

Communication Engineering
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Lecture - 16
Angle Modulation (Contd.)

We will continue our discussion on Angle Modulation today and if you recollect we had looked at this spectrum of angle modulated signal, when it is modulated. By modulating signal is a pure sinusoidal signal that was the point at which we finished last time, so let me recapitulate.

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The image shows handwritten mathematical notes on a chalkboard. At the top, the word "Spectrum" is underlined. Below it, the expression for a modulated signal $x(t)$ is given as $x(t) = A \cos(\omega_c t + \beta \sin \omega_m t)$. This is then expanded into a series of Bessel functions: $x(t) = A \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[(\omega_c + n\omega_m)t]$. Below this, a section titled "Some Useful facts about Bessel Func." is underlined. The first fact listed is $J_n(\beta) = (-1)^n J_{-n}(\beta)$, which is then expanded into a piecewise definition: $J_n(\beta) = \begin{cases} J_n(\beta) & \text{for } n \text{ even} \\ -J_{-n}(\beta) & \text{for } n \text{ odd} \end{cases}$.

We wrote an expression for x of t which was a modulated signal in terms of a series expression. We showed that this style of angle modulated signal which could have been either F M signal or a phase modulated signal. We could write this in terms of a series expression, in which the coefficients of the series are the Bessel function. And the functions involved here are sinusoids of frequency ω_c plus $n \omega_m$, so that is a series expression for the F M signal or P M signal.

And how did we arrive at this, we arrived at this by considering the Fourier series expansion of e to the power $j \omega_m \sin \omega_m t$. Then, considering the fact that the required signal is nothing but, the real part of e to the power $j \omega_c t$ into e to the

power $j\beta \sin \omega_m t$. So, we expand it to the power $j\beta \sin \omega_m t$ into its Fourier series and then took the real part of that and that is what you get.

Now, this is starting point for our discussion regarding the spectrum of FM signal. Of course please remember we are doing the spectrum discussion for the very special case. When the modulating signal is sinusoidal, not the general modulating signal MFT that is a much more difficult exercise.

So, what do we find first of all before we discuss any further of course, this is obvious from this expression that we have a very large number of frequency components. Unlike the AM phase, where the frequency components per ω_c , $\omega_c + \omega_m$ $\omega_c - \omega_m$ you have a very large number of Bessel components present here. And the behavior of the spectrum is dictated by the behavior of these coefficients, which are Bessel function values.

So, to understand the spectrum better, you need to understand little bit about the nature of this Bessel functions. Now, this is not a course on mathematics I will not really go into mathematical discretion functions. I will just give you few silent properties of Bessel functions, which are relevant to our discussion and I hope you learn it in more detail ((Refer Time: 04:40)).

So, let us review some useful facts about Bessel functions, first there is a kind of symmetry in these functions as a function of n you can in the series expansion you have n going from minus infinity to plus infinity. It can be shown that $J_n(\beta)$ is equal to $J_{-n}(\beta)$ for n even and $J_n(\beta) = -J_{-n}(\beta)$ for n odd. So, these are this is how the Bessel functions of order n and order $-n$ are related, what this means is, this is equal to $J_n(\beta)$ or $J_{-n}(\beta)$ for n even and equal to $-J_{-n}(\beta)$ for n odd.

So, alternative coefficients on the positive and negative side are symmetrical, but opposite in sign. So, $J_1(\beta)$ and $J_{-1}(\beta)$ they are opposite signs, but $J_2(\beta)$ and $J_{-2}(\beta)$ have the same sign and so on and so forth. So, that is one interesting property of Bessel functions.

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2.
$$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$$

3. For $\beta \ll 1$, $J_0(\beta)$ dominates
 $J_0(\beta) \approx 1$
 $J_1(\beta) \approx \frac{\beta}{2}$
 $J_n(\beta) \approx 0, n \geq 2$
: Narrowband angle modu.

The second one is this should be intuitively clear, but mathematically it can also be proved, that if you take the square of each of these functions and add it from minus infinity to plus infinity for any value of beta. This is equal to, it cannot be 0, we are adding sum of squares this is equal to 1 it should be obvious. Now, let us see why it is obvious look at these two expressions ((Refer Time: 07:05)) if we look into the power in the signal, what is the power here, A^2 square by 2 what is the power in signal here. It will be you know every sinusoidal that you have in the signal has the power A^2 into j n square beta by 2 and; obviously, they represent the same signal.

So, the total power must be the same, so it is obvious that this kind of property must be satisfied, irrespective of the F M phase of course, under which we are discussing carrying out this discussion, this can be shown purely as the property of the Bessel functions ((Refer Time: 07:42)). Of course, it is consistent with what we expect in our case, so that is one.

Thirdly for beta, remember what is beta? Beta is this index here ((Refer Time: 08:01)) that is present in the modulating signal and we have defined beta to be the, so called modulation index per angle modulated signals. Beta is equal to k_p times a_m for the case of phase modulation, in case of m times upon ω_m for the case of frequency modulation.

But nevertheless it is a factor which has somewhere important relevance to the present discussion which will come to very ((Refer Time: 08:35)), I will call it the modulation index. So, beta is the modulation index when the modulation index is very small that is much less than 1.

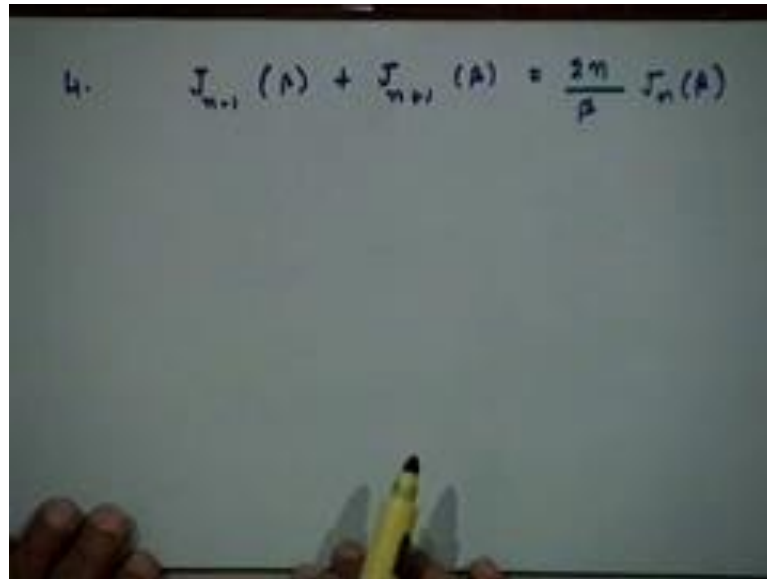
Then, we all the coefficients that we are referring to the Bessel functions that we are referring to a small argument. We are talking of $J_n(\beta)$ $J_0(\beta)$ $J_1(\beta)$ etcetera with beta argument being much less than 1 under these conditions the term $J_0(\beta)$ is the predominant term, it is a dominant term. So, this dominates or he dominates, what it means is let me be more specific $J_0(\beta)$ for beta very small is approximate y.

In fact, $J_0(0)$ is equal to y, so $J_0(\beta)$ is approximately equal to 1, $J_1(\beta)$ is approximately equal to beta by 2 and $J_n(\beta)$ is approximately equal to 0 for n greater than 2 greater than or equal to 2. What does this mean that when your modulation index is small out of all these infinite number of terms, this expression has only 3 terms will be the more significant one corresponding to n equal to 0, one corresponding to n equal to 1 and n equal to minus 1.

That means, the spectral components that we will see be primarily the carrier frequency which is for n equal to 0 and $\omega_c + \omega_m$, $\omega_c - \omega_m$ which is very interesting, because it is precisely these spectral components which are present in the A M signal. So, when the modulation index is very small, the spectrum of the F M signal is very similar to that of A M signal.

There are certain important differences which will come to, so we call this kind of a F M signal or P M signal as narrow band angle modulation. Anyway now I am discussing only the Bessel function properties we will come back to this point separately again. So, these are things that you prove that $J_0(\beta)$ is approximately equal to 1 beta very small beta much less than 1, $J_1(\beta)$ is approximately equal to beta by 2 and $J_n(\beta)$ is 0 for n greater than or equal to 2.

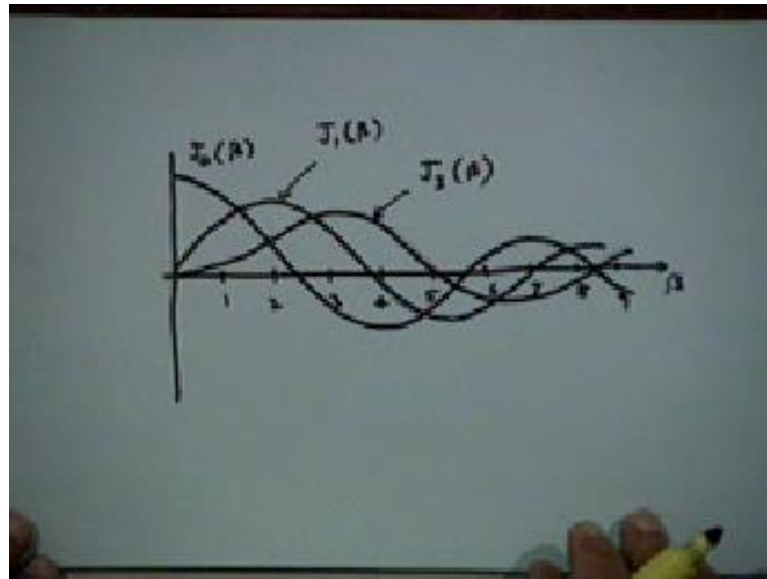
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$$4. \quad J_{n-1}(A) + J_{n+1}(A) = \frac{2n}{\beta} J_n(A)$$

Lastly ((Refer Time: 11:32)) may be there this is not a immediate physical relevance, but one important interesting mathematical property is that Bessel function satisfy it is an iterative relationship that they have with respect to each other. Bessel functions are of different orders satisfy our recursion and that recursion is given by this, which essentially means that if I know $J_n(\beta)$ and $J_{n-1}(\beta)$ if I give a value of β I can find out $J_{n+1}(\beta)$.

So, there is a recursive relationship that is satisfied by the Bessel functions, so which is helpful in the computation of the Bessel functions. But, still I think with the help of these properties you get some idea, but not the complete idea is what the Bessel functions actually look like, it will be interesting also to plot some of these functions $J_0(\beta)$, $J_1(\beta)$, etcetera. And see what they look like in that you start feeling better about this functions and if you have done that for you here.

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And this is how the plots look like, I have plotted here J_0 beta as a function of beta, J_1 beta as a function of beta, J_2 beta as a function of beta and similarly you can plot more. So, what you see, you see that these functions are oscillatory in nature, they are not monotonically increasing or decreasing, they are oscillatory in nature it is not exactly oscillation of constant amplitude. The amplitude decreases as beta increases and as beta becomes very large they all become very small.

Second the property that we discussed J_0 beta is approximately one this point is 1, in the neighborhood of 0 at 0 it is exactly equal to 1. And all other functions start from 0, it is 0 at beta equal to 0 and they are very small as you can see given beta approximately increases linearly at this point. Whereas this is not even linearly increasing this is more or less 0 for quite some fraction of value of beta. So, all the properties are just mentioned to you can be seen to be true here.

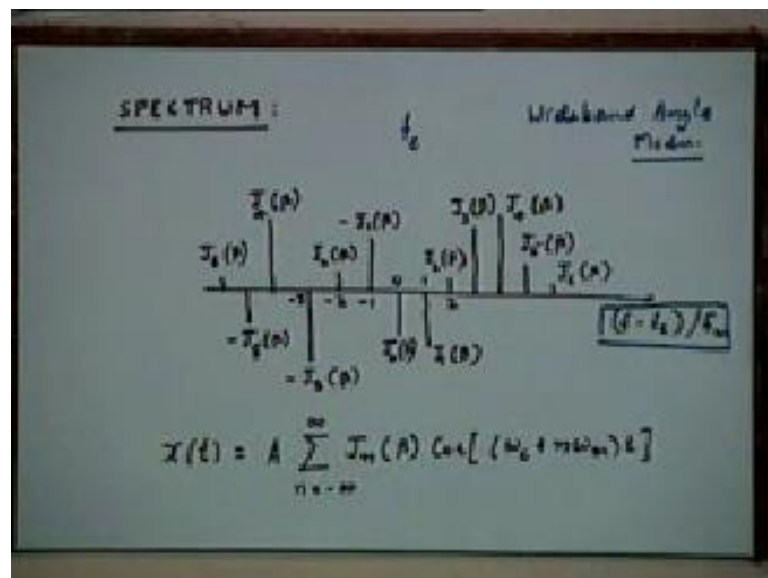
And because this function oscillatory in nature, the spectrum varies a lot for example, first look at this for a small value of beta the carrier component is the large stand there is of course, the first ((Refer Time 15:00)) that is ω_0 plus minus m components. But, suppose the modulation index is somewhere here, that is beta is equal to 2.4 or something for this value what happens J_0 beta, it becomes 0.

So, for a particular modulation index like beta is equal to 2.4, the carrier component becomes 0, J_0 beta is a coefficient of the carrier component. Remember the expression

that we had, look at this expression that n is equal to 0 you get the carrier component. So, $J_0(\beta)$ is the coefficient of the carrier component, this becomes 0 whenever β takes a value such that this becomes 0 here or for example, β is equal to 5 point something here.

So, for these values the carrier component disappears and you have all the energy present in the side bands only. Whereas, for large values of β energy is equally distributed in the various side bands and a very large number of sidebands, because they all decrease. So, that is I hope this picture you have a better feel for the Bessel function, what does it look like Bessel function β and if you were to use this in association with our discussion earlier.

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A typical spectral plot of an F M signal would therefore, look like this, this is the frequency axis, where I have chosen the frequency x is to be normalized one, I have looked at the frequency deviation from the center frequency F_c ; normalized with respect to the modulating signal frequency F_m . So, when I say this point to be 0; that means, f is equal to f_c here, so here this point we are looking at the spectral component at the point, where the deviation is F_m , $F - f_c$ is equal to F_m .

So, basically this component denotes $f_c + f_m$ and this component denotes $f_c - f_m$ I have written 0 and minus 1 here. But, remember that the center frequency actually

denotes the frequency; the point 0 denotes the frequency f_c , because I have normalized it by subtracting it from f_c .

Subtracting the value of f_c from f and then, so this denotes the component at the carrier frequency, this denotes the component at $f_c + f_m$, this denotes the component at $f_c + 2f_m$, $f_c + 3f_m$ and so on and so forth. Similarly, these are the components that $f_c - f_m$, $f_c - 2f_m$, so this is what the typical plot might look like. So, as you can see $J_1(\beta)$ and $J_{-1}(\beta)$ has opposite signs, but $J_2(\beta)$ and $J_{-2}(\beta)$ they both have the same signs $J_3(\beta)$ and $J_{-3}(\beta)$ have opposite sign of $J_3(\beta)$.

So, you see a very large number of spectral components present it is a wide band signal. So, in general when β is not small we call it wide band angle modulation, so this becomes wide band angle modulation.

So, although you had your modulating signal contained only a single frequency component f_m your modulating signal contains a very large band width any questions, so far. So, I think with this discussion you should be reasonably clear with what Bessel functions are like it is some of the important properties and the typical spectral plot of you appreciate now what is a typical spectral plot of angle modulation ((Refer Time: 19:55)).

Unlike only the 3 lines spectra that m signal has corresponding to the situation DFM or PM signal will have a large number of lines corresponding to just the presence of a single frequency modulating signal at the input. So, that is the important point any questions before I proceed further, no good let us proceed further.

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$$4. \quad J_{n-1}(\beta) + J_{n+1}(\beta) = \frac{2n}{\beta} J_n(\beta)$$

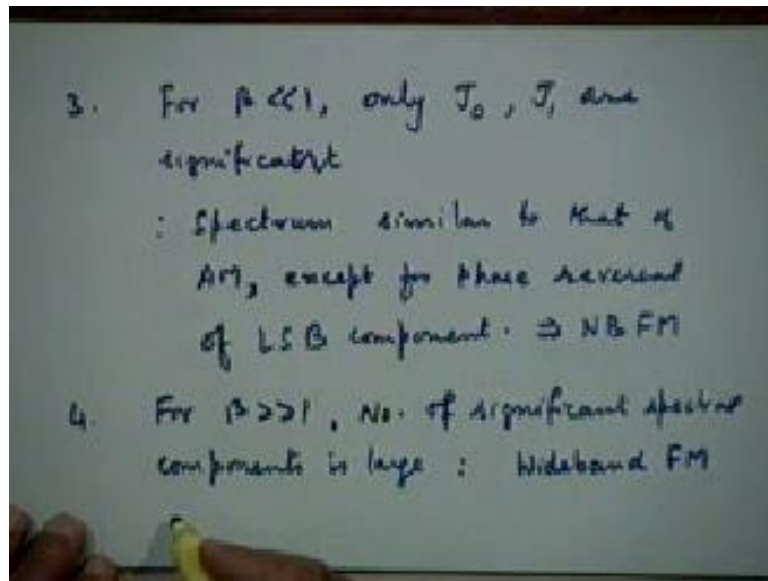
Spectral Properties

1. Carrier + infinite no. of sidebands
 $f_c \pm n f_m$ (AM: $f_c \pm f_m$)
2. Relative Amp & no. of significant spectral components

So, these are some of the important properties of the Bessel functions and this is what they look like, based on this let me summarize what are the important spectral properties we can derive. The spectral properties which are implied by the properties of the Bessel functions are as follows. So, first point is we see that the spectrum contains the carrier in general plus an infinite number of side bands. Now, we had 2 side bands like we had earlier both in the positive side as well as in the negative side irrespective of sign.

If I had number of sidebands I mean frequency components like f_c plus minus $n f_m$ unlike for the case of AM we had only f_c plus minus f_m . Second the relative amplitudes and number of significant side bands, number of significant spectral components what will they depend on would depend on the value of beta the modulation index. Like we saw for a small modulation index carrier has the largest component, other components are much smaller, but as beta increases there can be some values of beta where the carrier component is 0 and all energies contained in the side bands. So, which component has how much amplitude related to each other will be decided by the value of the modulation index, so depends on beta.

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Thirdly we have seen that for beta much less than 1 only the first 2 terms are significant J_0 J_1 are significant and the spectrum is similar to that of A M. While it is similar it is also slightly different that is the two side bands are phase reverse with respect to each other. The negative sign of one with respect to another $f_c + f_m$ has some amplitude where $f_c - f_m$ has the negative amplitude it means phase shifted by various degree.

So, except for the phase reversal of the lower side band component and this is called narrow band F M. So, narrow band F M, which is the F M defined to rise when beta is very small as a spectrum similar to that of a A M signal, in the sense that has only the 2 side bands $f_c - f_m$ with this difference. As against this when beta is large let us say very large the number of significant spectral components is very large.

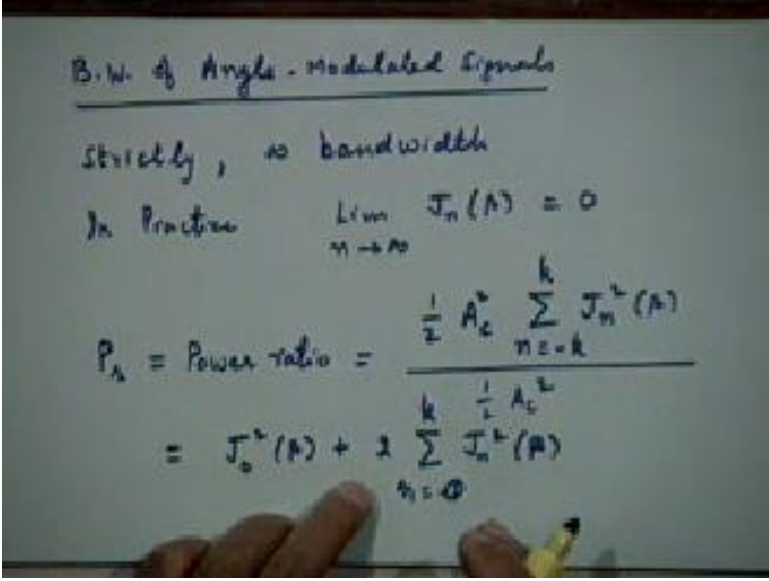
And such a signal is said to be wideband F M or wideband P M, it could be narrow band P M and wideband P M. So, large bandwidth, this implies large bandwidth for these descriptions of bandwidth are qualitative small or large is qualitative. A natural question arises can we convert this into a quantitative value can we say this is a bandwidth. Now, that is a slightly tricky situation here, why because theoretically no matter what the value of beta may be whether it is small or large, theoretically how many components are present in the series expansion infinite.

But theoretically the bandwidth is or quantitatively the bandwidth is always infinity, but for all critical purposes because we have seen those nature of the functions, these

functions keep on decaying as beta tends to 0. Similarly, if you look at the value of the functions for different values of n I have not got a plot against n. But you can see the trend here you can see j 0 beta starts with the largest amplitude, j 1 beta has smaller amplitude, j 3 has minimum smaller amplitude and so on and so forth.

Similarly, the other functions will be of much smaller amplitudes each, so you will also find that for larger and larger values of n the corresponding coefficients become smaller and smaller. So, therefore, it can be expected that beyond a certain value the amplitudes of the spectral components. And therefore, the energy contained in the spectral components will be small and therefore, they would not contribute significantly to the total energy of the signal. So, based on this fact we need to define the bandwidth in a particular way and then come up with a quantitative figure for the expression for the bandwidth.

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B.W. of Angle-Modulated Signals

Strictly, no bandwidth

In Practice $\lim_{n \rightarrow \infty} J_n(A) = 0$

$$P_n = \text{Power ratio} = \frac{\frac{1}{2} A_c^2 \sum_{n=0}^k J_n^2(A)}{\frac{1}{2} A_c^2 \sum_{n=0}^{\infty} J_n^2(A)}$$

$$= J_0^2(A) + 2 \sum_{n=1}^{\infty} J_n^2(A)$$

So, let us do some discussion on that, we would like to look at the bandwidth of angle modulated signals more quantitatively rather calling it large or small. So, as I said strictly speaking the bandwidth is infinity, but we I just said that in practice we make use of this fact that limit j n beta as n becomes large this tends to 0.

This again the properties of the Bessel functions which I perhaps should have mentioned when I made this list of properties, j n beta as n tends to infinity that is Bessel function of infinite order identically is 0. So, how do we define the bandwidth, to define the

bandwidth let me first define a ratio $P_{sub r}$, which I will call as the power ratio in this manner.

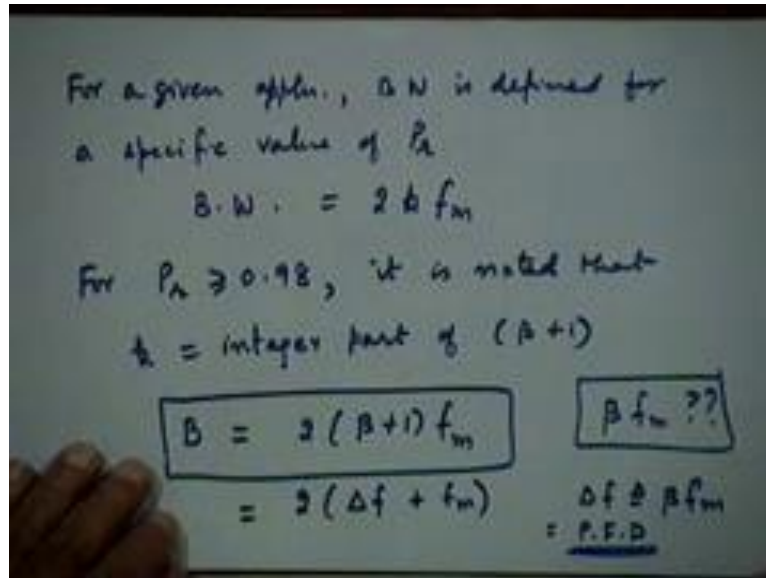
Suppose I consider k sidebands on either side of $f_{sub c}$, I look at the energy contained in this $2k + 1$ components from $f_{sub c} - k f_{sub m}$ to $f_{sub c} + k f_{sub m}$ look at the energy in this part of the signal. This is approximating the actual signal and takes the ratio of this energy with the total energy of the signal.

So, that I am calling either energy ratio or the power ratio actually power ratio is more appropriate term. So, what will be the power contained in $2k + 1$ I just mentioned. It will be $\frac{1}{2} A_c^2 \sum_{n=-k}^k \sigma_j^2 \beta_n^2$, n going from minus k to plus k and the total power is $\frac{1}{2} A_c^2$. So, as you see this you can write as $\frac{1}{2} A_c^2 \sum_{n=0}^k \sigma_j^2 \beta_n^2$ plus twice of $\sum_{n=1}^k \sigma_j^2 \beta_n^2$, β_n going from n going from 0 to k , because I have taken twice of this number of terms, I am sorry this should be 1 to k because here I have taken out, thank you.

So, now the way we define the bandwidth is as follows, we say we agree that for a signal to have the bandwidth whatever we define the bandwidth to be. It should have most of its energy in that band one thing we need to know quantify is we need to what is the definition of most. So, we can take some arbitrary figure and a generally accepted figure is 98 percent bandwidth. So, the way we define 98 percent bandwidth of any signal waves, particularly the signals which otherwise have infinite bandwidth is that, those band of frequencies over which the signal contains 98 percent of its power, that means the value of this power ratio is 0.98.

So, basically what we need to figure out is for what value of k , this turns out to be about 0.98, because this ratio can be at most one. So, the value of k for which this becomes about 0.98 would be the 98 percent bandwidth of the same.

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So, for a given application bandwidth is defined for a specific value of P_r and for whatever value that you select the band width will then become $2k$ times $f_{sub m}$. Whatever value of k would give you that value of P_r would then imply that the band width is $2k r$. So, for P_r to be greater than or equal to 0.98 that is the 98 percent bandwidth, now if you look at the table of Bessel functions every interesting fact comes to light that if you choose P_r to be greater than 0.98.

The value of k that you need, so that this becomes 0.98 depends on the value of β and then this is a very nice closed form. Of course it is not closed form in the medical sense its empirical sense empirically it is found that for a given value of β . If you choose k to be equal to $2\beta + \beta + 1$ just one more than the value of modulation index that value of k would lead to this being 0.98 or larger. So, this is something that you look at empirically from the table of Bessel functions.

I cannot prove it mathematically here for you may be it can be even proved, but I think we will just take it as that. So, for P_r greater than 0.98 it is noted that k can be taken as integer part as $\beta + 1$, so this is an empirical result. By looking at the nature of the Bessel functions and that tells us quantitatively the value of the 98 percent bandwidth.

So, this implies that the bandwidth is twice into $\beta + 1$ times f_m , because the bandwidth is $2k f_m$ and that is the expression for bandwidth of a F M signal approximately. So, as you can see there is a very simple closed results here, but this simple closed term result is for the special case when the modulating signal is a pure sine

wave, this result is well only for the special case, any questions can you give a let us give another meaning to this formula.

Can you interpret what βf_m is if you look at the expression for the F M or P M signal that you have can you give some physical meaning to the term βf_m . Let us go back to this expression, let us look at this expression this is the expression for the P M signal can you give some physical meaning to the term βf_m or $\beta \omega_m$.

How will it arise, if I differentiate the argument and what does that differential argument give me the instantaneous frequency, if I differentiate what I get $\omega_c + \beta \omega_m \sin \omega_m t$. Now, can you give a physical meaning, maximum value that the signal will have frequency deviation, because the frequency deviation expression becomes $\beta \omega_m$ into $\cos \omega_m t$ maximum value of $\cos \omega_m t$ is 1.

So, the maximum value of the frequency deviation would be $\beta \omega_m$, so βf_m is nothing but, the maximum or the peak value of the frequency deviation that the signal will execute. Do you understand the definition of frequency deviation with respect to the center frequency, the instantaneous frequency of both the F M signal and P M signal keep on varying or fluctuating about the mean value, which is the carrier frequency value at different times you have different instantaneous frequencies.

Remember the plots that we made for the F M signal at some point along the axis time axis the signal was highly compressed by the instantaneous frequency was large. At some other points the signal was much sparser and there will be instantaneous frequency smaller than ω_c . The peak value of the instantaneous frequency deviation is equal to βf_m , so please remember that.

So, therefore, you can also write this formula in a different way namely $\omega_c + \Delta f + f_m \cos \omega_m t$ let me call this $\Delta f + f_m$, where Δf is defined as βf_m and can be interpreted to be the peak value of the frequency deviation peak frequency deviation or maximum frequency deviation. Now, this analysis that we just done is valid only for sinusoidal signals. For more general signals it is approximately possible to argue out the band width of the F M signal. We will not go through that argument, because it is beyond our course here, but I will just give you the result.

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For a general $m(t)$:

$$D = \frac{\text{peak freq. deviation}}{\text{b.w. of } m(t)}$$
$$= \frac{\Delta f}{W}$$
$$\sqrt{B} \approx 2(D+1)W = 2(\Delta f + W)$$
$$B \approx 2(A+1)f_m = 2(\Delta f + f_m)$$

: CARSON'S FORMULA

So, we are now looking at the result for a general modulating signal $m(t)$, where $m(t)$ is not cosine $2\pi f_m t$, what can you say about the bandwidth and I am only giving you the result we are not going any discussion for this. To do that let me define D as the following ratio peak frequency deviation upon band width of $m(t)$, because band width, $m(t)$ is now general signal, it is not a sinusoidal signal, the only way you can characterize it is its bandwidth. So, let me call this as Δf on w Δf is the peak frequency deviation of the F M signal or P M signal w is the bandwidth Bessel signal $m(t)$.

Then it can be shown approximately the 98 percent of the band width signal or the F M signal or P M signal is now given by 2 into D plus 1 into w very similar to the formula that we had just a few minutes ago; which was 2 into β plus 1 times f_m and there also you can write this as Δf is it not. D into w is Δf plus w and here also you can write it as 2 Δf plus $f_{sub m}$, this is what we derived this is the generalization of this to arbitrary signals very similar.

These formulas for the bandwidth were given by a gentleman called Carson and these are known as Carson's formula for the bandwidth of an F M signal, any questions. Now, again in these general cases also you can see that whether you will have narrow band F M or wide band F M you will really depend on the value of Δf that is another way of looking at it.

When beta is large Δf is large and you have wide band F M. So, the important point is when Δf is large the band width will be governed largely by the peak frequency deviation and that mix lot of intuitive sense if you think about it. What you are saying is the band width will be essentially dictated by what is the highest instantaneous frequency in the signal, is it clear.

On the other hand Δf is small which will essentially happen when beta is small the modulation index is small the band width will be dictated by the highest frequency component present in the modulating signal, because then it is like a A M signal, is that clear. The signal is not like a A M signal I should not have said that, the signal is very different from that of a A M signal for example, the A M signal has amplitude variation this signal will never have amplitude variations.

But, when beta is small it is spectrum looks like a A M signal, so that is essentially a very deep idea about what the spectrum of an F M signal looks like. Any questions you can take a minute if you have any questions and solved everything that we have discussed.

Student: ((Refer Time: 43:12))

Frequency

Student: ((Refer Time: 43:19))

I am not clear about the question, his question this method suitable for frequency sensitive applications.

Student: ((Refer Time: 43:37))

Your question should I rephrase it as should F M or P M be used in frequency sensitive applications. I do not know what that question means, so I can t really ask may be

Students: ((Refer Time: 43:56)) so, what he exactly means to say is frequency sensitive because I cannot exact it and what is at frequency is at that point of time.

You see if you are referring to the process of modulation and demodulation the value of the modulation is some sense reflects sensitively that we are referring it to. But the sensitivity is with respect to the amplitude of the modulating signal, please remember

that. Our purpose is fully satisfied if our modulation is sensitive to the amplitude of the modulating signal, instantaneous amplitude and that is what we are dealing in F M or P M.

We are making the instantaneous phase or instantaneous frequency depends on the instantaneous amplitude of the signal. So, if we now we are looking at spectral properties we need to look at the demodulation of the signals, so that we can see whether not we will be able to recover the signals properly. Our concern should be with respect with the recovery of the signals and the recovery process will depend how you will demodulate.

So, I think your question therefore, is relevant to our discussion what we have done, now, but we are going to come to generation and demodulation.

Student: why are we using Bessel functions to analyze the signal, sir this is only evaluated when signal $m(t)$ is constant we cannot use it in general.

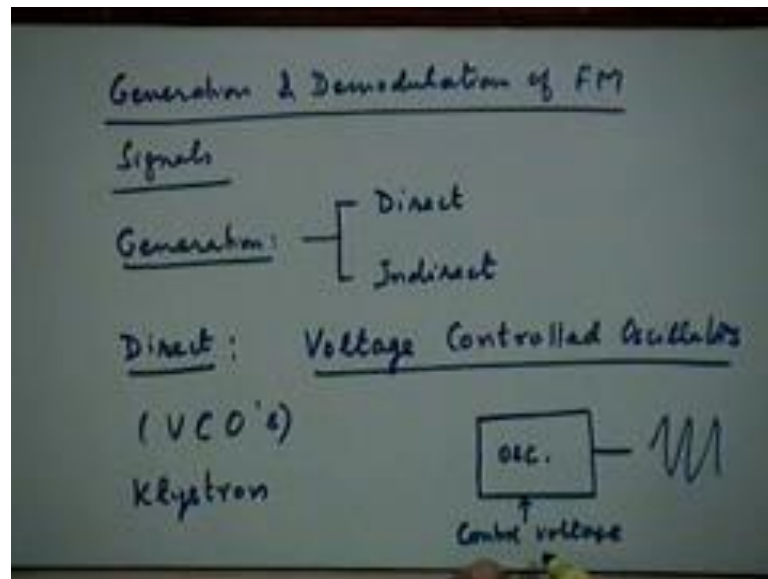
That is I mean we use Bessel functions deliberately to derive, the Fourier series expansion of $e^{j\beta \sin \omega_m t}$ and the Fourier coefficients turned out to be $J_n(\beta)$

Student: Sir, these Bessel signals are only integers it has to be an integer.

No Bessel functions can be defined for other values also which are not integer values, but now we do not need them here. For our discussion we do not need them here. There are various kinds of Bessel functions, this is only one kind various kinds of Bessel functions. So, I think for that I like you to refer to a book on mathematics.

Let me just proceed further there are no questions next thing what I like to take up is how do we generate and how do we demodulate angle modulated signals. Obviously that is the next thing that we have to discuss.

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So, generation and demodulation, so primarily there are 2 let us look at the generation issue, generation you can classify the various possible methods into two broad categories direct methods and the indirect methods. The direct method actually uses a device what do we want how can we generate F M signal just think about it.

How do we generate a sinusoidal carrier, what do we need for that, an oscillator of suitable frequency now what do we want. We want the frequency of this oscillator to depend on the amplitude of another signal. So, we need to modify this oscillator, if we can such that instantaneous frequency of this oscillator can be made to be depending on a voltage somewhere. And it is fortunately possible to design such circuits and when you are been able to design such a circuit we call it a voltage controlled oscillator.

So, the direct method depends on the design and construction of what are called voltage controlled oscillators or in short known as VCO's. Now, you have voltage controlled oscillators of different kinds which are being designed and fabricated. Now, depending on what frequency range working with you will have different kinds of designs, you can have voltage controlled oscillator based on IC components. You can have voltage controlled oscillators based or R C components which are called relaxation oscillators.

I do not know whether you have come across s table multi vibrators, in your either digital or analog electronics. If you have come across s table multi vibrators they are nothing but, voltage controlled oscillators. If you are working with microwave

frequencies, there are special devices like klystron, which can be used as voltage controlled oscillators and so on and so forth.

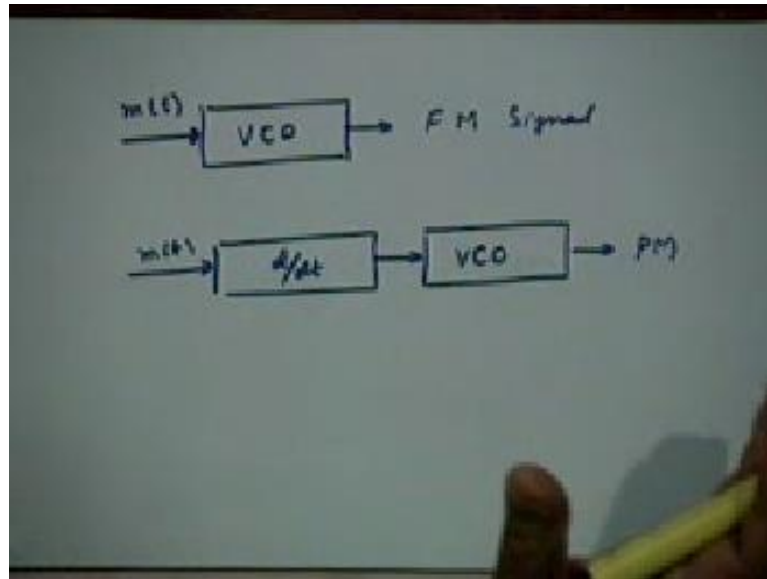
Now, since therefore, it depends on what frequency you are working on different kinds of devices. I am not going to go into a very detailed discussion of the design of the voltage controlled oscillators except to say it is possible to design circuits, devices, systems which can be essentially voltage controlled oscillators. And once I have such a voltage controlled oscillator to the control input basically what does it mean?

That you have an oscillator, I mean if you look at it purely from a block diagrammatic point of view. In oscillator there is normally no input except the power supply and it produces the output oscillation signal, here you have a control voltage and the frequency of this will depend on the voltage applied here.

So, if I apply to this where the control voltage terminal the modulating signal the output oscillations will depend on the input amplitude that is what these devices has do. How they are designed would be the subject would be the topics of subjects that you are doing subjects that you are studying or will study. So, we will not go into those we will assume that such things are possible may be we will discuss one such example slightly later.

This is for what kind of what kind of oscillations, what kind of system frequency modulated signal or P M modulated signal. If I use a voltage controlled oscillator this will give me F M or P M frequency modulation can I use it for generating P M just think about it.

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There is a message signal $m(t)$ very good suggested the answer, so if I apply $m(t)$ to a VCO I will get an F M signal, this is what we are saying. If I apply $m(t)$, but first differentiate it and then apply to VCO I will get a P M signal do you remember why. So, I do not have to repeat it.

But just complete the discussion, remember that the instantaneous frequency of a P M signal would be proportional to the derivative of the message signal is it not. It is obvious because instantaneous phase deviation is proportional to the message signal $m(t)$. So, the instantaneous frequency $m(t)$ will depend on the derivative of the message signal, because instantaneous frequency is the derivative ((Refer Time: 52:37)).

So, if I feed $m(t)$ directly to VCO I get F M if I feed the differentiated version of $m(t)$ to VCO I get the P M signal. Actually we will discuss for various reasons, what we call F M transmission is neither pure F M nor P M it is something in between what; that means, something we will discuss later. So, I will stop here for the moment and next I will discuss indirect methods.

Thank you very much.