Communication Engineering
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Lecture - 20
Demodulation of Angle Modulated Signals (Contd.)

Let us continue our discussion on Demodulation of FM signals or Angle Modulated Signals.

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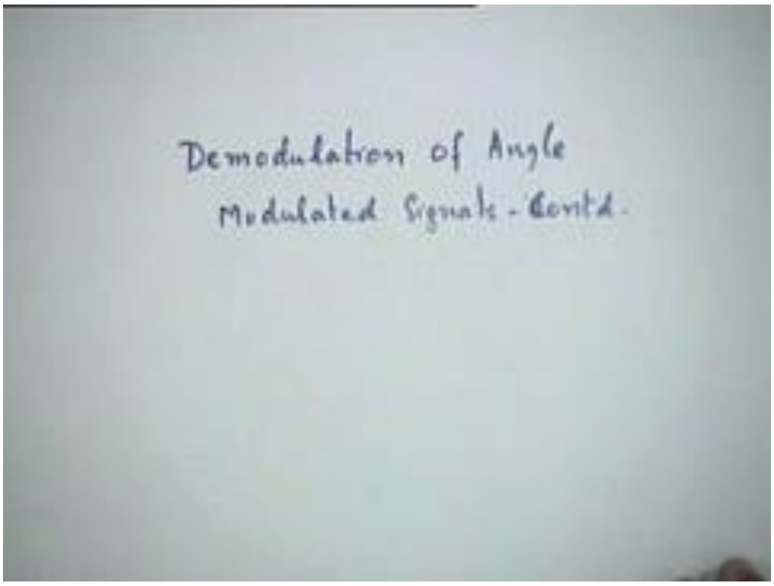

Let me very quickly revise for you, what we discussed last time, basically for demodulation of angle modulation signals. We need particularly FM signals; we need a device which produces a response proportional to the frequency deviation of the input signal.

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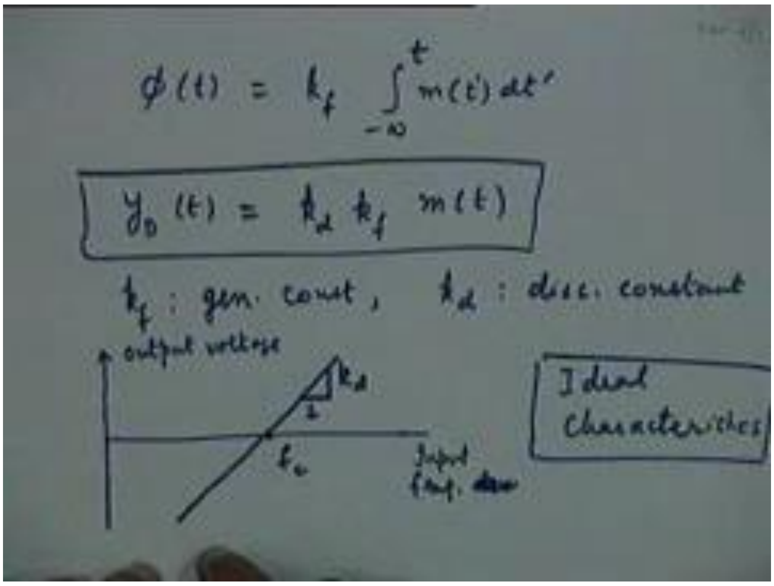

So, if you remember, we read ideal of a characteristic that we would like from, so called frequency discriminator will be something like this, it produce an output voltage, which is linearly dependent on the input frequency deviation. Now, such a device, we call a frequency discriminator and sometimes also called a slope detector. Because, basically we are following of this device, basically we are producing an output, a detector output, which is proportional to the slope of this line here. The approximate serialization of this, we said could be done by having a differentiator, I am just quickly reviewing, what we did last time.

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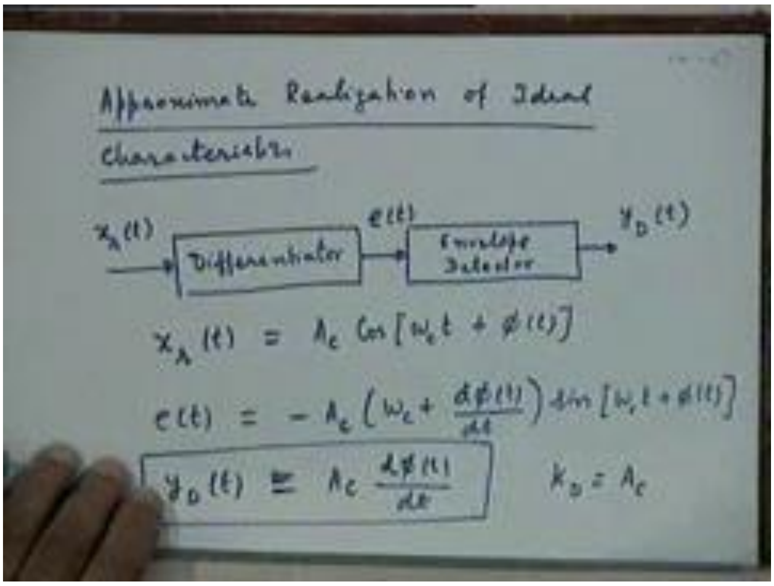

The differentiator followed by an envelope detector and we discuss, how this would approximately produce, what we want or in fact more or less, what we want.

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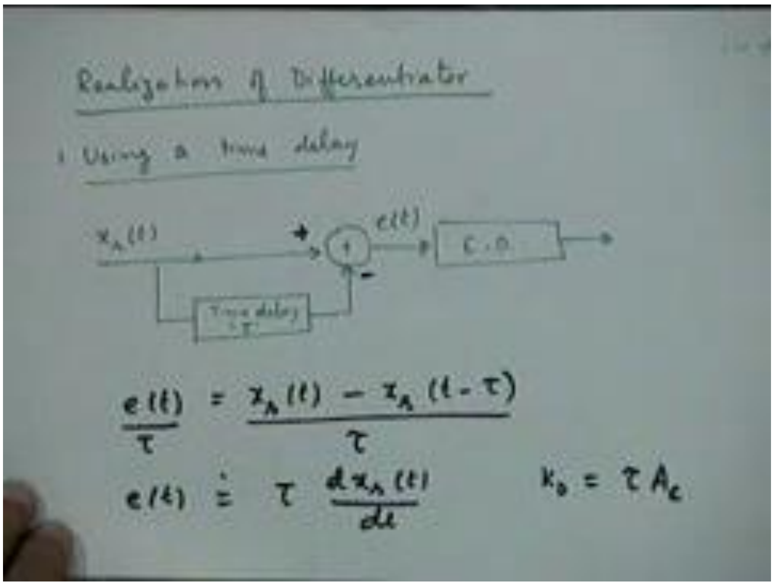

The only thing to discuss was how to realize these differentiators and we discuss two realizations of this differentiator, one using a small time delay that is using a time delay and other using a bank pass filter that is why we do it?

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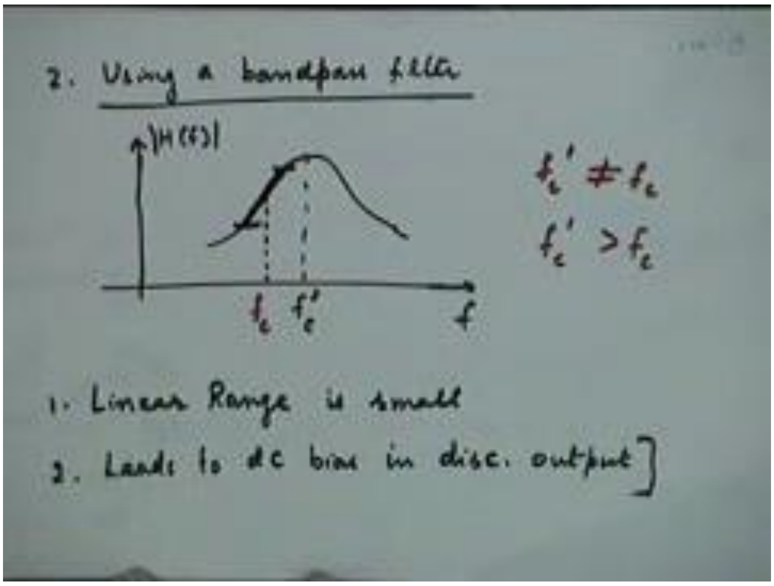

So, let me quickly come to the bank pass filter, we can use a bank pass filter, which produces a frequency sensitive response around the carrier frequency by choosing the center frequency of this bank pass filter to be away from the actual carrier frequency. So, we call it staggered unit that is the bank pass filter is tune to a frequency, staggered from the actual carrier frequency.

The only difficulty with this approach is, the linear range, the linear frequency sensitivity that we desire from such a filter is rather small. And the second difficulty is, it does not produce a 0 response and a carrier frequency accepts, which is what you like to have. And as to take care of these things, we suggested an alternative, which is what we are going to discuss now, things likely more detail and that is used bank pass filters.

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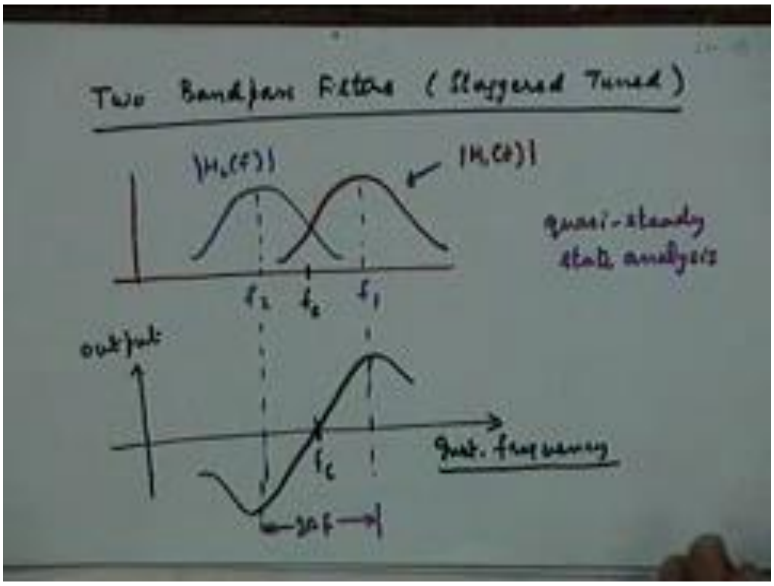

Each of which is tuned to a frequency, away from the carrier frequency of interest, one is tune to a frequency larger than the carrier frequency and the other is tune to a frequency smaller than a carrier frequency. And the envelope detectors, following this will produce an output, which is proportional to the output of these filters. So, this magnitude response that I have plotted here is also in some sense is representation of the magnitude of the voltage, that will be produced in the output of the envelope detector.

So, if you have an FM signal as the input, which will varies on frequency along this axis that as a function of time, the output voltage, the output of the envelope detector will vary accordance to this curve. Because, the response of this circuit will be in accordance to this curve, incidentally where I am making this treatment that is the slight approximation involved in this treatment.

I do not know, whether you appreciate that fact or not, but I come back to this point in minute. Similarly, this other twin circuit will produce an output voltage, proportional to this curve. Now, what is a approximation involved here, the approximation involved here is as follows, your input signal is we are really not mean, what is called a steady state analysis, we are not really make making a steady state argument here.

When, we talk about the output of a circuit as given by the input spectrum, multiplied by the transfer function that is the steady state statement. As the string, from a translucent behavior, but here we are slightly, we are working actually in a translucent domain. If you really think about it, is our input signal is constantly varying in frequency where, we are saying that the output will be proportional to whatever is the corresponding magnitude of that time.

Really, speaking your input is constantly changing it is never in a steady state in that sense. So, even if a slight approximation in the argument that we are giving here, but this is usually valid for the case of FM signals. So, because of variations, it is slow enough and we can assume that this steady state accrues; steady state is approximation is valid. So, as an input signal frequency varies, the output of this circuit will vary accordance to this curve.

So, please try think about this point that I making this is this kind of analysis is called quasi steady state analysis, because we are not ever in a true steady state, when with the FM signal. Because, here input signal frequency is constantly varying, which is different from input signal, which simultaneously has a large number of frequency components, but here we are saying that at different instance.

It is suppose to have a different instantaneous frequency, we are not looking at it in spectral terms, we are looking at it in terms of it is behavior at a specific point in time. And as that, is treating a frequency changes, we are saying immediate to the response or situations, that is the point, that we need and that is really this approximation, that I am talking about the Quasi steady state approximation.

So, nevertheless if you assume that this approximation is valid, we can see that if we take a difference between the two outputs, the two bank pass filter followed by envelope detector outputs, we will have this kind of s curve. And now, if you restrict your input signal to have your peak frequency deviation to lie in this linear range. If you this point, goes only up to our input signal has a frequency deviation only up to this point or on the other side up to this point.

Then our input signal will always be in the linear range of this characteristic and not only that we would have produced a 0 response and the carrier frequency as desired. So, this is a very good approximation to what we actually want to do, any questions on this.

Student: ((Refer Time: 8:30))

These are the outputs of two filters, followed by an, I will draw a diagrams to it soon; basically we want an output which is proportional to the magnitude of this response. And mind you, what are we looking at, we are looking at the frequency discriminator, which is consisting of a really speaking an ideal differentiator, followed by an envelope detector.

So, we now instead of an ideal differentiator, I have this kind of a circuit followed by an envelope detector. So, what will a envelope detector produce, it will produce an output, this magnitude output, envelope this magnitude is proportional to this and suppose the input signal frequency is this much, the output on the bank pass filter will have the bank pass modulated signal, whose amplitude is this much. And the envelope detector will simply be producing an output proportional to the instantaneous amplitude, the modulation will go.

So, basically that is what we are talking about, we look at a circuit this how, basically that differentiator followed by envelope detector is being replaced with two filters of this kind. Each followed by a corresponding envelope detector and the outputs of these two are subtracted from each other. So, basically the system that we are depicting here is something like this.

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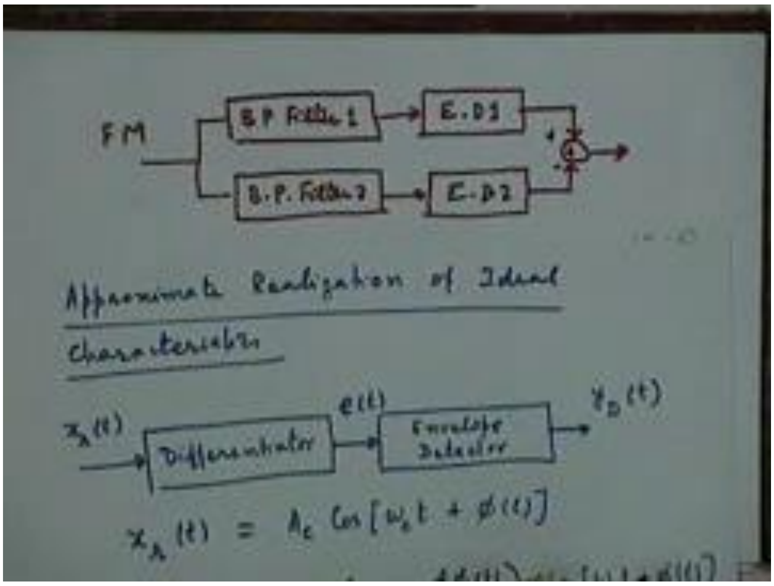

You have bank pass filter 1, corresponding to H 1 f, so the bank pass filter 1, corresponds to this H 1 f, this is followed by envelope detector 1. Similarly, you have a bank pass filter 2, followed by the corresponding envelope detector and we want the outputs of the 2 to be subtracted from each other. Why are we using, I will second question first, why we using an en envelope detector, what was that we are trying to do, we are trying to convert our input FM signal into an AM signal.

Because, there the variations in the frequency, due to the message signal are being converted to variations in the amplitude using the message signal. That was idea of using the slope detector in which we have a differentiator, followed by envelope detector, so please consider, what we discuss last time, this is something that we must understand.

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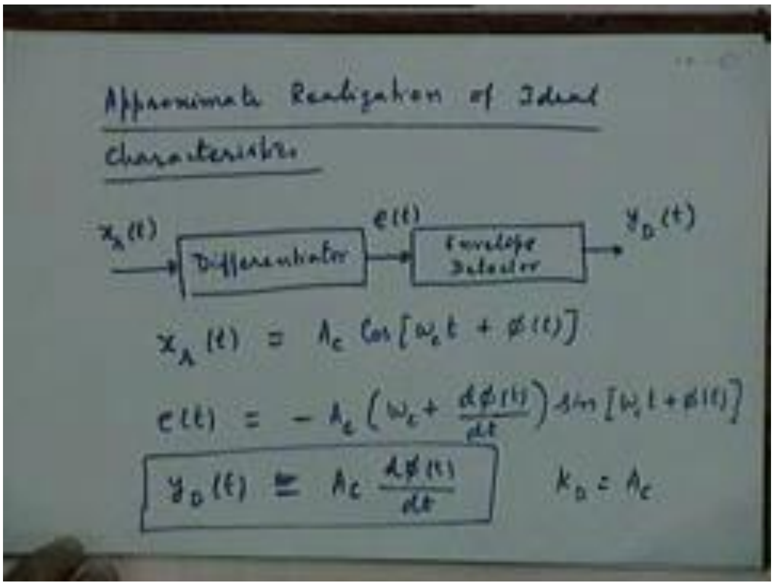

Well, here was your differentiator and this is what it was doing, it was converting this FM signal or angle modulated signal into an amplitude modulated signal. The only thing is we are replacing this; we are trying to realize this differentiator by different means, let me complete the answer. This differentiator, it was suppose to have an ideal frequency response of g omega is being approximated or being approximately realized by combination of bank pass filters.

Basically, I am replacing this differentiator or basic replacing this kind of a circuit with another equivalent circuit, which will do the same thing. How, is it doing the same thing, basically what we want is an output, we are not interested in this part. This is the carrier containing the model same modulation, we are not interested in this part, we are only interested in is taking this amplitude.

So, what will this bank pass filter do instead of differentiator, I have a bank pass filter, which will once again produce an output, whose amplitude will depend on the instantaneous frequency. That is a passive state argument, I am giving that at any given time instant, you have a certain frequency present the input signal and the bank pass filter will respond, according to the input frequency.

So, bank pass filter 1, for example suppose the institute frequency somewhere here, will produce an output, which is a sinusoid again at that frequency, sinusoid at this frequency, and let us see modulated sinusoid. But, the amplitude of their output will be the proportional of this and as the input frequency varies, produce an output proportional to this, but all the time, we are talking of the amplitude.

But, this signal is always still present, the carrier is always still there, the amplitude of the carrier at the output of the bank pass filter is varying in accordance with the filter response. And now, we want to remove this carrier itself, how do we remove the carrier, through the envelope detector that is the basic idea. So, hope it answers, both you your question as well as your question.

Quasi steady state is argument is being invoked, because we are assuming that as the input signal varies in frequency, instantaneous as the output magnitude changes in accordance with the response of this filter. That is a quasi steady state approximation that I talked about, so these are the two arguments that you need to understand. So, hopefully that we can realize these characteristics to the combination of two banks pass filters, each followed by a corresponding envelope detector.

Because, then you will get an output, which is proportional to this, in the first filter, in that first branch, proportional to this in the second branch, then you subtract the 2 outputs, you will get an output voltage. So, that we use a black pen here, think this as the output voltage, which is proportional to the which is given by these characteristics versus input frequency or frequency deviation or instant, I think best thing is to call it instantaneous frequency at the input. Though, it is no were impulses here, how does this question come to your mind

Student: Sir, I would saying that H 1 f and H 2 f are

They are the class's functions of the two bank pass filters; they are the frequency domain requisitions looking for bank a pass filter that is if you feed sinusoid at different frequencies, how your response did varies.

Student: ((Refer Time: 16:00))

So, the frequency response of the entire system becomes something like this, I think some of you have probably not understood what I am trying to say.

Student: ((Refer Time: 16:20))

So, that is the different question, let us try to understand, what we are doing, whether it is doing a job that we want it to do or not. As to why, we go in to many other things is something that I think at this point, it is difficult to explain, because there will be many such possibilities, which need to see, why they will not work, that will not work. Why, what you want is an output proportional to the instantaneous frequency, what we are getting from the top system, if please try to understand if this point is clear.

What we are getting from the top system, the top branch here, if you look are this output, the input to this envelope detector is an amplitude modulated signal. However, the amplitude of this is proportional to the input signal empty, only if amplitude lies in certain range. Otherwise this is amplitude modulation, but there is non-linear modulation, as input frequency varies, the output amplitude varies, according to this at this point.

But, you still have a modulated carrier present here; this is the important point to understand here, there were modulated carriers, which contain only frequency modulation. Here, we have a modulated carrier, which contains amplitude modulation, but the amplitude modulation is proportional to the message signal mt. Only in this interval and I mean suppose your input frequency varies beyond this, will contain some kind of a non linear amplitude modulation.

That is why, we do not want to go beyond this, that is why we want to keep make sure, that your input signal frequency lies in this feature that means you need to show your peak frequency deviation is less than f 1 minus fc on this side and fc minus f 2 on the other side. But, important point is you get an amplitude modulated signal here and all you want to do is check this out, detect out, amplitude of this output.

You have a modulated carrier; you want to get an output proportional to the instantaneous amplitude of the carrier. So, what should we do, use an envelope detector that is all, envelope demodulator, similarly here, you set for the 2, you get what you want, if you appreciate this, other questions will disappear. So, this is the thing to really appreciate, think about it, I think it will be clear, if you really think a little more about it yourself. But, if you have some specific question at this point, I am ready to stop for a couple of seconds and see what those doubts and questions are.

Student: ((Refer Time: 19:17))

2 delta f is approximately remember the constant formula is 2 delta f plus twice of f. So, approximately this is also proportional to bandwidth, but our argument is not based on of the bandwidth of the FM signal our argument is based on the peak instantaneous frequency deviation.

Student: ((Refer Time: 19:47))

I dint get that point, can you explain again

Student: ((Refer Time: 20:00))

Oh, yes, yes the each individual filter will be a small linear range

Student: ((Refer Time: 20:14))

But, I mean just look at this is precisely, why we are doing all this, it is because that each individual filter has a small linear range and using two staggered filters and the difference output reduces the characteristic like this. So, the range of this is let us say somewhere here, the range of this is somewhere here, but together the range becomes this.

So, if I choose by frequencies f 1 and f 2 in accordance with my delta f, I have done my job, I have increase my linear range. Suppose, I have a certain peak frequency deviation to work with as I say 75 kilo hertz, so what should I do, how should I design these two filters? I should choose the center frequency of f 1 to be 75 kilo hertz, away from fc similarly f 2, I should select 75 kilo hertz away from fc on the other side.

So, we have choose the frequencies f 1 and f 2 to take care of that peak frequency deviation that I will have on either side of fc, I would have got a linear range that I need.

Student: ((Refer Time: 21:27))

No, all the frequencies are passing through both the filters, but they are producing non linear amplitude responses, by subtracting the 2, I am making sure that over the entire range of interest, we get a linear response. Because, the filter is a filter which will respond to the input in accordance with it is characteristic, whatever frequencies are present in the input are being processed by both of them. But, this response to the input in a particular way, this response the input will be different way.

And when, two are combined, this is the net way in which they are together effectively seemed to be responding to the input. So, I hope with these questions and answers some of these things are clear, but we settle in it to interest that and think about it to make it absolutely clear to yourself.

Student: ((Refer Time: 22:24))

Actually, phase response is of no consequence, because we are only looking at the amplitude of the output, ((Refer Time: 22:35)) we are only looking at how the instantaneous amplitude varies as a functional type. As the input signal amplitude changes, we were like the output signal amplitude to vary with the input moderating signal amplitude linear, that is what we want.

So, first we have we converted the amplitude variations in frequency variations that is what frequency modulation is all about and now through the artifice of these two bank pass filters and converting those frequency variations back to amplitude variations. And then, detecting the amplitude variations through the envelope demodulator, think about it.

We look at the mathematics of it also very soon, first I watch it with the physics effect, now how do we realize this in practice, well we can realize it exactly a the way I have shown it.

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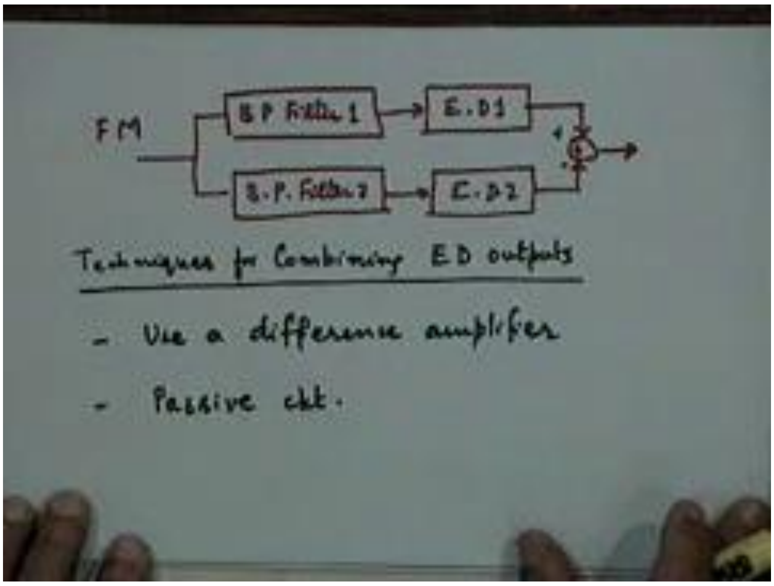

And finally, if we if we realize it like this, what you like to do is to have some kind of a some kind of a circuit, which takes a difference between the 2 output amplitudes and how can we do that, we can use a different amplifiers are something like that.

So, this techniques for combining the e d outputs, envelope detector outputs, we can combine them by using a different difference amplifier. But, there is a more popular passive alternative, the passive circuit is which will realize, this in one whole and not need a different amplifier, like to just draw that very quickly.

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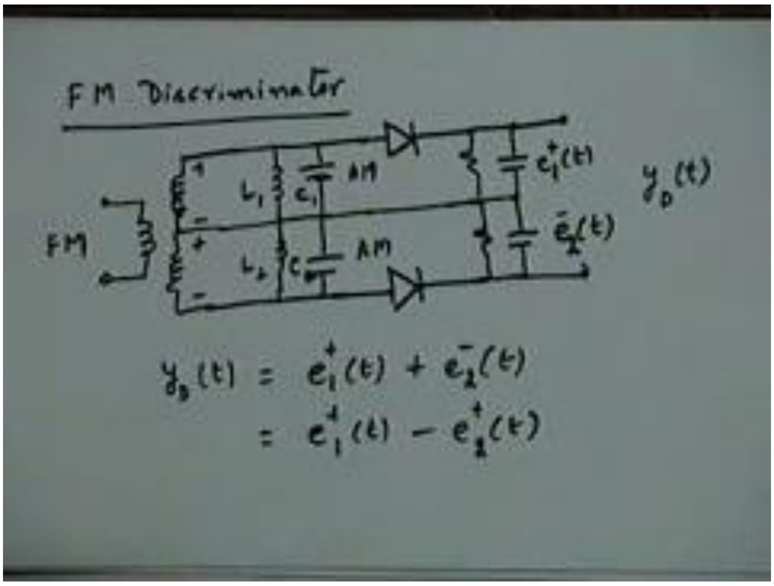

So, here is a based on this arguments, some kind of a circuit, which will work as a FM discriminator, the input modulating signal is coming in and through a suitable center tap rf transformer, it is fed to 2 circuit is which are operating in parallel. Each of them has a bank pass filter, which I am representing with approximately by a parallel LC circuit. This can be more elaborate than a simple parallel LC circuit, but this is the most common way of doing it and this is followed by an envelope detector.

So, this is one circuit and the envelop detector, if you remember contains a diode, followed by an RC circuit is in parallel. So, at any given time instant, suppose you have a certain signal here, FM signal coming in, is as a positive envelope and a negative envelope, so at any given time instant, the polarities will be like this. So, I have parallel similar circuit in the lower branch and a diode detector also present here and how I do is simply combine them like this, had the 2 outputs like this.

So, as you can see the output between these two points, this point and this center point here, may be proportional to the positive envelope that is coming here and the output between these two points will be proportional to the negative envelope that is coming here. Because, at any given time either this diode conducts, this diode will conduct from in the positive half cycles; this diode will conduct through the negative half cycles with respect to the input.

Of course, it has to be positive half cycle for the diode, but the posit positive half cycle of this diode will be the negative half cycle of this diode. So, this essentially this is the top circuit will be sensitive to the positive envelope and the bottom circuit will be sensitive to the negative envelope. And if you add the 2 outputs effectively, you would a subtracted what you wanted to do, you have carried out the subtraction that you really wanted to carry out.

Because, you are adding a positive thing with the corresponding negative thing, of course the two envelopes are symmetrical, we assume that the two envelopes, after the bank pass filter are symmetrical. So, here is the FM signal coming in here, is at this point at these two points you have the two amplitude modulated signals, one AM signal is here and the other AM signal is here.

This diode detector is positive to the positive envelope, this diode detector is positive to the negative envelope of the same AM signal, because the 2 AM signals are identical. And therefore, if you simply add the 2 outputs, you are able to reduce, so if I call this e plus t corresponding to the positive envelope. And this has e minus t corresponding negative envelope, y sub d t the detector output would be e plus t plus e minus t, that e minus t is minus of e plus t.

If you have carried out the difference between wanted to carry out, of course these two LC circuit is are tune to two different frequencies, they are not tune into the same frequency. What, we are saying is e minus t will be the negative of e plus t , so if I add the 2 is like subtracting the same thing with, I should call them differently, I will call this e 1and I call this e 2, I think that is a problem, otherwise this becomes identically 0.

So, this is e 1, this is e 2, e 1, e 2 thank you for that pointing out that mistake, because the two envelopes are slightly different, I mean the envelopes that we are looking at through the diode detector are different, because you have different tune frequencies here. This is tune to frequency f 1, this is tune to frequency f 2 and the responses are different, that is the whole point.

So, this is e 1 plus t, this is e 2 minus t, so the point that I meet earlier that the envelopes identical has to be modified. Envelopes are different, because the two bank pass filters are different and for that wrong statement and what you get here is this difference output that you want it.

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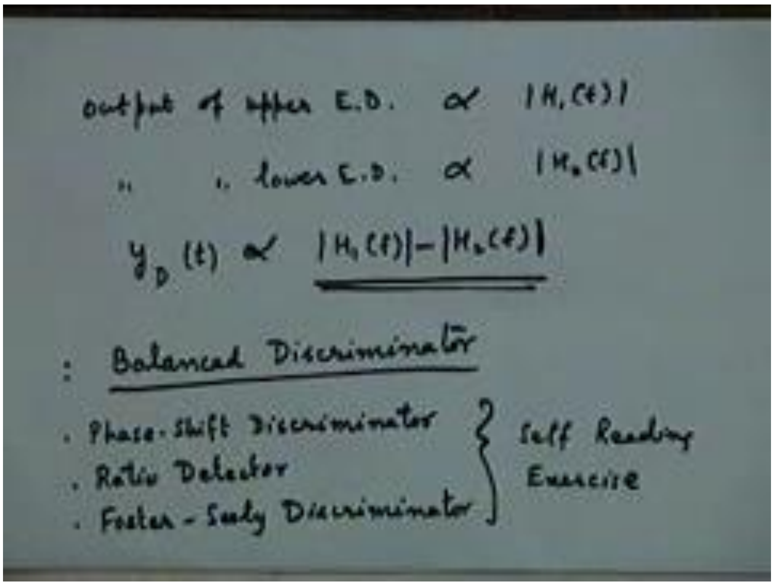

So, in another words let me just complete this discussion, output of your upper end is proportional to that, we just summarized what I have discussed, the amplitude of this is the important thing to appreciated. The output of the upper and lf detector here is going to be proportional to the magnitude of H 1 f, where H 1 f is the frequency response of this LC circuit.

Similarly, output of the lower envelope detector will be proportional to magnitude of H 2 f and the detector output yd t will be proportional to the difference between these two and this is what you want to do. This again the question that you phrased some time ago cannot be first do the subtraction and then envelope detection that is the answer to question no, but this is what you want to do. You do not want to do envelope of H 1 f minus H 2 f magnitude of H 1 f minus H 2 f.

We want the two magnitudes to be subtracted with each other that are the important point to appreciate it. Such device that, I have just indicated is known as the balanced discriminator, just like the balanced modulator that we discuss some time ago, so this is the balanced discriminator. Basically, what we are saying the term balanced is use to indicate that the response to the carrier is balanced out and the respond and for the final output to carrier frequency itself is 0.

The two circuit is will have identical response as the carrier frequency and the two, for the response of the carrier frequency, the final detected output due to the carrier frequency will be 0.When, an input signal is at a carrier frequency, then the output will be at that instant 0, because the two circuit is will behave identically at that time, so that is the balanced discriminator.

Now, there are several variations of this thing and they are number of variations of this practical implementation, which improve upon this basic principle and I will since we will not have time to discuss all those in the class. I will just mention the names of a few other discriminators, which improve on this basic idea further. So, I like you to read about them on your own.

Some of these are phase shift discriminator and there is a ratio detector and there is a foster seelyFM detector or discriminator. So, I like you to read more about them on your own, find out the suitable book in which they are available to read about them. Now, all of them would be discussed in your book I think, may be at this stage, I should put the entire query that we have discussed on a slightly formals fitting.

By considering precisely, how this discriminator works by developing a model for these discriminators, theoretical models. And then, looking at the output of response, I thing once we have that, then many of the questions that have been raised will disappear hopefully.

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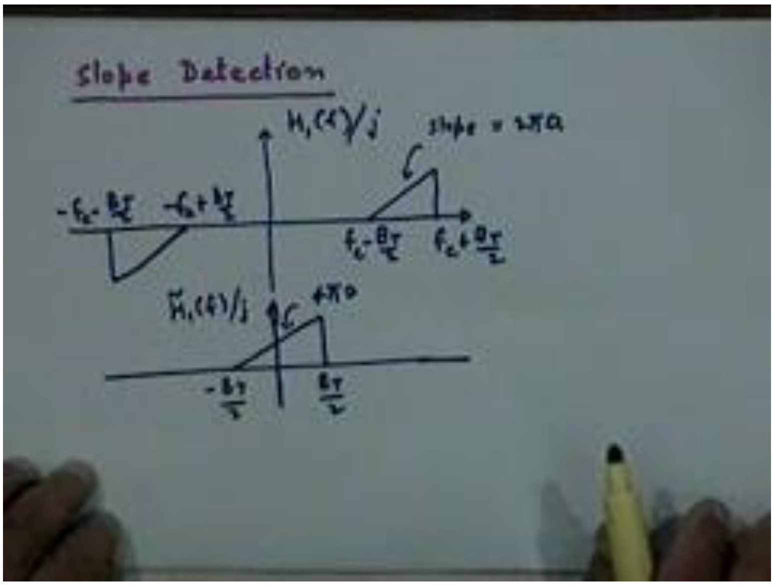

So, let us look at the mathematical theory of slope detection, basically that is what we are discussed so far, in developing a mathematical theory, we will work with slightly idealized filters. Basically to elaborate further as to what I mean by this idealization, in the two bank pass filters that we just discussed, we are really using only the linear portion, we are interesting in using the linear portion of the two bank pass filters.

So, in a way remember, these bank pass filters are motivated by what, what do we really want, you want a differentiator, to be approximately and that differentiator characteristic to be available around the carrier frequency. So, the g omega characteristics that we want, we want it to shift them to the frequency of interest the carrier frequency. So, really speaking let us look at the filter one of them, let us say H 1 f.

So, ideally we wanted filter characteristics like that, let me call it H 1 f upon j, from a single slope detector, this is the kind of thing I want. And since, I am plotting remember, I want a g omega kind of thing here, the differentiation and that is what I am plotting H 1 f by j, so what will I get on the other side, it will be this. So, a start from minus fc minus B T by 2 to minus fc plus B T by 2, where B T is your transmission bandwidth of a FM signal.

So, this is an idealized filter that who have served my purpose, corresponding to one of the two bank pass filters, basically by using any of the two bank pass filters, I am trying to realize this kind of a characteristic. At the moment, let us work with the case of a single bank pass filter, this idealization is for the case, when we are using the single bank pass filter.

Of course, the actual bank pass filter will have a complete response, but really speaking the portion of the bank pass filter is pass, which I am effectively trying to utilize is the linear portion and I am idealizing that as if I have filter with these characteristics. Basically therefore, we need to analyze the behavior of this filter followed by envelope detector.

If you understand this, you understand that, would you appreciate this model, I mean you go along with this model, because we need we do not need to look at the complete behavior of the bank pass filter. You are not going to use the characteristics of the bank pass filter, beyond the peak and we are representing the rest of it by linear approximation of this kind. That is what the ideally important, if you about to use a single bank pass filter, how this changes, how the picture changes, when you are using two bank pass filters, will take up subsequently, yes please

Student: ((Refer Time: 39:47))

Precisely, this is all I am saying is this is the approximation that I would get, if about to use a single bank pass filter case system, which we discuss earlier to get the 0 response at fc; obviously, I have to use the two bank pass filter system, which I will come to later. Let us, discuss first the single bank pass filter or single slope detector output, so you have a single slope demodulator.

Let us represent this, the characteristics of this filter by it is complex envelope, what will be the spectrum of it is complex envelope, what will that look like, let me do this brought down to 0, this will be between minus B T by 2 to plus B T by 2, sub t by 2. And let me say, the slope here is2 pi, some constant a, this is just an arbitrarily selected, this to be2 pi a, the a is a constant and the slope here would be 4 pi a, because the amplitude here would be double, when you consider the complex envelope.

So, this is a plot of H1 tilde f upon j, I am going to use complex envelope representation of both the FM signal as well as this filter to study the response at the output of this filter.

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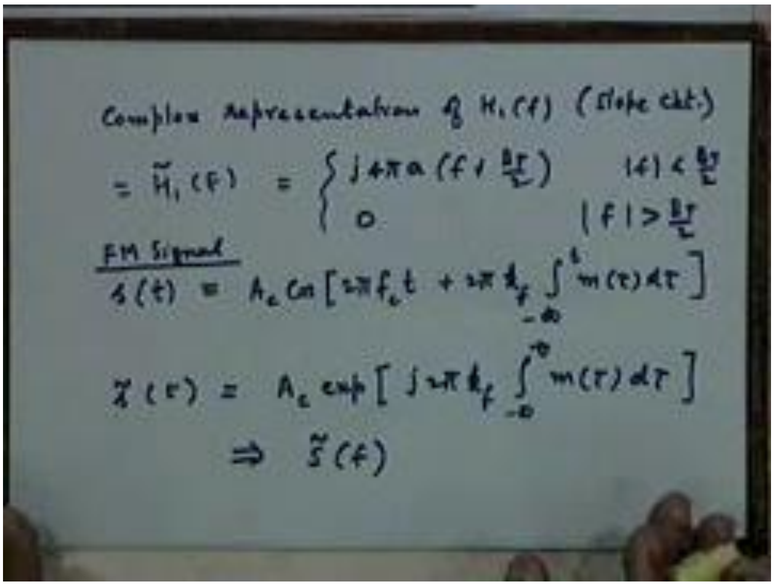

So, suppose I was to mathematically represent this picture, what we are saying this complex envelope representation in a frequency domain of H 1 f and a H 1 f. I will call it the slope circuit and no longer call it the bank pass filter. It is a slope circuit, which request particularly you raise the bank pass filter, which I am denoting by H1 tilde f which such that this is equal to just please look at this is equal to j 4 pi a into f plus b by B T by 2.

This is the linear equation of a straight line, the equation of the straight line can be written as j 4 pi a into f plus b sub t upon 2 in the range between mod of f line between b sub t by 2 just look at this carefully. So, this is f equal to minus B T by 2, this becomes 0, at f of equal to plus B T by 2, it becomes 4 pi a into B T.

So, therefore that is why, the slope is 4 pi a, this value will become 4 pi a into B T at f is equal to B T by 2 and it is 0 outside this. So, this is the mathematical representation of the complex envelope of the filter the slope circuit, that I am using, what is the input that you are feeding into it, the input that you are feeding into it is your FM signal. And it is a sub c cosine of2 pi fct plus2 pi kf; I am being consistent with my earlier notation that does not matter.

May be, I think the constant2 pi, which was not there earlier that does not matter, what is the complex envelope of this, pi inspection and if you assume, if you look the usual assumptions that we make about the representation of aero band signals. So, we can the complex envelope of this as ac into exponential of j2 pi k sub f integral m tau, d tau between 0, minus infinity to t also 0.

Let us say this has some spectrum S tilde f, this is the time domain representation, let us say the frequency domain representation is full transform is some function S tilde f. We do not need to find it, because we will not need it, we will come back to S tilde f will come back to time domain. So, to find the output, what do we need to do multiply this S tilde f with the H1 tilde f, that will give you the complex envelope of the output spectrum, rather the spectrum of the complex envelope are the are the output.

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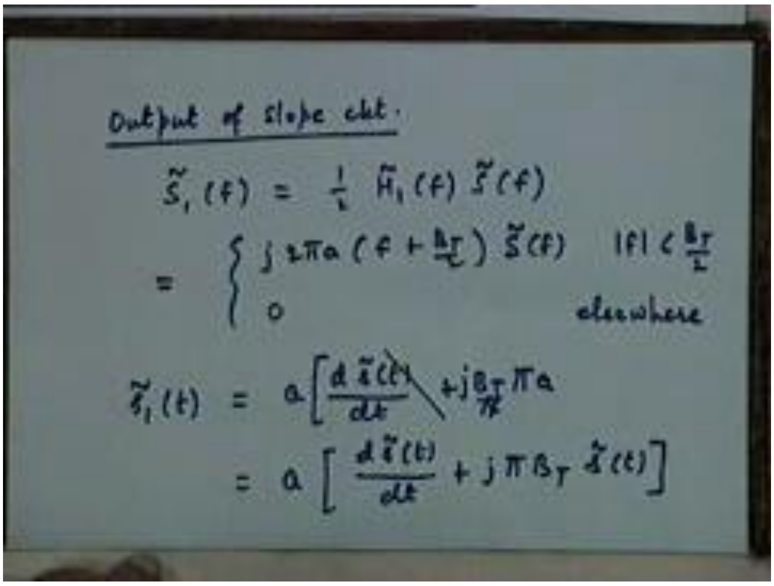

So, the output of a slope circuit, therefore would be S 1 tilde f, let me call this output spectrum on denoting the S 1 tilde f and that will be equal to half of H1 tilde half times S tilde f. Remember that, the output is half of the complex convolution of the two and the convolution in the frequency domain will be replaced by multiplication. So, if I substitute for H 1 tilde f that is given by j, which was 4 pia, so it will become j2 pia into f plus b sub t upon 2 into S tilde f. For mod of f lying between b sub t by 2 and will be 0 elsewhere.

Now, I do not know S tilde f, so what I will do is, I will take this equation back to time domain, take the inverse transform of this, what I will get, S1 tilde t, which is the complex envelope of the output. So, how convenient it becomes, we use a complex envelope representation everything becomes a simple, this is equal to how can you tell me, what the corresponding time domain equation would be.

The first equation j2 pi a f times S tilde f, this is the frequency domain relationship, what is the corresponding time domain use Fourier transform properties that is a times ds tilde f upon dt derivative of st, s tilde f, a is common, so I will keep it like this. I want me the second one this is the constant2 pi a into B T by 2 everything is a constant here, so I left with plus B T by 2, maybe I will make j2 pi.

So, this should be this two factor will go and then will be j there into pi a, a is outside have made a mistake of it. So, let me rewrite, this is a times d s tilde f upon dt plus j pi b sub t into s tilde f, that is the corresponding time limit equation. So, I have expressed the output in terms of the input, so what do we find the output is a sum of two things, the input itself plus it is derivative. So, this shows, that this circuit, the slope circuit that I have started with thus carry out the differentiation that I want you to carry out, but in addition, it also produces this additive component.

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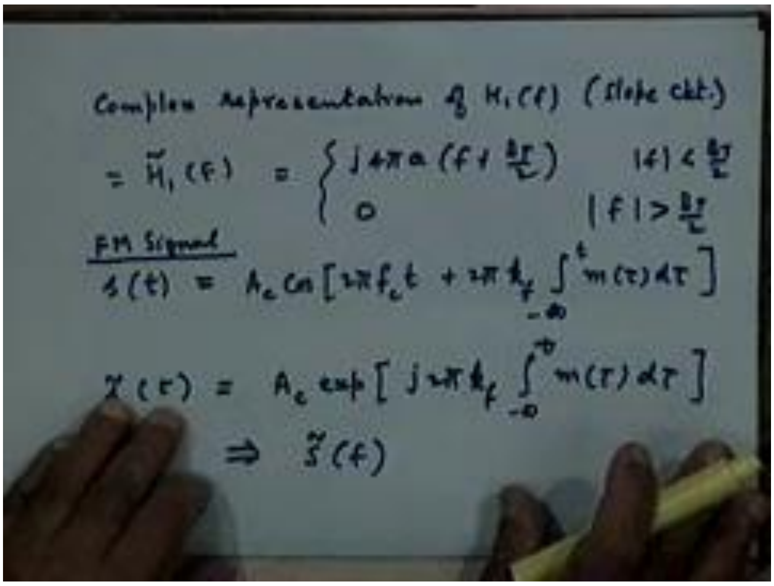

So, let us look at this again, let us substitute for s tilde t, remember what is your s tilde t, s tilde t is this complex envelope, and so let me substitute for this in the last equation. So, I will need to use this as well as it is derivative what will be it is derivative of this will become j2 pi k f into ac into mt. The derivative of the exponent which will be j2 pi k of mt time’s ac times exponential of the same argument and s tilde t is of course this.

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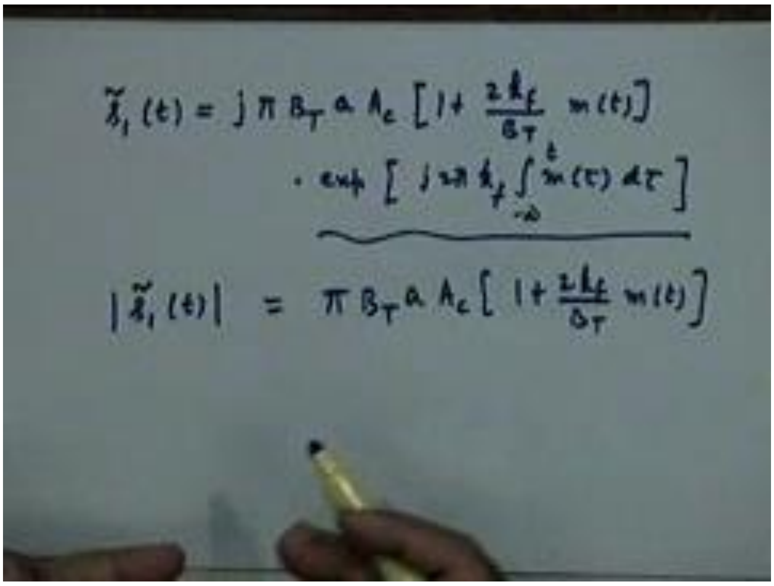

So, I need to use both this, in fact do that, it is a straightforward operation, I am left with the output s 1 tilde t is given by I am just writing the final expression, you can get it yourself the way I have explain it a few minutes ago. It will term out to be j pi b sub t a times a sub c into 1 plus 2 k sub f upon b sub t into mt into exponential of j2 pi k sub f integral m tau d tau, this is what you get when you substitute for s tilde t in this expression.

So, I substitute for s tilde t, find out the derivative, combine these two terms, simplify it keep that this expression, this was your s tilde t part, which comes outside as a common factor and this is what you get as the other factor and you are looking in a angle up of the output. How is the real envelope, relative to the complex envelope, just the magnitude of this.

If you want, you can first go to the bank pass signal, by combining it with e to the power j2 pi f not t and taking the real part of that and then taking the envelope in the usual way, but there is no need to do all that. The actual envelope of the signal s 1 t would be nothing but the modulus of the complex envelope, this is something that we already know and what that is going to be equal to what is the modulus of this; it is piB T a, ac into 1 plus 2 kf upon B T into mt.

So, that is what you get, you get an output, that the envelope of the output is proportional to ac signal plus some dc component same as before that we discussed. So, this is the analysis of the single slope circuit, based discriminator will complete this analysis. Next time, when you ask to consider the double slope circuit that just takes a minute.

Thank you very much.