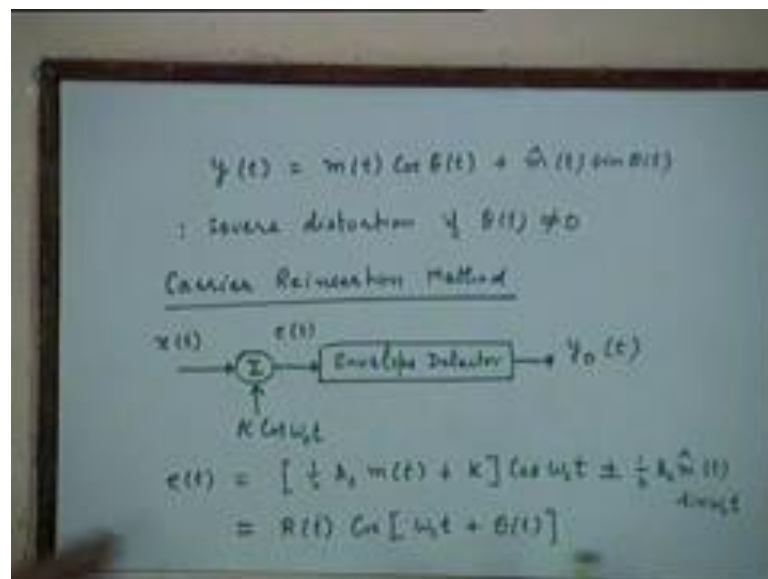


Communication Engineering
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Lecture – 11
Suppressed Side Band Modulation (Contd.)

If you recollect, we were looking at single sideband modulation yesterday and we learned how to demodulate single sideband signal, we also learned how to generate a single sideband signal using either the sideband filtering method or the phasing method. And the phasing method was based on a new representation of the single sideband signal that we discussed which involved the message signal empty with carrier cosine omega c t and quadrature carrier sin omega c t along with the Hilbert transformer of the message signal. So, that representation also led us to an alternating method for generating SSB signals.

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Now, we were also looking at the demodulation and when discussing the demodulation we found the demodulated signal when you do coherent detection is given by in the presence of let us say lack of synchronization between the local carrier and the incoming signal, incoming carrier. The demodulated signal would consist of a component due to m t plus a component also due to m hat t, so unless theta t is 0, this would lead to severe distortion and they are recovered by message signal, if theta t is not 0.

So, this is one small problem that you face in the demodulation process in this case, of course if there is perfect synchronization there is no problem, there is second method for demodulation of SSB signals and that is called the Carrier Reinsertion Method. This also requires a local carrier like the coherent demodulator, the standard coherent demodulator, but has some minor advantages over that in terms of technical implementation. Basically here what we do is, take the incoming modulated signal and instead multiplying this with the carrier, local carrier, we simply add it with local carrier.

Local carrier and when you add local carrier to the incoming signal and what do you think you will get now, well we will see that if it is not obvious. We will get a signal, whose envelope, approximately under certain conditions follows the envelope of the message signal. Now, it is not obvious at all, we will see how it becomes, so if this is indeed, so what we can do is follow this up with the envelope detector to produce a detected signal output, demodulated output $y_{sd}(t)$.

To see what happens, let us consider the signal $e(t)$ at the output of this adder, what does it look like, It contains the SSB signal coming in plus the local carrier So, the SSB signal coming in and the local carrier, if you do that if you remember $x(t)$ is $m(t) \cos(\omega_c t) + \hat{m}(t) \sin(\omega_c t)$ and you also have $K \cos(\omega_c t)$ added to that. So, $\cos(\omega_c t)$ has two parts, now the in phase part of the SSB signal was $\frac{1}{2} A_c m(t)$ this was multiplying $\cos(\omega_c t)$ plus you have a constant amplitude carrier K here and these together, now constitute the, so called in phase part of the SSB signal.

And what is the quadrature phase part, well depending on whether is upper sideband, we have $\frac{1}{2} A_c \hat{m}(t) \sin(\omega_c t)$. One of the very nice advantages of quadrature representation of this kind, remember we have discussed this quadrature representation for arbitrary live band signals. And it is interesting to see that various kinds of A_m signals we have discussed, each of them fall into this representation, for example the DSBSC signal has only one part, the in phase part.

Half $A_c m(t) \cos(\omega_c t)$, where d is the single sideband signal has both the in phase part, as well as the quadrature phase part, the quadrature phase part is the Hilbert transform of $m(t)$. So, this falls in the general class I mean this is in conformity with our discussion on the general representation of band-pass signals, which has an in phase and a quadrature phase part. Only thing is that there is a special relationship for the between

the in phase and the quadrature phase part for it to become an SSB signal that we have understood.

Now, one of the advantages of this representation is that you can easily identify what the envelope of the signal will be, by writing this expression. If you remember, what is the definition of the envelope, now the basic definition was when we wanted to find out it is a trace of the positive peaks of the carrier. So, to understand how we can identify the envelope If we one can rewrite this as an amplitude part $R(t)$, which is always positive into cosine $\omega_c t$ plus the phase part $\theta(t)$. This signal can always be written in this form, using trigonometric identities.

In general what will be $R(t)$, if you want to right this expression in this form what would be the value of $R(t)$, will be the square root of square of this plus the square of this and that becomes the envelope. What will the value of $\theta(t)$, will be the tan inverse of this upon this.

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The image shows a whiteboard with handwritten mathematical equations. The first equation is $R(t) = \sqrt{\left(\frac{1}{2} A_c m(t) + K\right)^2 + \left(\frac{1}{2} A_c \hat{m}(t)\right)^2}$. Below it, the phase $\theta(t)$ is given as $\theta(t) = \frac{\tan^{-1} \left(\frac{\frac{1}{2} \hat{m}(t) + K}{\frac{1}{2} A_c \hat{m}(t)} \right)}{\cot^{-1}}$. A simplification step shows $\frac{\frac{1}{2} A_c m(t) + K}{\frac{1}{2} A_c \hat{m}(t)} \gg \frac{1}{2} A_c \hat{m}(t)$. The final result for the envelope is $R(t) \approx \frac{1}{2} A_c \underline{m(t)} + K$.

So, here $R(t)$ is square root of half A_c , $A_{sub} c m t$ plus K whole square plus half $A_{sub} c m \hat{m} t$ whole square And $\theta(t)$ is tan inverse of half $m t$ plus K Half A_c , $A_{sub} c m t$ plus K upon half $A_c m \hat{m} t$, inverse of this. So, we can make this cot inverse.

Student: ((Refer Time: 10:17))

No I think you are not understanding the point that I am making, we are now looking at the envelope of this signal. The envelope detector will be the same, the envelope detector will remain the same, the diode detector, but we are discussing the envelope detector here. We are trying to understand the nature of $e(t)$ that is our concern here that is right, but please give me some time to explain how it is coming back, I am just explaining that give me time.

In any case, $\theta(t)$ is not relevant here, we are only looking at the envelope, so now let us look at the envelope. Let me complete my discussion then I will take the question from you, is there a mistake I am making.

Student: ((Refer Time: 11:13))

A cosine $\omega_c t$, I will come back to this point, that is good question. The question is, we are not really it appears totally avoided the phase coherence issue. We are adding a local carrier which is adding up to the cosine $\omega_c t$ part of the incoming signal, which essentially means you require a phase coherence, that issue remains to be discussed and we will certainly come back to it.

But first let us see how does it help us, so surely that is a relevant question to ask, so, now, let us look at this expression, expression for the envelope. I would like the envelope to follow the message signal, can I make some assumptions which will make it, so under what conditions will $R(t)$ approximately follow $m(t)$. When K is very large, correct, so that is precisely what we will do, will choose our carrier local carrier to be added in sufficiently large quantity with sufficiently large amplitude. So, this term dominates over second term.

So, that this can be ignored, in this squaring and square rooting process, so if we make the assumption that $\frac{1}{2} A_c m(t) + K$ is much greater for all time than $\frac{1}{2} A_c m(t)$ which can be easily done by choosing K to be a sufficiently large constant with respect to these other signals then we can approximate $R(t)$ as equal to $\frac{1}{2} A_c m(t) + K$ and therefore, if we ignore this constant, we got an envelope which is proportional to the message signal $m(t)$.

And if an envelope detector in that condition would produce an output proportional to $m(t)$, you agree with that, if the envelope follows the message then the detector would produce an output which is proportional to the envelope.

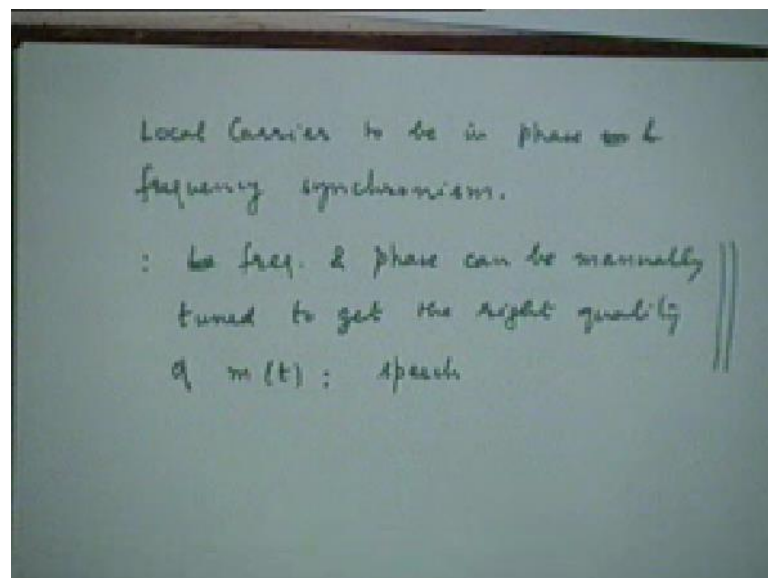
Student ((Refer Time: 13:41))

Yes please, whatever, $m(t)$ may be it will be $\cos(\omega_c t)$, it will not be $\cos(\omega_c t)$ it will be $\cos(\omega_m t)$ to be a low frequency signal.

Student: ((Refer Time: 13:56))

.Now, if we look at this total expression, this total expression will be much larger than this, that is the assumption we are making, so I think then we do not need to bring in that issue at all right, this will be still valid whatever the message signal $m(t)$, may be it will be still valid. So, the envelope will follow this, so therefore, this gives you an alternative method for demodulating SSB signals. Now, that issue that was raised a couple of minutes ago about the phase coherence of the local carrier in this case. It is true even in this situation.

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We would require the local carrier, to be in phase and frequency synchronism, however even though that is, so it is easier to manage than the local oscillator which mixes the two signals. It is easier to manage in the following way in the following manner then at least for applications like speech for example, one can adjust this phase and frequency

manually in this case. Till you get distortion removed and your, it is not as sensitive as it is as the other cases yes as the coherent demodulator is.

So, one can manually just like tuning the receiver you can tune the receiver exactly to have a phase and frequency synchronism and get till you get your speech quality back. It is easy to it in this case than the other case and therefore, this preferred over that system, but your question is very much valid, but this is the answer to that question. The local carrier frequency which is the frequency and phase in this case can be manually tuned to get the right quality of m t.

And this is particularly ((Refer Time: 16:56)) doable where m t happens to be signal which you can hear, like speech or music particularly in speech one does not use SSB for music ever. So, typically for speech till get your quality back and the intelligibility is good, we can keep on tuning it.

Student: ((Refer Time: 17:15))

That is I am saying theoretically it can be done, but that is a much more difficult thing to do than in this case, this in other words if you do some analysis you will find this is less sensitive to phase and frequency errors than the other systems. I am not doing this analysis here, but there are a slight phase off set will completely start distorting signal because multiplication of the terms cosine theta t and sin theta t, there will not be as sensitive as it is in that case.

That is something know by experience also it can shown by analysis, so see if you can work out what is the sensitivity of this new modulation to theta t, if you just attempt to do that and see what kind of conclusions you can reach, you will probably start agreeing with this. But if necessary we will take it up in our tutorial class I will make this statement that this happens to be the case that it is easy to at least tune it manually for applications like speech transmissions.

That is about single sideband modulation, single sideband modulation is good if you want to save bandwidth it is a little more tricky to demodulate like we have just seen. But these complexities one ignores if bandwidth is of very major concern to us, we want to save bandwidth we will live with the complexities, we will try to do whatever is required to make the receiver working very well, but if the receiver happens to become complex

in the process we do not mind, because bandwidth is important. it is important resource for us

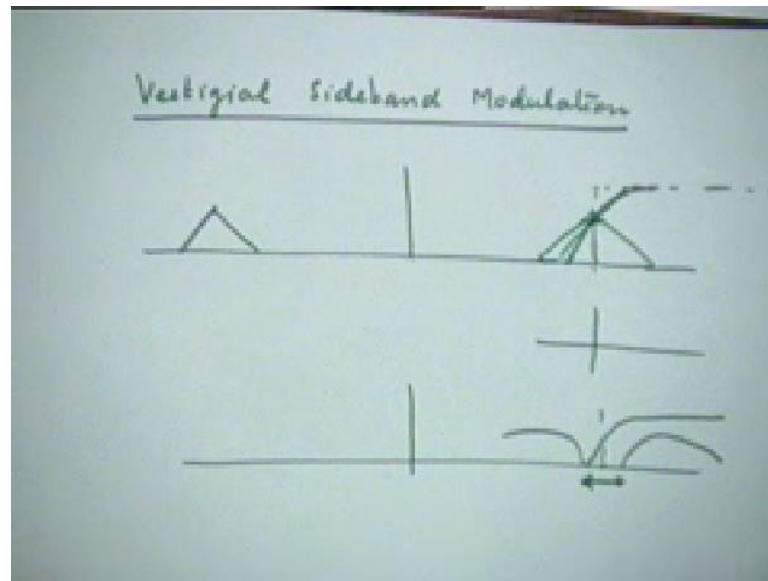
Student: ((Refer Time: 19:26))

Yes please, there are two different ways of saving bandwidth, they are equivalent ways depending on application you choose one or the other. So, if you have a application where you would like to actually transmit two messages on the same carrier, quadrature multiplexing is a good alternative. If on the other hand you are transmitting a large number of messages, maybe in the same overall frequency band.

It is much more convenient to put each of the put single sideband modulation use single sideband modulation and put them in large number of adjacent sidebands because the quadrature multiplexing can handle only two signals. If you want to handle more than two signals that philosophy will not be may not be useful directly for multiplexing, of course you can do quadrature multiplexing in each band, which is twice the single sideband that also can be done, so there are various alternatives that are possible.

We have a number of techniques available which technique you actually adopt will depend on the application, we will consider some applications where frequency division multiplexing will enable us to put a large number of messages in adjacent sidebands and we will view the single sideband modulation, but before coming to that let me discuss one more kind of amplitude modulation which is very, very pertinent in the context of suppressed sideband modulations.

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Now, let us discuss the motivation for that first, this new modulation I am going to discuss now. If you remember I classified the suppressed sideband modulation into two types the single sideband modulations and the next one which was vestigial sideband modulation So, I like to discuss this a little bit, first the motivation, why do we need such a thing, before even telling you what it is. The need is as follows single sideband modulation saves us bandwidth that is well understood.

It is however, has one problem. Suppose we already know that your a single sideband modulation is difficult to generate, because it either requires us to have very sharp cut-off filters in the frequency domain, around the carrier frequency or else you require equally slightly less difficult, but still difficult problem of arising wideband Hilbert transform and baseband, these are the two ways by which SSB can be generated. Now, when you try to try to do sharp cut-off filtering at the carrier frequency.

I do not know whether you have been involved in some filter designs, you know typically the amplitude characteristics are best in the middle of the band of any filter. Similarly, the phase characteristics also best in the middle of the band, but the movement you deviate from the band typical filters that you realize are synthesize or break will start to develop a non ideal characteristics. When you go away from the centre of the band, so around the edge the band you will have both non-linear phase characteristics as well as maybe not very nice amplitude characteristics.

If instead of remaining absolutely constant of course, they will have some oscillatory behavior or phase characteristics will become non-linear at that time and so if a message signal has a significant amount of low frequency component, you understand I am talking about baseband signal. If you have a significant amount of low frequency component in your message signal where will it go after DSBSC modulation, It will go around f_c . That significant energy content at low frequencies would translate to significant energy component, energy content at around the carrier frequency.

So, now if you have a sharp cut-off there and that sharp cut-off is associated with these problems, it will definitely distort low frequency components of a message signal. Because that is going to get affected by the cut-off characteristics near the f_c , near the carrier frequency, so if we illustrate by picture, so let us say your message band, message signal spectrum looks like this and. Ideally if you had sharp cut-off center like that, If you want to remove the lower sideband no problem, you can do that, But this is precisely the problem I cannot do that.

Typically, what will tend to happen is that I might get even if it is a good filter it might be like that, as far as amplitude characteristics are concerned and phase characteristics perhaps in this area will be highly non-linear because and sharper the cut-off the more non-linear it tends to become. So, phase characteristics tend to become highly non-linear which will imply some kind of a delay distortion or phase distortion of the low frequency components of the message signal.

Why low frequency because these frequency components actually belong to the low frequency component of the original message signal, so if your message signal has significant low frequency content then SSB will create all this problems for us. Because of the new for sharp cut-off, which we either we cannot do it or if can do it will be associated with these kinds of distortions, for certain kinds of signals this is not a serious issue.

For example for speech signals, we know that the low frequency content is not that high, I discuss with you some time ago at the beginning of this course that speech signal has significant content from, let us say 300 hertz onwards technically has no content between 0 to 300 hertz. So, in that kind of a signal, the spectrum after frequency translation would

perhaps look like that, there is a band here. In which there is no signal energy and that gives me flexibility in designing this filter, the filter design can be done.

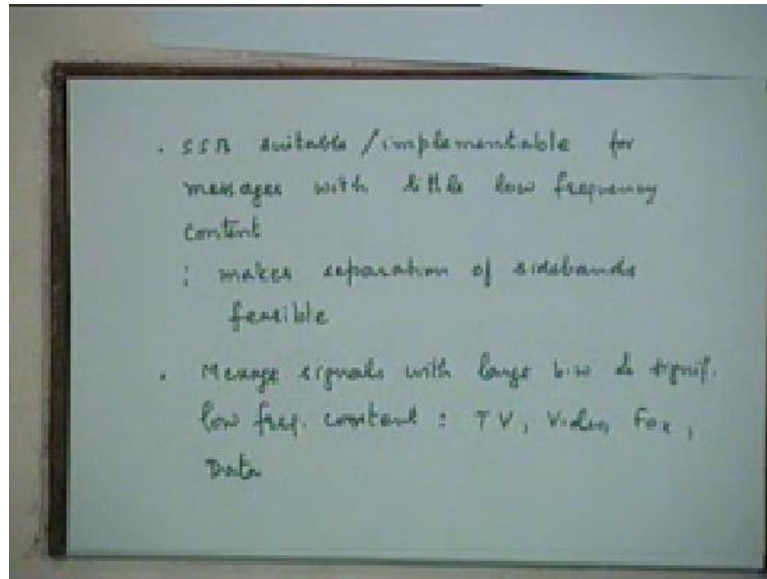
I do not need a really, really sharp cut-off filter mean in the sense of a big wall characteristic. Anyway there is a band available for which I can gradually roll off the characteristics of the sideband filter, which is not going to be available to me, if this band was not available too, you appreciate this point. So, in certain kinds and also in the second point, as we discussed early speeches are kind of signal in which even if some phase distortion occurs, It does not matter very much why, because our ears are insensitive to phase distortion.

The way the ear perceives speech, phase distortion or delay distortion is not very crucial, so therefore, for speech applications SSB is quite ok. SSB is durable In fact, in speech applications it is widely used single sideband modulation. However there are certain kinds of signals which have a lot of low frequency content and which are sensitive to phase distortion and one such signal is picture signal I mentioned to you. So, you have picture signals, you have signals which are also a kind of picture signals scanning the content of the page.

So, all these signals are very sensitive to phase distortion, which will occur if you try to implement in SSB model and at the same time such signals have very large bandwidth. For example picture signal has the bandwidth of the order of 5 megahertz we discussed that 4 to 5 megahertz and there is a therefore, a very strong case for saving bandwidth. The voice signal after all uses only 4 kilohertz bandwidth, so while there is need to save bandwidth if you want to transmit a large number of wide signals even one picture signal alone will consume, if you were to allow DSBSC 10 megahertz of bandwidth 5 megahertz on either side.

So, there is very strong need, a very strong case for saving bandwidth here at the same time we find that SSB kind of saving will not help, because it will create undesirable distortions. So, this is motivation to look for something different and that is the motivation for vestigial sideband modulation.

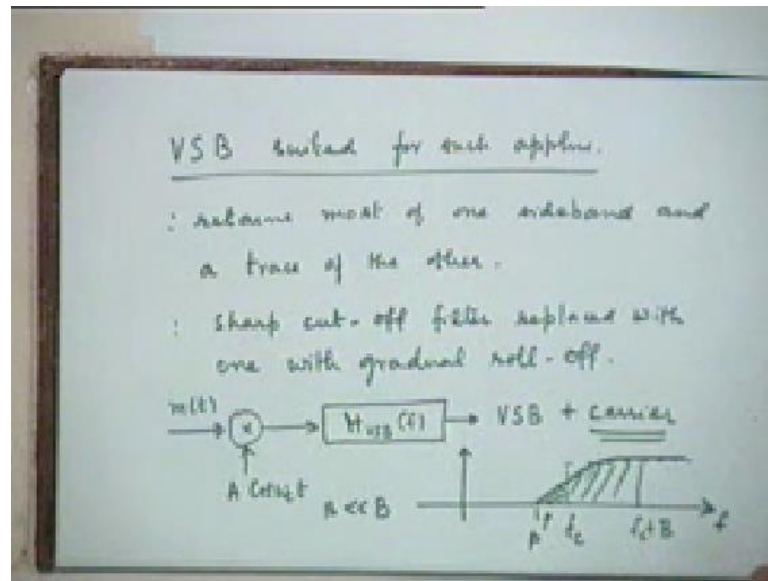
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So, basically let me summarize what I have said, what we have said is SSB is suitable and implementable, for what kind of messages, for those messages which have relatively less energy content at lower frequency, for messages with little low frequency and content. Because if that is the case the separation of the sideband, separation of the upper sideband and the lower sideband or vice versa becomes easy to do makes separation of sidebands possible This feature or this message signal makes the separation of two sidebands feasible or possible.

I have explained this I hope you understand this, now just summarizing, on the other hand message signals having large bandwidth and significant low frequency component content exist, like the t v signal, the video signals, the fax signals etc. The data signals, all these kinds of signal they have significant low frequency content, so this separation becomes more difficult and if try to force it will create all kinds of distortions, which are not acceptable.

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And it is for such applications that one thinks of VSB, so having motivated the need for a different modulation what is that VSB can do, which SSB cannot do. Which makes it preferred modulation for this case, the VSB's approach is like this. All right I will not be able to save the entire bandwidth, because of this very difficult situation the two sidebands are just meshing into each other and there is no gap between for me to design a graceful filter.

So, what if I allow, most of one sideband and the little bit of the other, so that the filter design becomes an easy job right and that is basically the idea of VSB. So. In fact, that is the meaning of vestigial sideband you allow most one sideband and a small portion of what is also called vestige of the other sideband. If you do that our filter design requirements become much easier to handle and all those distortions that were because of the sharp cut-off will not happen now. Basically that is the idea of a vestigial sideband.

It retains most of one sideband and very small portion or a trace of the other and that permits me replacing the sharp cut-off filter with a gradual roll-off kind of filter, gradually rolling-off filter. So, what we are saying is can be pictorially expressed like this you generate your signal, message signal you do DSBSC modulation on this by multiplying this with $A \cos \omega_c t$ and you have a sideband filter here, but it is not that sharp cut-off filter, we will call it some $H_{VSB}(f)$, which will yield the VSB signal.

Typically it will be, If required also sends some portion of the carrier if required, but that is not important right now. And this H VSB f Suppose, you want the hi the upper sideband It will not now be Filter which has ideal requirement It will indeed allow a gradual roller like that. So, you will be transmitting most of the sideband the upper sideband let us say the bandwidth is B So, this is f sub c , this f sub c plus B , but you are also transmitting this portion of the lower sideband, it could be the other way round.

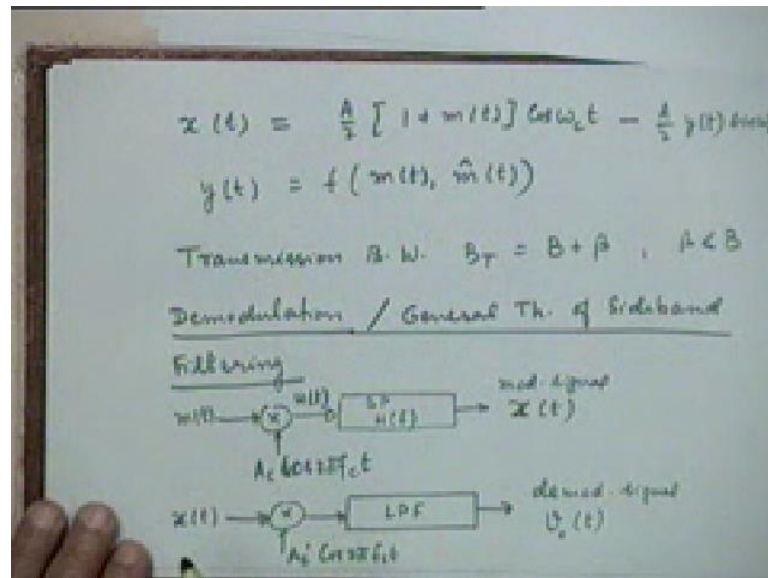
We could be transmitting the lower sideband most of the lower sideband and a small portion of the upper sideband could be either way, of course the similar thing will exist on the negative frequency axis. So, your bandwidth will now be suppose, basically we are going down on this side by a small amount the bandwidth B may be large for example, in the t v case it is 4 to 5 megahertz, 4.5 megahertz or something they will be an overflow in to the other direction by an amount let us say β .

Where β will be, typically much less than B , so you will be still saving a considerable amount of bandwidth, but you will be relaxing the requirements of your sideband filter, so sideband filter design becomes easy and you can Now, are any questions arising in your mind, when this kind of thing is suggested, what kind of questions now arise in your mind, to carry out our discussion forward, any question that arise, any other, yes the issue is can such a signal, how such a signal get demodulated and will it actually empty or will give something else.

Because we have done something funny, is this kind of filtering acceptable no matter what kind of filter you have here or we have some constrains, well the answer to that question is what we will now try to obtain we will find that the principle is fine, but you have to be careful in the design of this filter. If you want the demodulated signal, which is proportional to $m(t)$, which is a replica of your message signal. so that is what we will try to understand, how should such a filter be designed.

What should be the I am not getting in to filter design, I am getting in to the what should be the nature of the transfer function of such a filter. Such that we are able to reconstruct our message signal back, because that is the ultimate requirement. The two requirements one is we want to save the bandwidth which we have done, not by the whole amount of B , but more or less by fairly big amount, for example, for the t v signals this could be let us say 500 kilohertz, as compared to 4.5 megahertz, this is still a good saving.

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So, what we like to see is, what is the nature of the modulated signal $x(t)$, where $x(t)$, I am defining once again to be the signal here. So, we will soon see, you see in any case It is obvious that now we can write $x(t)$ in $x(t)$ is narrowband signal still It is a band-pass signal. There is a centre frequency carrier frequency and there is spectrum around the carrier, so it becomes a narrowband or a band-pass signal, if the carrier is large enough frequency and we know a general representation for any narrowband or any band-pass is the in phase and quadrature phase representation.

There is a in phase component which is a low frequency signal, so $x(t) = y(t) \cos(\omega_c t) - X(t) \sin(\omega_c t)$ that is quadrature representation, so obviously the SSB signal had a representation. The VSB signal will also have a representation, the only things that will change are the actual values $x(t)$ and $X(t)$, the actual nature of the in phase and quadrature phase components. So, we can certainly will be able to see that this signal that we have generated.

Will have this form, once again this carrier part, which I have introduced is because we want to be able to do envelope detection in the same manner that we did for SSB. So, this part will be the same as we have for the SSB plus of course, the carrier I am introducing here minus $A/2$ into some low-pass signal $y(t) \sin(\omega_c t)$, where $y(t)$ will be will soon will be a function of both the message signal and it is Hilbert transformer, will not be just Hilbert transform or $m(t)$.

So, we can have this kind of representation and if this kind of representation is possible, we can do perfect synchronous demodulation or approximate envelope demodulation. So, first we will discuss how this comes and then will discuss, what about the transmission bandwidth, It will be B plus β where β is typically less than B , most of the time much less than B . Now, demodulation is what we like to spend much time on because that leads to some crucial questions and answers.

In the process, we will also look at general theory of sideband filtering which will include SSB as a special case, VSB is a general case and SSB becomes a special case in which β is equal to 0. So, we will in this process also discuss general theory of sideband filtering and the general theory basically tries to answer the following question, the question is let us say I have this modulator, which we have just talked about $m(t) A_c \sin \cos 2\pi f_c t$.

Followed by a suitable band-pass or high-pass filter or low-pass filter depending on what kind of band, sideband you want and gives your modulated signal $x(t)$ and you get this $x(t)$ here, the demodulator multiply with $A_c \cos 2\pi f_c t$. And for demodulation what we normally do put a low-pass filter here that is your synchronous demodulation and you will get a demodulated signal. Let us call it $v_o(t)$ and the general theory that we are going to discuss addresses the following questions.

Question, What kind of band-pass filter should I use or what kind of a high-pass, low-pass filter should I use at the transmitter, what kind of $H(f)$ should I select. So, that $v_o(t)$ is proportional to $m(t)$ because we are suggesting some rather complicated filtering here in the VSB case, so what kind of filter transfer function suppose you want the upper sideband, so maybe high-pass filter But any high-pass filter or with some requirements with some specific nature, what should be the nature of $H(f)$, so that $v_o(t)$, so that is the question.

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Question: For what class of filters $H(f)$, will $U_o(t) = k m(t)$?

$$\begin{aligned}
 x(f) &= U(f) H(f) \\
 &= \frac{A_c}{2} [M(f-f_c) + M(f+f_c)] H(f) \\
 \Rightarrow U(t) &= A_c \cos(2\pi f_c t) \cdot x(t) \\
 V(f) &= \frac{A_c}{2} \frac{A_c}{2} [x(f-f_c) + x(f+f_c)] \\
 &= \frac{A_c A_c}{4} M(f) [H(f-f_c) + H(f+f_c)] \\
 &\quad + \frac{A_c A_c}{4} [M(f-2f_c) H(f-f_c) + M(f+2f_c) H(f+f_c)]
 \end{aligned}$$

Let me put it in the next page, question for what class of filters H of f will v not t v sub o t be proportional to the message signal, that is the question we like to address. To do that, Is the question clear to all of you, before we start answering it, before we start developing answer for this question. The question is you have some H of f here like the VSB filter which has this kind of characteristics, so we know, we want this filter to be having a gradual roll-off.

But roll-off can be designed in many different ways for example, will any roll-off be permissible or with only certain kind roll-offs be permissible. Because otherwise I do know that this kind of filter is quite fine, but is this fine for any kind of roll-off any kind of gradual, that is the basic issue we want to address, what should be the constraint on the filter, what should nature of the filter H f . So, that and if I at least do synchronous demodulation there should be no problem and v naught t should be equal to m t .

That is the question we are addressing, we do know one answer for example, if there is no roll-off is a perfect filter, ideal cut-off filter will get v not t equal to K times m t , that one answer is known, but we want to develop a more general answer that is we call it the general theory of sideband filter, to start doing that. Let us look at the spectrum of the modulated signal and we know that spectrum, let me call this m t into A c cosine $2\pi f_c t$ t I will denote this by u of t .

What is $x(f)$? It is $u(f)$ into $H(f)$ and $U(f)$. I know is, spectrum of the modulated signal DSBSC signal, so $x(f)$ would be equal to $U(f)$ into $H(f)$, which I know as equal to $A_c/2$ because $U(f)$ is $A_c/2$ into the message spectrum translated to f_c plus the message spectrum translated to $-f_c$ that into $H(f)$, that is the spectrum of the transmitted signal. What about the demodulated signal again at the demodulator I have called this the final output signal as $v_o(t)$, let me call this as $v(t)$.

The signal at the output of this modulator this mixer, so what will be the spectrum of $v(t)$ $v(f)$, so if I define $v(t)$ as $A_c' \cos(2\pi f_c t) x(t)$ this is what I am doing. Here $V(f)$ will be A_c' by 2 into transmission of this $x(t)$ the spectrum of $x(t)$, so $x(f - f_c)$ plus $x(f + f_c)$. But $x(f)$ is given here, so can I substitute that here how many terms will I get now four terms, I will shift each of these two terms, I will find out $x(f - f_c)$ I will get two terms.

Find out $x(f + f_c)$ will get another two terms will get four terms, so now out of these four terms can I identify the two terms first which will be the in baseband because this is the band-pass signal is in it. This $v(t)$ is a band-pass signal, your $x(f)$ is the band-pass signal that is the modulated signal when you are doing frequency translation of that. We will get a component translated around the 0 frequency and a component transmitted around the twice the carrier frequency, so can we identify those these two parts separately.

These four parts that this have four terms that you will have here, will contain two terms which will correspond to the baseband component and two terms which correspond to the $2f_c$ component. So, let us first identify the baseband component that will be A_c' into A_c' because we are substituting for $x(f - f_c)$ from here upon 4. What will the baseband part one component will come from when this becomes say I shift it to by plus f_c you will get $M(f)$ and that will become $H(f + f_c)$.

And similarly from here you can shift it by minus f_c , so I will get $M(f)$ into $H(f - f_c)$, so $m(f)$ will be common in these two terms and this will $H(f - f_c)$ plus $H(f + f_c)$ plus f_c is it clear. These two terms what about the $2f_c$ terms amplitude A_c' , A_c' upon 4, so what will you get here, we are shifting this to the by f_c , so this will become $M(f - 2f_c)$ do not think I will have a common factor, now $M(f - 2f_c)$ and what will the corresponding H factor.

H of f minus f_c plus M of f this will come from here and we are shifting it to the left again by f_c , so this will become M of f plus $2f_c$ into H of f plus f_c , please see this carefully and see that you are convinced. So, essentially what we are saying is this is the low frequency part of V of f and this is the high frequency part of V of f , which will be centered around $2f_c$ and you can see that this will always be centered around $2f_c$ because m of f was a low frequency spectrum, so we are shifting it to $2f_c$.

What about H of f minus f_c , H of f was centered around f_c and you are shifting it further to the right by f_c , so it gets in to around $2f_c$. So, this is center around $2f_c$ this will also center around $2f_c$ and everything is centered around $2f_c$, whereas this part everything is centered around 0. If it is all right then I can proceed further, so if I have that low-pass, now if we look at the output of the low-pass filter, in the modulator what will I see.

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$$\therefore V_o(t) = \frac{A_c A_m}{4} M(t) \underline{[H(f+f_c) + H(f-f_c)]}$$

$$V_o(t) = K m(t)$$

$$H(f-f_c) + H(f+f_c) = 2 H(f_c)$$

$$= A \text{ constant}$$

$$H(f_c) = \frac{1}{2}$$

$$\boxed{H(f-f_c) + H(f+f_c) = 1}$$

So, therefore V naught f , that is the output of the low-pass filter which term will you see, the first term, the first two terms, so you will get an output which is A_c , A_c prime upon 4 into M of into H of f plus f_c plus H of f minus f_c . That is what you will get and that provides the answer we are looking for what do we want, we want our v naught t to be proportional to m t , K times m t . So, what should be the what does that tell me what can you say about the filter H of f what condition should it satisfy, so that this happens.

This function of frequency which is coming from these two terms, which are coming from the filter transfer function, they should be equal to A constant value, they should

not be a function of frequency, that is what it means that is the condition we are looking for. $H(f) \cos(f_c t) + H(f) \sin(f_c t)$ should be a constant, let us for convenience, let us call it $2H(f) \cos(f_c t)$, because It does not matter what you call it should basically be a constant and for convenience I am just saying that let that constant be equal twice the value at f_c .

And if I specifically choose a response of Hilbert transfer function at $H(f)$ equal to f_c to be let us say equal to half, then this condition becomes is equal to 1, basically I am choosing the constant to be equal to 1 arbitrary I mean, so the filter should be such that this condition should be satisfied, if you want your demodulated output though synchronous demodulation actually is the replica of the message signal. We need to examine what this condition means and also we like to also address the questions of what is the actual representation of VSB signal like, so these are issues continue in the next class.

Thank you very much.