Communication Engineering

Prof. Surendra Prasad
Department of Electrical Engineering
Indian Institute of Technology, Delhi

Lecture - 15
Angle Modulation

Today, we will start a new topic, we will start talking about now Angle Modulation, now let me start with a brief general discussion, and so far we have been talking about amplitude modulation. And briefly how do you describe amplitude modulation some way of embedding the variations of the message signal on the amplitude of the carrier signal. So basically the amplitude of the carrier signal is modified in accordance with the amplitude of the message signal.

So, the information content lies in the amplitude of the carrier, now the other ways of embedding information into a carrier, amplitude is not the only way. In general you want some parameter of the amplitude of the message signal to modify some parameter of the carrier. What are the other possible parameters you can modify, you can modify phase, and instantaneous phase and you can modify instantaneous frequency.

Both these cases you are modifying the argument of the cosine wave, you have a carrier is nothing but, a cosine wave, cosine omega ct plus phi. So far we are modifying the instantaneous amplitude the carrier. Now, we are modifying something in the argument of the cosine function which constitutes the carrier wave and therefore, these two kinds of modulations that we just talked about namely phase and frequency modulation, they are also broadly known by the name of angle modulation. Because, they are basically one way or the other modifying the instantaneous total angle of the carrier, so we broadly call it angle modulation.

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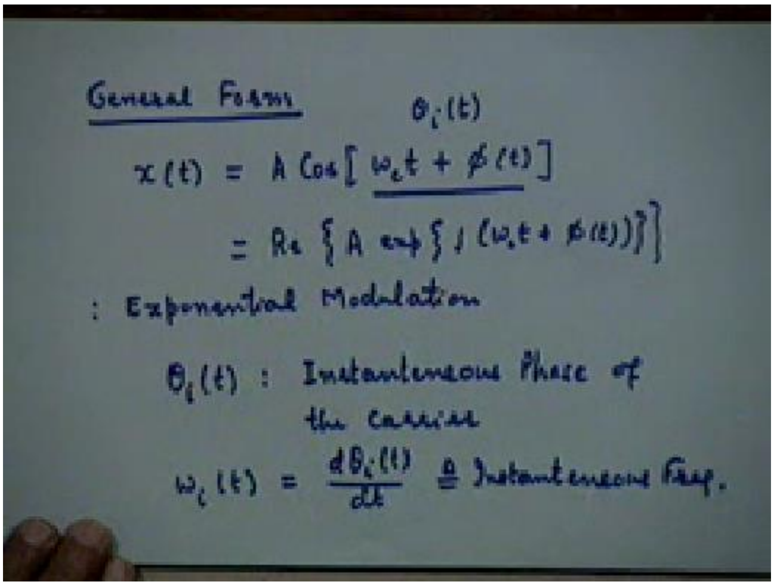

So, if you were to write down an expression for a general form of the angle modulation, you should say that the modulated signal x of t now is A cosine omega ct plus phase function which is time varying. So, you have basic carrier frequency is still omega c, but the angle of this cosine wave or the phase of this cosine wave is a time varying function at this function captures the information that you want to convey.

So, the modulating signal m of t somehow modifies this phi of t appropriately, now it will be convenient, because will now find it in many cases convenient to work with complex presentation figures. If you feel it makes complex presentation makes life pretty easier as far as the analysis goes, so it is also convenient to write this as real part of A times exponential of j omega ct plus phi t.

For this reason, sometimes this modulation or this set of modulation also known by the name of exponential modulation, so these are the various names, phase modulation, frequency modulation, angle modulation, exponential modulation they all refer to this kind of stuff. Now, this context let us define this total argument of the cosine wave or of the exponential function, let us denote it by theta sub i t and theta sub i t will be defined as instantaneous phase of the carrier.

Now, how is instantaneous frequency related to instantaneous phase can you say anything, suppose I ask you that at a particular time this quantity is theta i t, how can I decide the instantaneous frequency from the instantaneous phase, let me help you.

Suppose, phi of t was 0 it was constant, what is the instantaneous frequency, what is the instantaneous phase omega ct or omega ct plus phi.

Student: ((Refer Time: 06:24))

When I say carrier many cases I use the terms carrier and modulated signal interchanging it, because in this case carrier is not sent by itself, carrier is sent along with it is modulation, so we are referring to these things almost interchanging these things. So, let me come back to my question, suppose phi of t was a constant and so phase theta i t would be omega ct plus phi, what is the instantaneous frequency now, effective frequency is constant now.

What is the value?

Student: Omega c

Omega c, how do you take it from theta i t.

Student: ((Refer Time: 07:11))

Take the instantaneous derivative, the derivative of the phase this is very elementary, it from your school physics it is nothing but, the derivative of the instantaneous phase. So, I could define therefore a quantity omega c sub i t which is equal to d theta i by dt and that is defined the instantaneous frequency of the modulated signal, if you understand what I mean. And what would this be equal to in this case, suppose this is your instantaneous phase what will this be equal to.

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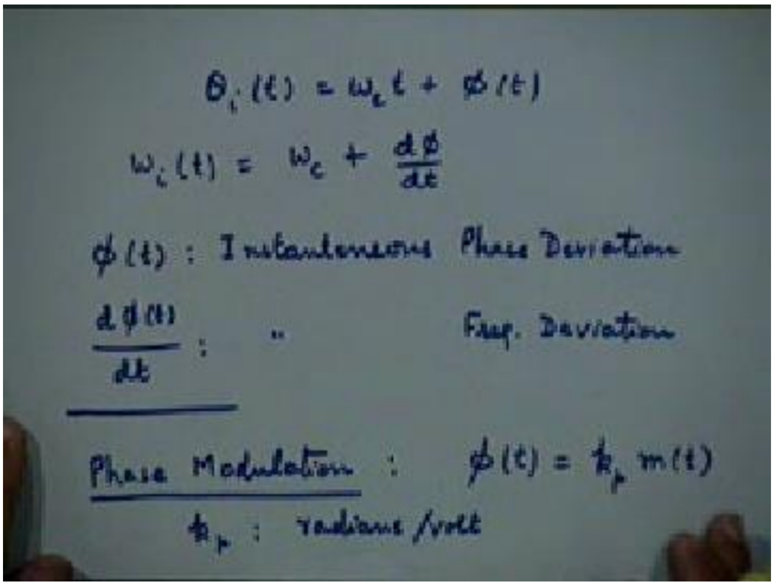

So, in this case your theta sub i t is omega ct plus phi of t, that means your instantaneous frequency is omega c plus d phi by dt. Now, let me introduce some more terms, whereas theta i t refers to the phase of the carrier, you should also give a name to phi of t. Normally when you talk about phase actually you refer to this, but zero speaking that is the precise meaning of phase.

So, what should we call phi of t, we call it phase deviation, if you had a pure sinusoidal signal phase would have gone like omega ct as a function of time. Now, on top of that phase variation you have an additional phase deviation ((Refer Time: 09:01)) phi of t. So we call it the phase deviation, so phi of t is also called phase deviation or instantaneous phase deviation.

Likewise, d phi by dt which appears there should be called instantaneous frequency deviation, so we have a concept of instantaneous phase, instantaneous phase deviation, instantaneous frequency and instantaneous frequency deviation. Because, we shall talk in these terms, so I would like all of you to understand these terms once for all, these are definition of these terms, all of you clear about these definitions.

So, now that we have worked with the basic terminology, we are defined the basic terminology, let us define what is phase modulation and what is frequency modulation, now we are ready to talk about these two kinds of modulation. Phase modulation we say that a carrier is phase modulated, any suggestions what should be the definition of phase modulation, suppose I allow you to define it yourself phi t is the...

Student: ((Refer Time: 10:39))

Phi of t is the instantaneous phase deviation should carry the message information, therefore it should made proportional to the message signal, that should be the obvious way of defining a phase modulated signal. So, in this case we say that phi of t is chosen to be some constant times the message signal, so the constant I am denoting by k sub p or the phase modulation constant or you can also call it the phase sensitivity of the modulator.

So, since this will be in gradients, phase in measured in gradients and this will be typically in volts, so the constant k sub p would have the dimensions of radians per volt. It is basically a phase modulator would reduce the phase deviation in proportion to the message signal amplitude at time t. And in the same token recursive modulation should be defined in terms of d phi by dt, the instantaneous frequency deviation should be made proportional to the message signal and that is precisely the definition of frequency modulation.

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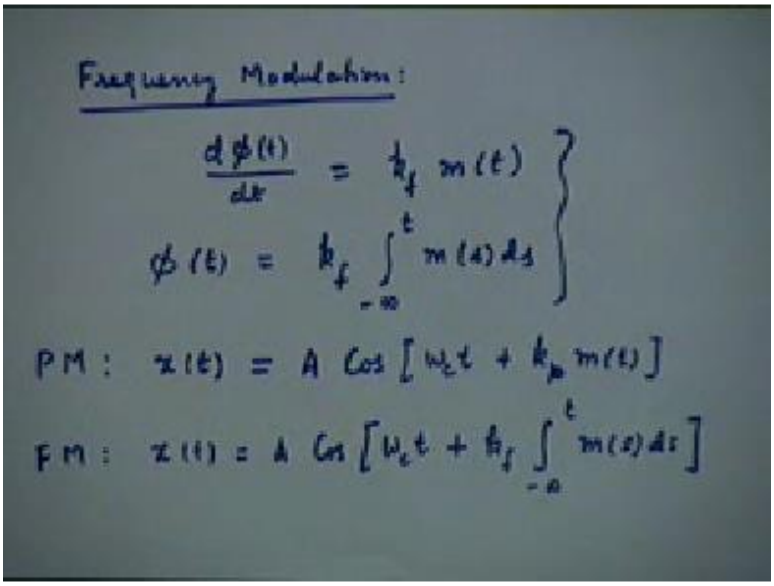

Actually since these two modulations are so similar to each other, we usually study them together rather separately and wherever there is a comparison, we discussed that comparison. Otherwise, many properties of the phase and frequency modulated signals are very similar to each other, so in this case d phi by dt is made proportional to the message signal and I am calling the proportionality constant now as k sub f.

So, k sub f will have the units of radians per second per volt or if you divide it by 2 pi 1by 2 pi on both sides even then k f is the same, what will be the expression for phase in this case. If you take the integral of this, this will become the instantaneous phase deviation equal to k sub f integral minus infinity to t m s ds, so these are the two ways in which you define frequency modulation.

Either you say that the instantaneous frequency deviation is proportional to mt or the instantaneous phase deviation is proportional to the integral of m. This basically follows like this by integrating both sides, these are very simple concepts, so far I do not think there should be any difficulty. So, if I were to write an expression for the phase modulated signal x of t is now very clear what it should be, it should be a cosine omega ct plus k sub p times mt.

Because, your instantaneous phase deviation is proportional to mt and the FM signal, the frequency modulated signal would be A cosine omega ct plus k sub f integral from minus infinity to t ms ds, I hope again it is obvious. We are just substituting for phi of t, phi of t in this case is this integral, in the case of phase modulation it is equal to k theta m sine theta, any questions.

It will be interesting to see what kind of waveforms come out of these definitions. What is the typical waveform for frequency modulation and what is the typical waveform for phase modulation.

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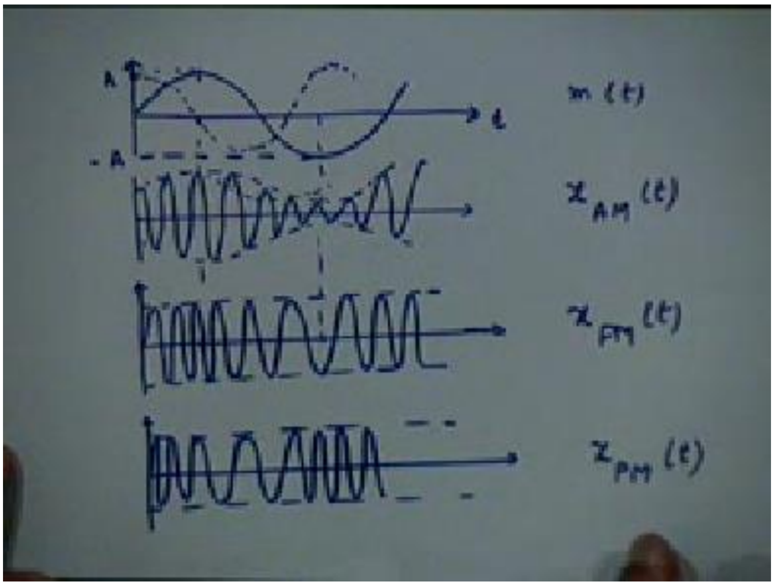

So, let me attempt to plot some waveforms, not exactly good at that, but I think you will try to, let me try to plot all the waveforms that we are discussing first. Let me take a sinusoidal modulating signal, so this is my mt, this is the time axis, this is instantaneous amplitude. Now, what could the amplitude modulated signal look like, I was afraid that I will not be able to do a very good job and that is what is happening, but does not matter I think we can still follow what I am trying to do.

The instantaneous amplitude of the carrier is made proportional to the message signal, that is what the waveform was right, sox sub AM t was like that, amplitude was dependent on the amplitude of the modulation signal. Now, what do we have, let us talk about frequency modulation first, slightly easier to imagine the waveform, the instantaneous frequency let us say you have some carrier frequency when the amplitude is 0, let us call it x sub c or omega sub c.

So, when the amplitude is positive the frequency will be larger will be more than fc or more than omega c, so the amplitude is negative it means less than omega c. So, as the amplitude let us say varies from some peak value plus A to some trough value minus A, the frequency will vary from fc plus some value let us call it delta f to fc minus delta f. And the frequency will be the largest at the peak value and will be smallest at this point, so now can you imagine what the waveform looks like, the amplitude of the waveform is not being changed, it is not being modified

So, the amplitude remains constant, so basically the cycles which constitute the cosine function, they are either occurring close together they are very short, either the cycles are very short in duration or the cycles are slightly longer in duration. Here the cycles will be most compressed the shortest, because the frequencies are highest, here the cycles will be the least compresses most rare, so the cycles will be slightly far apart from each other; so that is the way the waveform will look like.

So, if I were to capture this, at this point you expect the cycles to be very closely spaced, at this point you expect eh cycles to be sparsely spaced, so that you have to draw a waveform which like that, amplitude is constant, I am just calling this amplitude. So, it may look like this and this point they have to be rather sparsely spaced, I will explain again I have attempted to plot this, I do not know how accurately I have been able to do that, but let me repeat what I am saying.

Let me again go back to this reference waveform the modulating signal, you are making the instantaneous frequency proportional to the message signal amplitude, message signal amplitude is the largest here. Therefore, the instantaneous frequency should be the largest at this point in time, when if you the instantaneous frequency is large, that means the cosine wave varies rapidly that frequency at that time instant.

That means, your consecutive cycles are closely spaced, the time period is smaller, frequency is large means the time period is smaller, so the time period is smallest here. So, the waveform looks compressed at this point, where the amplitude is the smallest the instantaneous frequency would be the smallest and therefore, the cycles of the cosine wave they look farther apart will be more sparsely spaced. And therefore, that is precisely what you see, the time period is much larger here the time period of the cosine wave is much larger here, this is the rough plot and argument for this plot that are presented.

Yes please

Student: ((Refer Time: 20:37))

No, it has to be proportional to the actual value in the amplitude, because you want to capture the information from the instantaneous frequency, if I made it proportional only to the magnitude the sine information about the message will go away, which you do not want to go away. So, the definition was very clear, we wanted the instantaneous frequency deviation to be proportional to mt, not proportional to the magnitude of mt, very important.

So, the question was whether it was proportional to mt or proportional to mod of mt, answer is proportional to mt and not to mod of mt, except that is the case for any modulation. For example, even amplitude modulation you want the instantaneous amplitude to be proportional to the message signal, particularly in the case of DSP AM, because there we want to capture the or cover the message information by just looking at the envelope.

So, is that ok does this waveform intuitively appeal to you, that this is what it might look like, it look like a sine wave, after sine wave. But not with regularly spaced 0 cross x, the cycles of the cosine or sine wave would be either close by together or farther apart from each other depending on the instantaneous amplitude of the message single. So, that if you were to see the waveform on the oscilloscope, this is what you will see, the FM waveform.

Now, what about the PM waveform anything that I, you can say about the PM waveform actually these two waveform look very similar to each other. Because you are making the phase proportional to mt, but therefore making the instantaneous frequency vary with respect to m t, not proportional to mt, but somewhat dependent on mt. Let us go back to the definition of PM, this is the definition of phase modulation, what is the p, what can we say about the instantaneous frequency.

Student:((Refer Time: 23:06))

It is proportional to the derivative of the message signal now, so instantaneous frequency also varies once again, but the value of the instantaneous frequency follows the behaviors of the derivative of the message signal rather than the message signal directly. So, depend on what is the nature of the derivative of their message signal, as to what the instantaneous frequency in the phase modulation would be, so for this example I have taken sine wave as the modulating signal.

So, the derivative of the message signal is a cosine wave, so the phase modulated signal will look like a frequency modulated signal, which is modulated by cosine wave in this case. So, the peak frequency there will occur at pi by 2phase shift, with respect to this message signal, so instead of occurring here you can say it will occur here, so the message portion of the signal will come here and then, it will...

So, ((Refer Time: 24:27)) cosine wave and the wariness will occur here some were, again then this will occur here, I have not drawn the waveform right, but I hope you understand what is happening. So, this is x sub FM t and this is x sub PM t are you with me, so if you have understood these waveforms, you have basically understood the nature of FM and PM signals, any questions.

Yes please

Student: ((Refer Time: 25:38))

There is no concept of under modulation and over modulation here

Student: ((Refer Time: 25:49))

Yes

We can make it, you can choose this constant k sub f and k sub t as you like, small or large, there will be other implications of the choice of these constants which we shall look at. But not in terms of over modulation or under modulation, there will be some implications. Have you understood the nature of the waveforms of these two kinds of modulations that we are now studying, it is obviously now interesting to look at their spectral properties, that is the first thing you will like to understand. What are the spectral properties of these waveforms, so now what can we say about that some difficulty arises, because unlike...

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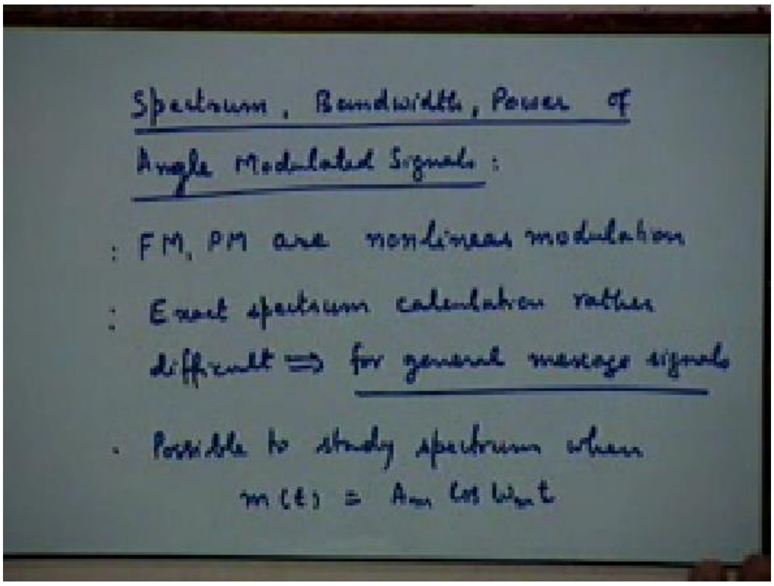

So, let me just see the topic of interest now is, we will like to understand the spectrum, because it is through the spectrum that we will understand what is the bandwidth requirement, that is the first thing that you will like to understand as a communication engineer. Bandwidth requirement if I go for FM, if I go for phase modulation, to do that I must understand what is the nature of the spectrum, so let us look at these issues, spectrum, bandwidth, etcetera.

And of course, the other parameters which is of interest is the power requirements, first difficulty that we must know it is, that unlike the amplitude modulation, the frequency modulated. And phase modulated signals in a way are non-linear modulation, we appreciate this, that is they do not satisfy the superposition principle which the amplitude modulation does, particularly the DSB AM it does, so these are non-linear modulations.

And because these are non-linear modulations, the exact spectrum calculation becomes very difficult, because the message signal is dependent on the modulated signal to the argument of the cosine wave. So, this is a highly non-linear function sine of some mt or cosine of some integral of mt, these are highly non-linear functions, so the spectral analysis now becomes very difficult.

So, exact spectrum calculation at least for the general case, where the modulating signal amplitude something arbitrary, at least for general message signals, this was not the case if I remember for the case of amplitude modulation. I could easily express, because what happened there all that was happening was frequency translation which is of course, one of the main objectives of modulation, it has to carry out frequency translation.

So, by choosing a carrier of some frequency f sub c, we are carrying out frequency translation even here, but whereas in the case of amplitude modulation of all kinds you are essentially carrying out frequency translation, this is something more than that. Because, you are modifying spectrum of the modulated signal in a rather complicated manner and that complicated manner is not possible to understand, when the message signal enters some arbitrary message signal.

However, it is possible to do this calculations for a very simple case and we will study that case only at the moment and that is when you choose the message signal just for the sake of the study to do a pure sinusoidal signal. So, possible to do this calculation, possible to study the spectrum when m t is equal to Am cosine omega m t, this is a very simplistic form of the message signal. But by understanding this, you understand a lot about what happens to the spectrum of the fm signal.

So, we will first go try to go for study the behavior of spectrum of the FM signal or the PM signal when the modulating signal has a simple form. We know that in general the modulating signal will not be of this kind, it will be some waveform, like speech waveform or video waveform or whatever you have, these results will not be directly applicable, but still you will have intuitive understanding through this.

So, we will look at that, I will make a more general statement later, so let us start with that, so let us take mt is equal to AM cosine omega mt.

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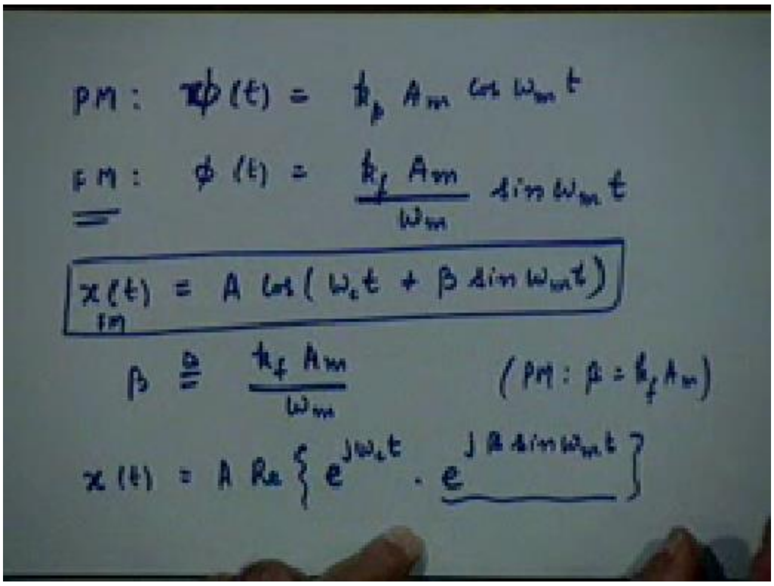

And phase modulated signal the expression for that will now become, basically your instantaneous phase will be given by k sub p times Am cosine omega mt and for FM this will become, what will be the value of the instantaneous phase is the case of FM, it will be the integral of this waveform. So, k sub f because the constant varies k sub f times, what is the integral of this A sub m divided by omega m into sine omega m t, so the instantaneous phase of the phase modulated signal would be like this.

For the frequency modulated signal it would be like this and let us consider FM first, so your waveform x of t would be let me write x sub FM t, when I do not write this I think the context will make it clear, whether I am talking about FM or PM. So, it will be A cosine omega ct plus, let us call it constant beta times sine omega mt, beta is defined as k f in this constant, k f Am upon omega t. For the case of PM this will become A cosine omega ct plus let us call it again beta cosine omega mt.

And the value of beta they would be k p AM, so for PM beta would be k p Am and of course, instead of sine omega mt it would be cosine omega mt it hardly makes any difference, you will see that. So, let us try to study this signal, now how can I study the spectral properties of this signal, can you suggest, is there a suggestion from the floor as the how we can study the spectral properties of the signal. Is there something about the signal that strikes you?

Student: ((Refer Time: 34:07))

Can you speak a little loud, so that everyone can

Student: ((Refer Time: 34:10))

Phasor representation, phasor representation does not help us study the spectrum very much, it helps us to study the behavior of the instantaneous envelope or instantaneous phase, but not the spectrum. Spectrum if you remember you have to go somehow in the frequency domain, to study the spectrum you have to go into the frequency domain, what is the most convenient frequency domain, representation that I should go for there.

Well you could say Fourier transform, but Fourier series would be more apt, why you could see its periodic signal and that was the reason why I chose the modulating signal to be a cosine wave if it that clear, here let me the complex representation. You can think of x of t as the real part of A times the real part of e to the power j omega ct into e to the power j beta sine omega mt, it is a real part of j omega ct plus beta sine omega mt.

I have separated that out, I write like this; can you see that this is the periodic signal, e to the power j beta sine omega mt sine omega mt is a periodic function, so e to the power j beta sine omega mt is a periodic function in the same period. So, we say this is a periodic function, it is complex periodic function of course, I can represent it by a Fourier series and therefore, I can study the spectrum by looking at the Fourier series of this function.

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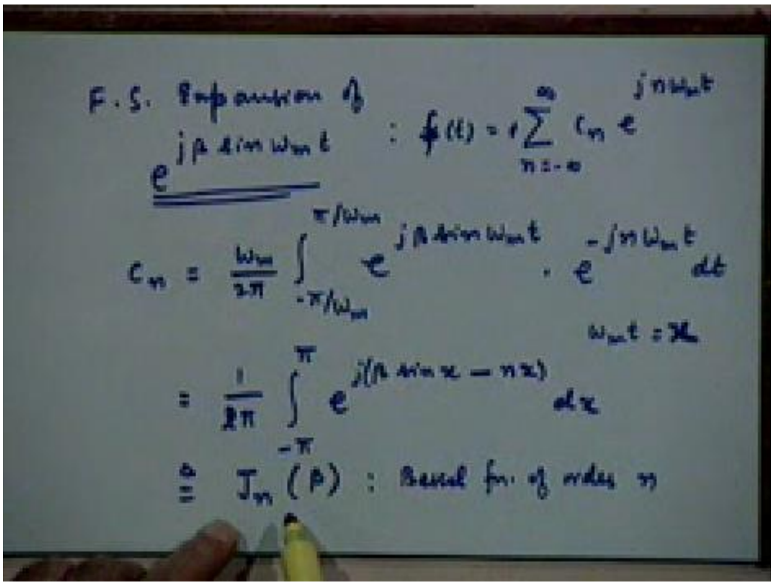

So, let us do the Fourier series expansion of e to the power j beta sine omega mt, what is the Fourier series expansion, we write x of t equal to n going from minus infinity to infinity. I will show you some other waveform, let me at this function, this function, I am calling it f of t. Some coefficient c sub n into e to the power j n omega m t, so the coefficient of Fourier series are c sub n, this is the n the harmonic, the fundamental is the omega m, so we need to find what is c sub n.

Integral from over the period and what is the period here, minus pi by omega m to pi by omega m, so the total period length is 2 phi by omega which I am dividing between minus pi by omega m to pi by omega m into the function ft which is e to the power j beta sine omega m t into e to the power minus j n omega sub m t dt divided by the period and how much is the period.

Student: ((Refer Time: 38:16))

So, it will become omega m by 2 pi, reciprocal of the period, period is 2 pi by omega, so you are diving it by the period which is omega m by 2 pi to see what this looks like let me put omega mt equal to some variable u. So, this will be come 1 by 2 pi minus pi to plus pi e to power j beta sine x minus n x dx, it should be u, let me make this x, I am putting omega mt equal to x that will become this.

Anyone familiar with this function, you have done it in your maths course, you remember what this is, this function this integral not familiar, not at all familiar, not beta function it is the Bessel function. You have not learnt Bessel functions, if you say so I do not know that usually not the case at this time, everybody is familiar with Bessel function, does not matter.

Let us define this, this is the definition of the Bessel function which is denoted by j sub n, Bessel function of order n s denoted by j sub n and x is a dummy variable, what is the constant here, what is the parameter in the integral which will definitely be this result will be a function of beta, of course, comes here the dependence on n is already there c sub n is equal to this, so this is equal to, there is a dependence on n displayed here and the dependence on n displayed d here. So only the parameter left is beta, so this is called Bessel function of order n, the argument equal to beta.

So, these are very highly very well studied set of functions and it is properties are highly documented, readily available in books of mathematics. But unfortunately this function cannot be expressed in closed form, it is expressed only in terms of this integral. In spite of that it is properties are very well studied, it is values are documented in the form of table of Bessel function values, so for different values of beta you have a table available for j 0 beta j 1 beta j 2 beta etcetera for all values f beta.

So, these are available in the form of tables in standard text, similarly for j minus 1 beta, j minus 2 beta, etc and for all values of beta. So therefore, what you find is the nth coefficient of the Fourier series of e to the power of j beta sine omega mt is nothing but, J n beta. So, can we come back to our original purpose, our original purpose was to study the signal ((Refer Time: 42:09)), so instead I have first looked at this signal, for which I can reconstruct x of t, this I am writing in terms of it is Fourier series. So, let me substitute this in terms of it is Fourier series, this will be what kind of Fourier series is this, complex Fourier series.

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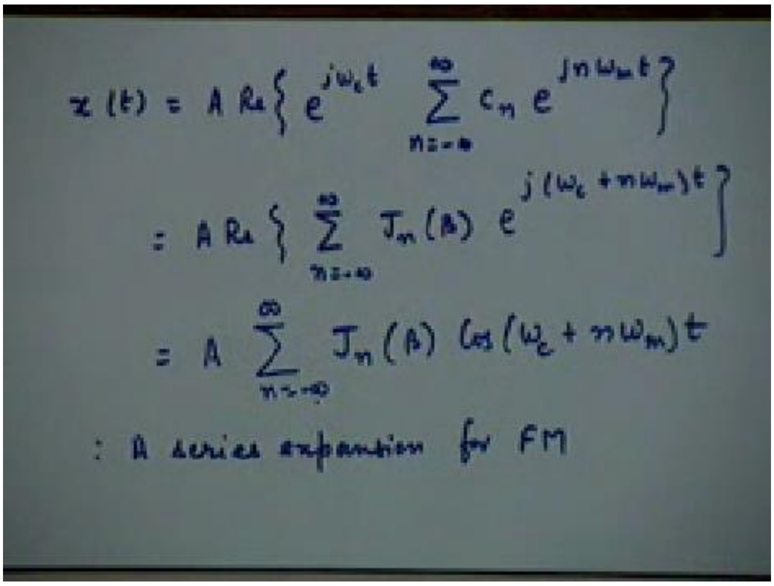

So, your x of t A times real part of e to the power j omega ct and what is the complex Fourier series that we have just derived c sub n e to the power j n omega m t and c sub n is nothing but, j n beta. So, this becomes A times real part of or let me combine this I can write lie this n going from minus infinity to infinity J n beta, if I combine these two terms what will I get e to the power j omega c plus n omega m t, I just come e to the power I have taken the term e to the power j omega ct inside this summation.

So, it will multiply every term and that is what you will get n th term omega c plus n omega m t and now if I take the real part this becomes, that is what you get, so FM signal I have obtained series expansion of the FM signal. But, this is not precisely a Fourier series expansion of the FM signal can you appreciate that, because the terms they are not harmonics of omega c and omega n, the terms they are omega c plus minus omega n, the harmonics are coming, because of the this component here.

So, in any case it is a frequency domain representation, I have expressed x of t in terms of cosine waves of different frequencies, so in that sense it captures the spirit of spectrum representations, any frequency domain representation. I have expressed x of t after all what is Fourier series, you are expressing a given periodic signal in terms of more basic signals which are the sinusoids of different periods, same periods through here x oft is comprised of frequency components which are present in this, in this series expansion.

So, let me just complete this, so let me complete this statement x of t, therefore we can say is comprised of recursive components omega c and omega c plus minus n omega n, n going from minus n 0 to infinity or if you are saying omega c plus and omega n from minus infinity to plus infinity. Yes please, what is the question?

Student: ((Refer Time: 45:42))

J n beta is a real value

Student: ((Refer Time: 45:46))

No, it turns out to be a real value, we will study the properties of J n beta, I will briefly I will not really study the properties of J n beta, but I will summarize the properties of the Bessel functions for you, if you have not done them anywhere, have not done them does not matter. The important properties that are relevant to our study we will enquire, we will not go into the study of the Bessel functions, but some of the things that we should know about them we will summarize that.

So, this tells us a few things about the FM signal, what are the things it tells, first of all our modulating signal was the signal of frequency omega m, let us keep the AM signal as pour reference point for our study. When the modulating signal is sinusoid of frequency omega m what was the frequency components in the AM signal, there was carrier component if you did not suppress the carrier omega c and there what is the components omega c plus omega m and omega c minus omega m.

What will you find here, you will not only have the components omega c plus omega m and omega c minus omega m, but omega c plus 2 omega m and omega c minus 2 omega m and so on, and so forth, add infinite term. You have still two side bands, one set of frequency components lying above omega c, the other set of frequency components lying below omega c, but each of these two side bands occupies a very large spectral domain, so it will keep going up to infinity.

And this is the case when the input signal contains only a single frequency component, so that is the spectrum of the FM signal is very different from that of an AM signal, that is the first thing to learn. The addition thing that you will like to learn is, theoretically it appears as the bandwidth of the signal is infinity, that is what it appears, but practically it turns out that this function J n beta as you increase the value of n it is magnitude decreases.

So, not all the components are significant, a certain number of frequency component are significant that will depend on the value of beta and how many component are significant will therefore decide, what is the effective bandwidth of the FM signal. Theoretical bandwidth is infinity, but practical bandwidth will be more reasonable, it will be some practical value which will depend on the value of beta, so bandwidth will now depend on beta which is incidentally have not defined what is beta.

I have defined beta in terms of the cost terms, but it is also called the modulation index in this case, so the spectral properties of the signal were therefore depend on the modulation index beta, so we will look at these things in more detail next time.

Thank you.