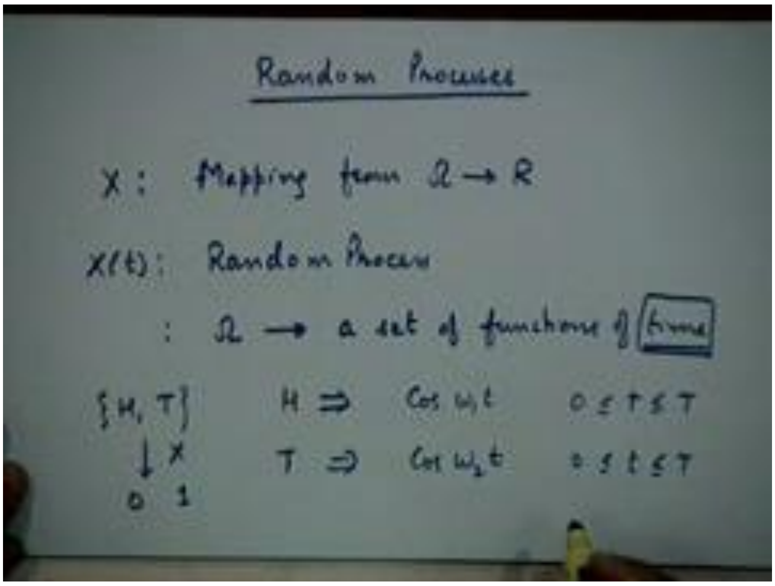

Actually, what is the need for studying random processes, as we know, as electrical engineers as communication engineers? What we will be dealing with, will be random wave, mostly random waveforms, for example, when we talk about noise that is one example of a random process for us. So, if you I do not know whether, any of you have looked at, the noise waveform on an oscilloscope, if this is not a regular waveform, which you can mathematically characterize, any kind of deterministic waveform that you might characterize.

You know for example, a sinusoidal waveform can be mathematically characterized. As a function of time. Similarly, an exponential function can be characterized mathematically these are deterministic functions deterministic waveforms, now when you deal with, a random thing like noise how do you mathematically characterize it. So, first ((Refer Time: 18:59)) there in order to do this, in order to carry out this mathematical characterization, we have this formal frame work, namely the frame work of random processes, in which we work.

So, let us try to understand, what is the frame work of random processes actually it turns out, that really speaking the concept of random variables and the concept of random processes, they are not really very different concepts, they are more or less the same concept, except for a very minor difference. Let us review, what did we say about the random variable, what was the random variable.

How did we define a random variable, ((Refer Time: 19:34)) recollect that, it was defined as a mapping from, the sample space to the real line, that was what a random variable was, so the random variable x is a function of mapping. Let us say, mapping from omega to R, where omega is the sample space, as we seen in the random experiment and R is a real line. So, for every point in omega, there is a corresponding point in R that is what we basically wants to do.

Now, you can also look at in random processes in precisely the same way, the only difference, being instead of mapping it from omega to the real line, you map it from omega the sample space, to a set of functions, functions of time not numbers, but a set of waveforms. So, for every point in the sample space, you generate a corresponding waveform and you say ((Refer Time: 20:41)), if that sample point has occurred, implication is that waveform is what you will see.

So, therefore, you can think of x of t, as a random process and you can visualize a random process, in terms of the same mapping frame work, that we discussed for the case of a random variable. So, you can think of this, as a mapping from omega to a set of functions, set of functions of time, the functions of time is what we called waveform, the functions of times is a waveforms for us. It may not be a function of time it could be function of some other variables, because random processing is not always, studied in the contents of time waveforms.

So, for example, in the case of an image, it will be a function of x y coordinates of the image. It could be function of any set of parameter any parameter, it, so happens that, in our content the parameter of interest is the time parameter, but there is nothing circumcised about this parameter at being time, this could be any parameter. So, basically that is what we do, to put the concept more strongly in a minds, what you do is let us consider the simplest experiments once again, maybe the coin tossing experiment and the two outcomes H and T need to be mapped.

If you remember for the case of a random variable, it could be very simple kind of mapping, like this being 0 and this being 1, or the other way around it does not really matter, the 1 could define many different mappings, to define a random variable. Now, we will say ((Refer Time: 22:38)) if H occurs, it implies or it is mapped to, let us say waveform cosine omega 1 t, over a period from let us say 0 to capital T, this for the sake of discussion

If tail occurs, we will map it to a waveform cosine omega 2 t, it is just this is some arbitrary definition could be something else, again over 0 to capital T. So, what is it mean, if H occurs, what is the waveform you will see, cosine omega 1 t, if tail occurs this is the waveform you will see. However, a priori you do not know what you will see, it will depend on the outcome of the experiment, so in that sense it is random, what you observe, is either this or that with some probability.

So, there is a, therefore, you can, if you look upon things in this manner, you can think upon, think of a random process, as the collection of waveforms, each waveform occurring with a certain probability. Just like, you could think of a random variable as a collection of values, each value occur in a certain probability or each value range occurring with certain probability, is that clear.

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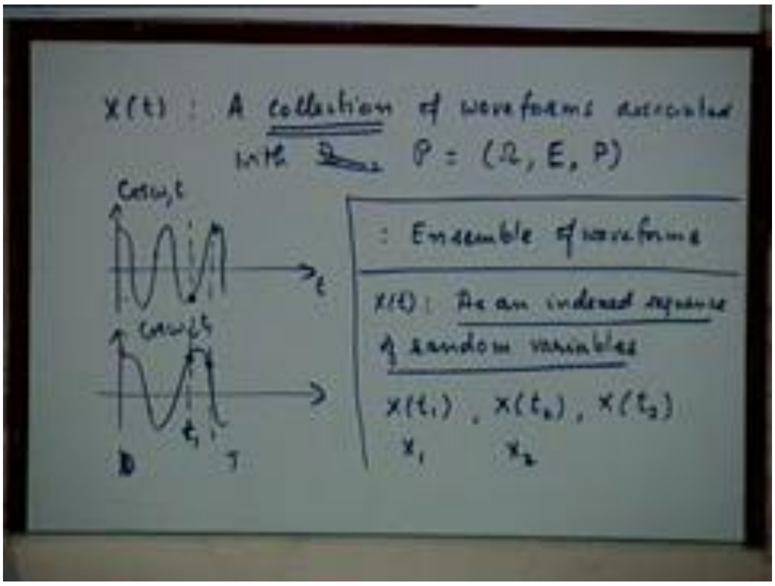

So, that is basically, what you can say, you can think of x of t in this manner, as a collection of waveforms, associated with omega or associated with more, more precisely associated in the probability space p, I think that is better where p if you remember, comprised of the triplet omega, the field E of events and the probability measure P. This triplet sometime is typically denoted by, the probability space p, so basically, this is one way of looking at it, and it is a collection of waveforms.

So, for the example that I have discussed, the collection was only of two waveforms, this was one and the other was this, this is cosine omega 1 t, this is cosine omega 2 t, over the interval 0 to t, 0 here and t here. In more general case this collection could be very huge, in this collection could be infinite and you would be observing, any one of these infinite set of waveforms, in at any given time, at any given instance.

So in as much as, you can think of the random process, in the same manner as the case as a random variable, you can therefore, look upon this random processes is a collection of waveforms, any one of these occurring with a certain probability. For example, for the, for this case, this occurs with probability half and this, occurs with probability half provided your coin is unbiased. If so, it is as much as you do not know rather you will see this or this, it is a random theory.

This is one view of a random process, this view is good because it is easy to, I could draw this view directly from the concept of a random variable, and also it basically tells us that random variable is, what and alternatively this collection is also given an alternative name is also called an ensemble of waveforms. There is an alternative view before I come to the alternate view, each function each waveform is called as sample function, and the collection of sample functions constitutes the random process.

Of course, the random process a law also will have an associative probability distribution is some sense, it is a little difficult to define, such probability distribution across the ensemble, we will come that in a few minutes. This view is not very convenient to ((Refer Time: 27:22)), when you want to mathematically characterize a random process. It is very convenient from the point of view of visualizing it as an extension of the concept of a random variable, but not mathematically very convenient to work with.

So, therefore, when alternatively defines a random process also in a different way sometimes, in fact, mathematically that is a much more convenient way, to motivate that, let us look at same example. Let us say, I am looking at, I am trying to see, what is it that, I have being seen, from the oscilloscope at time at some time instant t 1, what will you say, if this is the waveform that occur, you have to see this value of this time. If this is the waveform that is occur you will see, this value at this time, is it clear.

So, now suppose I were to just be interested in, what is the value that I will see at, this time instant t 1. Basically, I will see, one of these two values, which are fixed, this value is fixed, it is dependent on the value of t 1 that is all, at t 1 this value would be a specific value providing by this function, at t 1 this value also be a specific value, given by this function. So, basically it is like, head and tail having, occur and that head and tail having be mapped to either this value or this head being mapped to this value and the tail being mapped to this value, and what is that, a random variable.

And, at every time instant, I have a different random variable, because the mapping is different, is that clear. I have a random variable defined here I have another random variable defined here, so depending on a what the time instant, you sample the process is like I am sampling the process at time instant t 1. I will say one random variable, because I could say either this value or this value, or I could say this value and this value of time instant t 2, or some other pair of value at some other time instant.

So, in that sense, what I can also see is that this view of things could be modified in a different way, to say that basically at each time instant, the random process behaves like a random variable, I meant a different random variable. So, in as much as, I can have the infinite set of sampling instance, I have really ((Refer Time: 29:56)) a collection of, an infinite set of random variables.

So, if I view, if you if I look at things in this particular manner, I can define a random process more precisely, as a sequence of random variables, as an index sequence of random variables, index means as a function of time, so this is the alternate definition. Random process x t, can be defined as an indexed sequence of random variables, x is a random variable and t is the index, that is basically what I am saying. It will be a different time instant, at every time instant, we have a different random variable.

You have a random variable at time instant t 1 we have a different random variable at time instant t 2, we have yet another time random variable at time instant t 3. So, you can think of, this as x 1, for example, this x 1 here could take either this value or this value. So x 2 here could take this value or this value, the x 3 here similarly, could take some other set of values, but time of course, will assume moves forward, so t is in that sense, moving in one direction, so t is our index.

So, essentially it is a collection of a large number of random variable, so they are two different varied use of a random process, one has a collection of waveforms and the other has a collection of random variables, is it clear. And, the second is very convenient, to be automatically characterized, because we have already got the tools with us, to handle multiple random variables is not it.

Basically, what we are not saying is, that a random process, every observation of a random process is an every sample function of the random process is, essentially a sequence of some values that, have resulted from the occurrence of these various random variables. So, and we can characterize this waveform, in terms of a joint density function, of all this random variables.

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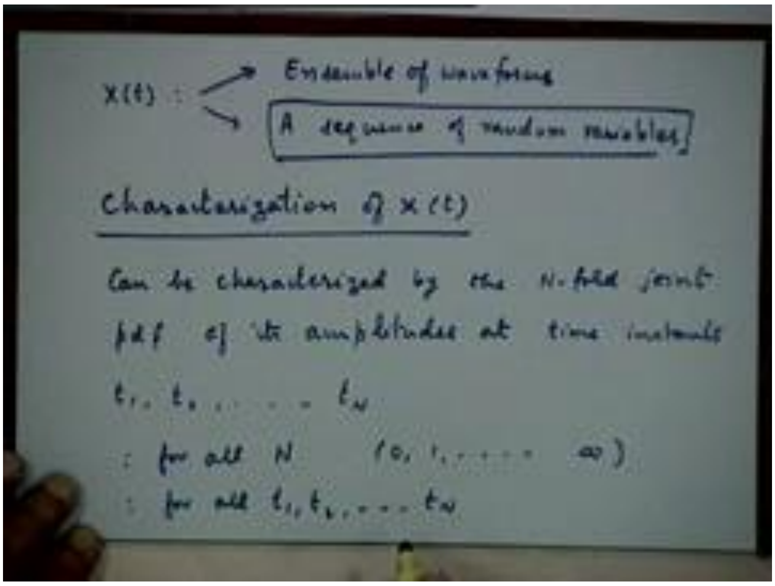

So, let me summarize, x t can be taught of either as an ensemble of waveforms or as a sequence of random variables, the sequence of random variables, could be a continuous sequence or a discrete sequence. For the example that I have taken now, correspond to a continuous sequence, because t is a continuous index, t could also be a discrete index. In this case, this would be a discrete index; it will be a discrete random process, the sequence of random variables.

So if it, if I take the second row, they are actually equivalent, one is one, one defines the other in some sense, they are there is nothing really different about them, except the way we describe it, that is all about it. So, now if we want to characterized a random process, how should we characterize it, for example take a same head, head tail coin tossing experiment. I cannot say that the waveform is cosine omega 1 t I cannot say the waveform is cosine omega 2 t.

If I were to describe it in a probabilistic framework, I cannot talk like that, I have to only talk in terms of language of probability theory and the language of probability theory requires to, talk in terms of probability density functions or probability distribution functions. And therefore, if I look up on, this random process is nothing but, a set of random variables, I can characterize the random process x t, by specifying the joint distribution function or the joint density function of, these random variables.

So, that is the proper characterization, however there is a difficulty, but before I come to the difficulty, let me summarize this. So, we can say that, a random processes is characterized, can be characterized, by the N fold joint pdf of it is amplitudes or of it is values, at time instance t 1, t 2, t sub N. And this itself will tend to the difficulty associated with the characterization of a random process, which time instance will you select, how many time instance will you select.

But, it will not, if it is a continuous random process, either if it is a discrete random process, then ((Refer Time: 35:52)) the waveform has a large number of points associated with them a continuous waveform has an infinite set of points. So, if I want to characterize it, in terms of joint pdf obviously, I have to sample the random process at some time instance. So, let us I am saying that you sample them at some time N instance, what should be the value of N, having determined the value of N, what should be the actual sampling instance.

There is a lot of choice, an every different choice will produce a different joint density function, you have, you are dealing with a set of N different random variables it is a really horrendous job. A complete characterization would therefore, require, that I be able to specify, this N fold of this, N variable joint pdf for every set of N points, that I could choose.

So, I should be able to do this, for arbitrarily for all values of N vary from 0 to infinity, I should be able to do this for all choices of arbitrary choices of t 1, t 2, t N and that is the difficulty. So, a complete characterization of a random process is really a very difficult job, it can be characterized, theoretically conceptually there is no problem, conceptually or in which you ((Refer Time: 37:30)) will sample the random process, it sufficiently large number of points and specify the joint distribution function.

Even here, there is another difficulty, you can have as I said many different specifications, I could choose N equal to 2, I could choose N equal to 3, I could choose N equal to 4 and so on and so forth, I could choose N equal to 50, depending on my requirement. But even here, you must remember that, if I choose N equal to 2 or N equal to 3, the joint density function associated with N equal to 2, and the joint density function associating N equal to 3, must be mutually consistent, together we just arbitrarily selected, what do I mean by this.

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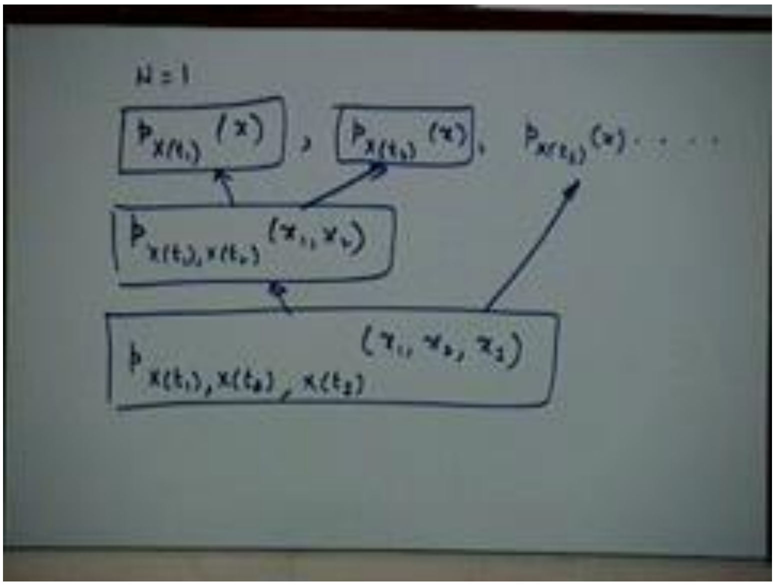

Let us talk about 1 and 2, suppose N equal to 1 that means, I am characterizing it, only at 1 time instant. So, why do joint density function associated with that, let me defined the corresponding time, corresponding random variable as x t 1. So, the density function, here is an, one dimension density function, so I will be specifying, the density function like this. How many different kinds of density function we are going to have here, infinite number even this density function, would be different at time t 1, from that a time t 2, from that at a time 3 and so on.

So, there are, so many first order density, these are called these are all first order density functions. I can have infinite, I need to have an infinite set of first order density functions to describe it, and that is only a first order description. Because, it only specifies the behavior of any one random variable at a time, it does not specify how two random variables behave with the relation to each other, in relation to each other, which constitute the random process.

If I sample the random process at two time instance, so I have two random variables, so the joint density function will not specify, the behave, the joint behavior of these two random variables ((Refer Time: 39:42)), is it clear. Now, not only that, suppose I initially choose, look at them individually, now looking at a look looking at them together, it is clear that, this characterization should be related to this characterization, in the sense the marginal density function associated with that, should be these.

Similarly, if I look at the characterization of three random variables together, this characterization is a joint density function of these three random variables should lead to this marginal density function. This joint marginal density function and should lead to these two and should lead to this, this should all be mutually consistent, you cannot just arbitrarily select or so it is a very difficult job, basically that is what I am trying to say. The complete characterization of a random process in terms of these joint distribution functions is a very, very horrendous job very difficult.

So, what do we do, how do, we characterize processes, unless you make some simplifying assumptions, we cannot really do much and what are the simplifying assumptions one can make here, any ideas.

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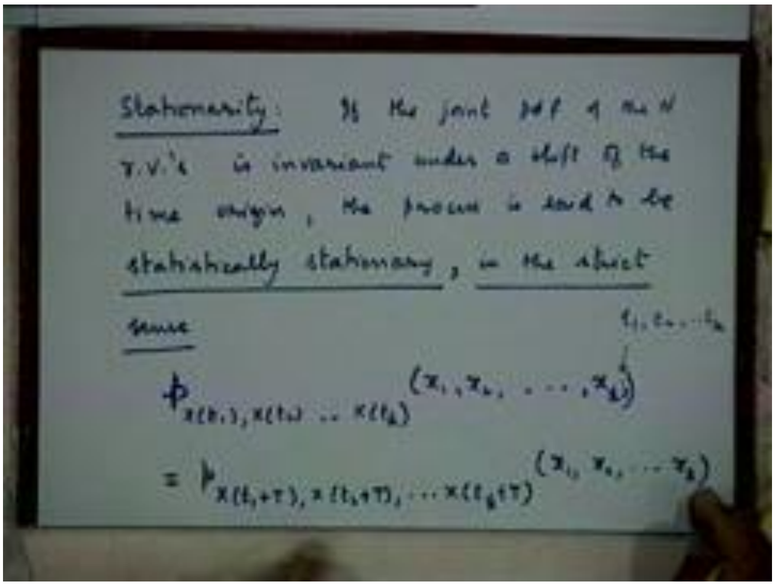

First assumption one can make is the concept of stationarity and that simplifies in the following way. Let us consider this first order characterizations, suppose it was reasonable to say, based on the physical considerations of the, physical understanding of the random process, that may not vary sample in the random variable, vary sample in the random processes, the probability distribution function remains the same, then this implies this step, is not it.

If all this were independent of t 1, the actual location of t 1 or the actual location of t 2 or then all this would be equal. So, instead of I need to specify infinite set of first order density functions, I need to specify only, one first order density function, because they are all equal, they are all the same, providing with the reason to assume this. Similarly, if I want to extend this idea, consider two time instance t 1 and t 2, if the joint density function of these two, were not to depend on, what is where, is t 1 and what is where is t 2, but only on, what is a mutual location, mutual distance between these two.

That is, it is independent of the time origin, it only depends on how far they are displaced from each other, timings t 1 and t 2 then again it becomes very simple. Because I just look at the time difference of the two random variables, I am able to characterize them, independent of where they are located and so on and so forth, you could do that or the higher order density function as well, if you could do this.

Students: ((Refer Time: 43:11))

Yes please.

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In this case, it will only depend on the mutual differences between t 1 and t 2, t 1 and t 3 and t 2 and t 3, that will be the case for third order, so that is the concept of stationarity. So, we say that, if the joint pdf, if the Nth order joint pdf, of the N random variables is invariant, under the shift of the time origin, that is the basic idea, time origin should not be of any consequence, where we are choosing the origin, really these three examples that I give, for examples of this particular, then the process is said to be, statistically stationary and this is stationary in the strict sense.

In the strict sense implies, where this happens for all density functions of all orders, first order density functions, second order density functions, third order density functions and so on and so forth, density functions of all orders are invariant to shifts of the time origin. If that is a case, you have a you have a simplified situation, it is still not simple enough, where it is a much more, it is much simpler then, the situation that we had to start with,

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Sorry.

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So, what you are saying is, yes let me mathematically specify what I mean by this, mathematically how do we write this. So, if I look at the joint density function of, let us say, k random variables, sample at t 1, t 2, t sub k, these are the corresponding variables. Inside, these are the names of the random variables, these are the value that they can take, and this is the joint density function associated with them. This should be equal to actually, many times the notation also with put a covalent here and specify the time instance here, t 1 t 2 t sub k, I am simplifying the notation.

And, here the time instance would be t 1 plus T, t 2 plus T, so then you do not, then the subscript can be compressed, we simply say x 1, x 2, x k and the time instance can be indicated in the argument there are many ways of denoting this. But, you get the essence of it, that is I have shifted the location of every random variable by T seconds, that is the shift of time origin, and the density function of these k random variables and these k random variables remains the same, I hope that clarifies the picture completely.

Now, just like we could characterizes, a any probability density function by gross characterizations like moments, first order moment, second order moment and so on and so forth. We could similarly characterize a random process by it is moments; because now characterization is what, characterization is joint density function of some order. We will have to decide, what is the order to which we are interested, that will depend on the application will depend on the example random process, which we are working with.

Somehow we have to, have, feel for to what order your description is required, for your purposes. Many cases, it is sufficient to work with, the first and the second order density functions, many, many cases, it is very rarely required to look at, higher order density functions. Even, when we are working only first and second order density function that means, we are working with density function of the kind p x t x and p x t 1 x t 2, x 1 x 2 we are working with these two kind of density functions. Even under this situation, you could characterize even these random variables, single random variables or this joint, random variables by the moments, rather than the complete density function.

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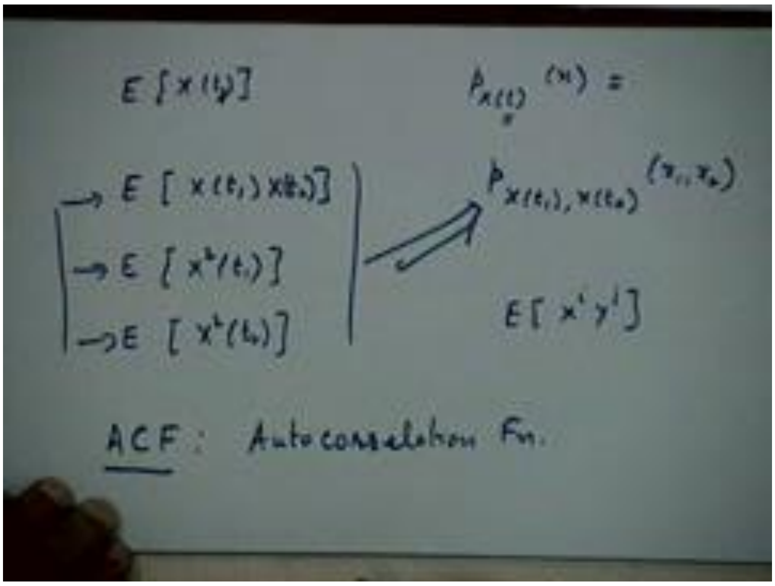

And, the moments are the same for example, you can have the mean value function, that is a, that is a gross characterization of this density function, is it clear. Corresponding to the first order density function associate with the time instant t, you have a corresponding expected value of this density function or this random variable. So, this would be a gross characterization of this, you can have this, if I have it for any value of t, for example if the process is stationary, then this is really independent of t.

So, in that case, this also will become independent of t, if the process is not stationary, this would depend on t and therefore, this also would depend on t, but this would be a gross characterization, this is a complete first order characterization. Coming to second order, ((Refer Time: 49:50)) the second order, second order moment characterization you could define, expected value of ((Refer Time: 49:58)) the complete characterization where is, the joint density function of x t 1 and x t 2, is that clear?

x t 1 is a random variable, sampled at time is in t 1, x t 2 is a random variable sampled at time is in t 2, these are joint density function. The second order characterization is essentially, the correlation between x t 1 and x t 2, of course there will be other second order characterizations for example this is another second order characterization. We know, what was the second order general characterization of, the general characterization is x to the power i, y to the power j.

If I take i equal to 1, j equal 1 I get this, if I take i equal to 2, j equal to 0, I get enormous second order characterization and we get another one, these are three second order characterizations, of associated with the second order density function. I could have higher order moment characterization with this second order density functions, these are here I am talking of order of moments here I am talking about the, number of random variables I am talking about, so the order of the new stiff slightly different things, in their two contacts.

This, these three functions are of great importance, this is nothing, but the mean square value of the random variable t 1, x t 1 and the mean square value of the random variable x t 2, if the process is stationary these two will be the same, if the process is stationary. And, this is called the cross correlation sorry, the auto correlation between x t 1, auto the word, auto stands for the fact that x t 1 and x t 2 are samples of the same random process, that is the word, that is what the meaning of the word auto. So, we defined therefore, this is a new term not a new concept, the concept is a same old concept, but it is a new term, auto correlation function of a random process.

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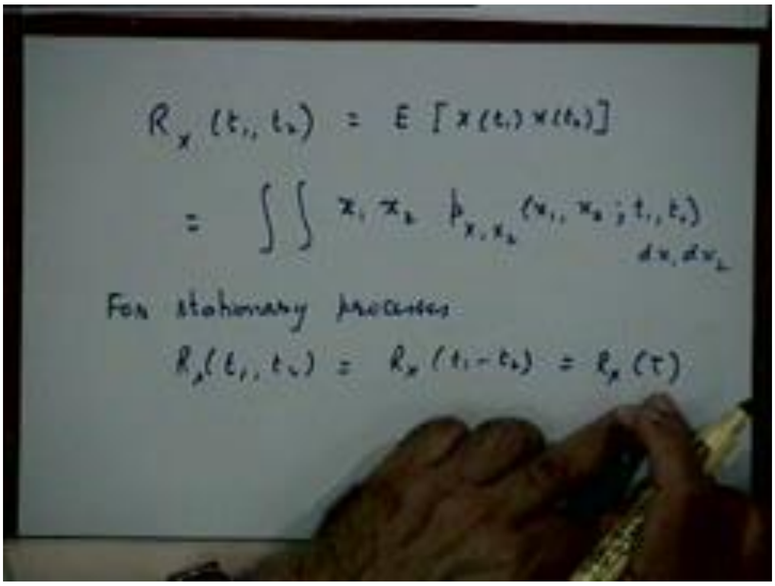

In essentially, will denoted by R x t 1, t 2 as expected value of x t 1 into x t 2, the average value of this product, where t 1 and t 2 are two arbitrary time instance, that which we are sampling around the process x t, the same random process x t. How would we compute it? To compute this you will look at the two random variables x 1 and x 2 multiplied by and take the joint terms simplifying the notation. Here, when I say x 1 please take that to mean x t 1, when I say capital X 2 takes that to means as x t 2.

If you want, you can characterize the time instance here, this has no significance as far as the integration is concerned it is only a notation d x 1 d x 2 is that ok. So, that is the meaning of, now clearly, this will be function of the time instance t 1 and t 2, if I, because the joint density function will depend on, the time instance t 1 and t 2.

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This is a general case.

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For stationary case, what will happen, in as much as this density function depends only on t 1 minus t 2, the auto correlation function will also depend on only on t 1 minus t 2. So, for stationary processes, this will be equal to R x, t 1 minus t 2 and since it really become the single variable, which is sometimes called the lag or difference variable. Usually denote that by separate name, separate notation tau, the tau is the difference variable given by, t 1 minus t 2, so this is the autocorrelation function for the case of ((Refer Time: 54:39)) stationary process, we will stop here and continue.