Applied Optics
Professor. Akhilesh Kumar Mishra
Department of Physics
Indian Institute of Technology, Roorkee
Lecture 60
Triffle

Hello everyone, welcome to this course of Applied Optics. Today, we are going to hold the last lecture of this course and this of course sixtieth lecture of this course. Today I will not cover something which is given in the syllabus. Rather I will talk about a few little things in which I found a student usually get confused and in this row, let us start with the expression of electric field.

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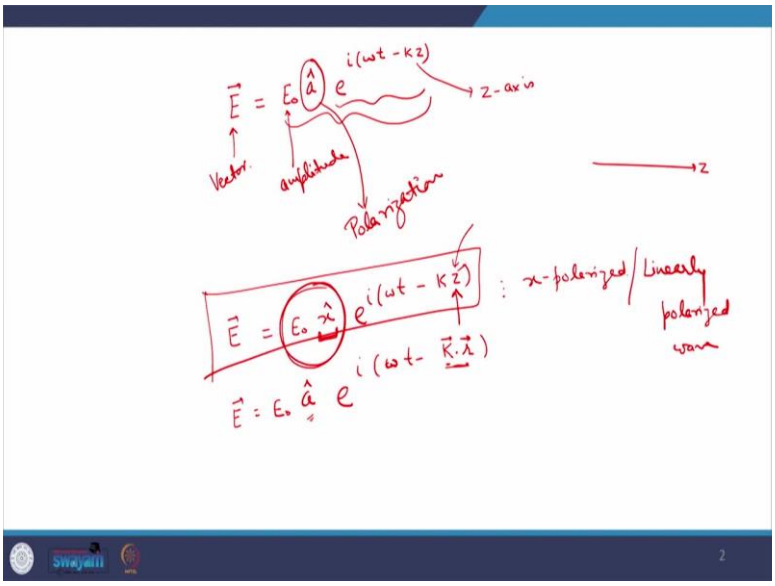

Usually we write field like this $\vec{E} = E_0\, \hat{a}\, e^{i(\omega t - kz)}$. Here in, on the left hand side it is a vector quantity and therefore, the right hand side must also be a vector, to make the right hand side a vector quantity, we have introduced $\hat{a}$ which is nothing but a unit vector, E_0 is amplitude of the field.

As you can see from this exponential part, the direction of propagation of the wave is along z-axis, wave is propagating along z direction and this is the direction of propagation. Now, $\hat{a}$ which is a unit vector it represents the polarization, it represents the direction in which electric field is oscillating.

Therefore, the vector quantity which is associated with the amplitude or which is multiplied with the amplitude is associated with the polarization, it tells about polarization. Say the same field is written like this $\vec{E} = E_0\, \hat{x} e^{i(\omega t - kz)}$. Now, in this case we will say that this field is x polarized and since the field is x polarized we can say that it is a linearly polarized wave. It is a linearly polarized wave. Do remember that polarization is always associated with the vector which appears here and direction of propagation appears here.

Now, again assume that we have a field which is polarized along certain direction and it is given by E and $\vec{k}.\vec{r}$. Here the two, k and r, they both are vector. Now k is not pointing along r, now the projection of k along the direction of r decides this value $\vec{k}.\vec{r}$ and these are the things which one must keep in his mind and usually people get confused like whenever I give this expression in the class then student say that the polarization is along z, but do remember this is direction of propagation polarization always appear with the amplitude here.

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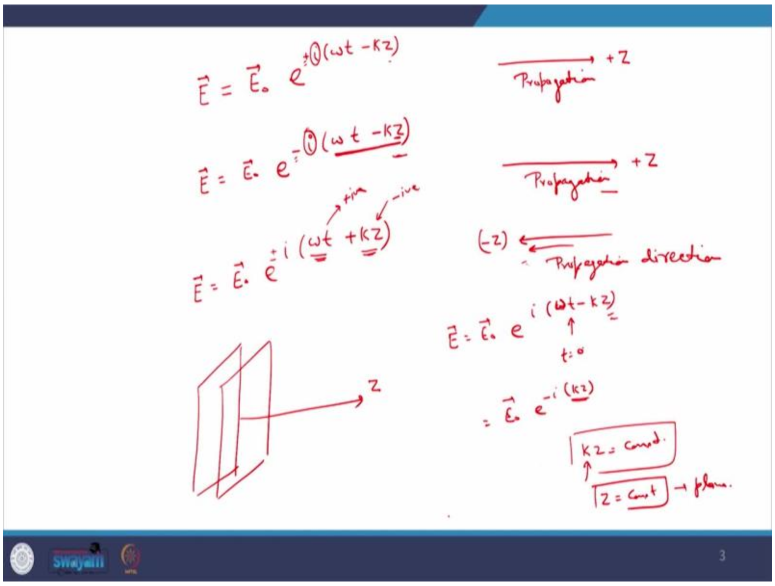

The next thing which I would like to add is that say the field is given like this E $= E_0$, say E_0 is a vector I observed the unit vector in the amplitude E_0 and then we have $e^{i(\omega t-kz)}$ it is a z propagating wave which is very much obvious you can say that it is a wave which is propagating in plus z direction. This represents the direction of propagation.

But what will be the direction of propagation of this wave $\vec{E} = \vec{E}_0 e^{-i(\omega t-kz)}$. Now, in this case do remember that as the waves propagate time always evolve in the positive direction you cannot go behind in time. Therefore, time is evolving in positive direction, time is always increasing.

And therefore, to make this quantity constant say it is a plane wave to make this quantity constant z also must evolve positively and therefore, this wave is also propagating in + z direction, direction of propagation is again here in + z. Then irrespective of sign before this

Iota the wave always propagates in positive z direction, the sign before i does not change anything.

It is a convention like depending upon the convention we sometimes speak $+i$ and sometimes $-i$, but the output which we receive out of this complex representation, it does not change. Now, let us assume the case where $\vec{E} = \vec{E}_0 e^{-i(\omega t + kz)}$.

Now, here t is evolving in positive direction this is increasing and to compensate this increase, now z must decrease, in this particular case z must be negative and this particular wave goes in (-z)direction. Now, again if irrespective whether we put $+i$ or $-i$ the direction of propagation here does not change it always propagate here in (-z) direction, it is a propagation direction.

Now, let us talk about the definition of a plane wave as I said in the classes a plane wave is defined as a wave in which the locus of points which are oscillating in the same phase is a plane and therefore, while representing plane wave we draw these planes which propagate in this direction say it is a z direction.

Now, assume that this is a plane wave $\omega t - kz$. Now, let us pick a time where t=0 and therefore, this expression reduces to e^{-kz}. Now, in plane wave this phase part must be constant, kz must be constant and k is a constant term we already know, therefore, from here we can write z is equal to constant and this represents a plane and therefore, this is an expression of a plane wave.

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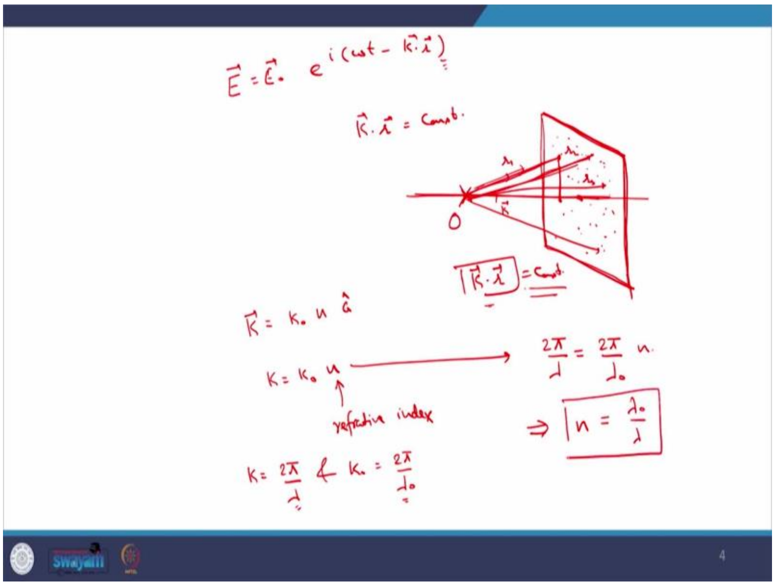

Now, if you pick another expression, a more generalized form is this $\omega t - \vec{k}.\vec{r}$. In this case too $\vec{k}.\vec{r}$ must be constant. Now say this is a plane and this is $\vec{k}$ and say this is your r_1, this is your r_2, this is your r_3, then the projection of different r along k direction would be constant this number because see, we will always get a same length the $\vec{k}.\vec{r}$ vector would be constant here.

If we choose a plane here and pick different points on the plane and then draw a line joining from some point on this horizontal line and then if we take different r from this point say O and then take the dot product of these vectors with respect to k, then $\vec{k}.\vec{r}$ gives constant it means $\vec{k}.\vec{r}$ is equal to constant is true only if all these points lie on a plane and this is why again this equation of a plane wave.

Now, the next thing which I would like to add here is that the k is nothing but $k = k_0 n$, it is a k_0 which is the wave vector in vacuum into n refractive index and some vector quantity. Let us remove the vector for a while let us talk in terms of the amplitudes only then $k = k_0 n$ where n is nothing but a refractive index of the medium and we know $k = 2\pi/\lambda$ and $k_0 = 2\pi/\lambda_0$ where λ_0 is the wavelength of light in vacuum and λ is wavelength of light in medium and n is refractive index of the medium.

Therefore, from here we can write $2\pi/\lambda = 2\pi n/\lambda_0$ and from here we get the expression of $n = \lambda_0/\lambda$.

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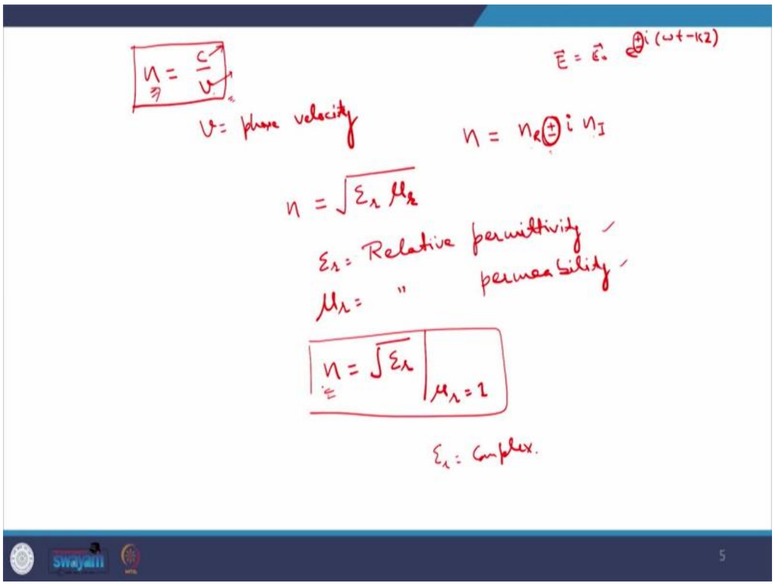

And we also know that refractive index is conventionally defined by c/v, where v is velocity of light in medium and v is usually known as phase velocity, but n is not always a real number. In media like metal, n assumes complex value, when n is complex, then we can write n as $n = n_R \pm in_I$.

Now, the $\pm$ which is appearing here it the exact sign whether it would be plus or minus it depends upon the sign before the i in the exponent here, in this expression where $E = E_0 e^{\pm i(\omega t - kz)}$, whether here before i whether it is plus or minus it decides the value of plus and minus between real and imaginary part of refractive index.

Now, coming back to the original definition of refractive index from here, we know that n will always be real because c is the speed of light in vacuum and v is the speed of light in medium and they both are real numbers and therefore, this definition cannot give complex n because, v cannot be complex.

A better definition of refractive index and from electromagnetic theory is derived as $n = \sqrt{\epsilon_r \mu_r}$ where ϵ_r is permittivity or more specifically relative permittivity, relative permittivity of the medium and μ_r is relative permeability. Usually most of the optical materials are nonmagnetic in nature and therefore, μ_r in those cases are taken to be unity and therefore, in most of the optical material this would be the definition of refractive index provided they are nonmagnetic.

We have already talked about permittivity and permeability, permittivity represents an extent to which a material can be polarized in presence of an external electric field. And similarly, permeability gives us an extent to which a material can be magnetized in an externally applied magnetic field. If with permittivity electric fields comes here it is related with the electric field and permeability is related to the magnetic nature magnetic field.

Now, here ϵ_r and μ_r they both can assume complex values. Now if we stick with this definition and if ϵ_r is complex or negative then we get complex value of refractive index and which satisfies like most of the requirements, but in few materials refractive index becomes direction dependent.

The example of these materials are quartz crystal, tourmaline crystal, these are birefringent material they are an isotropic and in an isotropic crystal refractive index become direction dependent and in those particular cases, we treat refractive index as a tensor or we treat permittivity and permeability as tensor quantities and in those material these quantities are represented by matrices.

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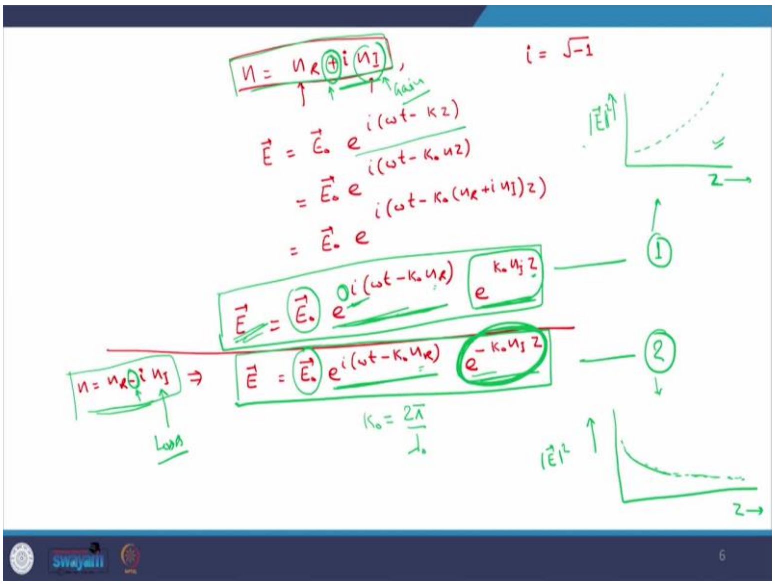

Now, if we assume $n = n_R + in_I$ where n_R is real part of refractive index and n_I is imaginary part of refractive index and this is i represents Iota, where $i = \sqrt{-1}$. Let us substitute this in this expression of field $\vec{E} = \vec{E}_0 e^{i(\omega t - kz)}$. Let us substitute it into this expression of field.

Now, this field can be represented as $k_0 nz$ which can again be expressed as $e^{i(\omega t - k_0(n_R + in_I)z)}$. Let us expand it further $e^{i(\omega t - k_0 n_R z)} e^{n_I k_0 z}$. But instead, if we put $n = n_R - in_I$, then this gives us the expression of the field as $\vec{E} = \vec{E}_0 e^{i(\omega t - k_0 nR)} e^{-n_I k_0 z}$.

Now, there are two expressions which are of prime importance to us. These are the two expressions which must be given extra attention. Let us say that it is equation number 1 and this is equation number 2. Now, this is the usual amplitude part with a polarization information the second term is common in both the expression $e^{i(\omega t - k_0 nR)}$, this is the phase part and in this phase part $k_0 = 2\pi/\lambda_0$, n_R is the real part of refractive index that its phase is similar to the initial phase.

Now, when imaginary part of refractive index is taken into account, then we get this term, this extra term. And this term in equation number 1 is exponentially increasing term while the same term in equation number 2 is exponentially decreasing term, it is a exponentially decaying, it is exponentially decreasing. Now, it means depending upon the sign before imaginary part of refractive index we get exponentially increasing or exponentially decreasing term in the expression of the field.

Now, if you plot equation number 1 and what you will get is that E is plotted here on the vertical axis and z is here on the horizontal axis. Then you will see that there is a phase term and since we are plotting mod, the phase term will go away. Let us put plot intensity instead mod square. Now, this is exponentially increasing term it means with z the field will increase, this would be the nature of the plot exponentially increasing field.

While if you plot equation numbers 2 keeping z on the horizontal axis and mod of E square on the vertical axis, then it will get exponentially decaying plot and this exponential decay appears from this second term, exponentially decaying term. It means that the oscillatory nature of the field owes its origin in real part of refractive index while the imaginary part of refractive index decides whether field will increase or decrease with propagation.

Now, in most of the material we get decrease in the field magnitude with propagation, why? because materials are lossy, and therefore, we associate losses with imaginary part of refractive index. But here in equation 1, we saw that the field magnitude or the intensity of the field is increasing with propagation and this is again due to the imaginary part of refractive index but here we have plus sign before imaginary part of refractive index. Contrary to the case in equation number 2, where we have minus sign before the imaginary part of refractive index.

Now, where we have plus sign of refractive index we will get increase in intensity and therefore, this term is responsible for gain, the similar kind of gain which we witnessed in case of gain medium of a laser. In laser 2, we saw gain and this is imaginary part govern these things whether the material would contribute loss or gain. Mathematically the sign before imaginary part decides this.

Further control also comes from the sign before Iota term in this exponential. If you put instead of plus minus sign, then the sign of gain and loss term would be exchanged, you can do it as your exercise put replace i with minus i here in the expression of field and then do all the calculation then you will realize that this expression of refractive index gives loss while this expression of refractive index gives gain. Therefore, the imaginary part of refractive index hold as much of importance as the real part of refractive index.

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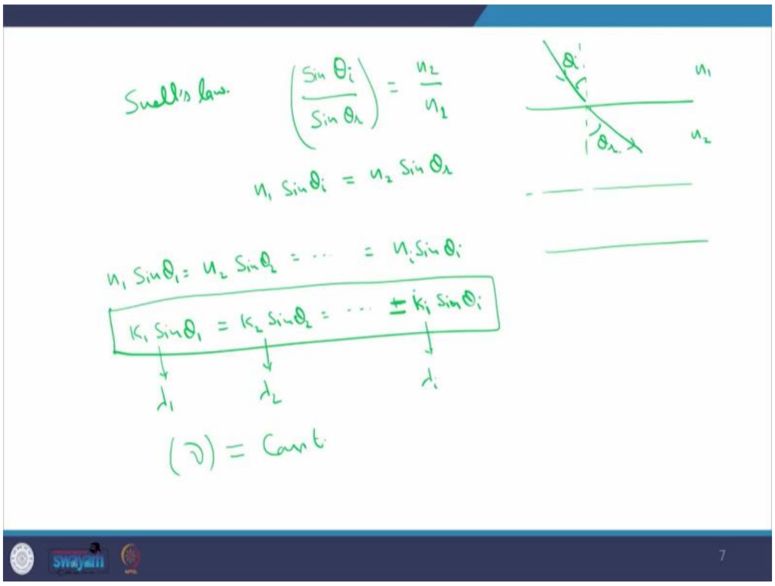

Moving to the next point, we know that the refraction is majorly defined using Snell's law which says that sine of angle of incidence and sine of angle of refraction this ratio is equal to n_2/n_1, n_2 is the refractive index of medium 2 and n_1 is refractive index of medium 1 and this is our Snell's Law.

This can also be written as $n_1 sin\theta_i = n_2 sin\theta_r$, where θ_i is this angle and θ_r is this angle. Now, if we have a multi-layer system then the same expression can be generalized as $n_1 sin\theta_1 = n_2 sin\theta_2 = \ldots = n_i sin\theta_i$. Or more generally we can write it as $k_1 sin\theta_1 = k_2 sin\theta_2 = \ldots = k_i sin\theta_i$. This is the most generalized form of Snell's Law, which states that tangential component of the wave vector at interfaces must be continuous.

Now, whenever we talk about refraction, we say that due to the change in refractive index, the phase velocity of the wave changes and therefore, we see a deviation in the path of light propagation or a ray propagation and due to the change in refractive index the wavelength changes here the wavelength would be λ_1 here and instead it would be λ_2 here it would be λ_i, the wavelength is changing if we change the medium, but the frequency of the wave it remains constant, it does not depend upon the refractive index of the medium.

It is frequency is medium independent quantity why because, the frequency is a property of source it is solely defined by source and it solely depends upon the properties of source. And it remains constant irrespective whether the light is undergoing refraction or refraction or diffraction, interference whatever. Frequency being a property of source it remains constant it does not vary.

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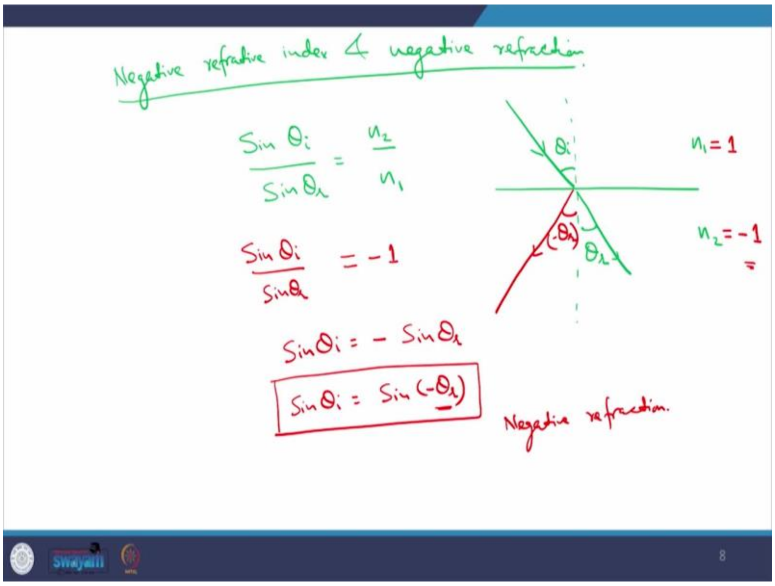

Now, last thing which I would also like to touch upon is negative refractive index and negative refraction, we know that $sin\theta_i/sin\theta_r = n_2/n_1$, this is n_1 and this is n_2 and this is a ray which is falling here and it makes an angle θ_i here and then after refraction it makes angle θ_r, but what will happen if $n_1 = 1$ and $n_2 = -1$, let us substitute it here in this Snell's law.

In this particular case, the right hand side would be equal to -1. Therefore, $sin\theta_i/sin\theta_r = -1$ or $sin\theta_i = -sin\theta_r$. This can also be written as $sin\theta_i = sin(-\theta_r)$, the angle of incidence is fixed because we are launching the ray at the interface between a positive refractive index medium and a negative refractive index medium.

But if the second medium is of refractive index -1, then we see that angle of refraction changes its sign, it means the refracted wave will now go in this direction, this would be θ_r now and this type of refraction is called negative refraction. Here we see negative refraction, in negative since, θ is changing its direction, the k also changes its direction.

The direction of energy propagation remains the same, but the direction of wave vector reverses in negative refractive index medium, group velocity which is associated with the direction of energy propagation, it remains the same, but the direction of phase velocity reverses.

Now, these were a few little things which I felt that it would be interesting to you, and it would be helpful in clarifying a few of your doubts. This is all for today. This was the last lecture of this course. And I hope that you must have enjoyed this course. Thank you for joining me.