Communication Engineering
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Lecture - 22
Feedback Demodulators - Phase Locked Loop

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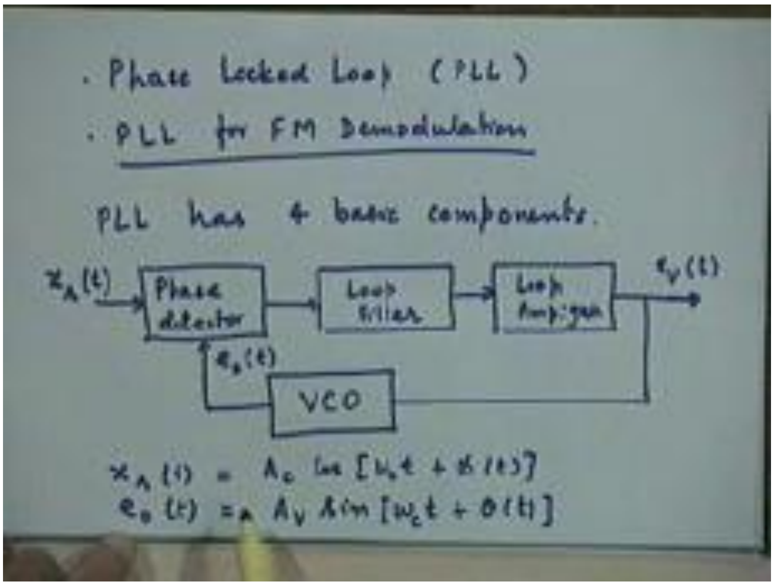

Let us now again look at the Phase Lock Loop which we started yesterday. This is the block diagram of the phase lock loop it has a phase detector, a loop filter, a loop amplifier and a voltage control oscillator. To proceed further let us assume that the input signal to the phase lock loop is an angle modulated signal of this kind, it has some amplitude, the carrier frequency and a phase modulation phi of t. And also assume that the VCO output is another sinusoidal signal with the same center frequency omega c and some arbitrary phase modulation, which will depend on what, is the nature of this signal.

Because, after all the VCO is given by it is input, the input it has some signal, so depending on the variation of the input it will have some modulation here. So, let us start from this onwards, you might notice that I have chosen the input signal to be cosine function and the receive output to be a sin function, that is deliberate and you will soon see the reason of why I have chosen like that. Now, let us proceed further, first of all how do we realize a phase detector, let us first discuss these issue a little bit and then we will look at the operation in more detail.

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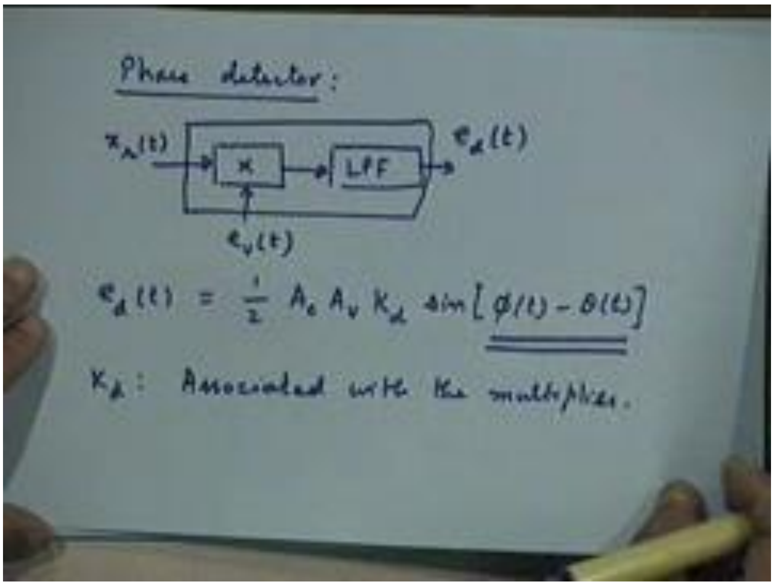

The phase detector a very simple realization of the phase detector is as follows, there are many ways of realizing the phase detector, but one of the simplest ways is to simply have a multiplier followed by a low pass filter. So, for example, if your input here is x r t, input here is e sub v t, then the output out as you can see the output here will contain the same frequency component, as well as the difference frequency component. ((Refer Time: 03:15)) When you multiply these two signals you will get sin into cosine and you can split it into some frequency component and difference frequency component.

And the difference frequency component will be a low pass component, since omega c t is same in both of them, the difference term will continue only the difference between the two arguments phi t and theta t. And the some will be at a frequency of center at two omega c, which will be removed by the low pass filter. So, basically if you look at the output of the low pass filter, let me call this e sub d t that is a phase detector output.

So, I am realizing the phase detector as a set of these two blocks and e sub d t will be half of A sub c into A sub v which was the two amplitudes of the two signals into a constant K sub d which is a constant of this circuit into sin of phi of t minus theta of t you agree with this. So, this combination of a multiplier followed by a low pass filter will produce an output, which is this and as you can see it is this output e sub d t is a function of the phase error, instantaneous phase error. If you consider phi of t as an instantaneous input phase and theta t as an output phase coming from the VCO, then phi of t minus theta of t is some kind of a phase error between these two inputs.

And you have an output here, which somehow depends on this phase error of course, it is not a linear dependence, this is in non-linear dependence, because of a function sinusoid function that is coming to the picture. But, nevertheless it is a dependence of course, when phi of t minus theta t is small, then this non-linear dependence will also become almost linear dependence. Because, you can approximate sin phi t minus theta t as equal to phi t minus theta t when the argument is small.

So, K sub d is a phase detector constant which is associate, in fact this whole thing is a phase detector constant, but K sub d is associated with the multiplier, multiplier in the phase detector. They are other ways of realizing phase detector, but purely this is a general principle you can of course, will other devices. So, this is how the phase detector of the phase lock loop will be characterized choice ((Refer Time: 06:00)) in our model here, because typically you will realize it using this combination.

Let us also look at, how the VCO can be characterized; because once I know the characterization of each block in this loop, then I can proceed with the analysis of this loop. That is what I am going to do get understanding that all of you are looking forward to.

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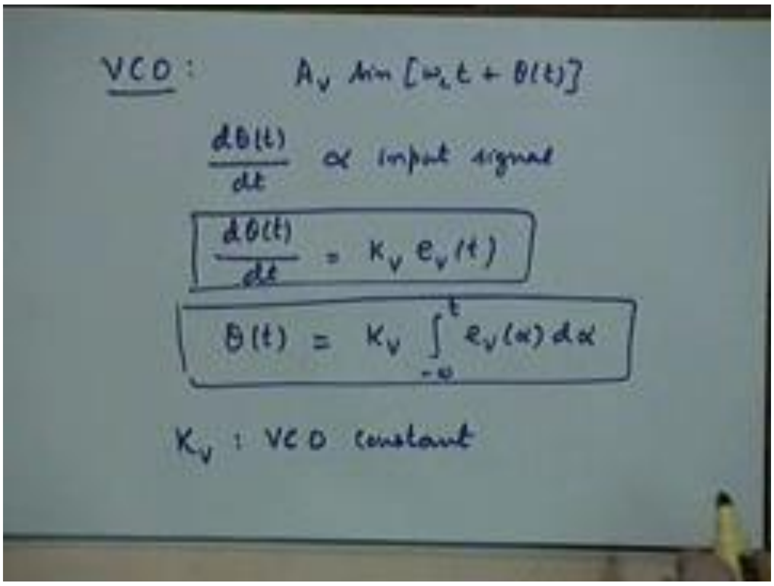

So, how do we model the VCO, the VCO can be modeled in the same way that we modeled an FM signal. Because, after all VCO is performing the same function it will produce an output whose frequency deviation depends on the input voltage that is the characterization of the VCO it is the same characterization. So, what is an instantaneous frequency deviation of the signal that I have just depicted, you are saying that the VCO is producing an output A sub v sin of omega c t plus theta of t.

So, what is a instantaneous frequency deviation, it is d theta by d t this must be proportional to the input signal and if you follow the notation that I have used in the diagram of the phase lock loop input signal to the VCO is e sub v t. So, d theta by d t is some constant let me call it K sub v as a VCO constant times e sub v t or if you want to model the phase itself theta of t is K sub v integral of e sub v alpha d alpha. So, these are the two equations which will model the VCO, the instantaneous phase of the VCO is governed by this equation, which is a standard FM equation really nothing special about it.

So, K sub v here is the VCO constant of course, you can work out what will be the dimensions of all of this constants that we discussed. So, we have now a characterization of every component that we need of course, we already know how to characterize a filter and how to characterize an amplifier. So, I do not have to spend time on that, the two new components that we are introduced were the phase detector and the VCO, which you have characterized now.

So, having got a characterization for each of them, let us develop a mathematical model which will serve as a framework for our discussion or our understanding of the working of the phase lock loop. In this mathematical model it will be nice if I can get rid of this omega c, omega c has no direct role to play in this discussion because, omega c is present in the input as well as in the output, effect for all practical purposes what is that I want I would like to do. I would like to see understand how the phase lock loop produces an output, whose instantaneous phase theta t is close to the input phase which is phi of t.

So, for all practical purposes I can consider phi of t as some kind of input to the system, think of this as a system whose input is phi of t unknown phase, instantaneous phase. And I want to produce an output in a close loop manner, such that the theta t that I am introducing follows phi of t. So I will like to produce a model of that kind, so if I about to do that I can redraw that loop that I have just discussed I have given to earlier in terms of an equivalent diagram like this.

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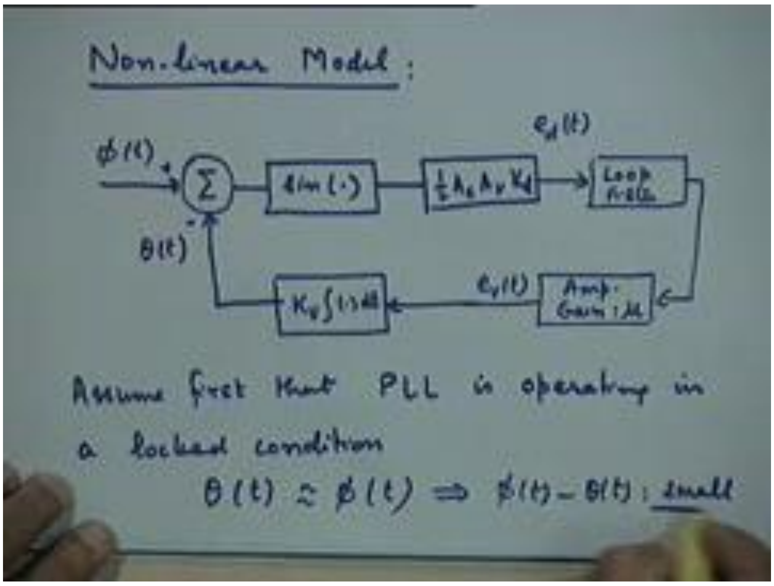

So, I am now talking about a model for the phase lock loop mathematical model, which will help us to do the analysis of course, it turns out to be non-linear model. We will see why it is a non-linear model very soon, what is it that we have. The input we have the unknown phase phi of t, the incoming phase phi of t. And I am comparing the phase detector compares this phase with the phase of the VCO output, which is theta of t.

And what does it do, it produces an output which is of a difference of these two phi t minus theta t and then produces the phase detector output is proportional to the sin of this difference. So, not sign, but sine the sin function, so there is a sinusoidal non-linearity here, this would model your e sub d t, the output here will be precisely what we are describe to be e sub d t is it not? Except that they will be also a constant which is equal to half A c into A v into K sub d t as if there is an amplifier of this scale. Just to recap if it comes from please remember that this was e sub d t ((Refer Time: 11:47)).

So, far I have produced sin of phi t minus theta t you have want to produce e sub d t the way I have depicted earlier I must multiply this with this constant. So, I am adding again of this much in the block diagram, this is your e sub d t at this point following this you have the loop filter, the output of the loop filter goes to the amplifier. Let us say the amplifier has some gain mu, and now what should I put between this point and this theta t to close the loop.

And how will the VCO be represent here, the VCO is not being represented as producing an output which is sin omega c t plus theta t, it is not being modeled as it producing an output which is equal to theta t purely a mathematical model for what the VCO is doing. So, how is theta t is related to this, this is e sub v t is not it, the amplifier output is e sub v t how are these two related through the integral relationship.

So, to close the loop all I have to put is an integrator here, the constant K sub v do you agree with this, I think this should be fairly clear from a development that ((Refer Time: 13:27)). So, this is a mathematical model which we can study to understand how this works and how precisely what it will be able to do and what it will not be able to do. Now, we will do our discussion in two phases, in phase one we will assume that somehow the loop comes in the lock, just to simplify our study somehow the loop is locked. And try to understand, when the loop is locked what is a nature of e sub d t that is phase one of our discussion.

Phase two of our discussion; obviously, should try to tell us, how the lock actually happens. So, should we do that will go through the discussion in two phases, rather than trying to understand everything in one part, so in the first instance assume that the PLL the Phase Lock Loop is in, is operating in a locked condition. You can simply say that the PLL is locked, which essentially means what is a meaning of this, this being in a lock condition, what we are trying to say by this is that theta of t is a good estimate of phi of t. That is what the lock condition means for us it may not be exactly the same, but it is a good estimate of incoming phase phi of t.

This will imply in turn, that the error between these two if at all there is a difference would be small that is the meaning of the lock condition. So, phi of t minus theta of t could be small in a lock condition.

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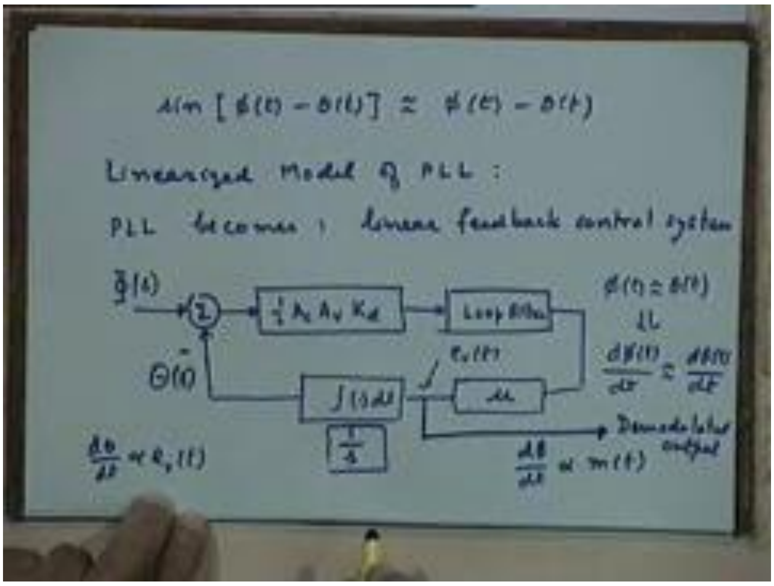

And if that is, so we can approximate sin phi t minus theta t by phi of t minus theta t do you agree with that, assuming that this is small. Now, when the loop is not in lock condition we cannot say that, because this may be considerably large in that case. So, when the loop is not in lock condition this approximation cannot be used, when the loop is operating in the lock condition, it is convenient to use this approximation. Because what it will do is, ((Refer Time: 16:13)) it will convert this non-linear model, why is it a non-linear model because of the presence of this function here.

Otherwise, everything else is a linear, if you look at the input output relationship it is a linear system, every block is a linear system, this is the simple amplifier, this is a linear filter, this again an amplifier, this an integrator, everything is linear, this is an adder everything is linear except for this block. Here, the input is something and output is sin of that, input is some phi t minus theta t the output is a sin.

So; however, if I use that approximation this also becomes linear, in fact this goes this is not even required, I am removing this and that makes it a linear model for the phase lock loop. So, that gives us what is called the linearized model of the phase lock loop and once you have this of course, I thing you do not have you have not yet gone through a cohesion control systems. Once, you have this it becomes the very simple case of a close loop a linear feedback control system, it does not matter we will not need that background we will whatever we need we will develop it here, we will use it here.

So, PLL becomes then a linear feedback control system and when you are working with linear feedback control systems, it will you will find it is very convenient to work not only in the time domain, that we are discussed. In fact, it is more convenient to work in the Laplace transform domain, rather than the previous Fourier domain, so very briefly will also work with the Laplace transform domain, all the most of the time we will not.

So, if I work with the Laplace transform domain, this linear model becomes something like this, with the Laplace transform domain I will represent every function that I have been, so for dealing with as function of s. So, instead of phi of t at the input I will consider it Laplace transform phi of s as an input capital phi of s is the Laplace transform of the phase function phi of t small phi of t. And here, you will have theta of s it is a capital theta of s, the sinusoidal non-linearity now goes I will not have been able to use this Laplace transform notation, if I had the sinusoidal non-linearity.

Because, I will not know how to characterize that using the Laplace transformation, I can only deal with linear systems using Fourier transforms and Laplace transforms, that is the convenient thing to do. So, that sinusoidal thing goes because of this approximation, you only have the next stage which is again half A sub c, A sub v into K sub d followed by the loop filter, followed by the amplifier let me simply call it again mu followed by the integrator the e sub v t is here.

Student: ((Refer Time: 20:11))

Yes, so this should have been simply 1 by s I should represent this as 1 by s, you are absolutely right. Now, since we assumed that the loop is in a locked condition, which means phi of t minus theta of t is small and some let us say constant value. Basically what we are saying is phi of t is approximately equal to theta of t, this would in turn imply that d phi by d t is approximately equal to d theta by d t.

But, what is d phi by d t, d phi by d t is proportional to the message signal m t if the input signal is a FM signal and d theta by d t is proportional to e sub v t. And since, these two are approximately equal what is it mean, that e sub v t would be approximately equal to the message signal m t. So, therefore, this output here is my demodulated output; this is a point that at the moment you need to understand e sub v t represents my demodulated output. The point therefore, I can summarize the discussion in the following way, when the loop is locked.

Since, this will be, so this will be, so and therefore, e sub v t would approximate the message signal m t. And therefore, the input to the VCO is the demodulated output that we are looking for is that clear. So we are therefore, finish the discussion on what I mean we have to still of course, understand the mathematical analysis of this view, which will do as we go along. But, right now even without going through any mathematics, if we assume we can conclude that in the lock condition the e sub the output e sub v t is proportional to the message signal m t that we are looking for. In FM demodulator and therefore, the phase lock loop in a lock condition works like an FM demodulator.

You all with me is there any question, any discussion at this stage, we have to still do the mathematics. But, without the mathematics this is where it is, with the no questions let us proceed further and the next step that we have to take up is how does the locking actually occur, you have to understand how does a PLL walk into a lock condition.

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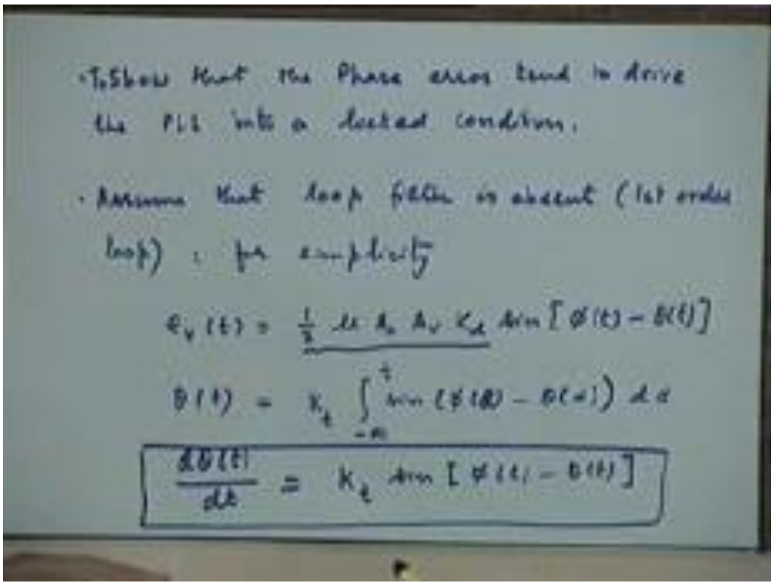

So, we like to show that the phase error that we have at any instant whatever phase error we have will tend to drive the PLL into a locked condition, that is the second stage of our discussion. We like to show that whatever instantaneous phase error the loop has or whatever error we have here. ((Refer Time: 24:37)) Assume by using this diagram I should be using this diagram, whatever error we have here at any time will serve will operate in such a manner that it will try to drive with serve to 0 or make it itself small.

The loop will work in such a manner that this error will tend to become small; the loop will tend to be in a lock condition. Remember, if doing that I cannot use the linear model, because initially the phase error may not be small. So I must now work with the at this stage work with the non-linear model, once a lock has been achieved we can work with a linear model, so first let us try to understand this thing.

To carry out this discussion I will again to simplify the discussion somewhat, I will assume that the loop filter is absent. Remember, there is already a filter which is inbuilt into the phase detector there is a low pass filter there, that we are assuming is still there, but the additional loop filter that I have put here is not there. When, the additional loop filter is not there we call this as a first order loop, as to why we call it a first order loop is something will become obvious later, but right now we will assume that this is absent.

So, let me state this assumption we will assume that loop filter is absent, first order loop and we will see we will find that the PLL works, even when there is no loop filter. And then, will try to understand why a loop filter in what manner a loop filter might help, at the moment we will assume that the loop filter is not present at all. So, this is for simplicity, so we have e sub v t equal to half mu times A sub c, A sub v, K sub t into sin of phi of t minus theta of t and theta of t phi of t is a input, theta of t is a output.

Let me denote this complete thing as K sub d as a constant K sub d, rather than carrying on all this constants let me throw out K sub t, this t of course, does not denote time you can just this t stands for total gain. You can think of this as some kind of a gain in the loop, this is a gain term, this is a gain term well these are amplitude terms, but effectively they are coming in this product. So, this is called the loop gain total loop gain, that is why I am putting K sub t, so do not confuse this t with time, this times for total I could have probably put capital t to simplify, but let us keep it small t.

So, K sub t integral of sin phi of t a phi of alpha may be minus theta of alpha into alpha is there an ((Refer Time: 28:06)) you see where is phase is coming from, you are looking at this output, this is the integral of this. Now, we because the loop filter has gone e d t is nothing but, equal to e sub d t except for a constant mu, so therefore, e v t is same thing as e d t except for including mu. I am able to do this, because I am assume ignoring the loop filter, because I am assuming that the loop filter is absent.

So, these two things becomes the same except for the constant mu is that, so that is why e sub v t is this, which is this and integral of this is the theta t is that or alternatively. I can write d theta by d t which is the model for the VCO, this is also model for the VCO can be written as K sub t sin phi of t minus theta of t, because this is from minus infinity to t and we are differentiating with respect to t.

And this is actually the governing equation which dictates the dynamics of the loop, you understand at any time instant phi of t is the input phase; theta of t is the output phase. The output phase at any time instant as a function of the input phase is governed by this non-linear differential equation do you appreciate this. So, this non-linear differential equation which we have now arrived at provides it needs to study, how theta of t will evolve as a function time.

If I know how to solve this equation, if I know how this equation will behave that will give me an understanding of how theta of t behaves as a function of time with respect to phi of t that is what I need to study. Unfortunately, because this is the non-linear equation it is also very difficult to study, so we need to evolve special methods of trying to get an understanding of how this works to do that… Any questions, so far before proceed further to do the discussion. Further, will do the discussion when phi of t is of a special is a special kind of function, rather than for a general kind of function.

To look at a general solution of this is a very cumbersome very difficult exercise to simplify the discussion further. Let us assume that phi of t has a very some special simple nature, what is it mean phi of t represents what represents a phase modulation of the input I am going to make that modulation very simple, that is all I am saying. Because study the effect and to study how the phase lock loop actually achieves lock.

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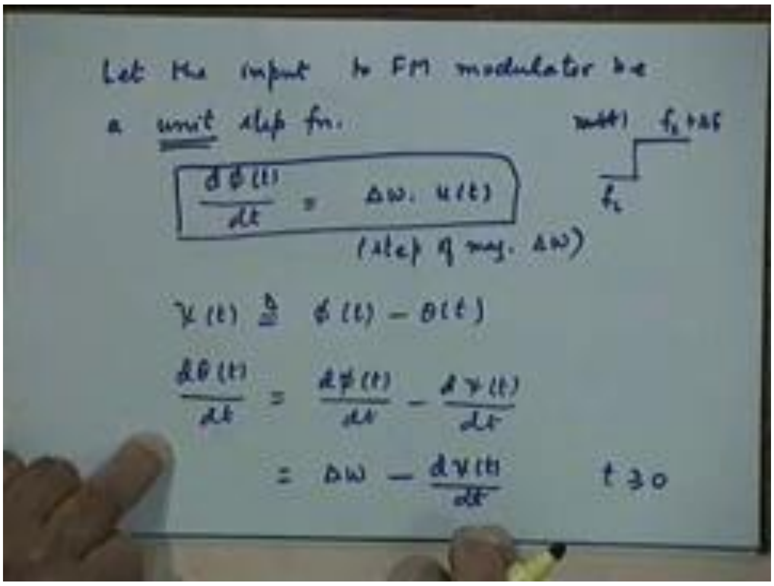

So, to do that what I am going to say is let the input FM signal, let the input to the FM modulator of the transmitter give a very simple kind of signal. And the simple kind of signal I am considering here is a unit step signal, unit step function not a general m of t, but m of t is equal to u t, that is the simplification I am bringing about. So, let the input to your FM modulator be a unit step function, what does it mean? That your d phi by d t is well let… I will not assume call it as proportional to unit step function is equal to delta omega times, that is the instantaneous frequency.

Let us say was constant let us say the input signal was having the carrier was the FM modulator was producing a constant frequency, carrier signal initially. And suddenly you provide a step change in the frequency that is equivalent to m t b step function is that, so you are producing the step change in the input signal. So, if you take your m t to be a unit step function, such that this change in amplitude of the input to the modulator produces a frequency change of delta omega, the frequency deviation.

And now it becomes it, so from f c it becomes f c plus delta f, the output this was m t. But now I am plotting the carrier frequency, initially the carrier frequency was f c now becomes f c plus delta f. So, this is a kind of simplification talking about, so this is a step of magnitude delta omega. ((Refer Time: 33:32)) Let me denote the phase error phi of t minus theta t by a symbol psi of t I am going to call the phase error.

So, let me define sin of t as the instantaneous phase error, and then I can write d theta by d t is equal to d phi by d t minus d psi by d t that comes from this equation. But, d phi by d t is modeled by this equation, this is delta omega I am writing the equation only for t greater than 0. So, that I do not have to work worry about the fact that is the unit step function involved at t equal to 0, so let me write this equation only for t greater than or equal to 0. So, this is delta omega minus d psi t by d t for t greater than or equal to 0 and what is d theta by d t equal to in turn remember your d theta by d t it is k t sin of psi t.

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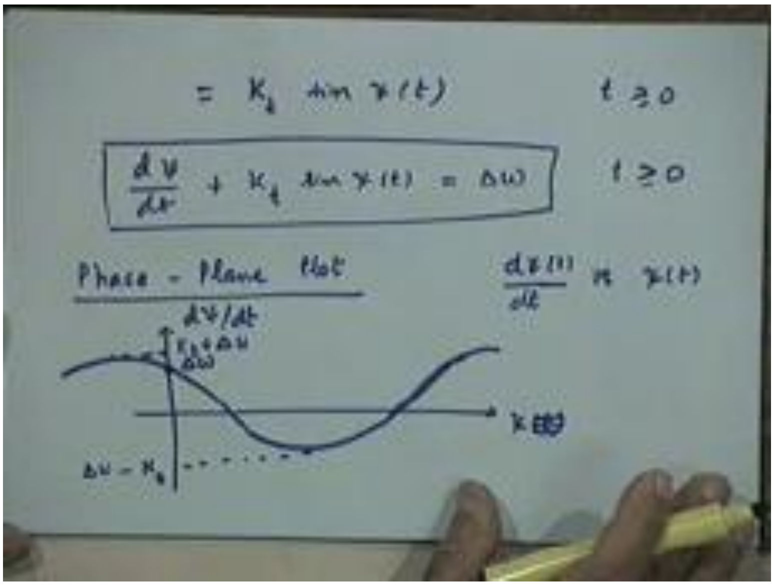

So, this is equal to K sub t sin c sin of psi t, so what do I have now for t greater than or equal to 0. Basically, what I have derived is this equation, if you look at these two together I have a differential equation for the phase error, which I can rewrite as d psi by d t plus. I am taking this to the right hand side plus K t sin of psi of t is equal to delta omega for t greater than 0.

So, for the special case when the FM signal is essentially a step change in a frequency at the input to the PLL, the governing equation for the phase error is this differential equation. And we like to understand, under what conditions will this differential equation imply a movement towards psi of t becoming 0, how the psi of t becomes 0 will when I want you become 0 should come out by stating this differential equation. Now, when you want to study non-linear differential equations, one pictorial one physically appealing way is to represent this behavior of this equation, pictorially by in a diagram called the phase plane plot.

So, we like to just see how that happens, so we like to try to study this through a device called the phase plane plot. The phase plane plot essentially is the plot of these quantities d psi by d t versus psi of t that is at any time instant how is the derivative of the quantity related to the quantity itself. We plot this against this think of this as x and this as y and plot that let us called a phase plane plot.

So, we want to plot this x axis is instantaneous phase error just you can remove the timer dependence. So, that it does not confuse us, just plot psi or d psi by d t versus psi what is the nature of this, think of this as y and this as x, so y is equal to y plus K t x is equal to delta omega. And you are plotting y against x, what kind of function is this, this is a sinusoidal function y plus K t sin x what is the value, let us say for psi is equal to 0 delta omega. And so you may have a function like that from that plot a good sinusoidal function, we can plot a better one this point corresponds to delta omega.

So, then this value will be equal to what is the peak value of the sinusoidal function, what is the peak value K t plus delta omega and what will be the value here delta omega minus K t, can you all of you see this, at these points d psi by d t becomes 0 at these two points that is the way to look at it. So, this point it will be equal to at one point it will be k t plus delta omega, at the other points will be delta omega minus k t, so this is the phase plane plot. So, look a study of this phase plane plot is very interesting and gives us all the insight that we need to understand how a phase lock loop locks itself. So, but before we do that is this plot alright, all of you understand this.

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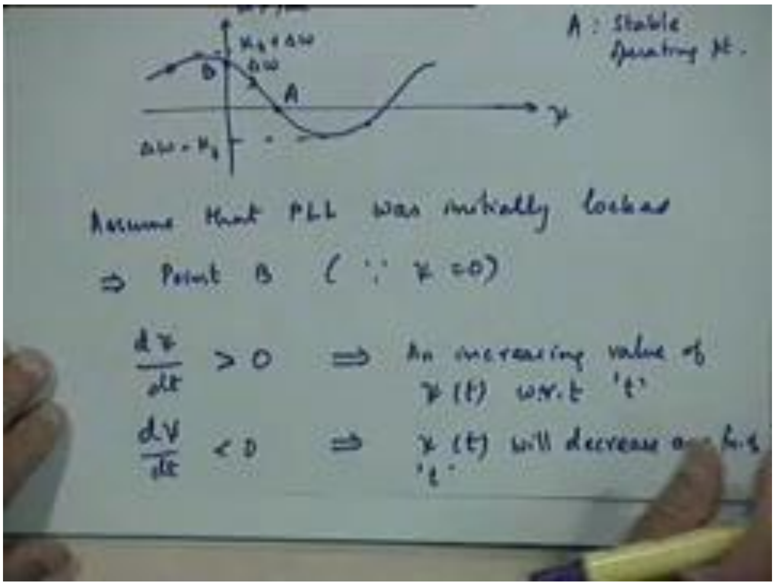

Let me redo the plot; so that we can have a better discussion, I hope this one turn will turn out to be better. So, this is d psi by d t versus psi, this is delta omega, this is K sub t plus delta omega, this is delta omega minus mu sub t what we are saying is the following. That the dynamics of the phase error will be such that d psi by d t and psi at any time instant must be consistent with respect to each other through this diagram. If d psi by d t is this value, the corresponding phase error must be this value fully it is a solution of an equation depicted diagrammatically in some sense.

So, the values of d psi by d t as psi must satisfy this relationship which is shown in this graphical form at any point in time. Let us assume to start with when the input signal of a constant frequency that is what we are done, we assuming that the input signal was at the constant frequency and suddenly there is a step gain in the input frequency. So, when the input signal was at a constant frequency for a long time let us assume that the loop was in lock earlier.

When a loop was in a lock condition, where was it psi was equal to 0, so you are at this point. So, assume for a discussion now that PLL was initially locked, so you are at this point let us call this point b, because at that point psi was 0, the phase error was 0. Now, what will happen that let us also understand another significance in the phase plane plot before you take the argument further. Let us consider at a certain point in the phase plane plot where d psi by d t is positive.

Now, once this is, so what will be the nature of psi of t as a function time, what can you say, can you make any statement remember d t is time increment in time, this will always be positive is not it, time only progresses forward it may never goes backward. Therefore, d psi will always imply the positive increment, so if you are at a particular point in this phase plane plot, where this is positive with the derivative is positive, which imply that a from this point onwards, the phase error will increase as a function of time.

Because, the derivative is positive, so psi of t must increase as a function of time slope is positive, if on the other hand. So, this will imply an increasing value of psi of t, psi of t increases as a function of time with respect to time, talking about with respect to time is this point understood is the very key point. We need to understand this point when d psi by now I am talking of the temporal behavior, time behavior of the signal, time behavior of the phase error.

Because, d psi by d t is positive at a certain point what we are saying is that will imply that the dynamics will be searched that the phase error will increase as a function of time. Similarly, on the other hand when d psi by d t is negative psi of t will decrease as a function of time what does it mean? That if you all in the positive side of this phase plane plot anywhere, we will move along this trajectory the d psi by d t and psi of t always have to be related by his trajectory, you can only move along this.

But, when d psi by d t is positive that is you are in the positive side of this plane, positive half of this plane, the phase error will try to move to the right, you will move along the trajectory to the right. Because, the phase error has to increase as a function of time, no matter where you are, you are in a positive side you will keep moving to the right, if you are in the negative side we will keep moving to the left along the trajectory are you with me on this.

Student: ((Refer Time: 45:46))

Yes, the time dependence is not obvious here, but the fact that this is positive, it will imply that psi of t has to increase as the function of time and where is psi increasing to the right. Suppose, you are here how can you are not showing exactly, what is a rate at which it moves, but it has to move in this direction that is all we are saying, we are not able to say at what rate it will move. But, we do know that is move on the right hand side to the right hand side, here it will move to the left hand side, because psi of t has to decrease when d psi by d t is negative psi of t has to increase.

So, in increases from this direction to the right side, decreases to the left side that is all we are saying. So, we do not know of course, at what rate it increases and at what rate it decreases, but the fact that it increases means that it will move along this direction at a certain rate, precise rate from this diagram we cannot figure it out. But, eventually where it will go we will be able to figure out, if this discussion is then we can proceed forward.

Because, this is the way we understand the dynamics of a non-linear system through the phase plane plot, mean what I am really telling you is how a phase plane plot should be interpreted to understand the dynamics, if you understood this then we can move forward alright. Now, suppose initially you are at this point b, then the loop is initially locked, so where will you start moving now, will move start going towards the right, till you reach this point what happens to d psi by d t at this point it become 0.

So, it stops to increase if d psi by d t becomes 0 it stops to increase and in some sense a lock has been achieved. But, lock has been achieved will not be the 0 value of the phase error, but with some finite value of the phase error, but we do not know whether it is a really a perfect ((Refer Time: 47:58)). Suppose, your let us say you are reached somewhere here by some chance, what will happen now will try to move to the left again to the same point.

So, even if you have deviated from this either in the positive side or in the negative side will tend to return to this point. So, you can think of this point let us call this point a as some kind of a stable operating point of the system, so point A is a stable operating point, you say that this will not be a stable operating point as you can see, this is a quasi stable operating point on stable operating point. Because, if you slightly move on either side you will move away from this point rather than, towards this point.

So, in summary in the upper half plane we will move along the trajectory to the right side in the lower half plane, it will move along the trajectory to the left side and that leads us to the conclusion that point A is a stable operating point of the phase plane plot. So, at this point when there is an attempt to move away from this the loop will try to return to this condition.

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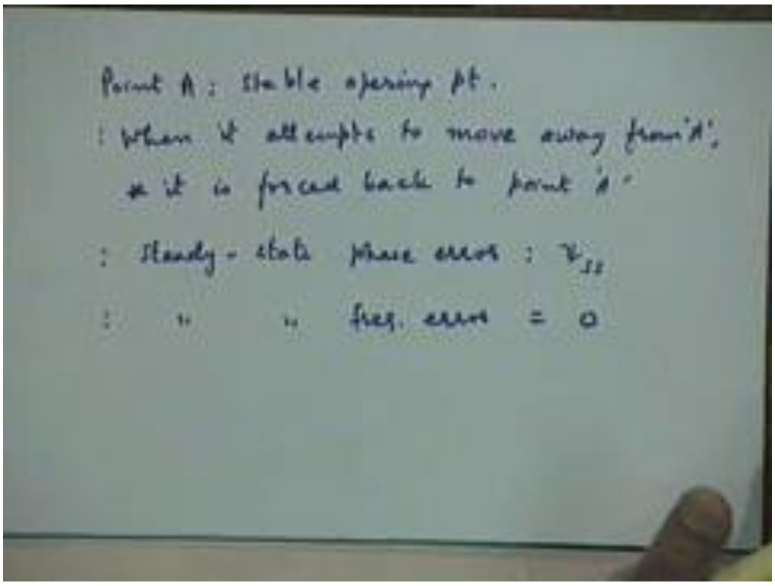

So, point A is stable operating point as I just mentioned earlier and the implication of that is when it attempts to move away from point A. And the operation attempts to move away from A, even by a small amount it is forced back to point A, we can also call it the steady state operating point. So, what we are basically trying to say is that in this condition ((Refer Time: 50:33)) the loop will try to arrive to this point, when you start with psi equal to 0 it will not stay at psi equal to 0.

Because of the step change in delta omega, the step change the input frequency, will now try to reach a point where there is a steady state error phase error, there is a steady operating point. But, at that point there is no frequency error, because d psi by d t is equal to 0 and think of d psi t when if psi t is the instantaneous phase error d psi by d t is the instantaneous frequency error.

So, the loop would have locked itself that is the VCO would have locked itself to the new frequency; there is no frequency error at this stage. But, that reduction of frequency error to 0 is associated with the finite phase error in the process; the phase error does not remain 0. So, a steady state phase error is called the psi sub s; s is some value ((Refer Time: 51:50)) that value is governed by this point, this is the value of psi s; s will write down an expression for this later. But, we can also see that the steady state frequency error how much is this equal to 0 in it is lock condition.

So, we will stop here try to study this project, try to write an expression for what is the steady state phase error and then try to understand what does the loop filter achieve. Under what conditions lock will occur, under what conditions it will not occur, we like to study all these things next time.

Thank you very much.