Communication Engineering
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Lecture - 10
Single Sideband Modulation

We will discuss, now we will continue our discussion, we will continue our discussion about Single Side band Modulation which we started yesterday. So, if you recollect what we said was that we do not have to transmit both side band, as we do in DSB SC or DSB AM. Because, information about the base band message signal is available in any one of the two sidebands, so we could decide to cut off either the lower sideband or the upper sideband.

And the basic advantage that we will achieve by doing so would be increase bandwidth efficiency and also power efficiency will be transmitting less power. So, well basically power efficiency is secondary, main thing is the bandwidth efficiency.

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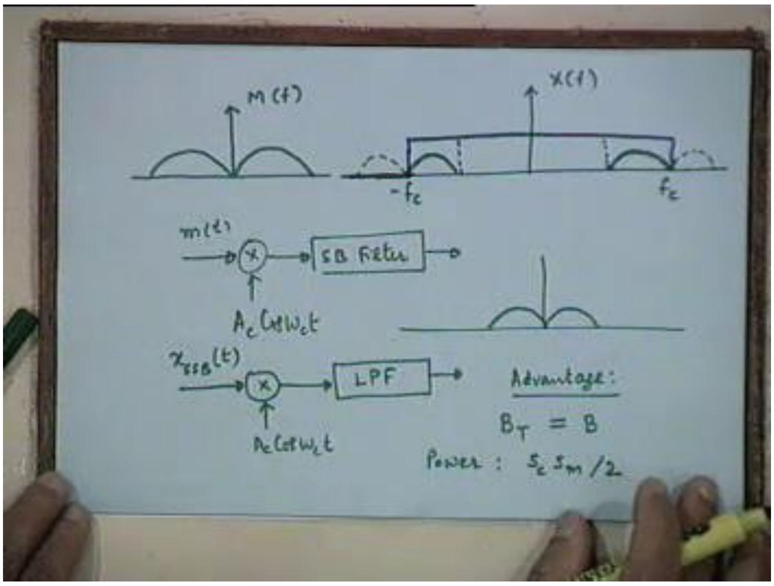

So, just to recapitulate once again in terms of frequency domain, if this represents the spectrum of your message signal, these are the two sidebands of the normal AM signal shown here in dotted lines. So, this is the normal DSB SC spectrum, what we are saying is we can remove one of the two side bands, for example we could remove the upper side band then transmit only the lower side band, so we are transmitting this will of course, come along with that, so this is around f sub c, this will be around minus f sub c.

So, this portion of the spectrum is being filtered out at the transmitter we will generate from here to the DSB, first generate the DSB SC signal and then by a suitable side band filter like we discussed yesterday. You go through the normal modulation process first and through a suitable side band filter eliminate one of the two side bands. If you eliminate the upper side band you are transmitting the lower side band and this for eliminating upper sideband you can use a low pass filter or a band pass filter, whose pass and is from fc to fc minus B.

If you want the, if you want to transmit the upper side band you have to use a high pass filter or a band pass filter, whose transfer function is non zero between fc to fc plus B. So, that is what that is how you will choose a sideband filter and like we discuss the reconstruction is possible by the same process that we have for the DSB SC signal. So, here is your SSB signal coming in, so this is your SSB spectrum, this is the SSB signal in time domain you again multiply with the carrier and pass it through a low pass filter, just like what were you doing for the case of DSB SC.

In the frequency domain what basically we are saying is that as a result of this multiplication the same frequency translation is happening again this part of the spectrum will come here. And this part of the spectrum will go here and because of the symmetry it will be precisely be the same M f that we started with, so of course, they will also be translation of two fc and minus two fc.

But that will be eliminated by the low pass filter, so the reconstruction process in the time domain as well as in the frequency domain follow the same principle that we had for the case of DSB SC, the double side band suppressed carrier modulation. Now, so far, so good, but suppose, let us look at some, now before I come to that what are the advantages that you have mentioned of doing this, the main advantage is for a bandwidth efficiency.

Your transmission bandwidth BT is now not equal to 2 B which was the case for the two previous modulations we discussed, but simply equal to B. So, in the same bandwidth now you can transmit twice as many messages, as we can transmit two messages in the same bandwidth rather than in the 2 B around fc rather in a single message. That is the main advantage, what is the power that will be transmitted in the SSB signal what was the power that was being transmitted in the DSB SC signal, if you remember the expression.

It was s sub c into s sub m which was the carrier power into the message power, so in this case how much it will be divided by 2 because; you are removing one of the sideband. Let us discuss briefly the issue of practical realization of the SSB waveform, the practical realization would require that you have a suitable side the additional thing that you require on top of what you do in DSB SC is the side band filter. Now, this is a non trivial operation that you want to do, this is what you need to understand, you require a side band filter with almost ideal characteristic, suppose you decide to retain the lower side.

So, what kind of transfer function should the filter have, let us say you do it through a low pass filter. So, what kind of the transfer function would a low pass filter have, it should have this kind of transfer function. It should have a perfect transmission up to the carrier frequency fc and 0transmission immediately, after that and you know that such an ideal low pass filter is almost impossible to realize very, very difficult to realize.

In practice of course, you will not realize a low pass filter you will realize a band pass filter, which will have this transfer function, but it will also associated with the same difficulties. One can approximate such filters, basically you require what are called high Q tuned filters you require because typically these carrier frequencies are very high and this band width will be very, very small fraction of the carrier frequency, so in narrow band high Q tuned filters will do the job.

You understand the quality factor of a tuned circuit, quality factor determines how much bandwidth you have around fc, the larger the quality factor the lower the bandwidth. A large quality factor a high Q filters will meet a very sharply tuned filter for an ideal tuned filter the q is infinity, but as q becomes finite s becomes smaller and smaller the bandwidth increases. So, you require a q of a suitable value and the typically the process of fact you require a very sharp cut off you require fairly high Q filters.

You require very high Q filters and designing high Q filters, high quality factor tuned filter using conventional passive components like inductors and capacitors is extremely difficult. They would be typically very large order filters and remember no matter what you do how, ideally you construct your inductors and capacitors there will be losses, there will be losses due to the resistance of the inductor or various kinds of the losses in the capacitor. And these losses is what, reduces the quality factor, if we do not have this losses you can have the ideal tuned filter.

Because they are losses the frequency response gets flattened and we do not get a very sharp filter. On the other hand there are devices, there are materials which enable the realization of this high Q filters.

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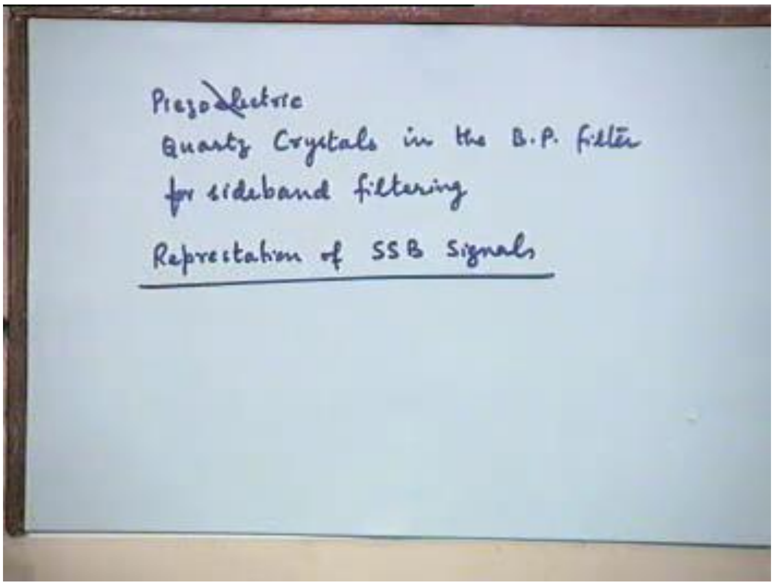

For example I will just mention the name here for your information, you could use what are called piece of electric crystals or rather piece of electricity is not important here. So, I think I should remove this, this again quartz crystal is the more appropriate point. You can use quartz crystals and interestingly they have some very useful electrical characteristics. And they can be used in the resonant circuits of tuned circuits to provide you very sharp frequency response.

So, you could get the kind of the filters that you want to design for the SSB generation by using quartz crystals, so you will use quartz crystals in the band pass filter of the for the side band filter. More about this later, if we get the time at the moment, I just want to mention this fact because I do not want to get into discussion on how quartz crystals work in this connection, yes please.

Student: ((Refer Time: 11:08))

No, this side band filter is at the transmitter, this is the transmitter we are putting the sideband filter before transmission please understand that at the receiver I am doing exactly what I was doing for the DSB SC signal. The receiver is, no longer, not different at all. That if that was not the case then there will be no advantage. So, this is the transmitter, so you are putting the sideband filter at the transmitter, please understand that.

So, one can use a specially designed, properly designed quartz filters, quartz crystal filters, which have this very interesting properties of providing you very sharp cutoff. A very high characteristic, but we will discuss more about that later if the time permits. Now before we proceed further, let me ask you a question suppose I ask you tell me what will this waveform look like in time domain.

Can I write a mathematical expression for this waveform in the time domain the answer is not that easy anymore because I can write the expression here. That is simply, the product of these two waveforms or what is the kind of waveform I have after this single side band filtering is not, so obvious and we will take that, we will take that up as next job to do. And when we do that, we will find that once you have this mathematical expression, it also provides us another method of, it also gives us another method of generating SSB signals.

Student: ((Refer Time: 13:11))

Yes please between fc minus B to, so what is the question? We cannot transmit anything during this in this yes because of using the bandwidth from fc to fc minus B.

Student: ((Refer Time: 13:49))

Let me, clarify the doubt I think the question is can I use this bandwidth or not, can I use this message bandwidth or not yes of course, I can. I can put another message in this bandwidth. Suppose. In fact, this is when I discuss frequency division multiplexing with you later we will see that this is precisely what you do. These are the advantages that you derive from SSB, you can put different messages, and each of them occupies only a bandwidth of b close to each other, adjacent to each other.

I can put one message here, another message here, a third message on this side, fourth message on this side and, soon and so forth. I can put them side by side at a spacing of only b hertz there are methods to do that and I can do that precisely because each of this messages requires only b hertz, so that is the advantage of SSB. And if I if there is an application we should use that as well right otherwise what is the point if I am not going to use it then what is the point of course, the whole idea of seizing it is use it somewhere else.

Student:((Refer Time: 15:01))

How there is an interference, I am eliminating this from this message I am putting another message in that spectrum yes, so there is an interference. Obviously, at the receiver you will have to separate them out. Okay I understand your doubt, now at the receiver if I see you want what is the idea is to be able to share the same physical medium in terms of bandwidth. So, that you do by putting multiple signal at different adjacent bands.

Obviously at the receiver you must make sure that you demodulate only the right signal for you have to do the separation corresponding separation filtering at the receive. So, that you only see one message signal at a time. Otherwise there will be; obviously, interference, I hope that answer is your question all those things will become clear as we proceed for these are these are question that you must ask and so that things get clarified. So, let us return to the issue that I just raised and that is representation of single side band signals, so let me start with fresh on a fresh page.

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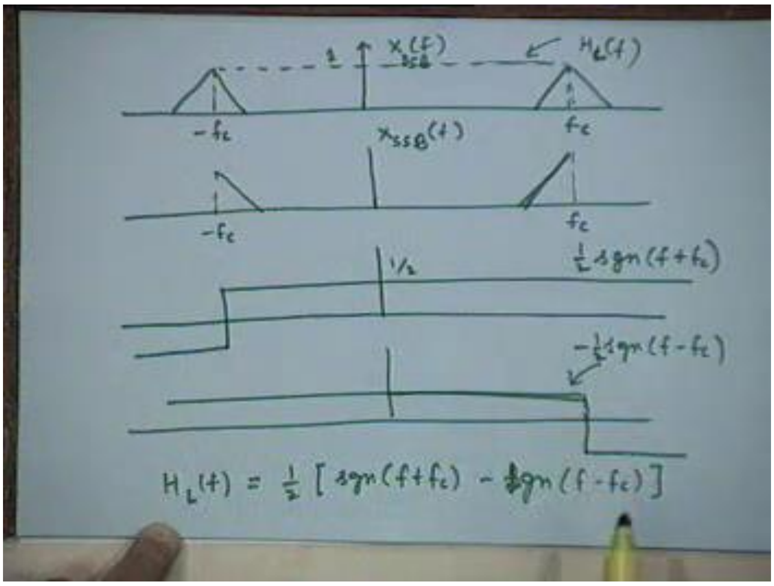

And let me draw few diagrams, so that we can work with these diagrams, I want to write an expression for the SSB waveform in the time domain for to do. So, I will start with the frequency domain because SSB signal is very easy to understand in the frequency domain not, so easy to understand in time domain. So, let me start from where it is easy to understand and let us say as we know, we can generate an SSB signal by starting with a DSB SC signal and then low pass filtering the.

So, I must multiply the message spectrum which looks like this, let us say the transfer function of an ideal low pass filter which looks like this and that will give me lower side band signal. So, this will generate, so this is X DSB in the frequency domain the corresponding X SSB spectrum would be therefore, look like this after this low pass filter. So, this is the process through which the SSB waveform is generated in the frequency domain.

Multiplication of the DSB SC spectrum would be transfer function of this kind of an ideal low pass filter. So now, let us look at this ideal low pass filter, I can represent this ideal low pass filter as the sum of two signum functions, this is minus fc this is plus fc. Can you see that I can represent this rectangular function which specifies the ideal low pass filter as the sum of a function like that and a function like that? This is if you have to express this in the mathematically in terms of the signal functions, how will you write this, signum of where is the signum function look at it here minus fc.

So, this will be signum of f plus fc and this will be correspondingly, signum minus signum of f minus fc, if I add up these two functions of course, let us say take the amplitude as half. So, that after addition this will give you 1 here, so this will give you filter, the ideal low pass filter that you are looking for where the transfer function value equal to 1 between minus fc to plus fc and 0 below minus fc and 0 above plus fc. So, add up these two functions like this I get this, now doing this just to do some simple manipulations.

So, that I can write down the corresponding expression in time domain, so it is clear that I can write this spectrum as the product of this DSB SC spectrum with this function plus with this function because that will be equivalent to product of this with the ideal low pass function.So, let me put it down more clearly, so what we are let me first write this mathematically what we are saying is that the ideal low pass filter H this should be H sub lf is equal to half of signum f plus fc minus signum f minus fc. So, this is basically what I am saying this transfer function is the sum of these two.

Student: ((Refer Time: 21:32))

It is because this half is there that are, so this is half signum m this is another half right I am adding these two functions and getting this function that is all I am saying.

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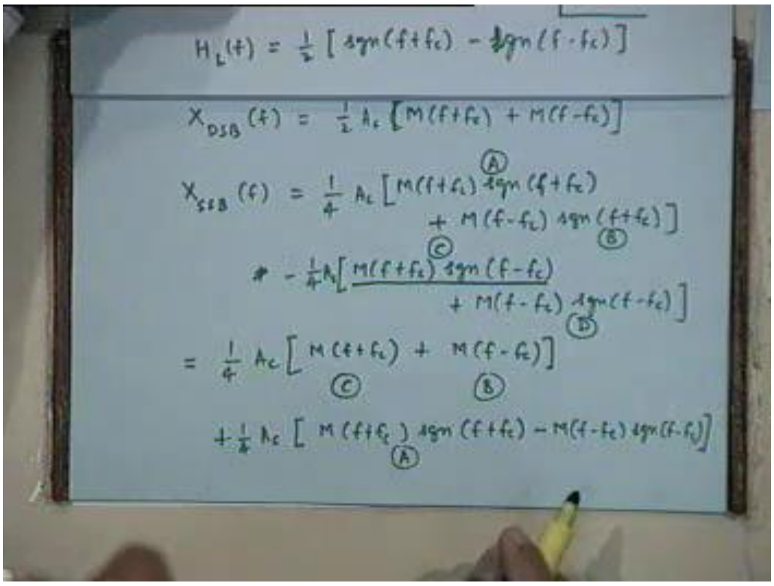

So, what is the expression for the DSB spectrum that is half Ac M f plus fc plus M f minus fc, so what are you doing, you are multiplying this spectrum with this transfer function to produce out SSB output agreed. If I multiply this transfer function with the DSB SC spectrum which is this I will get my SSB spectrum. So, therefore, my expression for the SSB spectrum in the frequency domain can be now written in a closed from. It will have how many terms, you are multiplying two terms in one expression and two terms in another expression will get four terms and these four terms will be 1 by 4 Ac.

So, may be you can is it visible both these terms, so we can manage with that, so, if I multiply this first with M f plus fc we will get signum f plus fc minus or plus M f minus fc. So, I am taking these two terms and multiplying with this signum f plus fc plus minus 1 by 4 Ac Mf plus fc into signum f minus fc please check it out that I am doing things right, plus M f minus fc into signum f minus fc. These are the four terms that you will get any questions on this.

Now, if you look at it carefully, I can collect these terms into two groups is there a term corresponding to just M f plus fc alone, which one is this, this term or which one. Suppose I call this A, call this B, call this C, call this D is there one of these terms is equal to M f plus fc, it will be this is the term C you see this is minus half M f plus fc into signum f minus f minus half signal f minus fc is plus 1.

So, this term this is the term C similarly can you identify a term which corresponds to Mf minus fc, so that is this term and the other two terms, now you have deliberately collected these two terms first. Because this together will be the same double side bands spectrum that I started with plus the other two terms, which you can write, so we are taking care of C and we are taking care of B. The two terms which I have left are plus 1 by 4 Ac M f plus fc signum f plus fc and which is the other term left minus M f minus fc that is, so this is the term A and this is last term is the term D signum f minus fc.

So, please look at it carefully and see that is with you, basically what we are saying is that this term, look at this M f plus fc is which that is this component M f, this is your M f plus fc part. That one multiplied with this part will produce this portion of the spectrum, so what we are saying is this multiplied with minus half signum f minus fc, in this entire interval this function is plus 1, so this into this would be this itself that is all we are saying, so that is the term C.Is that clear?

Similarly the term B, M f minus fc is this part; this into this function at this region could be again this itself, which is M f minus fc. So, these two terms one can take out and combine them after then put them like this and the remaining two terms are here, the term A and term D. So, I have explained that again, so is that is fine we proceed further.

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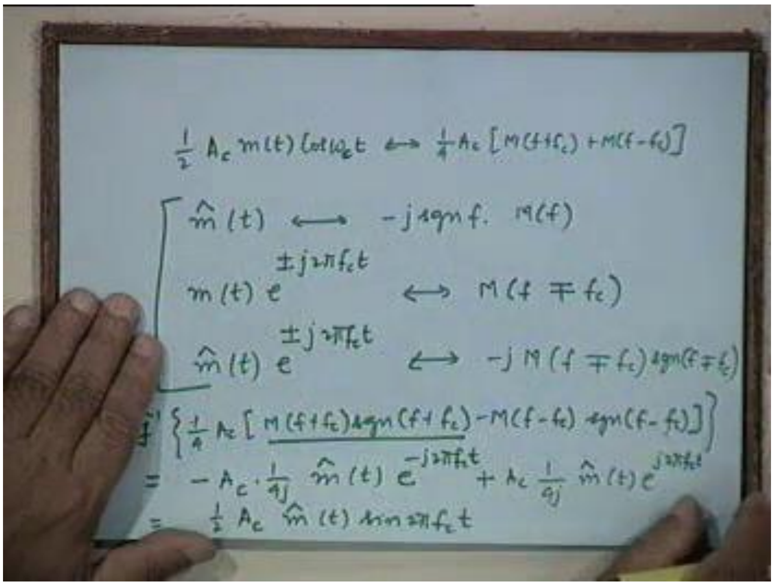

So, now look at the first two terms, we know that those first two terms will correspond to can you give me the time domain expression for the first two terms. The sum of these two terms, what is the corresponding time function half Ac mt cosine omega ct, this is the DSB SC signal that is all. So, this is half Ac mt cosine omega ct. So, this is nothing, but 1 by 4 Ac in the frequency domain M f plus fc plus M f minus fc; however, what the second two terms will amount to is not very clear right, now what will be this part of the function is not very clear.

So, let us try to understand this part, this part is clear in the time domain, so now let us start with this is where a Hilbert transform comes in very handy remember Hilbert transform is defined in terms of the signum function in the frequency domain and we are having signum functions there. So, there is a relevance here of the Hilbert transform use of Hilbert transform to start with we call that in the frequency domain, this is the same as minus j signum f into M of f .

The inverse of a transform would be nothing, but the Hilbert transform of mt you multiply the spectrum of mt with minus with the function minus j signum f in the frequency domain and that is nothing, but the Hilbert transform of mt r. So, we start from here, now also recollect the basic frequency translation theorem suppose, you multiply mt with e to the power plus minus j 2 pi f ct, what is the corresponding frequency domain relation, when it is mt plus e to the power plus j 2 pi f ct this will be f minus fc.

When it is minus it will be f plus fc. So, this is f plus minus plus f sub c, this is your basic frequency translation theorem similarly now if I look at m hat t e to the power plus minus j 2 pi f sub c t. Now, can you tell me what will this be equal to given that m hat t has this many transform or first let us talk about the frequency translation this will produce the same spectrum around if it is plus here around minus. So, this will give you minus j M f minus plus f sub c into signum of f minus plus f sub c, basically the same spectrum on which this frequency translation around is applied.

So, now keeping these three basic results in mind let us look at the inverse Fourier transform of the last two terms in this expression, what do they correspond to. So if I look at the inverse Fourier transform of the terms 1 by 4 Ac into M f plus f sub c into signum f plus f sub c minus M f minus f sub c signum f minus f sub c which is the last two terms. Now, please tell me term by term, what do they correspond to what can I say about this term look looking at these relations, can I say that this is minus A sub c which result will be applicable this one divided into 1 by 4 j.

This one by j will come because I do not, I do not have minus j here, so1 by 4 j into m hat t e to the power minus j 2 pi f sub c t and what about this term now same thing plus A sub c 1 by 4 j into m hat t plus j 2 pi fc. Are you all with me, so now, if I combine these two terms, what do you get take m hat t outside as a common factor you are left with e to the power j 2 pi fc t minus e to power minus j 2 pi f sub c t divided by 2 j, so this is nothing, but let me raise it slightly. So, that you can see it properly half of A sub c what will it be equal to m hat t into sin 2 pi fc t.

So, I have a closed form expression for the inverse Fourier transform of the last two terms. The first two terms are equal to half Ac mt cosine omega ct and the last two terms are equal to half Ac m hat t sin 2 pi fc t, very interesting.

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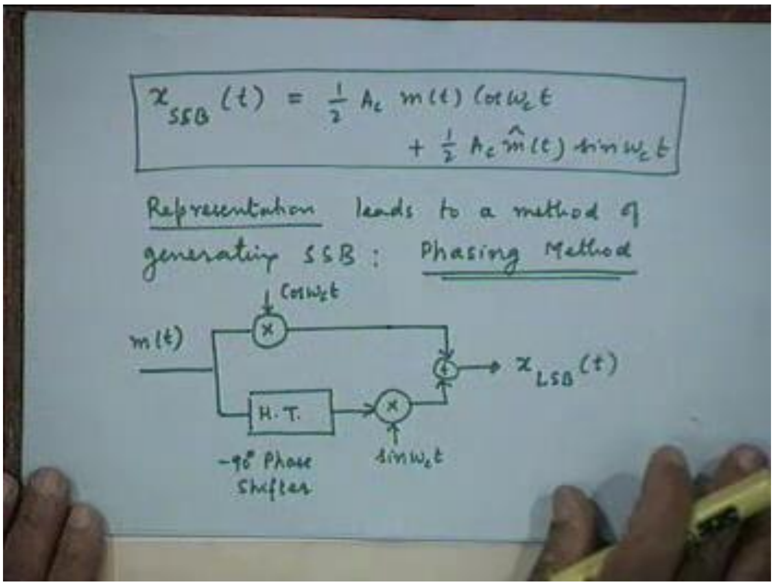

So, that gives me the closed form mathematical expression for the SSB signal, the single side band signal in terms of the message and the carrier just like we had for the DSB SC. And that is equal to half Ac mt cosine omega ct plus half Ac mt m hat t sin omega ct, very neat and interesting expression, any questions. I hope you followed the derivations it is fairly straight forward basically makes use of the properties of Fourier transform and the properties of the Hilbert transform.

And what kind of a SSB signal it is, it is a lower side band signal; it is a very simple exercise for you to go through the whole process again and show that if instead of plus here I have minus here will get the upper side band signal. It is a very interesting and useful expression, mathematical closed form expression for the SSB waveform, it is not easy to visualize this waveform because to visualize this waveform in you can not only have to visualize this plus, but also you have to visualize this.

Now, for arbitrary signal one does not know what m hat t will look like, for certain kind of signals one can work it out for example, for cosine omega mt I can say that Hilbert transform is sine omega mt. But for an arbitrary waveform very difficult to see what the Hilbert transform will look like. So, in general it is very difficult to visualize it, but for specific kind of waveforms it may be easy to visualize.

Now, this representation, so this is the representation that I was talking about mathematical representation of the SSB signals this representation leads to a second method of generating SSB waveforms that is called the phasing method. And it basically implements this equation that all, so what you need is you start with a message waveform mt and you want to generate this waveform, you just go through this the terms of this equation.

So, we have two branches mt you have a carrier component cosine omega ct and that is one branch, the lower branch before you do this multiplication you have to do a Hilbert transformation, which is what. It is a 90 degree minus 90 degree phase shifter for all the frequencies present in mt and that is followed by a second product modulator in which the carrier is sine omega ct. And we simply add up these two to produce your lower side band signal.

If you subtract one from the other we will get the upper sideband signal, so looks very neat and very simple, the requirement of, so this is the second method in the first method you require high Q tuned filters around the carrier frequency to remove one of the side bands and this method it looks very simple. But actually the difficulty still remains the difficulty has been passed on from realization of a high Q band pass filter to an ideal Hilbert transform.

Remember an ideal Hilbert transform will have to effect in 90 degree phase shift a minus 90 degree phase shift for each and every frequency component in the bandwidth of the message signal starting from f equal to 0 to whatever is the bandwidth to f equal to B. But that is also such an ideal Hilbert transformer is also not easy to realize; however, since the whole operation has to be done around base band, this Hilbert transform is operating on the base band signal is much easier to approximate. Such a Hilbert transformer to realize the high Q tuned filters that is required for SSB filter.

Both the methods are used in practice, this one as well as the side band filtering method, both have their pros and cons, but this gives you an alternative. So, this is one a second way of generating the SSB waveform, which is called the phasing method. Basically we call it the phasing method because this requires the implementation of a 90 degree phase shifter. We require a 90 degree phase shifter not at a single frequency realizing at a 90 degree phase shifter at one frequency is a very trivial job.

You can you can think of a very simple circuit consisting of only Rl resistance and capacitance to do that. But in fact, ideal capacitance would produce a 90 degree phase shift. But to realize that for a band of frequencies it is not, so easy over the entire band same constant 90 degree phase shift, so that is about the SSB waveform representation and its generation.

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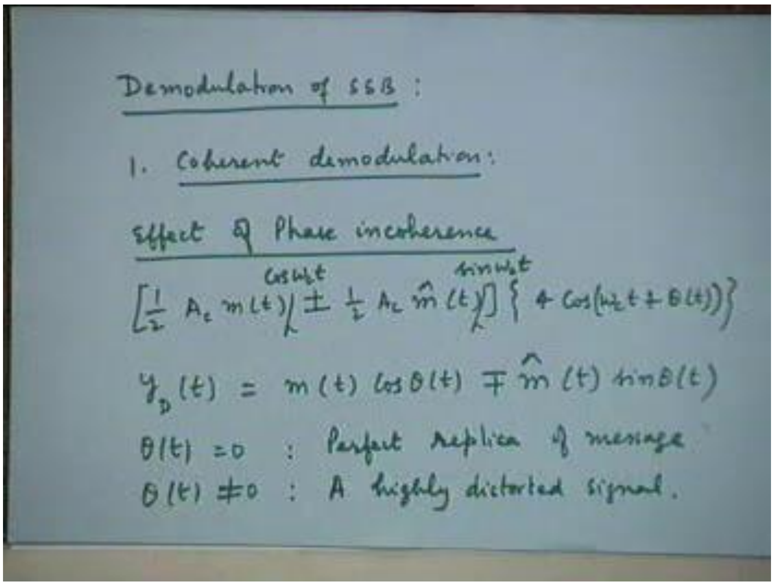

Obviously, we have already talked about demodulation of SSB waveform to some extent. But I like to come back to this issue, the demodulator that I have discussed, so far at the beginning of today's class and also mentioned earlier. What kind of a demodulator is that, what we call it, the synchronous or coherent demodulator which would require us to have a local carrier in phase synchronism and frequency synchronism with the carrier of the incoming signal.

So, one demodulation process is a coherent demodulation that we already discussed, so this we already discussed. Let us consider what will happen in this case, when it is not coherent. Remember we did this carry out this discussion for the case of DSB SC signal and we understood that due to a phase offset, the signal will be attenuated and due to a frequency offset we will see, so called warbling effect.

The further modulation of the received signal in time, message signal in time, some where some time amplitude will go up and sometimes it will come down. The rate of this depending upon delta f the frequency offset between the transmitter and the receiver carriers. What will be the case in SSB, let us look at that, so let us look at the effect of, if suppose you are using a coherent demodulator, but there is a phase in coherence, what will be the effect.

So, to do that now we have a mathematical tools to handle, it we know that I can represent the SSB waveform like, this is the expression for the SSB waveform with the plus sign for the lower side band and a minus sign if I want the upper side band. So, either of these two is fine, in a coherent demodulator you will take this SSB signal and multiply it with cosine omega ct that is what the coherent demodulator does and pass it to a low pass filter.

For convenience lets us multiply with 4 cosine omega ct, so this spectra of half of the trigonometric functions go away, let us also include phase offset. In fact, let me consider this as a time varying phase offset, which actually can model both phase offset as well as frequency offset. The frequency offset for example, is a time varying phase offset, so more general expression is to write omega ct it should have been the 2 carrier frequency plus theta of t which models both the phase offset as well as frequency offset.

So, if I do this exercise multiply these two pass it through the low pass filter it is a very simple trigonometric exercise, I will like it to do that yourself and you will see that the detector or the demodulator output. Now after low pass filtering reduces to this expression, simple trigonometry and removing the two omega c terms. When you multiply these two there is something missing here, no one has pointed out, this should be here cosine omega ct here and there is a sine omega ct here, so you are multiplying these two removing the two omega c terms and seeing what is left.

This is what you will have, that is the result, if theta t is 0 that is with no frequency and phase offset your demodulated output will be same as mt which is what you want. If you have a perfect synchronism, perfect coherence output would be equal to mt the required output, but when theta is not equal to 0. So, in this case you will get a perfect replica of mt of message. When theta t is not equal to 0, you have problem, you have a highly distorted signal you not only have something the effect that we observed in the DSB SC. But to that you are adding something else the second component and this will lead to a highly distorted signal

So that shows that if, I use a coherent demodulator for demodulating SSB signals such a demodulator will be even more sensitive to frequency and phase offset than the DSB SC demodulation using the same demodulator. So, that is something that we have to keep in mind, now fortunately it is possible to effect this demodulation with better tolerance to phase and frequency offset, which can be of course, by manual tuning by another method, which we shall discuss later.

Thank you.