**CMOS RF Integrated Circuits**
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**Module - 10**
**Oscillators**
**Lecture - 29**
**Phase Noise in Oscillators**

Hello and welcome back to cmos RF integrated circuits today's is the twenty ninth lecture and we were discussing oscillators and as part of that discussion today I plan to talk about noise in oscillators. So, in the last class we saw couple of oscillator architectures right and these are fairly standard architectures this kind of oscillator is used almost all the time in RF CMOS circuits c mos radio frequency circuits.

This kind of these oscillators are used all the time the quality factor inductors of course, is not terribly high you get on in on in integrated circuit you will get q of five little bit higher may be depending on how high frequency your how high frequency your planning to oscillate at the frequency of operation is typically decided by the resonance frequency between the inductor and the capacitor right.

So, remember the half circuit right how we went from the full circuit to the half circuit whe/where was that be that over here right over here right. So, L and 2 c are going to resonant which means that the resonant frequency is going to be 1 by square root of 2 L c that is omega and then this divided by 2 pi will give you the frequency in hertz.

Fine now if you are planning to oscillate at a particular frequency you do not exactly have too much control over the value of the inductor over the exact value of the capacitor right. I mean you layout some capacitor and and unfortunately because of parasitic effects because of process variations temperature variations the actual value you finally, end up with is going to be somewhat different from what you wanted to start with similarly the value of the inductor is also not well controlled.

So, the actual oscillation frequency is not necessarily [clapping] going to be the frequency that you want. So, if the tolerance on the capacitor is ten percent if the tolerance on the inductor is 20 percent then the actual frequency is going to be ten plus twenty actually square root of that. So, 15 percent in error. That's a lot of error for a frequency if I pick up remember our discussion on the precise frequency etcetera etcetera if I pick up a quad crystal oscillator the error in the frequency over there is 10 parts per million right 10 parts per million is something like 1 in 10,000 no 1 in 10,000. So, that is 0.001 percent in error. So, for a quad crystal the tolerance in the frequency is 0.001 percent for our LC oscillator is 15 percent that is a huge amount of error that we have to deal with.

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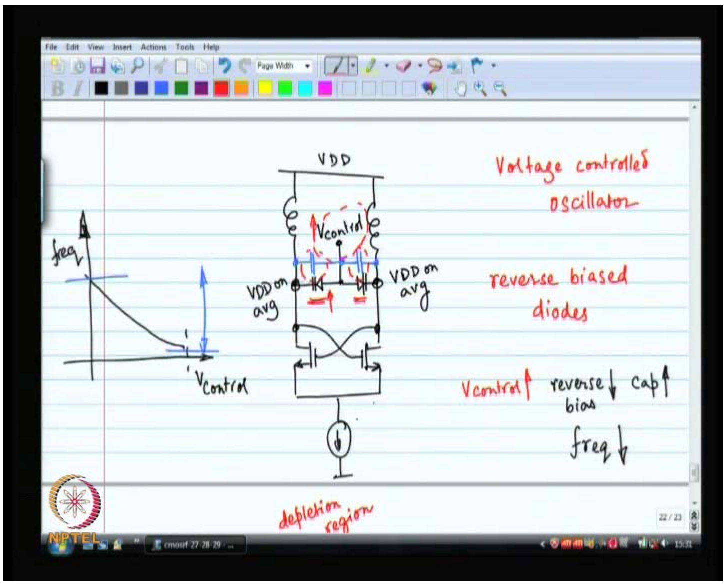


So, what is solution solution of course, is not to have an oscillator like that let us make an oscillator that is controllable. So, if somehow I can vary this capacitance value suppose I am able to vary this capacitance value if I can vary the capacitance value I can cover a large range of frequencies and 1 of those frequencies will be the 1 that I want. So, this is basically the idea instead of being able to oscillate specific frequency I will make variable capacitor over there and let us [clapping] find out let us turn the nub over there and figure out exactly let us tune the oscillation frequency right. So, this is standard how do you tune this oscillator.

Now, in the old hand tuned radios you remember there used to be a radio even a 2 in 1 box like this and there used to be a nub over there this nub was basically tuning the frequency. It is tuning this particular variable capacitor because you do not know what the value of that capacitor is it has to matched to the value of radio station to the the the frequency of the oscillation has to match the value of the frequency of the desired radio stations. So, that is why you are tuning it adjusting it and catching the precise radio station we are listening also. So, there is some sort of feedback and when you get the loudest noise or when you get the clearest sound that is when you stop tuning right.

So, even older radios there used to be a course tune fine tune etcetera etcetera right we do not want to be tuning this by hand special on a cell phone you do not want to tune the receiver by hand there is no nub you want to do it electronically and more importantly you want it to be seem less that is you want it to be controlled by the base station the base station ask you go to this frequency and you should immediately go to that frequency.

We should be no such tuning business. So, to start with this particular capacitor should be able to vary electronically. So, we use a voltage control we want a capacitor whose value can be controlled by a voltage. So, this becomes a voltage controlled oscillator the capacitance value depends on the voltage. So, as the result the oscillation frequency depends on the voltage alright.

How do you make a capacitor whose value changes as a function of a voltage well 1 of the easiest ways to do that is to actually use p n junction diodes what is diode got to do with the capacitor well the symbol actually like this be call it a varactor, but it is basically a p n junction. So, let understand this first first we understand that at d c the voltage at these 2 nodes is VDD what that also means is that even these 2 voltages are varying the average value of the average voltage at these 2 nodes is VDD if you look at the average over time value is VDD exactly equal to the VDD because of the inductor. Right. So, if I have V control as any voltage between ground and VDD which is usually the case. So, V control is a voltage between ground and VDD what does it mean for this diode is it forward biased or reverse biased  they are both reverse biased alright.

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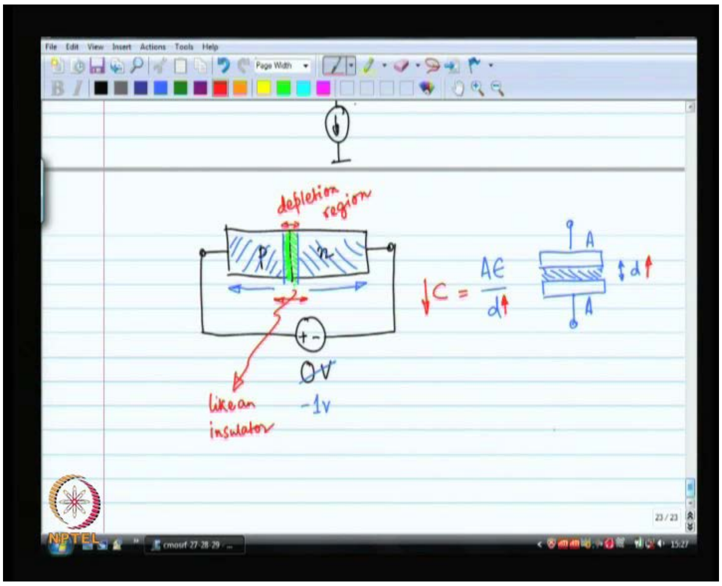


Now, think about what happens when you make a p n junction whenever you make a p n junction even before you apply or reverse bias voltage let us say the voltage you have applied across this is 0 volts even at that situation what is going to happen is that some holes from the p side are going to recombine with some electrons from the n side. Rather some free electrons from the n side are going to cross are going to defuse a cross the junction and are going to fall into the holes that are there on the p side and this happens along the junction and what you are going to find is that along the junction there is some region where there are no free carriers this is called the depletion region.

Alright. So, this happens even without applying any voltage. So, you have formed the depletion region now there are no carriers in the depletion region what this means is that that particular region the depletion region acts like insulator. Right you cannot have electricity going you cannot have electrons moving across the depletion region right run or carriers in the depletion region no carrier allowed as soon as carrier comes in its fall into a hole or falls into an electron recombines with a hole. So, carriers in the depletion region this means that current cannot flow through depletion region.

So, the depletion region is like an insulator. So, what we have over here are 2 regions that are full of conductors which are full of conductors free conductors approximately which are separated these 2 regions are separated by a depletion region which is like an insulator whenever you have 2 plates separated by an insulator. This is some dielectric in between it is like a capacitor alright the capacitance is a times epsilon divided by d where a is the area of the 2 plates its parallel plate capacitors this is the classic parallel plate capacitor alright. So, the capacitance is a function of the distant between the 2 plates now when I reverse bias this let us say I make this minus 1 volt.

So, this portion is 1 volt the inside is 1 volt above the p side. So, n side is positive what happens the electrons get sucked or get attracted towards the positive voltage and the hole side is negative the p side is negative the holes which are positive charges get attracted towards the posit[ive]- towards the negative voltage. So, effectively what happens is that the depletion width increases.

So, the more you reverse bias set the more is the depletion width right. So, if you reverse bias set more d increases if d increases the capacitance decreases agreed. So, there you are we have got a capacitor that is a function of a voltage. So, the capacitance is a function of how much you have reversed biased that particular junction. So, here we have 2 and both of them are identical. So, if I increase V control V control goes up then the reversed bias decreases if the reverse bias decreases the value of the capacitance is going to increase if the value of the capacitance increases if the value of the capacitance increases then the frequency is going to decrease.

All of these are not proportional right. So, the amount of the capacitance is not porpo[rational]- not proportional to the V control do not even think that it is a proportionality over here no it is not I am not saying that its proportional the frequency is not inversely proportional to V control it just goes down as V control goes up. So, over all what you going to see is if you plot on the x axis V control on the y axis you come you plot the frequency of operation of this.

Then what you are going to find is that there is some sort of negative behavior in any case you cannot have V control more than VDD right. So, this is basically the idea now the other things that you have to take care of over here are first of all you need a appropriate diode size. Secondly, you may be do not want such a high variation in frequency I mean this is the variation in frequency may be you do not want this much variation in terms of frequency may be you want some lesser amount of variation.

In that case you put a fixed capacitors instant with this will get lesser variation in terms of frequency. So, your net capacitance is now the fixed capacitor plus the variable capacitor which will reduce the freque[ncy]- control dependence of the frequency right. So, there are number of these ways. So, this is commonly used technique this is an analogue voltage controlled oscillator popular modern oscillator sometimes do not use this analogue control.

Uh lot of time they used digital control. So, they are no longer voltage control oscillator they will be a digital code controlled oscillator. So, they will be a digital code and that will switch some switches on and off. And as you switch some switches on and off you create more or less capacitance whatever you want over here and accordingly the frequency of oscillation is determined.

So, those are called d c os[cillator]- these are also quite popular d c o is something a jolt. This is quite popular used routinely alright now important what is the noise in an oscillator. So, we have talked about noise in the past right we understand the mosfet lot of these noise components these noise because of channel of the mosfet these noise because of the gate of the mosfet to some extent right.

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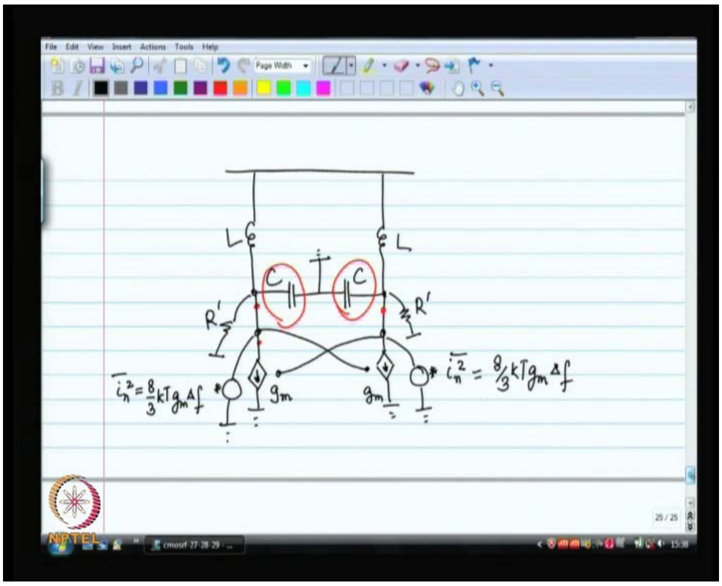


There are other noise components like flicker noise or 1 over f noise what is the noise of an oscillator. So, how we do this is as follows let us make an attempt . So, this is my standard oscillator alright and let us say both of these mosfets under quotient operation they have a certain amount of g m. Its use the extremely basic small signal model of the mosfet is got g the g m and there is R d s right; however, that is not really correct right there is c g d now c g d can be lumped in to the discrete capacitor we have got over there . So, c g d is really between these 2 terminals and I can lump it inside the capacitonces that we already have over there.

So, I need not bother about c g d right what else there there is c gate to source c gate to source is like a capacitor on the load and if you recall the small signal half circuit model of this and that also the the this particular node is actually signal ground if you recall the small signal half circuit model of this. And this is capacitor between the output node and signal ground. So, it is actually coming insant with c. So, I do not need to worry about c g s either what else is there there is c source to source to bulk its irrelevant because source is at signal ground bulk is also at signal ground there is c drain to body drain to body capacitance once again looks exactly like the node c.

So, drain to body also looks exactly like the load capacitance and as a result I should not be bothering about it. So, all of these parasitic capacitors are really not of much consequence they can be lumped all of them can be lumped inside these 2 capacitors alright. So, then in that case our crewed small signal model of the mosfet which is basically a g m and n R d s actually not a bad 1 quite a good small signal model. So, let us plug in that particular small signal model.

Right also you have to recall that the inductor is not a pure inductor it is something like an inductor insant with a resistor. So, I am going to all of that right over here alright let us call this lets start of calling it R parallel R d s I am going to call it R prime I think that is what I did in the previous class yes way now the important part now is the noise source. So, I have got a noise source because of the channel there is an additional amount of noise current over here what is going to happen to this particular noise current right this is the issues that is of concern right now. So, let us saw redraw our circuit in a simplified manner

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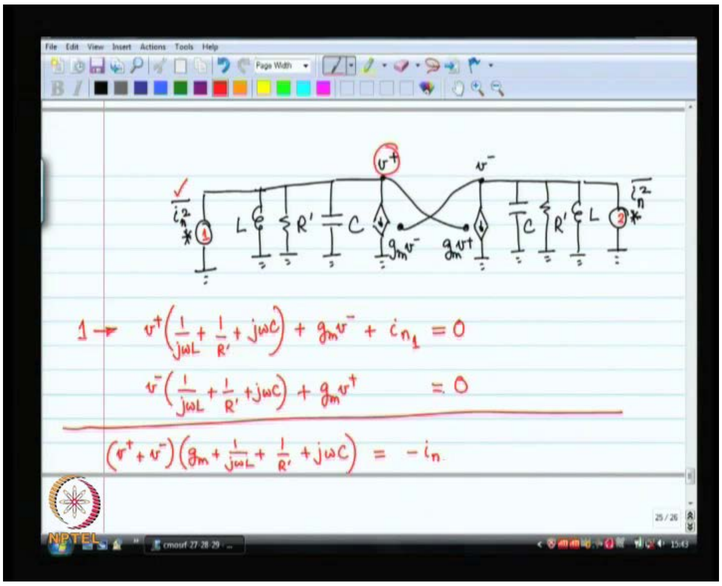

So, I have got L to ground R prime to ground and c to ground c is not really the actual capacitance it is the capacitance insant with all the parasitic capacitance I have got another c another R and another L all of these are to ground. and I have got this noise current source on the both sides as well alright. So, this is my complete small signal model and the way we deal with noise is that we take 1 source at a time and we add the effect of both together at the output.

So, let us talk about what happens to V plus and we are going to select 1 noise source at a time. So, let us first pick this particular noise source call this noise source number 1 and let us call this noise source number two. So, we will pick source number 1 and see what is its effect on the output. So, first of all what do I have over there I have V plus times 1 by j omega L plus 1 by R prime plus j omega c that is the current I am basically trying to write a k chopes current k chopes node equation for V plus. So, this is what I have got at that particular node and if you examine the node V minus then you have got more or less the same thing without the noise source.

no i'am not yet ticking noise source number 2 and 2 account we are going to use super position later on we will deal with my source 1 then will deal with noise source 2 that is the way you talk about noise great.

So, we are interested in finding out V plus right you have got 2 equation you have 2 unknowns should be able to find out what is V plus and what is V minus how do you do that well it is not very difficult you add the 2 equation you have got V plus plus V minus times g m plus 1 by j omega L plus 1 by R prime plus j omega c equal to minus I and 1. Ok

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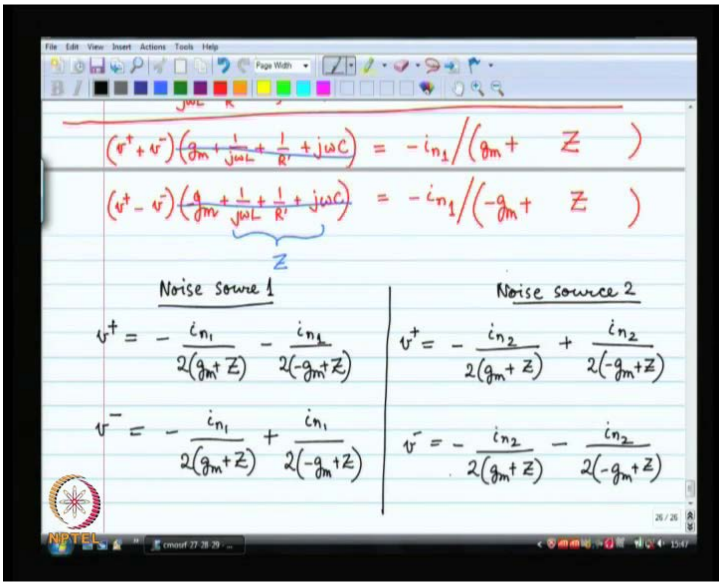

And next what you can do is you can subtract the 2 equation 1 from the other and you are going to get V plus minus V minus times minus g m plus 1 by j omega L plus 1 by R prime plus j omega c is equal to and you can transfer 1 of these terms in common to the other side to the right hand side. So, instead you are going to get that is what I am calling z over here and then finding out V plus is a trivial affair you add the 2 of these and divided divide net result by 2 right. So, this is your V plus.

What about V minus V minus is also not terribly difficult you subtract the 2 and then divided by two. So, this is because of noise source number 1 now because of noise source number 2 I will get basically a flip of signs and I will basically get more or less the same thing just that this addition and subtraction will now become different. alright. So, this is what I am going to get because of noise source number 2 now what you have to do is you have to square both of these and add them right. So, you have to square the contribution of noise source 1 to V plus you have to square the contribution of noise 2 to V plus and you have to add this 2 squares that will give you the mean squared noise voltage at V plus similarly for V minus. So, let us do a squaring operation and adding operation just for to see for ourselves what is going on here

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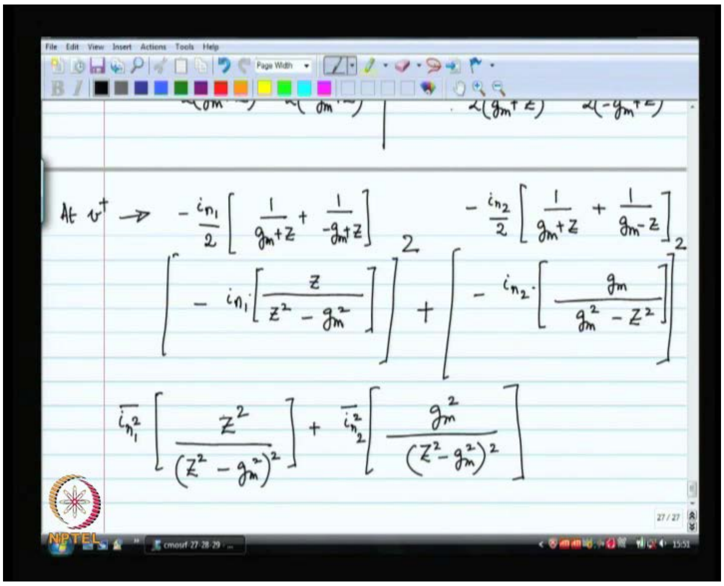

So, first of all what happens to 1 by g m plus z times plus 1 by minus g m plus z. So, this works out to  if I am not mistaken. So, the 2 is cancel out and this is what you end up with and on the other side you get if I am not mistaken this is correct yes and this is what you end up with now what I am saying is you add the square of these 2 things. Right and I n 1 squared is now going to be a mean squared noise voltage I n 2 squared is also a means squared noise voltage and in the denominator I have got  alright.

Now I n 1 and I n 2 are I n 1 squared and I n 2 squared R basically both going to be more or less equal because they are coming from noise voltages coming from noise currents of 2 different identical devices. They are matched devices. So, the amounts of noise contribution from these 2 devices are going to be more or less the same the values of this is eight by 3 k t g m delta f if it is strong inversion and saturation. So, that hole factor comes out and alright now the mistake that you have not pointed out it is not a really a mistake mistake.

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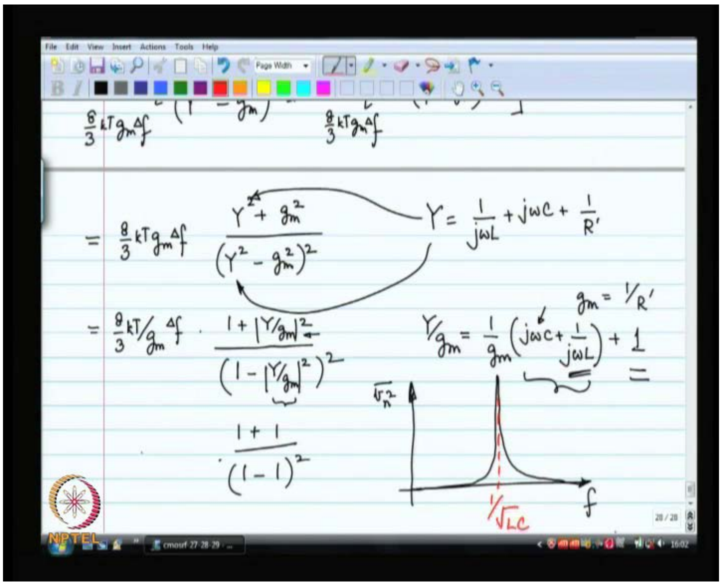

It just that some dimensional problem over here I have g m n z, but I should not really have called it z I should have called it y. So, let me call it y instead of z I am sorry over here just call it y instead of z still the same result should have pointed this outerior  this is not a really mistake this is just notation right rather call it y. Y is what 1 by j omega L plus j omega c plus 1 by R prime this is y alright. So, this is what I have got over here now important the noise is a function of frequency surprise surprise does not come to me as a big surprise, but. So, why has got to be replaced by what you have over here and that makes the noise a function of frequency.

Alright next thing to observe what happens at the resonant frequency what happens at the resonant frequency. So, let us say at the resonant frequency not at the reso[nant]- let us say that this g m quantity is equal to 1 over R prime right that was the case we started of with and said that we get beautiful sinusoid oscillations when this is a true. When g m equal to 1 by R prime then the poles are precisely on the j omega axis which means that I am going to get beautiful sinusoid oscillation alright. So, let us say that g m squared or rather g m is same as 1 over R do you get noise is this quantity of noise large or small.

So, to assess this what I am going to do is I am going to divide numerator and denominator by g m to the power four I put modulus over there because really that is what it means I mean it is a complex number you’re squaring it you should really be talking about modulus over there. So, what happens in this case. So,  this is what I have y by g m is 1 plus a complex quantity right now at resonance this come this imaginary quantity is going to be equal to 0 at resonance agreed at resonance this imaginary quantity is going to be 0 which means that all I have is this real one.

Which means that I have got 1 plus 1 squared which is 1 divided by 1 minus 1 the whole square times some quantity eight by 3 k t by g m delta fthis is the noise voltage mean squared noise voltage the output which means I have got infinitely large quantity of noise at the resonant frequency. This is fantastic right this means that at the resonant frequency the noise is going to be amplified small amount of noise is going to be amplified and the circuit is going to oscillate this is really something very very nice that I have going over here right what happens if I am little bit off from that resonant frequency.

I will still see of very very large amount of noise right we still see a very large amount quantity of noise what happens let us say at d c omega equal to 0 at omega equal to 0 what I have got as far as y by g m is concerned y by g m is something which is infinitely large 1 by 0 omega is 0. So, y by g m is something which is imaginary, but infinitely large agreed. So, I have got a quantity over here which is infinitely large squared. So, I have got infinitely squared square of that. So, I have got infinity to the power four in the denominator I have got infinity squared in the numerator which means that I have got effectively infinity squared in the denominator and which means that the noise at d c is 0.

There is no noise at d c. So, there is noise at d c is huge quantity of noise at the desire frequency of oscillation what about at let us say infinitely large frequencies at infinity large frequency omega tense to infinity this quantity goes to infinity right.So, again y by g m is j in finity y by g m is j infinity means again I have got infinity squared divided by infinity to the power four which means that I have got no noise at infinitely large frequencies. So, all these are wonderful.

What does this mean is if you plot as a function of frequency the amount of output noise that you are seeing you will see that at d c there is no nothing at large frequencies these nothing and the amount of noise is infinitely large at the desired frequency of oscillation right and then it drops off great. So, what this tells us is that this oscillator will work because of noise. Right because I mean let us say there is no startup let us say we did not make g m more than 1 by R prime right let us say g m is precisely equal to 1 by R prime then the poles are precisely on the geo mega axis. So, when the poles are precisely on the g omega axis will the oscillator work at all that was a question raised in the previous lecture right.

I mean the question was if at all oscillates what will be its amplitude why not 0 amplitude. So, this is why it would not oscillate with 0 amplitude it will oscillate with significantly large amounts of amplitude even when R prime is exactly equal to 1 by g m will gate perfect sinusoid oscillation, but because of noise alright now unfortunately what this means of us is that the output of the oscillator will look like this

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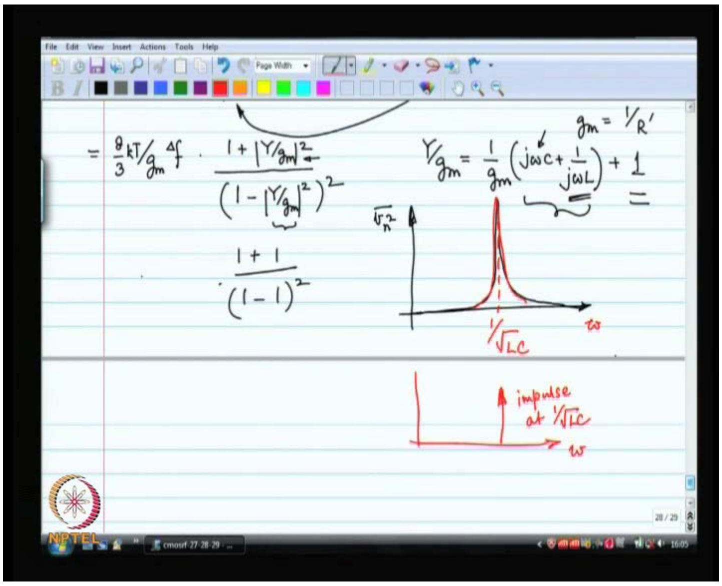


So, what you had hoped for was the spectrum of the oscillator to look like an impulse this is what you wanted to make instead because of noise channel noise in the mosfet I did not even talk about noise in the resistors right. But There is going to be noise in the resistor there is no real resistor over there its a resistor because of the loses in the inductor, but that will create noise loses in the inductor means that you have got noise over there anything which is lousy will have noise.

Right. So, this is what we have got we have got a system which will have which will show response like this all of this is really called phase noise in a real oscillator all of these extra stuff is called phase noise why why is it called phase noise because there is no noise in the amplitude. The noise is only in the phase the amplitude is amplitude of square wave this nothing to be said about the amplitude amplitude is 0 and V d t or the amplitude is positive and negative brain whatever it is.

Right you have generate a squared wave there is no amplitude noise there is only phase noise all that noise is over here all of this this entire scort is basically phase noise. So, this is a way of figuring out the phase noise in an oscillator analytically what we just did in this lecture. So, far is method of figuring out the phase noise in an oscillator analytically unfortunately this method is not correct. So, here I am am delivering a lecture and in the end of the lecture I am telling you that the all that I talked about in the lecture is not correct this is ridiculous right the fact of the matter is that this issue of analyzing phase noise has been dealt with in several research papers right.

There are several techniques talking about how to compute the phase noise in an oscillator unfortunately they are all they all have some or the other now you are going to ask where is the deficiency in this technique the deficiency in this technique is in the fact that we did a small signal analysis. The very fact that we did a small signal analysis is wrong because of 2 reasons reason number 1 if the poles are on the j omega axis then the system is marginally stable you cannot possibly be doing a small signal analysis with j omega laplace transform and. So, on.

That's number 1 if the poles are outside the j omega axis on the right half lane then you have got large signal oscillations why are you even doing a small signal osci[llation]- small signal analysis. So, these are the reason why this analysis is not correct; however, it does give an engineer intuition into what is need to be done to reduce noise for example, right.

So, this is the reason why we did this analysis for that purpose this analysis works mostly works if you ask a simulator to simulate phase noise for you even the simulator there are some deficiencies in that also. So, there are lot of issues with precise computing of phase noise it still a research topic this is 1 way in which we can analysis phase noise alright with this I am going to close this particular lecture in the next lecture we look at other different kinds of oscillators and we are going to move on

Thank you.