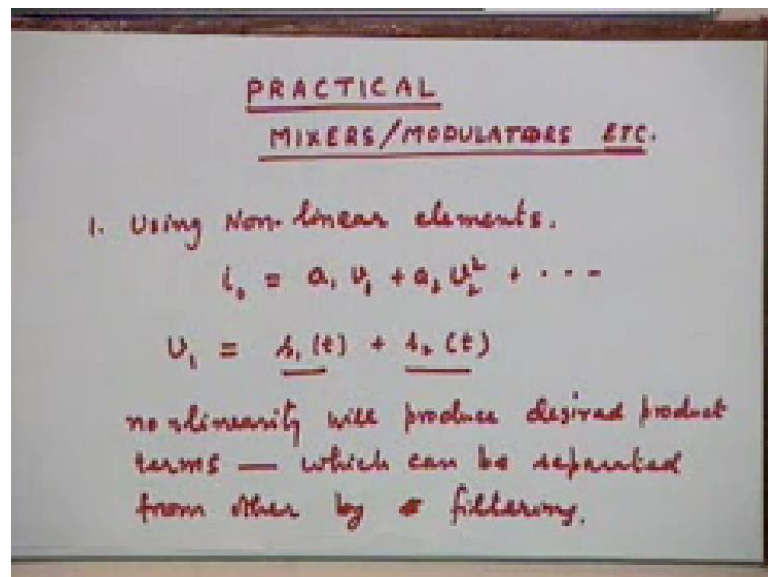


Communication Engineering
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Lecture - 14
Practical Mixers
Effect of Tonal Interference in AM

Today, we will continue our discussion that we had started yesterday on Mixers, if you may recollect we said that as far as this component multiplication is concerned.

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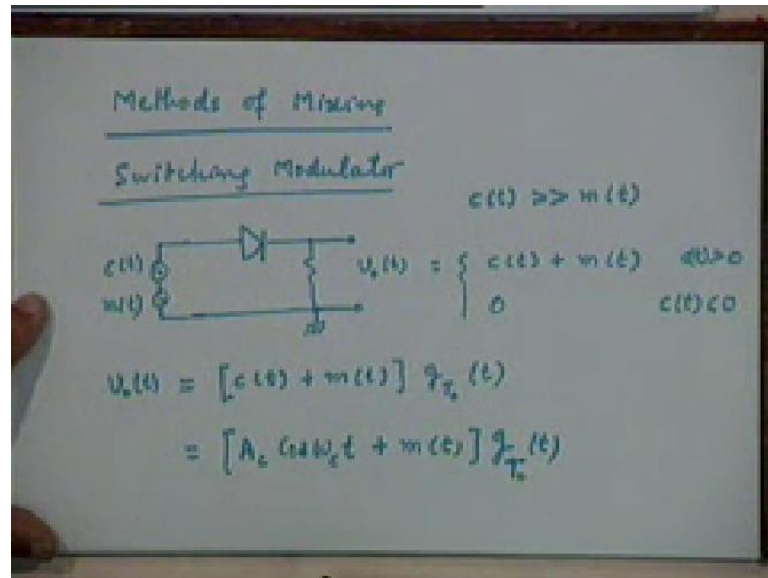


And this is the basic component multiplier, this is the basic component required from modulation and mixing, this can be realized in several ways and one method was to use in non-linear device which has some non-linear characteristics. For example, the device in which the current voltage characteristics have a non-linear behavior, like we have in junction diodes or the PN junction of a transistor. So, based on this principle we can see that, if you apply the input voltage which is the sum of the voltages, which you want to multiply basically.

The cardinal nonlinearity present the output current will have some product terms and if you can ensure that there is suitable filtering you can take out only the desired product term, and reject all other components, you have your the purpose is achieved. So, I will leave at this point for practical circuits based on this principle I like you to refer to

books. The second kind of mixers that we can have is based on the switching principle, which also I believe you have some idea of and we will quickly first review what you might already know.

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So, we are looking at some methods of heterodyning today, a mixing and in particular we look at first the switching modulator which is very simple, is the simplest form consists of an ideal diode followed by a resistance. At the input to which we apply the sum of the two voltages which we want to multiply a mix, so one of them let us say is a carrier c of t and other is the message m of t . Since the diode is considered to be ideal, if you look at the behavior of the output voltage as a function of the diode connecting and not connecting, basically the diode will go through the two phases of operation.

In the normal mode, in the forward bias mode it will act as perfect short between input and output, and the output voltage v_{output} would be simply equal to the input voltage, at that time. So, when the diode is perfectly conducting, the output is simply the input and which is nothing but, the sum of the two voltages that we are applying the carrier and the message.

When the diode is reverse biased the output voltage is 0, let us say if this is connected to ground which essentially means that, now basically we have to now see when does the diode conduct and when it does not conduct. To ensure that to get the required multiplication factor that we want, what we impose we will impose a constraint that the

carrier is most of the time much larger than the message signal; to carrier amplitude is much larger than the message signal.

If that assumption is true, then it is clear that whether the diode is forward biased or reverse biased will be essentially dependent on the carrier voltage. So, whenever carrier is going through a positive half cycle, the diode will be forward biased and the output voltage will be nothing but, equal to the input voltage. When the carrier wave form is going through it is negative half cycle it is less than 0, the diode will be reverse biased and the output voltage will be.

So, this kind of switching action, basically what we are doing is you are getting an output voltage which is a product of the input voltage and a 1, 0 sequence. In this half you are multiplying the input voltage with 1; in this half you are multiplying the input voltages by 0. So, it is as if your output voltage $v_{\text{out}}(t)$ is the product of input voltage wave form $c(t)$ and $m(t)$, of course this is not the product what you want or apparently want, but actually it leads to the same effect.

This is being multiplied by a switching function or a square wave function which we call $g_{\text{sub T}}(t)$, which looks like this before I say about $g_{\text{T}}(t)$, let us substitute for carrier which could be of this kind plus the message wave form $m(t)$.

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The whiteboard contains the following content:

- A square wave function $g_{\text{T}}(t)$ with period $T_c = 1/f_c$.
- The Fourier series expansion:
$$g_{\text{T}}(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} \cos[2\pi f_c (2n-1)t]$$
- A block diagram of a Band Pass Filter (BPF) with center frequency f_c . The input is $U_i(t)$ and the output is $U(t)$.
- The resulting AM signal equation:
$$U(t) = \frac{A_c}{2} \left[1 + \frac{4}{\pi A_c} m(t) \right] \cos 2\pi f_c t$$
- The text: "= AM Signal"

Where $g(t)$ is a periodic wave form which looks like this, so for half the carrier cycle you are multiplying with 1, so $g(t)$ is 1 during that half cycle, for the other half cycle it is 0, then again it is 1, again it is 0. So, basically this is the kind of function with which you are multiplying the input voltage which is $c(t) + m(t)$, this is the nature of where the period of this wave form is T which is equal to $1/f_c$ the carrier period.

Now, this does not conclude our discussion, because we do not want this ((Refer Time: 08:07)), we want a product between these two, fortunately that is possible when we realize that this periodic square wave can be expanded in terms of a Fourier series. And without going for the details all of you know what the Fourier series of square wave should contain only the odd harmonics. Since this is a square wave which goes to an excursion between 0 and 1, it will also have a DC component it is not a zero mean square wave.

So, there the DC component which is equal to $1/2$, this is the Fourier series which I am not completing here, but which all of you know and easily compute. These are the coefficients of the Fourier series $1/(2n-1)$ and this is cosine wave of frequency f_c into $2n-1$. This is the $2n-1$ odd harmonic, so it will have harmonics at the fundamental corresponding to n equal to 1, n is equal to 2, because we are going from 1 to infinity, third harmonic corresponding to n equal to 3 and so on and so forth.

So, if you now look at this product $v(t)$ is the product of ((Refer Time: 09:39)) this into this Fourier series and as you can see, this product if you look at, if you were to have a band-pass filter following this system, suppose you follow this up with a band-pass filter, so your $v(t)$ is coming here. You follow this up with band-pass filter and the output $v(t)$ will contain only those terms which are allowed by the band-pass filter, let us say band-pass filter is tuned to the to pass frequencies in the neighborhood of f_c .

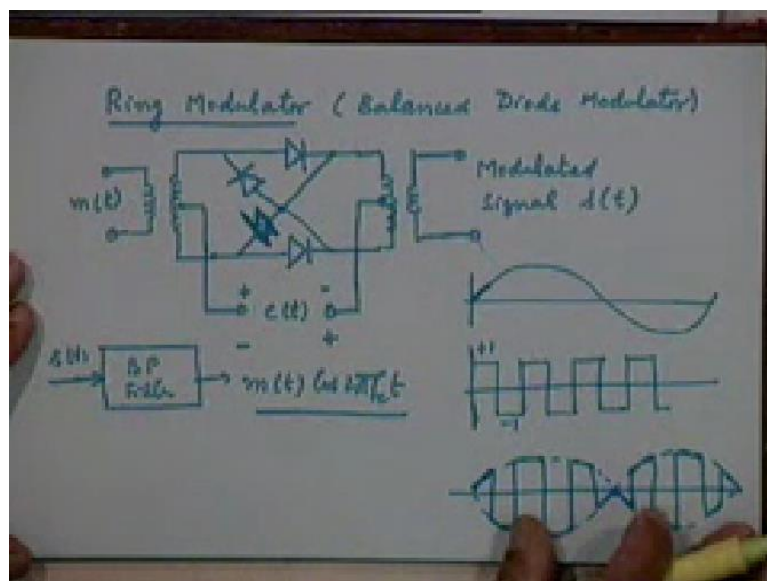
So, the output $v(t)$ in this case would be look at the product terms which will be at frequency f_c , there is a constant half which will multiply with $A_c \cos(\omega_c t)$ and there is this message $m(t)$, which should then multiply with the first term of the Fourier series will produce a component at ω_c or around ω_c . All other components in this term when multiplied with either this or this would lead to frequency components

which are multiples of ω_c , $2\omega_c$, $3\omega_c$ whatever, then all the rejected by this band-pass filter.

So, the final output $v_o(t)$ would therefore be nothing but, we can write this as $A_c [1 + 4\pi A_m t \cos(2\pi f_c t)]$, this is a desired modulated signal. This contains the carrier component plus the DSBSC basic component that we are looking for, the modulation index is $4\pi A_c$ in this device, so this generates an AM signal, so this is fine.

Now, suppose we wanted a pure DSBSC signal without the carrier component, we can still apply the switching idea the only difference we need to somehow create is that instead of multiplication of alternate half cycles with 1 and 0 we need to multiply the input wave from a plus minus 1 to make this wave form zero mean. The DC component of this wave form should become 0, so that this factor half does not appear which multiplies with the carrier component in the other term.

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And that is precisely done by a device called the ring modulator or the balanced diode modulator, the typical circuit will look something like this, this should have been like this, so this is the balanced modulator, it is also called ring modulator. The reason being that these diodes all face in the same direction, as we trace the diodes their all point in the same direction. So, if you go along these diodes they form a ring in some sense, so

we call it a ring modulator, so we applied the center tap these transformers here and apply the carrier voltage c of t at these points.

And your modulated wave form m of t is applied here and this is your output voltage the modulated signal, let us call it s of t , the principle is more or less the same, when c of t is positive half cycle, two of these diodes conduct and two of them are reverse biased. Two of them are forward biased, for example when let us say this is in this cycle to this terminal is positive with respect to this that positive half cycle which diodes will conduct these two, and these two will be reverse biased as you can see.

Because, this end will be at higher voltage than this end and therefore, will be reverse biased similarly for this, remember that we are assuming that for this discussion that these transformers are short circuited in that sense. So, more or less we are assuming that this point is directly connected to this point, so we are assuming that the impedance is offered by these diodes at the carrier frequency is very small.

Similarly, so when is these diodes conductive basically what you are going to get is this voltage, which is sum of once again the carrier voltage and the input voltage appears across this and therefore, they across the output directly. In fact, more or less you can say that the m t directly appears in this case, because c t is actually balanced out, there is no net carrier applied at from here to here. As you can convince yourself, the carrier voltage appearing between these two transformer term.

If the transformer is perfectly balanced, if this center term exists precisely in the middle of the binding, there will be net 0 voltage of due to carrier here and similarly net 0 voltage due to the carrier here, across these two terminals. So, basically this voltage m t assuming this ((Refer Time: 16:57)) whatever will appear across this output and therefore, at the output. So, you will get m t multiplied with 1 at the output in this case, when the carrier is in the other direction, now which diodes will conduct, these two diodes will conduct, and these two will be switched off.

What is happening now instead m t reaching this point here, the basically the ((Refer Time: 17:27)) the connections are reversed, so as if we are changing the sign of m t , the positive voltage is going down in a manner of speaking and negative voltage is... So, effectively the connection between the input and output is crossed and therefore, that is

equivalent to multiplication of $m(t)$ with ± 1 . So, if you want to plot this wave forms, basically what we are saying is something like this, maybe this is a better place to plot.

Assume this is your modulating wave form and to show the switching action, basically what we are saying is we are multiplying with this wave form with these positive and negative pulses or this square wave form which goes between plus 1 and minus 1. The output will be a wave form like that; of course this switching will be at a much higher rate than I have shown in the picture, because this is a carrier frequency, if this switching is taking place at the carrier frequency which is a high frequency, so you will get something like that.

So, during this time we are multiplying with plus 1, so whatever is appearing across $m(t)$ here is goes across like that, in this interval we are multiplying minus 1, so you get something like that, in the same messages form appears with the negative side and so on, and so forth, etcetera. So, you get a modulated wave form which looks like this, of course once again the carrier here does not appear to be sine wave, it appears to be square wave.

So, what should I do? If you remember the basic theory remains the same, $m(t)$ is multiplied with the square wave, square wave contains this fundamental component and it is harmonics, odd harmonics. So, you want only the component which is the product of this $m(t)$ with the first harmonic of this, so what should we do simply pass this to a band-pass filter tune to the carrier frequency f_c .

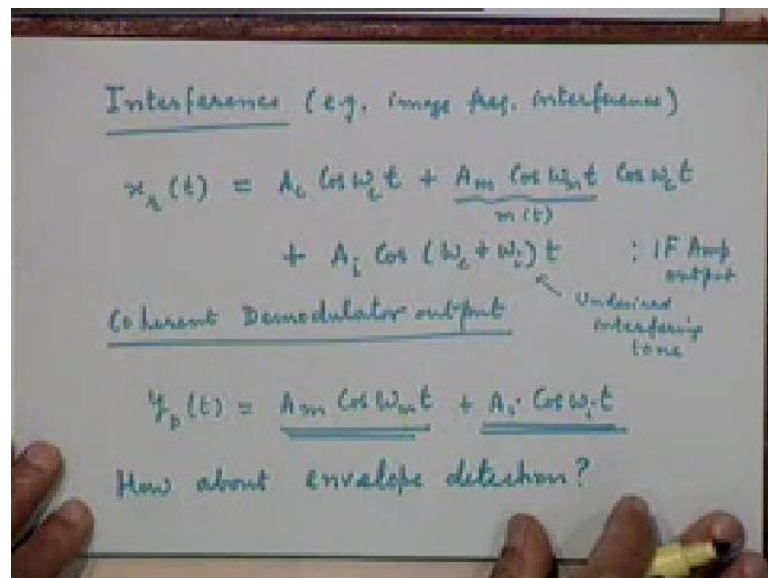
So, this output, this $s(t)$ will pass through the band-pass filter will produce the perfectly balanced $m(t) \cos(2\pi f_c t)$, that you want with a scaling factor of suitable value, so that is a balanced modulator for us. For this to be balanced this transformer need to be perfectly balanced, so that the carrier component does not appear any output, so that the net voltage if the carrier here and here is 0 and also this diode should be identical.

If they have finite, if they are ideal there is no problem, but if they non ideal they should have more or less the same value of the forward resistance, so those are the requirements for making them balanced. We can similarly have other circuits I think I just wanted to give flavor of the kinds of things that we can do to generate both AM signals, as well as DSBSC signals. Having generated any one of them, one can then generate either the SSB signal or the VSB signal by appropriate sideband filtering.

Any questions, if not then the last topic that I am going to take with amplitude modulation for the time being, I will return to envelope detection once again, remember envelope detector is one of the most commonly used detectors for detecting amplitude modulated signals. Commonly used, because most of the when we use amplitude modulation, particularly in broadcast applications we are using it in with the carrier, it is DSBAM, which has a carrier component.

So, as to make the envelope proportional to the message wave form m of t and the idea is to have a very simple demodulator using a diode and 2 r c one resistance and one capacitance the diode detector. Now, when you use this device for detection we must be aware that there are some limitations of this device and that limitation is what I want to talk about right now, that limitation typically arises in the presence of interference. If there is an interfering signal present along with the desired signal, then under certain conditions the interfering signal can take over, the output rather the desired signal. And that is something that we must understand, so I like to spend some time on the effect of how the diode detector works in the presence of interference.

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So, at the moment I am not taking interference to be noise, noise analysis is something will do separately, at the moment I am talking about another interfering signal as the interference. For example, let us say just to put the interference idea in a proper perspective, we discussed the super heterodyne receiver yesterday and we said that

unless you design your RF filter properly, there is a possibility of the image frequency component coming through at the input of the IF filter.

And if that does, if that happens, because it will happen in that image frequency component will nearly appear at the IF intelligent frequency itself and the intelligent frequency filter will not be able to reject it. Because, it is in the neighborhood of the intermediate frequency, so maybe you have desired signal ω_c , desired signal of frequency which is a modulated signal around the frequency ω_c .

Along with that we get an image frequency component, if your RF filter is not good enough that is likely will happen, so for example it can be the image frequency component. Of course this will not happen, if your RF filter is designed to reject at least the image frequency, but even if this was not the case, there can be situations where an undesired transmitter is located very close to the frequency of interest.

Let us say when you are trying to receive a transmitting station at a certain carrier frequency and there is an interfering transmitter which is also located in the same neighborhood or in the neighborhood of receiver. It may not be in the neighborhood of the transmitter, but it may be in the neighborhood of your receiver transmitting something else, it may be an undesired transmission, sometimes you have a RF equipment, it is not supposed to transmit, but there is a leakage.

You have a receiver and there is a leakage coming from this equipment, which works on that radio frequency which has an internal operation at the radio frequency. And some components may radiate some power and that power will be getting in to your receiver, and that will appear in the form of interference, so that is why you call it a tonal interference. Tonal, because there is a sine wave or cosine wave of a frequency close to the carrier frequency, which is directly coming in into your receiver along with the carrier frequency.

So basically what we are saying is that we have a receive signal x_{rt} , which contains the carrier of interest, it contains the modulated signal of interest and for the purpose of discussion here, I am taking the modulated signal to be $A_m \cos(\omega_m t)$. As for this is $A_m \cos(\omega_m t)$ into $\cos(\omega_c t)$, these two are the desired components out of which eventually I want to get output at the envelope detector output I want $A_m \cos(\omega_m t)$.

I want something proportional to the message signal, I am taking this to be message signal $m(t)$, but along with these two things which is a carrier the DSBSC part, this for example this could be the A_m signal that I am receiving, these two together. Along with this I am getting an interfering signal of amplitude $A_{sub i}$ into cosine of ω_c plus ω_i , that is a pure tone cosine wave of frequency ω_c plus ω_i .

Where ω_i is close to ω_c and is therefore not rejected either by the RF filter or by the IF filter, so where is this interference present, at the output of the RF filter mixer this will produce a tone at the frequency, think of this as the output of the IF, intermediate frequency stage. So, this is IF amplifier output. In this case basically what we are saying is that this ω_c we are talking about here is the intermediate frequency.

So, in any case there is a very closely located signal which is not rejected by the IF filter and that is coming along with your desired signal and what will the stage following IF stage, the diode detector will try to detect your message signal by passing this through a diode detector. So, we now like to study what happens if the input to the diode detector is not this wave form which could have been the normal thing, but this corrupted with this interference, is that clear what we are trying to do. So, before we talk about diode detector, let us talk about coherent detector, what will the coherent demodulator produce, so remember this is the undesired interfering tone.

Student: ((Refer Time: 29:39))

No, think of this ω_c as ω_{IF} , normally at the output of the IF filter, you were only got the carrier at the higher frequency and both as well as here, but along with that you are also getting a close by tone which the IF filter cannot reject, think of ω_c here as ω_{IF} . So, this interfering tone is close to that ω_{IF} and therefore, cannot be rejected, so can you say what will be the output of the coherent demodulator, the coherent demodulator will produce an output corresponding to this which will be equal to...

So, y D t we are going to in a details, because these are things that you now understand very well, you are multiplying this with cosine $\omega_c t$ and then, low pass filtering the result. You will get $A_m \cos(\omega_m t)$, maybe there is some scaling factor, which I am ignoring here. But, you will also get something here

Student: ((Refer Time: 31:03))

We will get something proportional to $A_i \cos(\omega_i t)$, so even in the coherent demodulator you will get an output corresponding to this interfering frequency passing through the input signal. So, even the coherent detector will not work perfectly in that sense, not only you get your desired signal, but you get your tone, the interfering tone of frequency ω_i . However, the amount of this interference that you will see at the output will be directly proportional to the amount of interference present in the input; this is something that we accept as communication engineers.

We call this a graceful behavior, the output is as bad or as good as the input let us put it that way, input is bad your output is bad; we cannot do very much about it. So, is this output interfering component is directly proportional to the input interference component, so that is what the coherent demodulator will do. Now, how about the envelope detector, to see this, it is useful to take the course to the phasor diagram, phasor diagram representation of this signal that we are talking about here.

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$$\begin{aligned}
 x_r(t) &= \text{Re} \left\{ A_c e^{j\omega_c t} + A_i e^{j\omega_i t} \cdot e^{j\omega_c t} \right. \\
 &\quad \left. + \frac{1}{2} A_m e^{j\omega_m t} \cdot e^{j\omega_c t} + \frac{1}{2} A_m e^{j\omega_m t} \cdot e^{-j\omega_c t} \right\} \\
 &= \text{Re} \left\{ e^{j\omega_c t} \left[A_c + A_i e^{j\omega_i t} + \frac{1}{2} A_m e^{j\omega_m t} + \frac{1}{2} A_m e^{-j\omega_m t} \right] \right\}
 \end{aligned}$$

Phasor diagram: A vector $R(t)$ is shown in the complex plane. Its projection onto the horizontal axis (representing the carrier) is labeled as the "ED output" or $R(t)$, which is proportional to $A_m \cos(\omega_m t)$.

And let us first look at the phasor diagram, first let us write the whole expression, let us write this in the form of complex exponentials, because for phasor diagram it is better to work with complex exponentials. So, you can think of them receive signal x_r , x sub r t as a real part now the first term was $A_c \cos(\omega_c t)$, so is the real part of $A_c e^{j\omega_c t}$ that is the carrier component.

Then, there is this interfering component which is at frequency, $\omega_c + \omega_m$, the cosine $\omega_c + \omega_m$ that is $A \cos(\omega_c + \omega_m)t$ real part of $e^{j(\omega_c + \omega_m)t}$ into $e^{j\omega_c t}$, where this is $\omega_c + \omega_m$. And then, these are modulated component, there are two sidebands present here, one at frequency $\omega_c + \omega_m$, the other at frequency $\omega_c - \omega_m$, so what are the corresponding phasor components.

So, this is $\frac{1}{2} A e^{j(\omega_c + \omega_m)t}$ plus $\frac{1}{2} A e^{j(\omega_c - \omega_m)t}$ plus $\frac{1}{2} A e^{j\omega_c t}$ into $e^{j\omega_c t}$ plus $\frac{1}{2} A e^{j\omega_c t}$ into $e^{-j\omega_m t}$, so the real part of all this is that is it, which you can write as the real part of take $e^{j\omega_c t}$ term out common. Because that is present in every one of these terms $A \cos(\omega_c + \omega_m)t$ plus $A \cos(\omega_c - \omega_m)t$ plus $A \cos(\omega_c t)$ plus $\frac{1}{2} A e^{j\omega_c t}$ plus $\frac{1}{2} A e^{-j\omega_m t}$ plus $\frac{1}{2} A e^{j\omega_m t}$.

And we like to draw represent this $x(t)$, in terms of phasor diagram, you know how to do that I am sure all of you know that, but just to recapitulate, first of all let me be convenient to only look at this square bracketed term. You can ignore this $e^{j\omega_c t}$, because you can think, that your phasor diagram is been drawn in a reference plane which is rotating at the frequency ω_c . So, every term is rotating at that frequency, so you can now think as if, as far as these components are concerned you are working in a stationary ((Refer Time: 35:53)).

So, if everything is rotating then with respect to this it is stationary, so we will only therefore look at these components and draw the phasor diagram corresponding to these, to understand what is happening. Let us first draw the phasor diagram without the interfering component this is the interfering component. So, let us see what is happening to received signal, what is the nature of received signal as represented on the phasor diagram without the interfering component.

So what will you get, you have this carrier phasor with amplitude $A \cos(\omega_c t)$, then you do not have this, because you say this is at the movement. We are not considering this we have these two side-band phases with respect to carrier phasor, this will actually rotating at ω_c ; if the whole frame is rotated it will appear to be fix. This is rotating faster than the carrier phasor, so with respect to the carrier phasor which is having a rotation at the speed of ω_m radians per second.

Let us say in a counter clockwise direction, so plus ω_m let us say moves rotating in count clockwise which I will denote rotating like this, it is going round and round, around the carrier.

Student: ((Refer Time: 37:26))

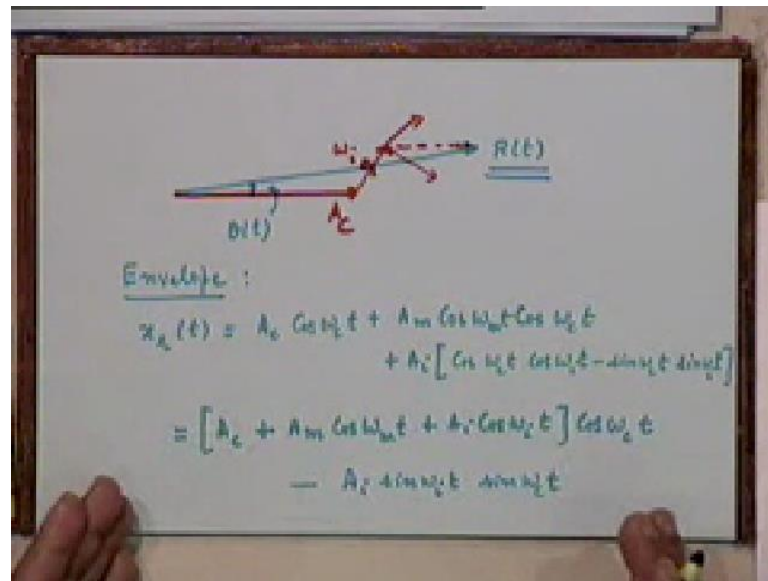
Let me take this to be minus ω_m and then, it does not you can take any one of them as clockwise is a matter of rotation, this is what you will get, one phasor rotating like this and the other phasor is rotating like this. And what you will see the actual signal will be the result of all these three phasor, now no matter where this rotation is it is symmetrical with respect to the carrier phasor at any time t . At any time t this phasor will make an angle $\omega_m t$ with respect to the carrier phasor, this phasor will make an angle minus $\omega_m t$.

So, there are symmetrically placed with respect to carrier phasor and therefore, the resultant wave by a parallelogram vector etcetera will appear like this R of t , the resultant will always be along this line. The resultant R of t will fluctuate around A_c on both sides for example, at this position the resultant will lie here. So basically what you see that the carrier wave is being modulated in amplitude around A_c , it is going up and down in proportional to the message signal $\cos \omega_m t$, so that is how the whole thing is working.

And envelope detector will actually follow this, you can take of this R of t the resultant amplitude as the envelope, this is the envelope that you tried to trace through the envelope detector, if you are sensitive to this envelope amplitude, envelope value. So, the envelope detector output here is proportional to R of t and since R of t here will be proportional to $\cos \omega_m t$, this will be proportional to $\cos \omega_m t$, as we expected it to be, there is no problem, I hope all of you are with me on this.

Now let us see how things change if we also introduce A_{sub} ie to the power $j \omega_m$, how will you modify the phasor diagram any one would like to comment, when this component also has to be considered.

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So, we start with the same starting point that is you have a carrier component $A \sin \omega_c t$ carrier phasor and then, what do you have $e^{j\omega_c t}$ to the power $j\omega_c t$, so it is rotating let us say like this at a frequency of ω_c radians. And now at this point you superimpose the two sidebands which will look like this, because with respect to the carrier they still have the same configuration. They are $\omega_c + \omega_m$ and $\omega_c - \omega_m$ they will form an angle of $+\omega_m$ and $-\omega_m$ with respect to the carrier, not with respect to this.

So, they will look like this and their resultant is something here and the overall resultant is where, overall resultant could be this waveform and this phasor $R(t)$, so your $R(t)$ the overall resultant plus this one. So, what are the things that you can see, that now the resultant phasor is not along the carrier phasor, therefore it is not undergoing only amplitude variations, amplitude variations will be there as all these phasors rotate as this rotates and these two rotate, the amplitude also keeps varying.

But, in addition there is also a phase variation $\theta(t)$, so these two variations are being in a truth, so the issue is what is the nature of this envelope, what can we say about this envelope, it is very difficult in general to visualize what is happening, but it is not that difficult to visualize under some simplifying assumption. So, let us study the behavior of the envelope under some simplifying assumption, by envelope I mean the length of this vector the amplitude.

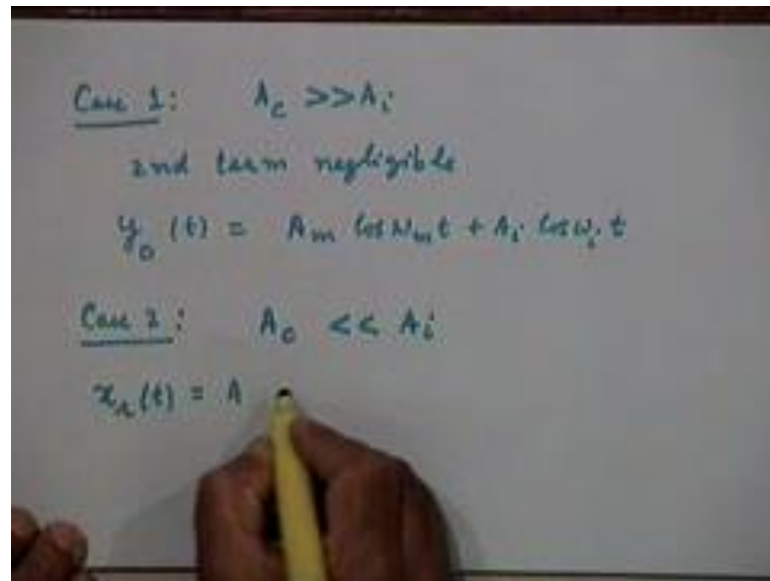
The amplitude is what will be important when we are looking at the output of the Envelope detector, because the envelope detector will produce an output proportional to the instantaneous amplitude of the carrier wave. So, to do that let me turn to my original expression, $x_{r,t}$ is equal to A_c with all I did all this just to give a feel for the phasor representation of the signal, which is important as we see later, it is very important particularly when we want to do the noise analysis, both for AM as well as for FM signals.

So, the phasor representation is something I wanted to introduce at an early stage so that you get used to it, so let us come back to your old expression $A_c \cos(\omega_c t) + A_m \cos(\omega_m t) \cos(\omega_c t)$ plus, the interfering term. Also let me write as, if you remember it is $\cos(\omega_c + \omega_i)$, which you can write as $\cos(\omega_c t) \cos(\omega_i t) - \sin(\omega_c t) \sin(\omega_i t)$. You know why I did this, what is the motivation for writing like this, because I like to express this receive ((Refer Time: 44:35)) quadrature representation.

Any narrow band signal we know it you can write in quadrature form, there is an in phase part which is multiplied with $\cos(\omega_c t)$ and there is a quadrature phase part which is multiplied with $\sin(\omega_c t)$. If I have it in the quadrature form I can easily identify the nature of envelope by taking the square root of modulus the thing. So it will be what is the in phase part, it will be $A_c + A_m \cos(\omega_c t)$ or $\cos(\omega_m t) + A_i \cos(\omega_i t)$, this whole thing multiplying $\cos(\omega_c t)$.

I have collected all the terms containing $\cos(\omega_c t)$ together, minus the quadrature part which is $A_i \sin(\omega_i t)$ into $\sin(\omega_c t)$, so we all like to understand the nature of this envelope, and at least some simplifying assumptions, in general it is little difficult to appreciate.

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What can we say, let me the simplifying assumption that, let us take case 1, that $A_{sub c}$ is much greater than $A_{sub i}$, this is simplifying assumption, we are saying that $A_{sub c}$ is much greater than $A_{sub i}$. Now we have had some discussion of the behavior of the envelope detector under these kinds of condition, do you remember we discussed this in some other context, in the context of demodulation of SSB and VSB signals by the carrier reinsertion method?

So, is the same kind of situation here, if this is much greater than this, the envelope will be dominated by which term, the first term, because we take the square of this and the square of this add them up and take the square root. But, the square of this will be much smaller than the square of this you can ignore it and therefore, essentially becomes the square root of the square of this term.

Therefore, in this case the second term is negligible and your demodulated output, if you forget the DC component $A_{sub c}$ will be DC component of the demodulated output which is of no consequence we can ignore that will be $A_{sub m} \cos \omega_m t$ plus $A_{sub i} \cos \omega_i t$ which is perfectly fine. If this is precisely what we got, when we use the coherent demodulator, if you remember some time ago we discussed that this is precisely the output that was produced by the coherent demodulator, so no problem.

So, under the situation that the, but the only thing is here we have obtained this result and then, assumption there was no assumption that is the difference, the assumption here is

that the interfering component is much smaller than the signal component, the carrier part. So, the carrier we had to make no such assumptions for the coherent demodulator, which will always produce this output, no matter what is the value of A_m and A_i , if A_i is large of course the output will be large this will be large, if A_i is small the output will be small.

This two is given here, but for this to work like this, what we are saying is the carrier component much greater than the interfering component, that is what we are saying, to appreciate the significance of this assumption, let us take the opposite situation. The opposite situation is that A_c is much less than $A_{sub i}$, now we look at ((Refer Time: 49:14)) this expression what will you get; it is difficult to say directly from this.

So, what we do is we slightly manipulate this expression once again, this expression for $x_r(t)$, difficult to say because $A_{sub i}$ is appearing here as well as here, that is why it is difficult to say what is happening. If it was appearing in only one of these two terms it would have been very easy to say. But since $A_{sub i}$ is appearing both these terms A_i is very large, it is difficult to say directly from this, so what we do is slightly re-manipulate this expression as follows.

(Refer Slide Time: 49:59)

The image shows a handwritten derivation on a whiteboard. It starts with the word "Envelope:" followed by the equation:
$$x_x(t) = A_c \cos \omega_c t + A_m \cos \omega_m t \cos \omega_c t + A_i [\cos \omega_c t \cos \omega_i t - \sin \omega_c t \sin \omega_i t]$$
This is then rearranged into:
$$= [A_c + A_m \cos \omega_m t + A_i \cos \omega_i t] \cos \omega_c t - A_i \sin \omega_c t \sin \omega_i t$$
A horizontal line is drawn below this. The final result is:
$$x_x(t) = A_c \cos (\omega_c + \omega_i - \omega_i) t + A_i \cos (\omega_c + \omega_i) t + A_m \cos \omega_m t \cos (\omega_c + \omega_i - \omega_i) t$$

I am writing the first term cosine omega c t as $A_{sub c}$ cosine, this is trivial kind of thing that I am writing, but it has a significance which we will soon see I am writing like this omega c plus omega i minus omega i. Then we have the interfering term which was A_i

cosine omega c plus omega i t. I think very soon you will all see what I am trying to do and then, A m cosine this is the modulated part A m cosine omega m t into cosine omega c plus omega i minus omega i t same thing.

Everywhere, then I have only cosine omega c I am replacing it with omega c plus omega i minus omega i, I think now probably already know what I am trying to achieve, I am going to write an expression in which the carrier is not omega c, but omega c plus omega i that is basically what I am trying to do in this and the reason will become very clear. So, if you expand this now as cosine A plus cosine A minus c t etcetera and then, simplify maybe I can skip that trigonometry and give you the result directly, because that is straight forward, you get finally this expression.

(Refer Slide Time: 51:42)

The whiteboard shows the following derivation:

$$x_r(t) = [A_c + A_c \cos \omega_c t + A_m \cos \omega_m \cos \omega_c t] \cos (\omega_c + \omega_i) t$$

$$+ [A_c \sin \omega_c t + A_m \cos \omega_m \sin \omega_c t] \sin (\omega_c + \omega_i) t$$

The final result is boxed:

$$y_p(t) \doteq A_c \cos \omega_c t + A_m \cos \omega_m t \cos \omega_c t$$

Below the equations are two frequency spectrum diagrams. The left diagram shows a carrier frequency f_c with two sidebands at $f_c + f_i$ and $f_c - f_i$. The right diagram shows the carrier frequency f_c and the modulated sidebands at $f_c + f_m$ and $f_c - f_m$.

You get x sub r t is equal to A sub i plus A sub c cosine omega i t plus A m cosine omega m t cosine omega i t into cosine omega c plus omega i t plus A sub c sine omega i t plus A sub m cosine omega m t sine omega i t into sine omega c plus omega i t. So, once again I have quadrature representation, so please assume this is correct this is just coming out of, important thing is instead of writing it in terms of quadrature representation using omega c I used omega c plus omega i.

Why do I do that, now that will be obvious from here, as you can see this second term now does not contain A sub i, because earlier A sub i was present in both the terms. Now, this is present only in the first term and therefore, we can say that the first term

will dominate the envelope detector output, because this is much larger than A_c . We are saying that A_i is very large interference, so this term will dominate the output and therefore, we can say that your envelope detector will produce an output $y_D(t)$ which is approximately equal to...

If I forget the DC part A_i is some DC component $A_c \cos(\omega_c t) + A_m \cos(\omega_m t) \cos(\omega_c t)$, now this is very interesting if you look at it carefully what are the conclusions you can draw from this. First conclusion from this discussion is as the envelope detector, now thinks that the carrier is not ω_c , but carrier is the carrier is $\omega_c + \omega_i$. So, any way the stronger of the two components capture the envelope detector output, capture is the receiver, if the interfering component is stronger it captures the receive.

It will dictate what the output of the envelope detector would be ((Refer Time: 54:40)) along the signal component to carrier component to dictate, and once it dictates that the output has no resemblance to the message signal, that is the second conclusion. The output now you see the message signal was containing a component ω_m , so the input was there was a message component at frequency f_m and there is the interfering component at frequency f_i .

If at the output I got these components no problem that is what the coherent detector does it produces both these components of the output, because input contains interference output should contain interference. However, what I am getting here is I do not get f_m at all, what you are going to get is this component f_i there is a component $f_i - f_m$ and $f_i + f_m$.

We do not even have a component at the desired signal frequency, message frequency was f_m , there is no component in the output at the frequency f_m , interfering tone and plus minus f_m , these are all the frequency components present in the output. Therefore, we have no resemblance to the actual input signal, this output signal will be quite distorted, so any way the presence of a large interference signal output to the extent as if modulating signal is, actually modulating is not coming out, but it is modulating the interfering tone ω_i .

So, I hope you understand then, that the envelope detector works very nicely as long as the interference is small, but if some situations the interference becomes large this does not work well, that will be your conclusion for today.

Thank you very much.