Communication Engineering
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Lecture - 19
Demodulation of Angle Modulated Signals

Today, we will take up Demodulation of FM signals, Angle Modulated Signals, we looked at the methods of generating FM signals, FM and phase modulated signals, now let us look at, how we can demodulate. Now, what do we need for demodulating FM signals, we need a system or a device, whose output is sensitive to the angle of the incoming signal to the phase of the incoming signal, that is what you are looking forward to.

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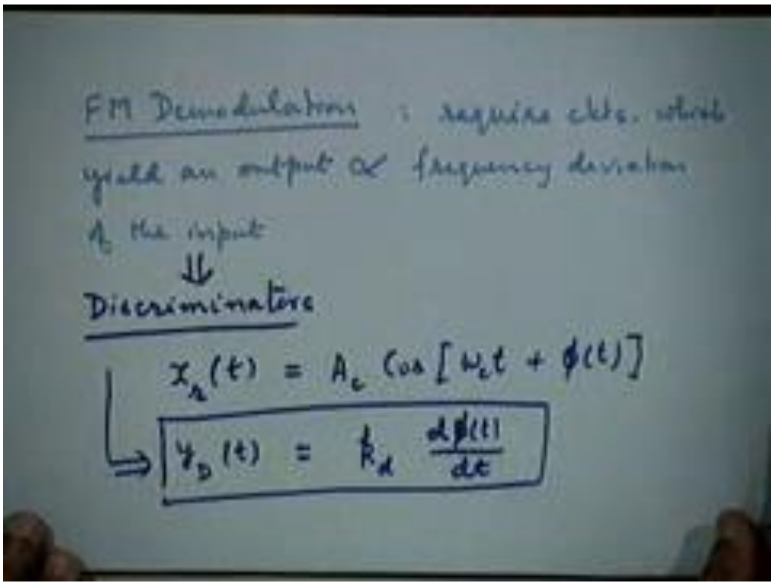

So, FM demodulation, I may talk about FM demodulation of FM signals, but really speaking, whatever we discussed for FM is with some modifications can be used for phase demodulation. So, basically we require circuit, which are sensitive to and therefore, which yield an output proportional to the frequency deviation of the input.

After all, where is the information residing in the FM signal, the information is hiding in this particular frequency deviation, as the input frequency varies as the instantaneous input frequency varies, you would like the output of this device, this circuit. The output amplitude to vary according, let us put it in paper, devices are circuit is, which can do this job are called discriminators. So, these kinds of devices are called discriminators, more precisely, they are called frequency discriminators, because they are able to discriminate the incoming signal.

So, if you see it mathematically, suppose you received modulated signal, let us call it x sub r t, sub r is denoting for the received signal is the angle modulated signal of this kind. Then, you need an output from the discriminator, the discriminator output let me call this y sub d t. The discriminator will produce an output y sub d t, which should be proportional to let me say that discriminator k sub d into d theta by d t.

Because, frequency deviation is proportional to theta, pardon me, I will just call this phi, because theta I am going to use something else later, so let the instantaneous deviation be phi of t. Therefore, the discriminator output, either the discriminator should be having the input output characteristic like this.

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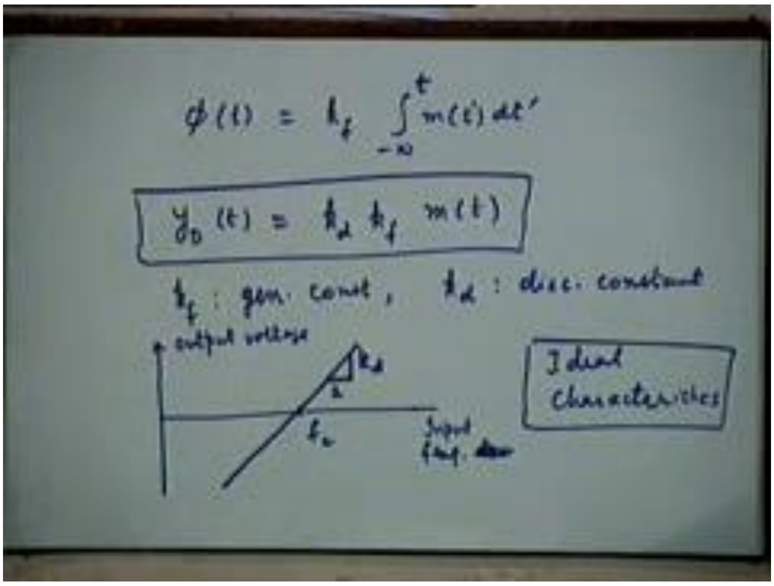

And as you can see for an FM signal, your phi of t is k sub f times integral of m t d t or m t prime, d t prime from minus infinity to t. And therefore, the discriminator output it is proportional to the derivative of this will be k sub d times k sub f times m t, which is what you want. We want an output proportional to the message signal, in order to get your message back, after demodulation.

So, k d, if you remember k sub f was the FM generator constant, that was the generator constant, proportionality constant and k sub d is the discriminator constant. So, if you want to express this fact graphically, on the x axis, suppose I show the input signal frequency deviation input frequency deviation. What kind of characteristics you want or let me say, let me not call it deviation, let me call it input frequency.

So, when your input frequency is equal to the carrier frequency itself, f sub c; that means, that instant the message value is 0, so at the carrier frequency except c, the output should be 0, this is output voltage. I am plotting output voltage versus input frequency characteristics, you would like to have a frequency discriminator and the input signal is f sub c output voltage should be 0.

And it deviates from it; deviates from f c, the output voltage should be proportional to the deviation. So, it should be a linear function around f c, when it becomes less than f c, it should produce a negative output, it becomes more than f c; that means, there is a positive output in proportion to the magnitude of the signal. So, you are looking for characteristics like this, and the constant k sub d is nothing but the slope of the straight line.

So, I will call this as k sub d, when this is 1, when the input frequency deviation changes by one unit, the output voltage changes by k sub d unit is. So, these are the ideal characteristics of a frequency discriminator, this is for the demodulation of the FM signal. Of course, now you are looking at the principle, as to what do we need to do in order to demodulate.

The question of how to do it still remains, but there is something that address, but before we do that, before we take up the question how to realize such characteristics. Let us also consider the situation, when the input signal is a phase modulated signal. Now, if I have a frequency discriminator that I use it to demodulate for demodulation, as what to be integrated.

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That is why is, if you remember how do you realize a phase modulator using the frequency modulator, first differentiate the input signal and then pass that to the FM generator. So, therefore, the FM generator output that you get will be proportional to d m by d t, and therefore, if you want m t back you must integrate the output. So, if you have a frequency discriminator, but you are working with a phase modulated signal.

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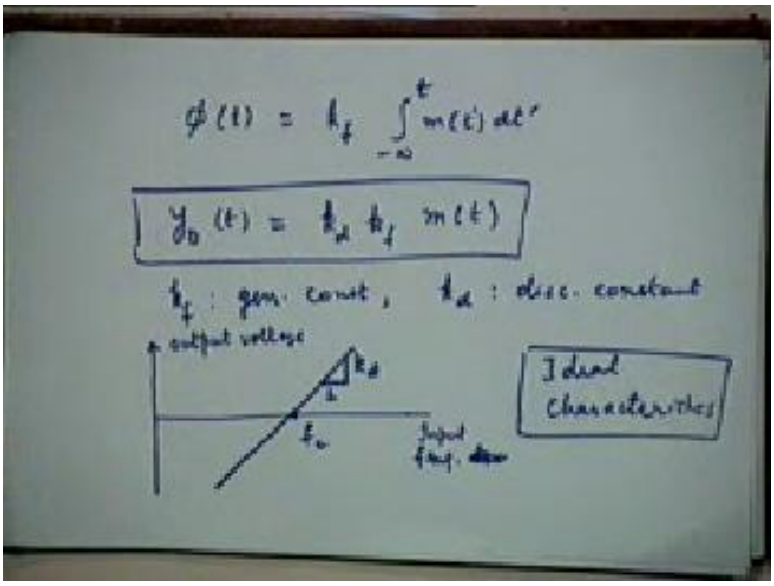

So, demodulation of PM is as follows, you have a phase modulated signal at the input and a frequency discriminator, the output y of t is here is not proportional to m t, but proportional to d m by d t. So, you must pass it through an integrator to get something, which is proportional to. So, this is your the detective output now and you can say that y of t, as far as y of t is concerned, this will produce a output k sub t times k sub p into d m by d t.

Where, k sub p was the phase modulated cross check and y sub d, the detective output, the demodulated output, the final demodulated output is going to be equal to k sub d times k sub p into m t, when you integrate. So, if you have a frequency discriminator, you can use it to demodulate both FM signals, as well as phase modulated signals. Now, the question of how to realize it, you are looking for some circuit or some system, which can do this job, produce an output voltage proportional to the input frequency.

Actually, input frequency deviation as this is proportional to input frequency deviation from the carrier frequency f c, there are many ways in giving this and we will consider some of them here. And all of them unfortunately will not realize the ideal characteristics that you want, but they will try to approximate as closely as possible. The ideal characteristics, that you want is clear to us, but if I raise them you will not be able to get them.

Because, you will have to use some physical device and there is no physical device, which will directly do this. You will have to design something and see, whether it comes close to it the required operations on that.

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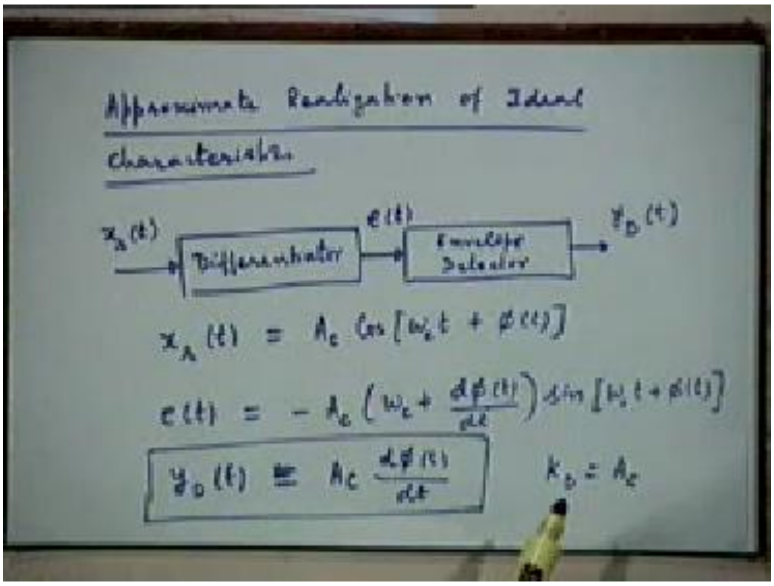

So, let us look at some approximate realizations of the discriminator of the ideal characteristics that we have discussed. So, one very simple approximation can be obtained as follows, take your FM signal x sub r t, pass it through a differentiator could you have thought of this yourself. Whether, come from this characteristic for example, you know, you have a system here, whose output linearly depends on the input frequency.

Suppose, you think of the ideal differentiator d y, d t, what is it is frequency response slide, if you have a system which carries out differentiation is equal to g omega, linearly depends on the frequency. So, it makes sense that somehow this characteristics could be realized by using a differentiator, the only thing is the differentiator has this characteristic not around f c, but around the zero frequency, which is not we are looking for, we are looking for these characteristics of f c.

So, we could call this device as some kind of a band pass differentiator, the differentiator which gives you linear characteristics are also carrier frequency, around some center frequency, f sub c that is what we are looking for. So, you start with a normal differentiator, just d by d t operation, so is not a band pass differentiator by itself now.

But, this output of the differentiator, if I call this e of t, now I pass this through an envelope detector, that is the change combination, but it does the job that we are looking for and that can be taken as the detected output or demodulated output. Let us see, how it works, how this can possibly work, let us start with our received signal which is A sub c cosine omega c t plus phi of t and differentiate this.

To generate e of t, what will you get, when you differentiate, this you get minus A sub c into the derivative of the argument, which is omega c plus d phi by d t times sine of omega c t plus phi t. Now, can you see some interesting features of this waveform e of t, what can you say about the waveform e of t, any comments you would like to make any remarks, that is right.

You can think of this as some modulating signal, which has constant value times, plus the time varying value, think of this as a modulating signal, the modulating signal is d phi by d t here and this produces amplitude modulation of this carrier. We can ignore this negative sign, because you can always absorb this negative sign by putting a plus pi by 2 or minus pi by 2 or whatever is here.

So, you can ignore this, essentially it becomes amplitude modulated signal, the only difference between the normal amplitude modulated signal and this one is there a difference. What is the difference, the carrier also has a phase variation, the same phase variation which was there already, nothing new, it is still there. But, if I look at the envelope, they would not bother us, the phase variations do not bother us, because the envelope, if you remember, what is the envelope, envelope is the trace of the peaks of the carrier.

Phase variations do not bother us at all, so if I produce an output, which is proportional to this instantaneous envelope, what will I get. I will get omega c plus d phi by d t or d phi by d t, because envelope produces a varied output proportional to the message signal m t. So, the detector will produce a y sub d t, which is equal to not approximately, which is equal to A sub c d phi by d t.

So, in this case, we have detector constant k sub d becomes equal to A sub c equal to the input carrier amplitude itself, so here k sub d is equal to A sub c is it clear. So, in principle by having a differentiator, followed by an envelope detector, I can realize approximately, what I want to this is what I wanted and this is what I thought. Is there any difficulty with this, see I think the point, that I perhaps, I am making this is the point, that to make sure that, this is always positive; that means, d phi by d t is always magnitude of d phi by d t is less than omega c, which is not a big deal.

Because, omega c is typically very large, sod phi by d t, so the normal condition required for end of deduction will be satisfied. So, there is a condition required here and the condition required is well this is the point that we are trying to make as here.

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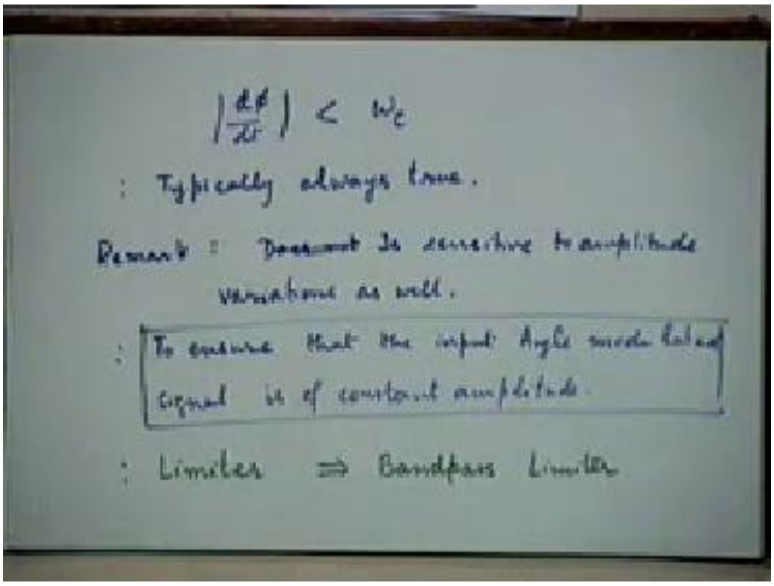

So, magnitude of d phi by d t should be less than omega c that is, what you are looking for and this are always typical in the case, typically always true. But, there is still one more difficulty, yes one is the difficulty of realizing ideal differentiator that is one issue that we need to discuss. Because, it is not very clear, how to realize that, the second is and this is what I want to discus very briefly at this moment. Is the fact, that you are ignoring in this discussion, the fact that the input signal may have some amplitude variations as well.

Although, at the transmitter when you send this signal, you can generate the signal there are no amplitude variations. But, the channel has noise, which we are not discussing here, but still we must be sensitive, the channel at noise and as a result of that the received signal will not have only amplitude phase variation or frequency variation, which should be the case for an ideal FM or PM signal. But you will also have amplitude variation and if it does the differentiator compresses sensitive to both the things.

It not going to be sensitive only to the phase variations, it is also going to produce, it is going to depend on phi that you have input amplitude is no longer a constant, but a time varying thing. So, you have product of two varying functions in that case and you differentiated output is then going to be proportional to this. Because, the amplitude variations will also affect what you finally, get as a demodulated output that is signal e of t.

So, that is something that we need to return, so this is a remark that I would like to make at this stage, but the discriminator as you analyzed. This is what an ideal discriminator, an ideal discriminator should only look at the phase and even if there is an amplitude variation it should ignore them. So, as much as the above system does not or is sensitive to, since it is sensitive to amplitude variations as well, it is therefore, not an ideal discriminator in that sense.

So, to make it behave like a ideal discriminator, what you need to do is, you have to make sure that the input FM signal that you present to it, must have a constant amplitude. So, we need to ensure that the input angle modulated signal is a constant amplitude, it is alright, if it is not a constant amplitude, we will have an output y sub d t, which will have dependence not only on your d phi by d t, but also on the time variations a sub t a of t.

Suppose, the input signal amplitude varies, then those variations will also produce an output, they will be proportionality, because of that, there will be component, because of that. But, this kind of thing can be done by this requirement can be met by a device called limiter. So, this is taken care of by the device called limiter, more precisely it is called a band pass limiter, let me tell you, what band pass limiter looks like.

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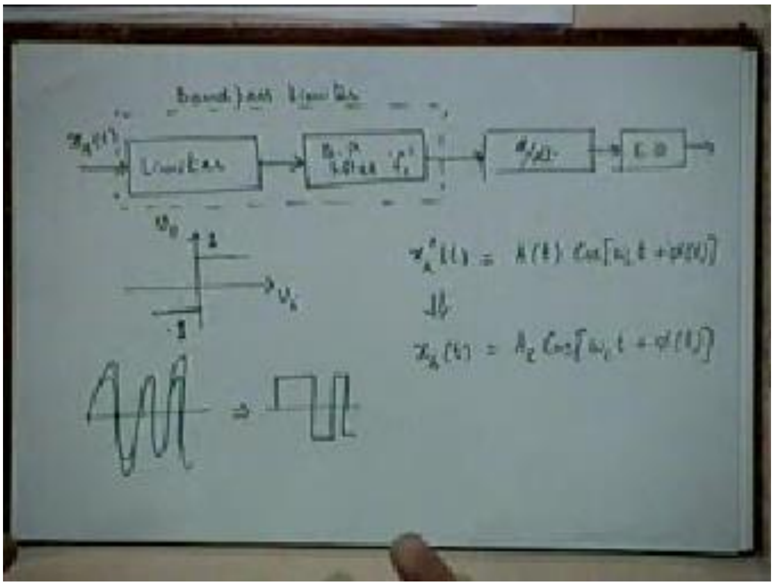

So, you are talking of a complete system diagram like this, the received signal coming in, the first block is, what I call the amplitude limiter. Now, amplitude limiter has an input output characteristics, which are like this, if this is the instantaneous input voltage v sub i, this is the output voltage v sub o is like this. So, what does it mean, at any moment the input amplitude is positive, the output is fixed amplitude signal, output is a fixed voltage which is 1, let us say plus 1.

If it is negative, it produces an output, which is minus 1, can you see what is happening, we have a FM signal coming in which has some amplitude variations, what will be the output of such a device, if the FM signal passes through this. The sine wave the moment it becomes positive, it crosses 0 towards the positive side, what will this device produce, when there is constant amplitude from there onwards, as long as it remains positive.

So, the entire positive half cycle will become a constant value equal to plus 1, similarly the entire negative half cycle will become a constant value equal to minus 1.So, what kind of operation you are performing, this is called a hard limiting operation, because they are basically converting smooth sinusoidal signal into a spiral kind of frequency modulated signal.

The input waveform is that is like that, this limiter at the output will produce the signal like that, is it clear. During this period, it will produce a plus 1 output, during this period; it will produce a minus 1 output and so on. So, that is hard limiting, actually this is therefore, this limiter is also called as a hard limiter, because it converts smooth variations into just hard plus minus 1 variations.

As you can see the frequency variation will be captured still by the zero crossings of the square wave you get at the output, it is not a pure square wave, it is a modulating square wave or some kind, but that is what you have. But, this is not what you want; you want a pure FM signal, the way you had retrieved mathematically a few minutes ago. To do that, what you have to do is, pass it through a filter a band pass filter with center frequencies f sub c and this combination.

This combination is called a band pas limiter and this it can be shown mathematically, this will produce an output here finally, at the output of the band pass filter. The pure FM signal, the pure angle modulated signal, without the amplitude variations. Even, these signals has amplitude variations, this signal will have, because amplitude variations are craved by this limit, you can see that, whatever amplitude variations are there in the signal.

Suppose, this cycle is of larger amplitude on this of smaller amplitude, all those amplitude modulations will disappear, once you have this hard limiting operation that was the reason, why we put the limiter there. However, to convert it back into the sine wave that we started with modulated sine wave that we started with we must pass it to band pass filter of center frequency of sufficient bandwidth. So, that all the FM components are passed through.

That is the basic purpose of band pass filter and together we call it a band pass limiter, I will take up the mathematical analysis of this device a little later. But, I think the purpose of keeping it is very clear now and approximately how it works is also clear, may not be precisely, but approximately we know what is happening. Well, killing the amplitude variations first in the limiter and then converting back into the FM signal with sinusoidal carrier by using a band pass filter that is the basic purpose.

Clearly, this is a non-linear device, limiter is a non-linear, and I mean it is highly non-linear input output characteristics. So, this entire thing is a non-linear system, now we are ready to use our combination, which we discussed. But, the ideal differentiator d by d t followed by envelope detector, this system comes pretty close being an ideal discriminator, ideal frequency discriminator.

Any questions, if I assume that, let me spend a minute on, yes please

Student: I have A c cosine function with omega c plus phi of t; now I am passing through a band pass limiter, so if I change.

Basically, what we are saying is, if your x r t, let us see what is the function of this dotted box. The function of this dotted box is combination of a limiter followed by band pass filter, which we are calling this band pass limiter, is if your input signal is not a constant amplitude, but has some amplitude variations A of t, which we know is a undesirable thing. When, as far as this combination is concerned plus if you feed this to this directly, the output here will not be proportional to only d phi by d t.

It will also have some dependence on this time function A of t, which is what we do not want. So, what we need to do is to convert this, let me call this r prime t and you want to convert this to our good old x sub r t, which is equal to constant amplitude A sub c cosine of omega c t plus phi t that is our objective. We want to go from this signal to this signal, the pure FM signal or pure angle modulated signal that we assumed earlier this is the input of this device.

So, this is what we are actually getting from the channel and this is what we hope to do by passing it through a band pass limiter is that clear, as top precisely mathematically, how do we kept that is something I will discuss later. But, intuitively I think it is clear, first objective is to kill the amplitude variation that may be there. Suppose, the input signal has the amplitude variations of this kind, it is amplitude is large here, small here and large here again.

It would not matter anymore, because the moment, it becomes positive as long as it remains positive, the output is a constant fixed value plus. It is not this envelope, which I am going to detect; this envelope is going to be converted to a constant amplitude signal.

Student: ((Refer Time: 33:30))

This is what, I am getting at the input, that is nothing, it is confused about, what is happening in the limiter, please try to understand what is the limiter is doing, limiter is just limiting the amplitude of the input signal. No matter, what the input amplitude is it produces an output, which is a constant value, over the positive half cycle. Over the positive half cycle, look at the input output relationship, as long as the input is positive, the output has a constant value equal to plus 1.

During this entire period, the input is positive, so during this entire period the output has the value plus 1.During this entire period, irrespective of the actual amplitude variation, I will get an output which is minus 1, irrespective to the fact that this is large amplitude or small amplitude or always get only a plus 1 or minus 1. Similarly, plus 1 and minus 1, so on and so forth.

When, this signal is passed through the band pass filter, you will get back your old modulated signal back without the amplitude variations, the mathematics of it will be discussed separately. So, if you have confusion about the mathematics, please wait for some time, till we come to that, but if you have confusion about the physics by the physical picture, you can discuss it now.

Student: ((Refer Time: 34:54))

Exactly, it will move amplitude variation, it may be also due to noise, and it may also be due to some changes.

Student: ((Refer Time: 35:20))

Actually, you are coming to now, how the FM signals behave in the presence of noise, we are not doing that kind of discussion now, and we will do it separately. Now, our limited goal is how to convert a non constant amplitude angle modulated signal into a constant amplitude modulated signal. So that, you can feed it through a combination of d by d t and detector to produce the modulated signal that was the limited objective of this discussion. We will discuss the third effect of noise separately; we have to wait for that, yes please, yes

Student: ((Refer Time: 36:10))

No, how can the output of a band pass filter be square wave, please think about that, it cannot be, especially what we discussed mathematically also, but intuitively the band pass filter is a resonant circuit tuned to a certain frequency of certain bandwidth. You cannot produce a square wave, where it is output, which has very rich in harmonics, which will have large number of frequency components not only around f c, but also around 2 f c and 3 f c and so on and so forth.

It will only pass the components around f c and later it will produce an output, which was similar to the origin which we wanted, the band pass filter cannot produce a square wave, no filter can produce a square wave at it is output. The filter may smoothen out the square wave, it will the components, the higher harmonics components of whatever is not being passed by the filter.

Is that the answer to that question, now this differentiator, we are talking about somebody mentioned that, it is difficult to realize that. The ideal differentiator and that is really a key point, how to realize the differentiator, particularly we are looking for a differentiator, which works nicely for an input signal, that we are feeding.

We are feeding a base band signal here, the input to the system is not a base band signal, and it is a modulated signal around some carrier. So, I cannot use a simple operational amplifier and use it as a differentiator. Well, theoretically one can, if your operational amplifier can work up to that frequency, suppose your carrier frequency is 100 mega hertz or around that number.

You will find it very hard to find operational amplifiers, which will help you realize that, the differentiator around that carrier frequency. So, realization of differentiator at those frequencies of interest is nontrivial exercise.

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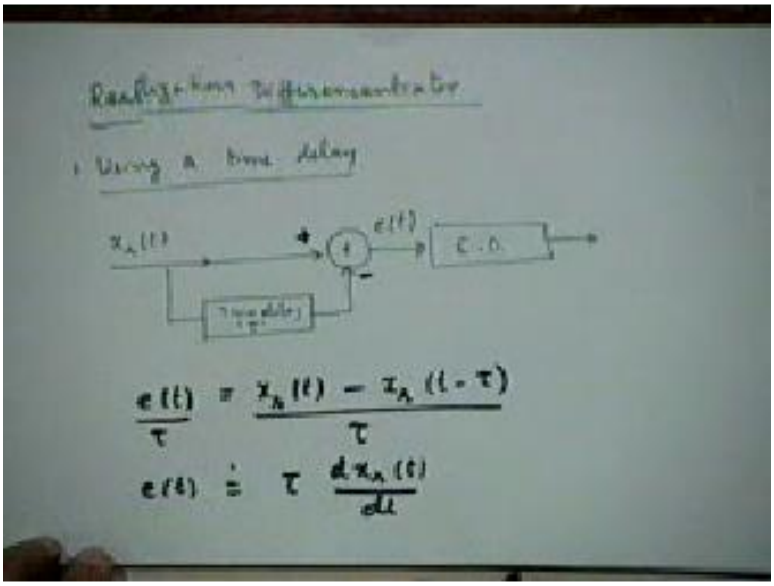

There are two ways of realizing a differentiator in this context, there are many ways of realizing a differentiator otherwise, but realization in the context, we are just mentioned can be done in two ways. I will be coming back to the band pass limiter later, let me just finish the discussion regarding the demodulated structure. One way is, what is called using simple time, delay a small time delay, basically literally taking the derivative operation as, what we you know, how we defined the differentiation in mathematics.

So, take x t minus x t minus tau divided by as tau tends to 0, that is the definition of a derivative, so literally try to realize that. So, your input signal is coming in, you also have a small delay line of some kind, which introduces a time delay, which is very smalltime delay and take the difference between these two outputs. In the direct output, x sub r t and this delayed output and that you do not have to physically divide anything.

Because, after all you are diving by constant, now there is some costing value, simple as well ignore, you take it as a constant of proportionality, that is you can take it as your e t, passes through an envelope detector and that becomes a discriminator. So, basically what we are doing is, this if this is plus 1, this is plus, this is minus, so your e of t here is going to be x sub r t, minus x sub r t minus tau and if you were to divide by tau, you see what you would have got.

This would have been the approximate value of the derivative, so since you are realizing only this much, your actual e t that you get is tau times, approximately tau times d x r by d t. As small as practically feasible, but it obviously, has to be non zero value, but that is the design, we will look at the design issues separately, but you will; obviously have to choose not too small not too large value.

Because, too small value, you may not be able to control exactly, too larger value will not produce a good approximation to the derivative, also since e t is proportional to if you look at the output, output is tau times this. If tau too small, what will happen, if tau is 0, we will really subtract out the signal, nothing will be there at the output. So, your amplitude will depend on the value of tau, so you must choose a very large value reasonably large value of tau, so that reasonably you have good amplitude.

At the same time, if you make too larger too larger value of tau, this is not a good approximation to what we want. So, one has to make physical compromise on that basis, what is the demodulator caused in this case, so case of d, will be here basically tau. Actually, A c into tau, because this differentiator itself will produce, this x r t itself in terms of component of A c, so A c into tau.

This is one method of realizing the differentiator, using a small delay lines, the delay line at the carrier frequency is not too difficult to realize, you can just wind a small inductor and the inductor will produce the delay. So, delay line is easy to realize in these frequencies.

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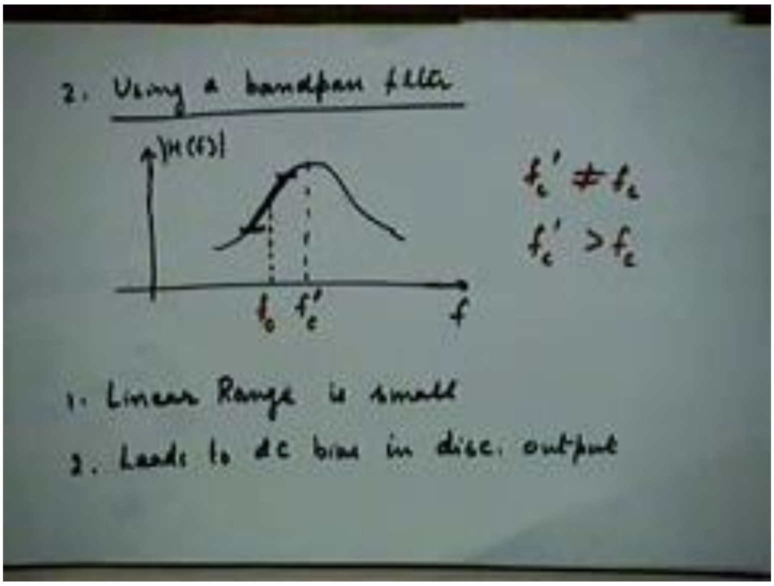

The second method is using a band pass filter itself, this is interesting, you can use a band pass filter input output characteristics, the frequency response characteristics of a band pass filter to realize a differentiator. Well, that we look at a little straight, let us see what do I mean by this, what is a typical band pass filter transfer function, this is what it is I am plotting the magnitude here, peaks set it is resonant frequency.

Plotting h of f against f, mod of h of f against f and it has a certain band width around this carrier frequency, can this be used as a differentiator, what I want for differentiation. I want a linear characteristic, linear mod of h of f, should be linear function of the frequency. Is there a portion in this transfer function, which is approximately linear, this portion?

If, I use this characteristic, this portion of the band pass characteristics and not the entire band pass characteristics, then I have got an approximate differentiator. So, I will use the center frequency f c prime, which is different from an actual carrier frequency, the actual carrier frequency will be somewhere here. The AM signal has a carrier frequency, which is somewhere in the middle of straight line.

So, approximately realize the linear dependence of the response on the input frequency deviation. Obviously, the band pass filter center frequency f c prime will be not equal to f sub c, it will be typically greater than f sub c, we will choose f c prime, which is greater than the actual carrier frequency. So, this is one way of doing it, however as you can see the obvious problems in this approach, what are the obvious problems, one is the linear range will be small.

So, linear range, if you want to demodulate wide band FM, which has a large frequency deviation, then this may not be a very good idea, that is one problem. And the second thing is, it will not be what in terms of response with f c, what is the desired response to f c, derived response should be 0. This is not producing a 0, response leads to a d c bias, a discriminator output, which might itself is not a big deal. Because, disc output is not removed, but you see, it is preferable to have that is, I do not have to remove any d c.

It should not produce a d c of it is own, in fact that is one of the advantages of an FM signal, then it has a good response to dc component of the input signal. If the input signal has a dc component, it will come through, because that component carries out a suitable frequency deviation, which will come through.

But, if the demodulator itself is the dc bias, then I would not be able to differentiate in this dc that is introduced by the differentiator, by the demodulator, by the one, which is coming through the original signal. I will remove both of them, which is not very nice, so that dc bias mentioned character of the Fm signal, which is an advantage of the FM signal, would not get lost unless I do something about this.

So, that does not mean, we have to give up this one, it is only we have to slightly modify this and will briefly discuss the basic idea behind the modifier, I am only at the moment discussing this conceptually, we will convert this concept into mathematics later.

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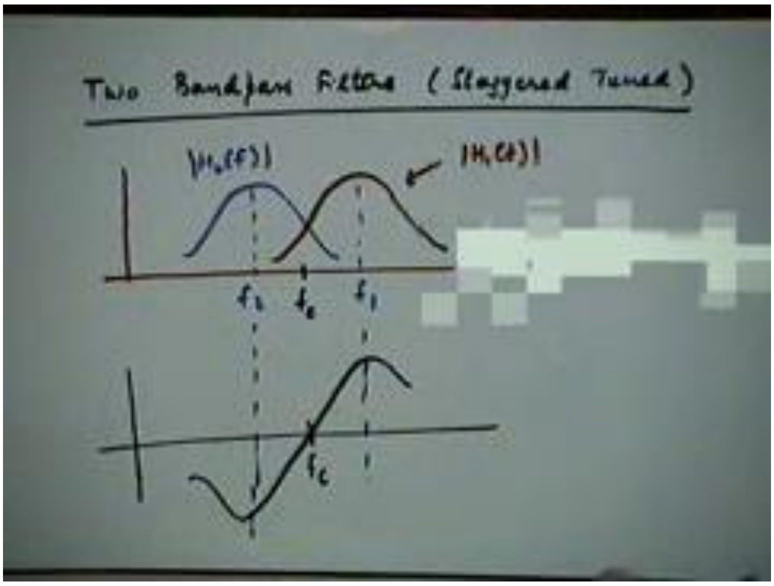

So, what can we do about this, so what we can do is something very instead of having one band pass filter, have two band pass filters, both of which have center frequencies side band away from the desired carrier frequency, differently. So, we have two band pass filters, so we can realize the differentiator or the discriminator using two band pass filters, what I said to staggered tuned band pass filters.

Let me say, this as one band pass filter, whose amplitude characteristics are H1 f and this is the second band pass filter, they are just going to different center frequencies. So, this is H2 f, this is tuned to some frequency f 2, this is tuned to some frequency f 1and if somehow, I can produce an output. So, I have different filters I feed the same input to this filter as well as this filter.

I choose the center frequency somewhere here, the carrier frequency somewhere here, if I can now produce an output, which is proportional to the output amplitude of these two, amplitude differences of these two outputs. That is produce the output, which is proportional to the different outputs at this filters, what will I get, I will get the net result, which will subtract out these two, responses.

And you are likely to get a frequency response, which is like this; it is linear response f c and the linear response, which has a much larger range. The linearity will be in larger range and that will serve as a nice, ideal discriminator. You can have two staggered band pass filters, the center frequencies of each of which are chosen to be away from the desired carrier frequency. You can realize a differentiator is closely to a fairly good one, as a practical example. So, this is the principle, we will discuss this idea little further, corresponding circuit and all that and we will discuss the entire thing mathematically.

Thank you very much.