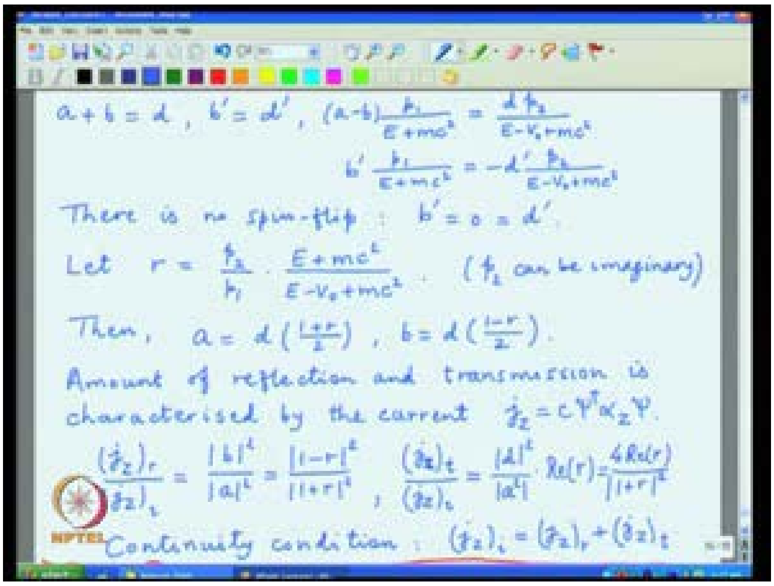
Relativistic Quantum Mechanics
Prof. Apoorva D Patel
Department of Physics
Indian Institute of Science, Bangalore

Lecture - 11
The Klein paradox, Pair Creation Process and Examples

In the last lecture, I worked out the solution of Dirac particle reflected form a step function barrier. And today we will try to understand its interpretation in terms of the process involved. And we will quickly see that we will have to extend our language to incorporate all the things that go on.

(Refer Slide Time: 00:47)



$a + b = d$, $b' = d'$, $(a-b)\frac{p_1}{E+mc^2} = \frac{d\,p_2}{E-V_0+mc^2}$

$b'\frac{p_1}{E+mc^2} = -d'\frac{p_2}{E-V_0+mc^2}$

There is no spin-flip : $b' = 0 = d'$.

Let $r = \frac{p_2}{p_1} \cdot \frac{E+mc^2}{E-V_0+mc^2}$. (p_2 can be imaginary)

Then, $a = d\left(\frac{1+r}{2}\right)$, $b = d\left(\frac{1-r}{2}\right)$.

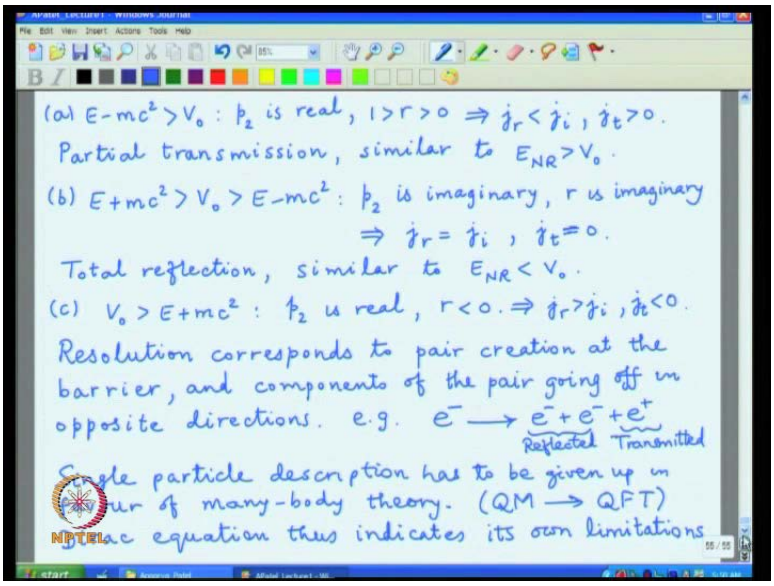
Amount of reflection and transmission is characterised by the current $j_z = c\Psi^\dagger\alpha_z\Psi$.

$\frac{(j_z)_r}{(j_z)_i} = \frac{|b|^2}{|a|^2} = \frac{|1-r|^2}{|1+r|^2}$, $\frac{(j_z)_t}{(j_z)_i} = \frac{|d|^2}{|a|^2}\,\mathrm{Re}(r) = \frac{4\,\mathrm{Re}(r)}{|1+r|^2}$

Continuity condition : $(j_z)_i = (j_z)_r + (j_z)_t$

So, the process is characterised in terms of the transmission and reflection amplitudes. And last time I calculated them as the expectation values of the currents for each of the 3 terms. And now we would like to evaluate the values of these ratios of currents for different values of the potential height.

(Refer Slide Time: 01:21)



(a) $E - mc^2 > V_0$: p_2 is real, $1 > r > 0 \Rightarrow j_r < j_i$, $j_t > 0$.
Partial transmission, similar to $E_{NR} > V_0$.

(b) $E + mc^2 > V_0 > E - mc^2$: p_2 is imaginary, r is imaginary
$\Rightarrow j_r = j_i$, $j_t = 0$.
Total reflection, similar to $E_{NR} < V_0$.

(c) $V_0 > E + mc^2$: p_2 is real, $r < 0. \Rightarrow j_r > j_i$, $j_t < 0$.
Resolution corresponds to pair creation at the barrier, and components of the pair going off in opposite directions. e.g. $e^- \longrightarrow \underbrace{e^- + e^-}_{\text{Reflected}} + \underbrace{e^+}_{\text{Transmitted}}$

Single particle description has to be given up in favour of many-body theory. (QM $\longrightarrow$ QFT)
Dirac equation thus indicates its own limitations

So, let us look at the first case which is the kinetic energy E minus m C square is larger than the height of the barrier; in this case the transmitted momentum is real. And the values of the ratio parameter I defined last time lies between 0 and 1. And these 2 properties; then allow us to calculate the range of values available to the reflection current and the transmitted current. And we will easily obtain that the reflected current is less than the incident current. And the transmitted current is positive and as has to be true; the some of the reflected and transmitted currents matches with the value of the incident current.

So, this is perfectly legitimate situation; it corresponds to partial transmission. And it is similar to the non relativistic case; where, the energy is larger than the height of the barrier; the second case we can easily look at is now increase the height of the barrier. So that it becomes larger than the kinetic energy and interval which I want to first look at corresponds to the barrier height; being between E plus m C square. And E minus m C square which means, the kinetic energy is less than potential energy. But the difference is smaller than 2 times m C square.

In this particular case we can easily see; that the transmitted momentum becomes imaginary, this just is a simple chat from the dispersion relation. And so r is also imaginary; and the consequence then is that the reflected current is equal to the incident current; and the transmitted current vanishes. So, this is again consistent with the conservation of the current it corresponds to total reflection; the wave function on the positive side of z axis where; the barrier is exponentially decays with z. Because the p 2 is imaginary it is not a propagating solution. And this situation is similar to the non relativistic case; where the kinetic energy falls below the potential energy.

So, these things as easily understandable; but we are left with the third possibility which is navel as far as the relativistic quantum mechanics is concerned. And that is the situation where; the potential barrier is higher than kinetic energy. But the difference is larger than 2m C square. And in this particular case again; the dispersion relation will tell you the p 2 is real; and we will have the convention that p 2 corresponds to a wave travelling along the positive z axis. And in that particular situation; the value of r becomes negative; and this as a peculiar result that the reflected current is proportional to 1 minus r; and r is less than 0.

So, it becomes actually larger than the incident current which had only 1 plus r and. So, j r is greater than j i; and the transmitted current which is proportional to r now is negative. So, this is something which has no analogue in the non relativistic case, it would seem that there is a extra contribution somehow of a wave coming from the positive z axis side. And going to the negative side corresponding to this negative value of the transmitted current; but that kind of situation does not make sense physically. Because we have the principle of locality and relativity; you cannot have information travelling faster than the speed of the light and correlations between casually disconnected region.

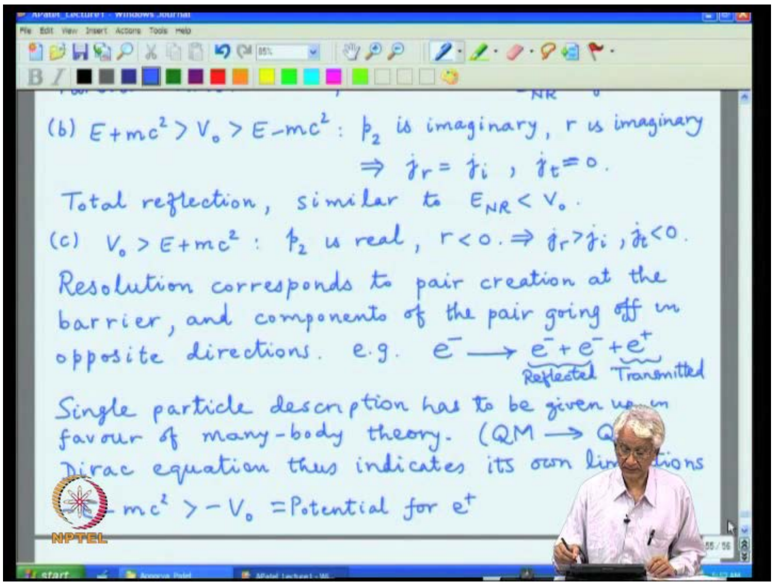
So, there is no way of the transmitted part of the space to know what is coming from the incident part? And so one cannot have simultaneously; something coming from the left as well as, something coming from the right. So, the interpretation has to go beyond just a simple minded single particle description. And that feature is what basically is labelled as a paradox it is a Klein paradox; that you send a current from the negative z axis side; you obtain a reflected current larger than that. And that is; because there is a current coming from the positive z axis side and adding on to the incident current; and the together they give a larger value for the reflected part. So, the way out is not to try to make current come from positive z axis region. But interpret at the negative value of the current, but still propagating in the plus z direction indicated by this value of p 2.

So, there is a wave going; but that wave somehow corresponding to negative value of current; and the wave which can happen is the direction of propagation remains the same. But the charge of the particle is opposite and that will flip the charge of the current which gets transmitted. So, the resolution corresponds to the process known as pair creation; and the components of the pair going off in opposite directions. And for example, if you are talking about electron wave function what happens is the incident electron came from the negative z axis tide. But it produced a pair e minus and e plus. These 2 electrons get reflected with making j r bigger than j i and the positron gets transmitted. And because it has a opposite charge; it will correspond to a negative value of the current relative to the sign of the electron current.

So, this is the peculiarity; but we have to pay a price. And the price is that we can no longer talk about the whole process in terms of a single particle we immediately have to go to a many particle description. And this is a necessity required by relativistic dynamics there is no way around it. So, the single particle description has to be given up in favour of what can be called a many-body theory. And in some sense these labels correspond to the quantum mechanics changing over to quantum field theory. And this is a unique phenomena of merging quantum mechanics with relativity. And so the Dirac equation thus; indicates its own limitations, we started formulating the Dirac equation as a single particle equation; we worked out various consequences etc.

But then we run into these navel processes; which we can only explain going to a many-body description and. So, the Dirac equation is not self sufficient; in the sense in which Schrodinger equation is when we are dealing with problems involving Schrodinger equations, everything can be solved in that specific context. And you do how no need to give up; the 1 particle description in favour of a many-body theory. But Dirac equation necessitates that. And this is a important component, that relativity forces one to go beyond a fixed number of particles; theory to what is generally labelled as quantum field theory. And that is; what we will have to address in more detail little bit later. But one can easily see that the process is quite legitimate of a positron.

(Refer Slide Time: 14:58)



(b) $E+mc^2 > V_0 > E-mc^2$: p_2 is imaginary, r is imaginary

$\Rightarrow j_r = j_i\ ,\ j_t = 0$.

Total reflection, similar to $E_{NR} < V_0$.

(c) $V_0 > E+mc^2$: p_2 is real, $r<0. \Rightarrow j_r > j_i\ , j_t < 0$.

Resolution corresponds to pair creation at the barrier, and components of the pair going off in opposite directions. e.g. $e^- \longrightarrow \underbrace{e^- + e^-}_{\text{Reflected}} + \underbrace{e^+}_{\text{Transmitted}}$

Single particle description has to be given up in favour of many-body theory. (QM $\longrightarrow$ Q

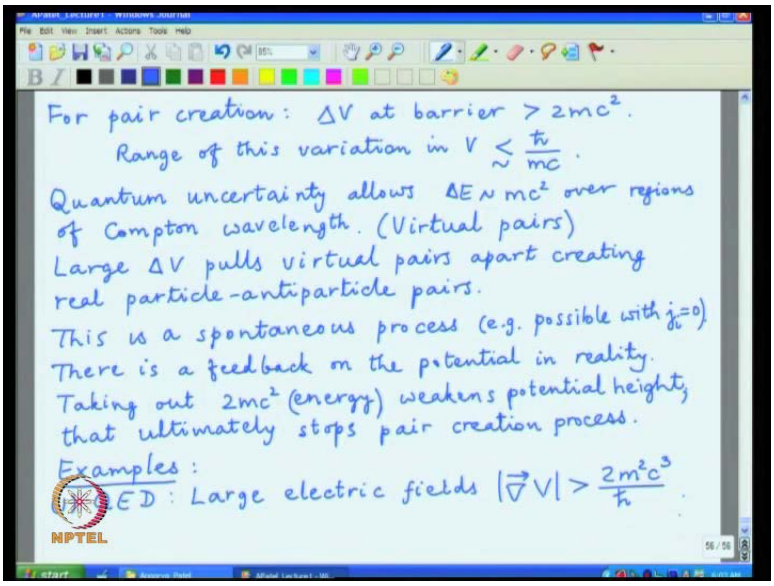
Dirac equation thus indicates its own lim[illegible]tions

$- mc^2 > -V_0$ = Potential for e^+

So, the conditions which I had is this V 0 greater than e plus m C square, can also be written as minus c minus m C square; which corresponds to the kinetic energy of a positron, with energy minus e. And this is less than minus V 0 sorry, it is not less than; it is greater than; that means, the potential height felt by the; this is the potential for e plus. And because the positron has operative charge its potential felt by is also opposite, and its kinetic energy exceeds the potential.

And, you can have a genuinely propagating wave; that wave runs off on the positive z axis type to infinity, that is; the interpretation of j t. So, this is a caveat which one has to understand in case of a relativistic quantum mechanics. And we have a resolution only if we invoke this peculiar process of pair creation whenever, the barrier is large enough.

(Refer Slide Time: 16:50)



For pair creation: ΔV at barrier $> 2mc^2$.

Range of this variation in $V \lesssim \frac{\hbar}{mc}$.

Quantum uncertainty allows $\Delta E \sim mc^2$ over regions of Compton wavelength. (Virtual pairs)

Large ΔV pulls virtual pairs apart creating real particle-antiparticle pairs.

This is a spontaneous process (e.g. possible with $j_i = 0$)

There is a feedback on the potential in reality.

Taking out $2mc^2$ (energy) weakens potential height, that ultimately stops pair creation process.

Examples:

QED: Large electric fields $|\vec{\nabla} V| > \frac{2m^2c^3}{\hbar}$.

So that, it can provide the energy needed for pair creation which is 2m C square. So, there is change in the potential at the barrier which has to be greater than 2m C square and another feature which is not. So, obvious from the previous analysis; because we took only a step function barrier, the barrier was concentrated in a very small space region. But suppose if you take a smoother potential, then the barrier certainly has to exceed in this particular range. But also we need in a second condition; which is this range of variation that has to be less than the Compton wavelength, this is a rough inequality not.

So, precise and this can be understood either by doing a full calculation; but having a gentle barrier or even by appealing to the uncertainty principle; we know that we cannot localize the particles arbitrarily without its momentum becoming very much uncertain. And when the particle is localized to a region of the order of Compton wavelength its momentum become equal to m c. And so the energy corresponding to that particular uncertainty is uncertain by m C square and so whenever this region is very small, we can have quantum fluctuations of in energy which are of the order m C square.

So, this quantum uncertainty allows delta E of the size of m C square over regions of Compton wavelength. And we can then have pairs coming out because this energy is comparable tom C square. And this pairs are often referred to as virtual pairs in the background often called the vacuum. So, quantum vacuum will always have this virtual pairs hanging around over very small distances; what the external potential can help is that, if the potential is changing very fast over the size of whatever, this virtual pairs are then these virtual pairs can be pulled apart and converted to real pairs. And once these pairs become real, than we will see consequences as in the case of paradox; there will be a value of current which becomes observable.

Hence, so one needs this particular condition, unless that you need a large variation of the potential over a size; which is of the order of Compton wavelength, if that is; available then we will have this virtual pairs coming out of this particular vacuum. And one can even look at this as a spontaneous process. So, this is a this one can be understood in the previous derivation, that we can have no incident wave at all; you can put j i is equal to 0.But you can still obtain non 0 value for the reflected current, as well as the transmitted current; pairs are just popping out of the vacuum one of them runs upon one side; and the another one on the other side.

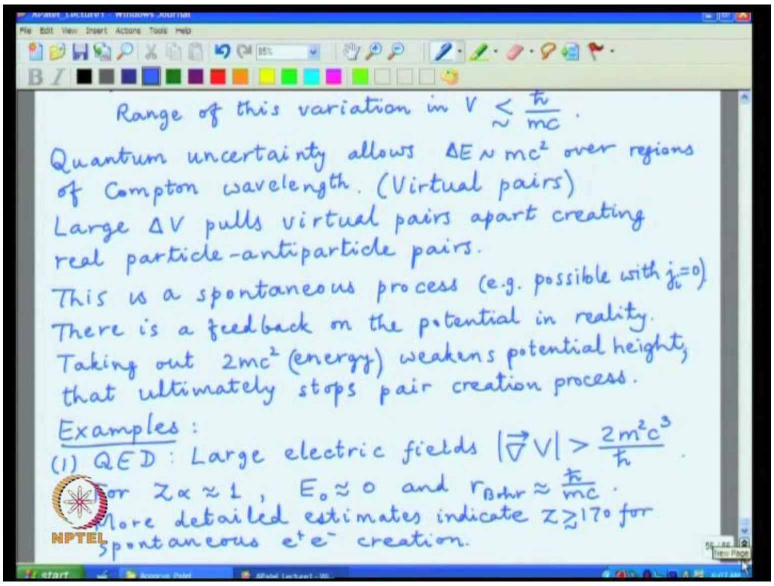
And, this limit of the calculation with j i equal to 0 is a smooth limit. So that is, indication of a spontaneous process; one can also observe a 1 more feature, that is; the pairs are getting produced. But somebody is paying the cost of this 2m C square energy; when you treat an external potential, we do not consider any variation. And we but in reality there is a feedback; and that is taking out 2 m C square energy weakens; the potential height and that ultimately stops pair creation process. So, it is a spontaneous process; but every time the pair comes out the potential loses energy.

And, so it becomes smaller in height and ultimately it will become small enough in height. So that, no further pair creation will takes place. So, the procedure in real world with a feedback does have a ultimate limit in the sense, that pair creation cannot just go on forever; that is just a simple consequence of energy conservation. So, this is a general background and its interpretation.

And, now we can look at some simple examples, how we can see this particular process; one example is to just look at quantum electrodynamics; which is what we have been solving using the Dirac equation. But we want such large potentials or equivalently; large electric fields. And that is the ratio of this how much potential is needed to the distance over which is needed. And you can work it out this it can be expressed as a gradient of potential has to be larger than the ratio of these 2 particular quantity 2m C square and h cross by m c. So, we need the electric fields which satisfy this particular condition.

Now, such large fields are not that easily available; we need special configurations to produce the m. And that requires getting a large charge in a small enough region; one way to look at this condition; is to just look at the solution of the hydrogen atom which we have worked out earlier.

(Refer Slide Time: 27:30)



Range of this variation in $V \lesssim \frac{\hbar}{mc}$.

Quantum uncertainty allows $\Delta E \sim mc^2$ over regions of Compton wavelength. (Virtual pairs)

Large ΔV pulls virtual pairs apart creating real particle-antiparticle pairs.

This is a spontaneous process (e.g. possible with $j_0=0$)

There is a feedback on the potential in reality. Taking out $2mc^2$ (energy) weakens potential height, that ultimately stops pair creation process.

Examples:

(1) QED: Large electric fields $|\vec{\nabla} V| > \frac{2m^2c^3}{\hbar}$.

For $Z\alpha \approx 1$, $E_0 \approx 0$ and $r_{Bohr} \approx \frac{\hbar}{mc}$.

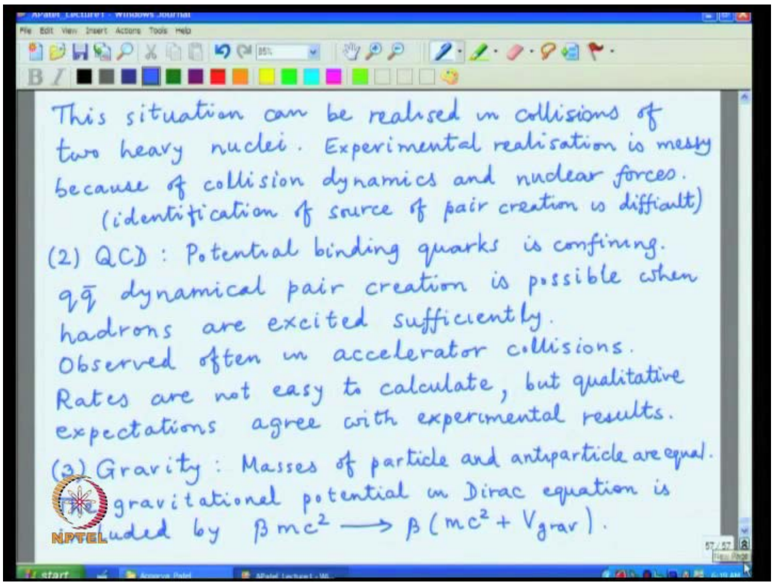
More detailed estimates indicate $Z \gtrsim 170$ for spontaneous e^+e^- creation.

So, in that particular case we have roughly that for nuclear charge becoming of the order of 137; we have the total energy for the ground state of the hydrogen atom roughly equal to 0. And the so called Bohr radius becomes order of Planck constant divided by m C which is nothing but the Compton wavelength. So, we have the condition roughly satisfied; though the potential is large enough. So, the binding energy is of the order of m C square; that is essentially what is available to the electron.

And, that energy is available over the Compton wavelength. Now, you may argue, that you need a little more energy in the sense that it; this will give m C square and not 2m C square. So, then you can say that well, we will have to go to nuclear charges; which are larger than 137;that is a more detail, estimate of what nuclear charges will be needed to see this particular process; but roughly this is the case.

So, in this kind of situation; if you had a nuclear charge with large enough value, there will be spontaneous e plus, e minus creation coming out right from the innermost orbit of the electron. And that will be easily visible because the atom will suddenly start producing; this radiation e minus will fall on this nucleus reducing the particular field; and e plus will be emitted as in the case of beta decay. And we will have a spontaneous signature. But this particular condition; that nuclear charge is of the order of 1 over the fine structure constant is not easy to obtain in practice. So, more detailed estimates indicate that you need the desired greater than about 170 for and we cannot have a single nucleus of this particular charge.

(Refer Slide Time: 30:51)



This situation can be realised in collisions of two heavy nuclei. Experimental realisation is messy because of collision dynamics and nuclear forces.
(identification of source of pair creation is difficult)

(2) QCD : Potential binding quarks is confining.
$q\bar{q}$ dynamical pair creation is possible when hadrons are excited sufficiently.
Observed often in accelerator collisions.
Rates are not easy to calculate, but qualitative expectations agree with experimental results.

(3) Gravity : Masses of particle and antiparticle are equal.
The gravitational potential in Dirac equation is [illegible]uded by $\beta mc^2 \longrightarrow \beta(mc^2 + V_{grav})$.

But what has been argued as this situation; can be realised in collisions of 2 heavy nuclei. Then we need the nuclear charge on each 1 of the 2 heavy nuclear of the order of 80; which is certainly available in practice. But this collision will not be a static situation; it will be only for a short duration, these two things have to come together. And in that particular duration you will have this instability in the potential; and you will have particle e plus e minus productions and that is somewhat messy. And also we only included the electromagnetic interaction in this particular calculation once; you start colliding to nuclei lots of other process also start happening because nuclei forces themselves come into play.

So, experimental realization is a messy we can certainly collide 2 nucleuses. But then identifying the signature out of all the things which go on; that is a part which is messy. And the reason is the collision dynamics and nuclear forces. So, certainly you can see lots of things going on. But identifying that this particular e plus e minus creation corresponded to the kind of feature we saw in the Dirac equation; that is not clear enough or rather some ambiguities are left inside allocating various species to various kind of interaction. So, this is 1 particular situation not easy to construct and not easy to interpret because of certain caveats. So, identification of source of pair creation is difficult.

So, now let us look at a different example, which corresponds to the theory of Q C D in this particular case the interaction is the strong interaction; that of quantum chromo dynamics or which binds the protons and neutrons together; and also produces then nuclear process the interactions is strong. So, it certainly produces strong fields what is seen in processes of Q C D; is that the potential which binds the various components together the components are the quarks. So, potential binding quarks is confining by confining it means, that the separation of the components become large the potential goes to infinity. And that certainly is the situation required for satisfying Klein paradox the potential does go to infinity.

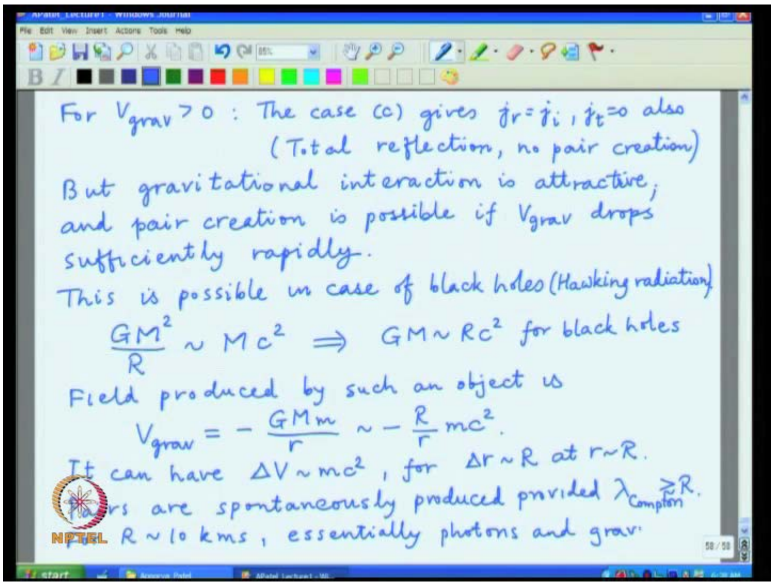
So, you will have regions where you can produce 2 m C square of energy. And so the particle antiparticle pairs; that region to be small enough. But that is possible because the coupling involved in the Q C D is certainly large. So, you have large gradients available and then the process; which is q q bar, quark. And anti-quark dynamical pair creation is possible; when the binding system is perturbed by certain kind of excitation; you just push it high enough of the energy by say hitting it with some external radiation. If that kick is large enough, you will have this spontaneous pair creation possible. And the bound states in Q C D are called hadrons.

And, when they are sufficiently excited you have possibility of creating quark, anti quark pairs. And this is actually seen very often in many collision process; which occur at different kind of particle accelerator. And we have easy identification also about the quarks and anti quarks coming out. And you can actually see them as a certain jets coming from that collision region. So, here the identification is actually much straight forward. What is not clear cut is that this is the theory with strong interactions. So, the rates of how frequently this process occurs they are not very easy to calculate quantitatively, that the pair creation occurs.

But you can ask how frequently it should occur based on the fundamental equations involved? And because the potential is very strong one cannot use the standard machinery of perturbation theory to calculate these rates. And that is the sort of limiting part of the verification. But there are so can make some models and construction qualitative expectations and they agree with experimental results. So, this is a second example I would go further. And give a third example also which corresponds to theory of gravity this has a certain difference compared to the previous cases; for 2 reasons, 1 is we do not have a complete quantum theory of gravity. And another thing is the gravitational interaction behaves differently than electromagnetic or Q C D interactions. And that difference comes because of the fact that in Q E D or Q C D the particle and anti particle have opposite charges. But in case of gravity the particle and anti particle have same value of mass.

So, one has to include the effect of gravity. But as a slightly different operator in the Dirac equation here; the masses of particle and anti particle are equal. And so once we include this potential it has to behave the same way as the mass term in the Hamiltonian behave. So, the potential appears in the Dirac equation by the replacement, that you have the term beta m C square, going to beta m C square plus V gravity. And this is the slightly different prescription compared to electrodynamics potential change; the Hamiltonian without this extra factor of beta. And because of that the shift of energy used for particle and antiparticle who is in opposite direction in case of gravity; the shift is in the same direction. And change in the mathematics is this extra factor of beta; the potential will couple in the same way as the mass term is there in the equation.

(Refer Slide Time: 42:49)



For $V_{grav} > 0$: The case (c) gives $j_r = j_i$, $j_t = 0$ also (Total reflection, no pair creation)

But gravitational interaction is attractive, and pair creation is possible if V_{grav} drops sufficiently rapidly.

This is possible in case of black holes (Hawking radiation)

$$\frac{GM^2}{R} \sim Mc^2 \Rightarrow GM \sim Rc^2 \text{ for black holes}$$

Field produced by such an object is

$$V_{grav} = -\frac{GMm}{r} \sim -\frac{R}{r} mc^2 .$$

It can have $\Delta V \sim mc^2$, for $\Delta r \sim R$ at $r \sim R$.

rs are spontaneously produced provided $\lambda_{Compton} \gtrsim R$.

$R \sim 10$ kms, essentially photons and grav.

Once, you now understand this particular caveat you can go back to the reflection from a barrier problem. And see what this kind of potential will do and here; you see a slightly different feature whenever the potential of this particular type is positive. And the case C which I considered for pair creation; in case of electric potential it gives the same result as case B. And what this implies is there is total reflection and no pair creation in this particular situation. So, a positive potential barrier does not do anything navel with the potential is a gravitational potential. But we already know that gravity is a different kind of a interaction; the potential of gravity happens to be actually negative. And then we can have the conditions for pair creation satisfied. Because the potential can drop by 2m C square over a certain region, that can give sufficient energy to be released for pair creation.

So, gravitation is attractive and the pair creation is possible if the potential drops sufficiently rapidly. And then the 2 m C square can be released over a distance scale of the order of Compton wavelength. And now we can make an estimate about what is the condition or how strong this field has to be? And that can be done qualitatively looking at large changes of the order of m C square of the for the potential gradient; and that corresponds to the situation which arises in case of black holes.

So, this is possible in case of black holes where the field is sufficiently strong. And the pair creation is possible if V gravitation sufficiently, rapidly; then the 2 M c square can be realised over a distance scale of the order of Compton wave length. And now you can make the estimate wave length what is the condition and how strong is this field has to be? And that can be done qualitatively looking at large changes of the order of m C square of the for the potential radiant. And that corresponds to the situation which arises in case of black holes. So, this is possible in case of black holes where the fully efficient strong.

And, pair creation of this particular type has a special name it is called hawking radiation. So, we can now easily workout the estimate; the condition for a formation of black hole is that the gravitational potential energy which is G M square by R is the radius of the black hole; and M is the mass has to be of the order of the rest mass energy, these are just order of magnitude estimate. And that implies the mass and the radius of the black hole are related by this simple relation mass is proportional to the radius.

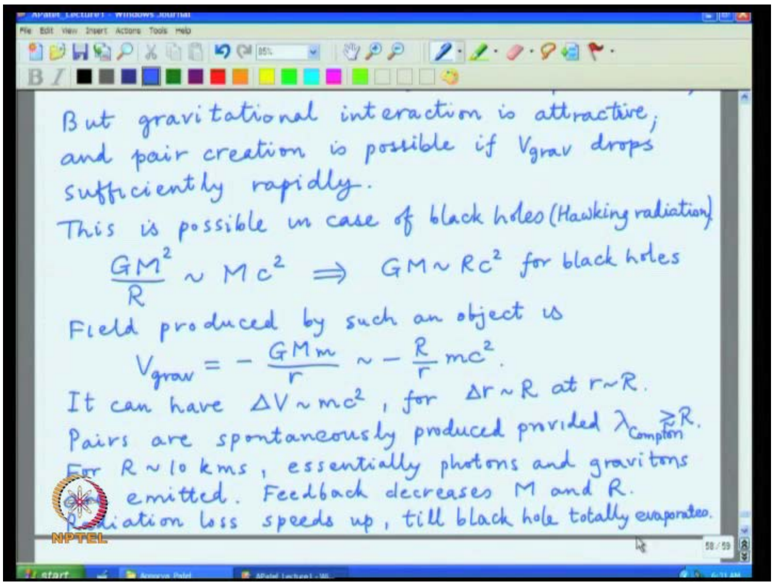
Now, we want the second condition also which has to be related to how fast the potential is changing. So, what is the gravitational potential in this field produced by a black hole? So, field produced by such a black hole it is given by V gravity is minus G mass M is the test object and divided by its distance. And once you plug in this order of magnitude estimate then it is ratio of the black hole radius to the position of the test particle. So, it can have this delta V of the order of m C square when the change occurs over distance of the order of the radius; and the of course value of the radius is that of the black hole.

So, this condition is indeed satisfied and in that particular case. So, pairs are spontaneously produced, provided this lambda Compton is then larger than the black hole radius. We needed large variation over a distance of the order of Compton wavelength we saw that the variation is available at the size of the black hole radius. And so the particles which will have Compton wavelength larger than; that will certainly get emitted by this pair creation processes. And that now gives a generic picture of this hawking radiation; that the standard black holes of the so called just produced black holes from neutron stars have radius which are of the order of 10 kilometres. And the only particles which we know of Compton wavelength larger than this are the mass less particle.

So, they are photons; and nothing much else we do not anything resembling a the electron positron pair at this particular case. But again because of the peculiarity of gravitational interactions you can now see what will happen with the feedback of this radiation onto the black hole itself? So, certainly some energy is going away that will decrease the mass of the black hole; it will also decrease the radius of the black hole because of this condition that G M is proportional to R C square, and so as radiation comes out the black holes start shrinking both in mass as and in radius.

And, the spontaneous pair creation condition; then becomes possible for particles with smaller and smaller Compton wavelength. Once, the radius has shrunk to sufficiently small size there may be, more than just photon and graviton which will have wavelength larger than particular size and they will get emitted. So, the radiation due to pair creation actually accelerates R shrinks. And more and more particles start gets emitted until at the end the black hole just disappears; all the energy which has now is part of the radiation. And it is all gone and you are left with nothing at the end.

(Refer Slide Time: 53:32)



But gravitational interaction is attractive; and pair creation is possible if V_{grav} drops sufficiently rapidly.

This is possible in case of black holes (Hawking radiation)

$$\frac{GM^2}{R} \sim Mc^2 \Rightarrow GM \sim Rc^2 \text{ for black holes}$$

Field produced by such an object is

$$V_{grav} = -\frac{GMm}{r} \sim -\frac{R}{r} mc^2 .$$

It can have $\Delta V \sim mc^2$, for $\Delta r \sim R$ at $r \sim R$.

Pairs are spontaneously produced provided $\lambda_{Compton} \gtrsim R$.

For $R \sim 10$ kms, essentially photons and gravitons emitted. Feedback decreases M and R.

Radiation loss speeds up, till black hole totally evaporates.

So, the feedback decreases both M and R and the radiation loss speeds up with more and more particles getting emitted even the ones with larger masses; once the radius becomes sufficiently small till black hole totally evaporates. So, this is again a sensible limit for case of a feedback; that the radiation ultimately stops. But the radiation actually accelerates. And then stops quite unlike what happens in the electro dynamic case? So, this is the illustration of this pair creation processes in various kind of different interactions different backgrounds. And their peculiar features certainly; we can hope to get more details if we do have a more quantitative analysis what I have presented are just the qualitative estimates and order of magnitude pictures.

But they are quite sufficient for understanding the general principles involved; exact results are actually not available to the detail which we want for various reasons in case of Q C D; we do not have ways to solve equations exactly in case of gravity, we do not have a complete theory of gravity in the quantum region. So, we do not actually know literally what happens at the end points of the process; it has to rely on quantum gravity which we do not know. But in the so called in between region which is called semi classical or qualitative understanding of the mechanism the behaviour is expected.

And, it works in several cases when we can design experiments to see them; of course the hawking radiation is something which we have not observed in any particular science. But efforts are underway to construct systems in condensed matter of physics; where the same kind of situations can be created. And then we will be able to see some of these particles, anti particle, pair creation process. And those are again very interesting examples I will describe an example of recent interest that is the property of grapheme; where Dirac equation plays a very important role in a later lecture; before that I will continue with more properties of Dirac equation in the next lecture.