Applied Optics
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Lecture 57
Basic Concepts and Ray Optics Consideration of Optical Fiber

Hello everyone. Welcome back to my class. In last two classes, we covered basic concepts of holography. Today, we are going to start a new topic.

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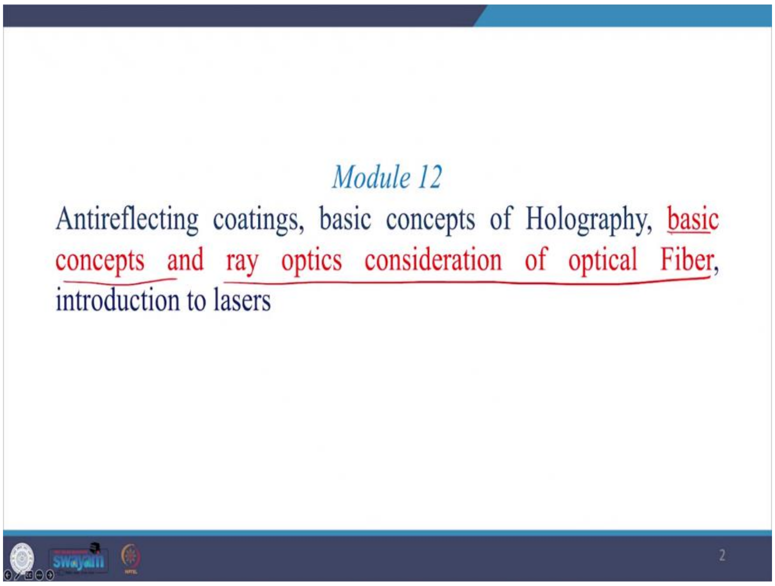

Which is basic concepts and ray optics consideration of optical fiber. Now, optical fiber is a waveguide. Now, the first question which comes to our mind is that, what is a waveguide? As the name suggests, the waveguide is a device which guides a wave. Suppose, you couple wave inside a fiber at one end, then the wave would be emitted from the other end of the fiber and this fiber maybe a few kilometer long or a few 1000s of kilometer long and then guide through very smoothly.

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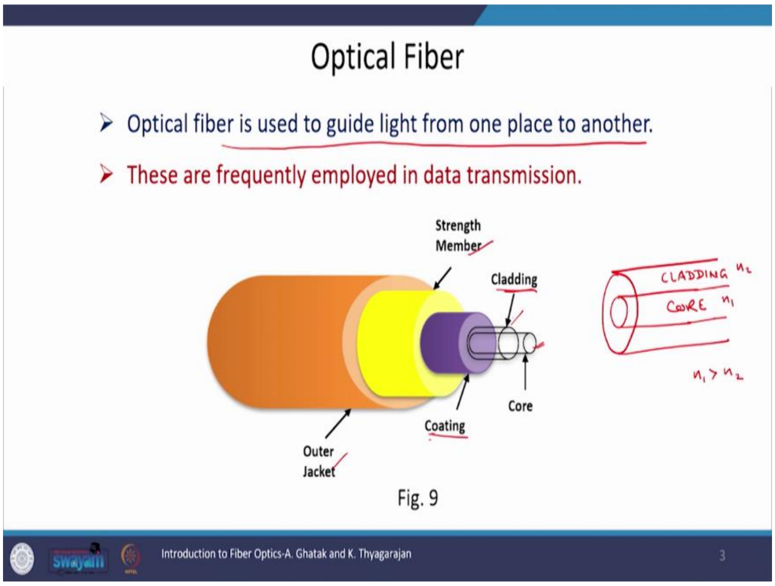

Therefore, the optical fiber is used to guide light from one place to another and these are frequently employed in data transmission. Whatever communication we do long distance of course, long distance communication like we use mobile phone and then the data from our mobile it goes to some antenna and from antenna it get coupled to a waveguide.

Like optical fiber and from this optical fiber it goes to very far away like USA and from the air again goes to some antenna and from that antenna it again goes to the mobile and then we can hear we can talk to the person who very far from us. And this is the most important use of a waveguide.

In particular, optical fiber is made up of core which is very thin and then covering core there is another layer which we call cladding and then on the cladding there are several other coating which we call the strength membrane outer jacket. But majorly it consists of a core and then covering the core there is a cladding, the center portion is called core and the upper portion is called cladding.

The refractive index of core is larger than that after cladding say n_1 is refractive index of core and n_2 is refractive index of cladding then $n_1 > n_2$. So, this is a basic introduction to an optical fiber.

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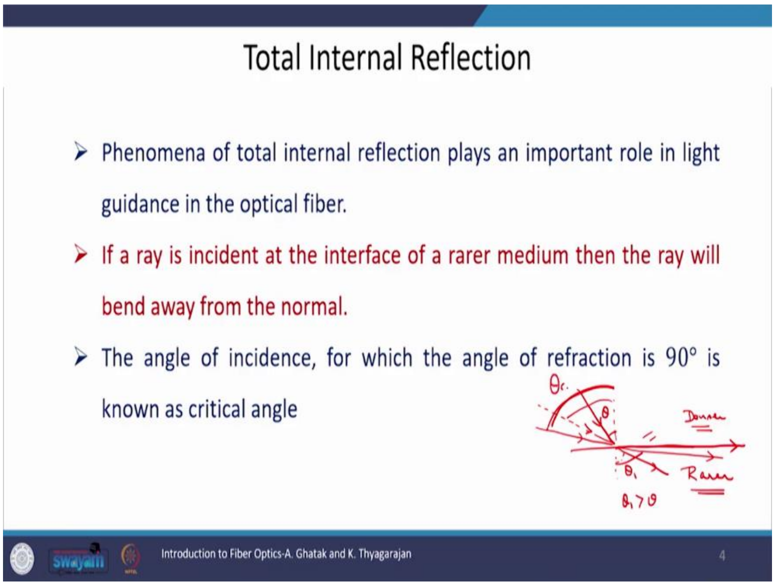

Now, the mechanism which enables an optical fiber to guide light is total internal reflection and the phenomena of total internal reflection therefore, plays an important role in light guidance. We know what a total internal reflection is, if a ray is incident at the interface of rarer medium then the ray will bend away from the normal. This is the usual refraction which we know.

I repeat suppose this is a denser medium and this lower one is a rarer medium if a ray falls on this interface say at angle θ, then it bends away from the normal, the θ_1 would be larger than θ if the lower medium is rarer as compared to the upper one, this is the usual law of reflection, on the Snell's law we know it from our junior classes.

Now, if we keep increasing the angle of incidence say we launch the light at this angle, then this would bend away, the refracted ray will bend away further. If we still increase the angle of incidence, then a situation may arise in which the refracted ray goes along the interface. Now, this is called total internal reflection, the angle of incidence for which the angle of refraction is 90 degree is known as critical angle and the phenomena is called total internal reflection. This angle for which the angle of refraction is 90 is called critical angle θ_c, and this phenomena is called total internal reflection.

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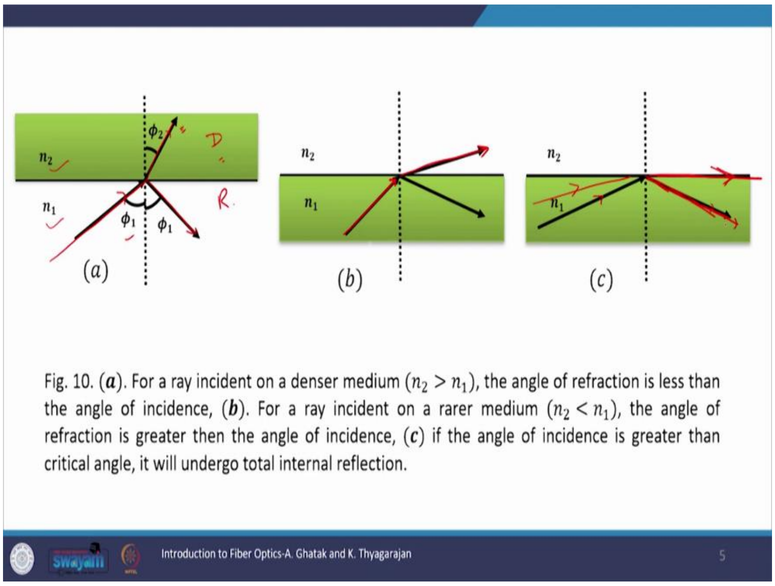

Now, this can be understood from this figure also, here, there are two media of the refractive index n_1, n_2 respectively, a ray of light is being launched and angle φ_1, then a part of the light gets reflected and a part of light get transmitted this is what we know. Now, this is a denser medium therefore, the ray bends towards the normal in the green shaded medium and this lower one is the rarer medium. But if light is launch from the denser medium to the interface of rarer medium, then it will bend away from the normal as you can see here.

Now, if you increase the angle of incidence then it may so happen that light comes in the first medium itself and at a particular angle the light will follow this path. It will be refracted along the interface which is called total internal reflection and the angle of incidence is called critical angle. Now, if you launch the light at an angle which is larger than critical angle, then the refraction would happen in this direction only, the light rays being lost and it is getting refracted in the first medium itself. The same phenomena is being exploited in optical fibers also.

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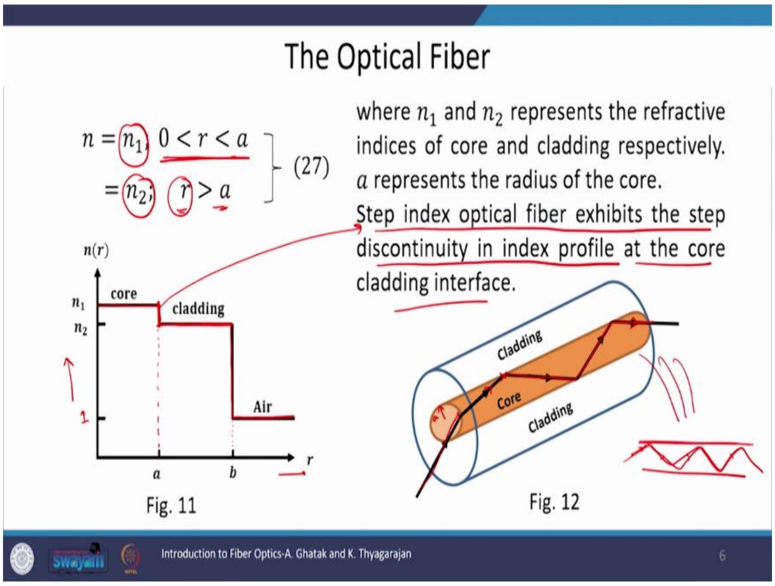

In optical fiber as I said before, the core refractive index is larger than that of the cladding. Therefore, if you launch a light which is larger than critical angle for refractive indices of core and cladding, then it will suffer total internal reflection at core cladding interface and this way it get guided along the length of the fiber.

Now, say the radius of the core is a, then as long as the radius is between these two limits between 0 and a, the refractive index is n_1, and when r is sorry when r is radius is larger than a, the refractive index is n_2; this how refractive index profile in step index fiber is defined. Now, if you plot r here in the horizontal axis and refractive index n in the vertical axis, then the core has largest refractive index, then there is a step at the core cladding interface here, this is core cladding interface and then cladding starts and cladding has a slightly lower refractive index as compared to the core.

And after cladding say there is air then there is a bigger step and then here comes the refractive index of the air which is equal to 1. And cladding ends at position where $r = b$ here. Usually, cladding is thicker than core. The step index optical fiber exhibits the step discontinuity in the index profile at the core cladding interface which is represented by this line here, there is a step discontinuity in the refractive index.

Now, this can also be understood by taking two glass plates, if you shine light then it may suffer multiple reflections. It will go through like this, the similar thing is happening here in cylindrical geometry. So, it is a planar geometry here. And while here in the optical fiber it is in cylindrical geometry.

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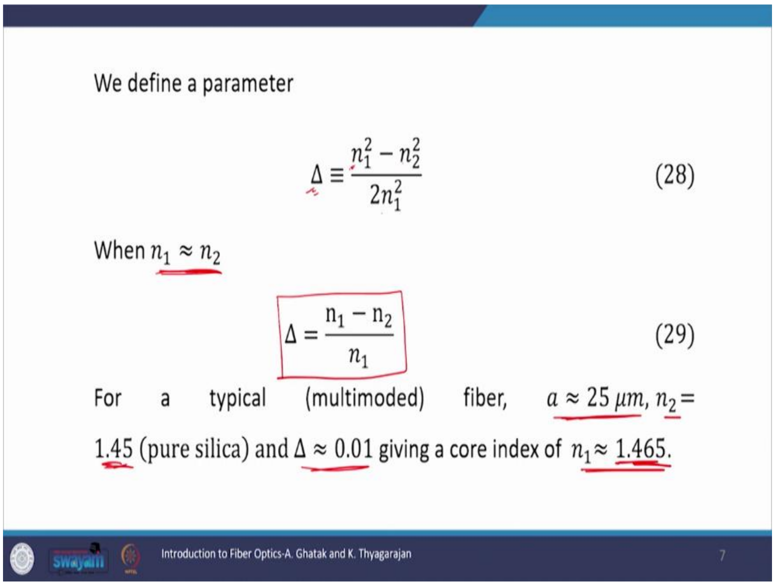

Now, moving ahead, while analyzing an optical fiber, we define a parameter which is called Δ which defines relative refractive index difference between core and cladding, which is defined as $(n_1^2 - n_2^2)/2n_1^2$. But usually in fiber, n_1 is very close to n_2, this is done to reduce dispersion which we talk about in coming slides.

And in this approximation, the new definition for $\Delta = (n_1 - n_2)/n_1$. And for a typical fiber, a is particularly multimoded is usually 25 micrometer, n_2 is around 1.45 and Δ is 0.01 which gives core refractive index as 1.465. Now, you see the difference between core refractive index and the cladding refractive index; core is 1.465 while cladding is 1.45, very little difference is there.

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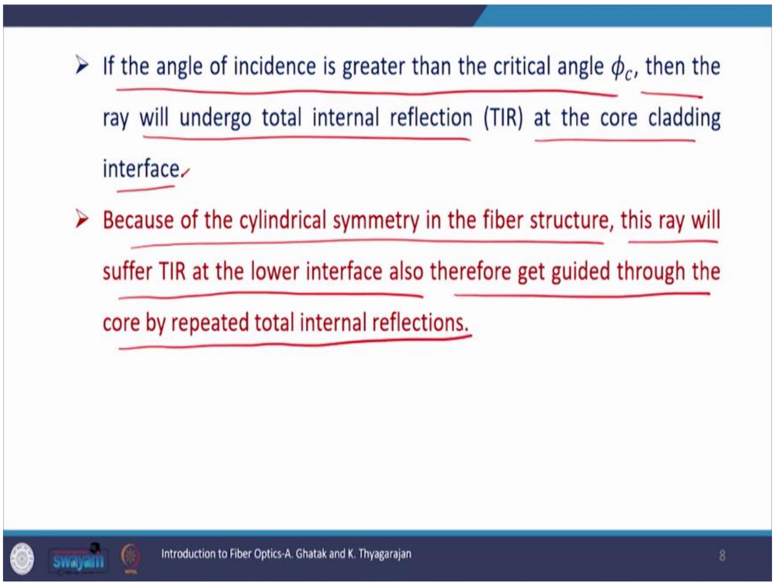

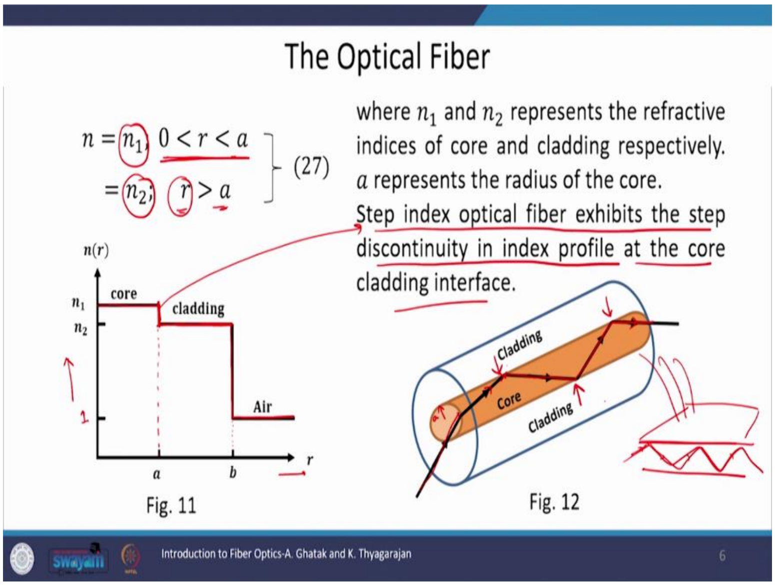

Now, if the angle of incidence is greater than the critical angle, then the ray will undergo total internal reflection at the core cladding interface and because of the cylindrical symmetry in the fiber structure, this ray will suffer TIR at lower interface also and therefore, get guided through the core by repeated a total internal reflection, which is what we saw in this figure. The total internal reflection is happening at this interface, then it here, then here, repeated total internal reflection is happening and the ray is getting guided. It is the rays propagating along the length of the fiber and this is how waveguide guides light.

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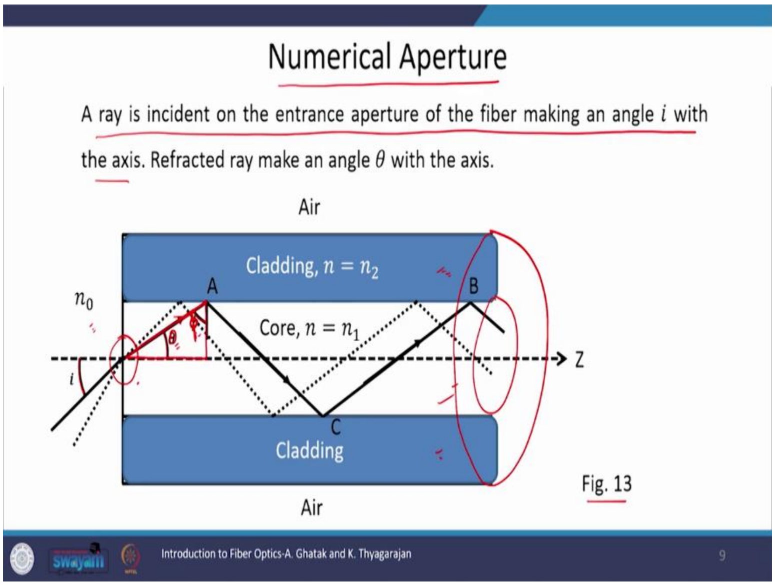

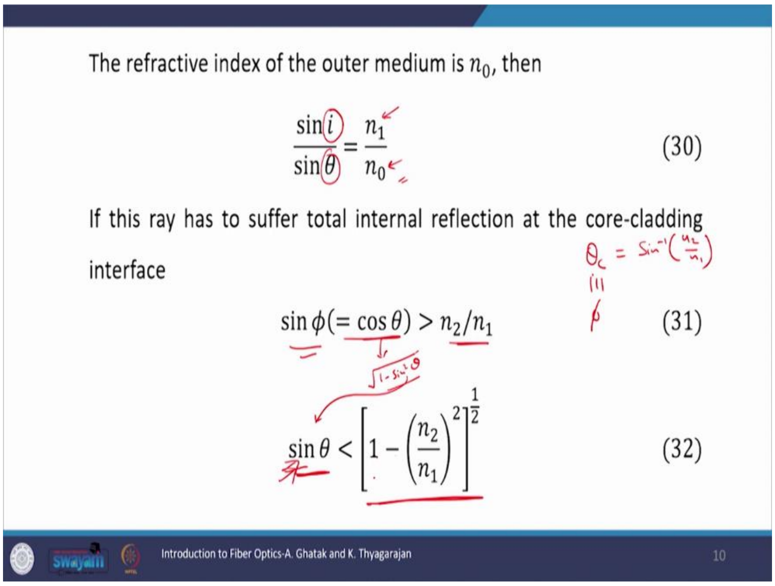

Now, we will define new terminology which is Numerical Aperture. Now, in this figure, figure 13, we present a cross section of an optical fiber where this white region represents core and this shaded region represents cladding. Basically, it is cylindrical, but I am showing you the cross section of this optical fiber.

Now, ray is incident on the entrance aperture of the optical fiber making an angle i, with the axis, the refracted ray says making an angle θ and therefore, this refracted ray within the core it incident at the core cladding interface at angle φ here. With this assumption, the Snell's law for the first refraction which is happening here at the interface of outer medium.

And the core, it can be written like this, where i is the angle of incidence at the core and outer medium interface and θ is the angle of refraction, the refractive index of the core is n_1 and that of the outer medium is n_0. Now, if we want this ray to suffer total internal reflection at the core cladding interface, then this angle sin of this angle this must be larger than n_2/n_1.

Because we know for critical angle $\theta_c = sin^{-1}(n_2/n_1)$, but here θ_c is nothing but φ. Therefore, $sin\varphi$ must be larger than n_2/n_1 for total internal reflection to happen. But we know that $sin\varphi$ as you can see in this triangle $sin\varphi = cos\theta$. Therefore, we can relate it, we can illustrate right $sin\theta < [1-(n_2/n_1)^2]^{1/2}$ because $cos\theta = \sqrt{1-sin^2\theta}$.

And from there we can calculate the expression of $sin\theta$, here what we did is that we write $1-sin^2\theta$ instead of $cos\theta$ we wrote $\sqrt{1-sin^2\theta}$ and from here we calculated the expression of $sin\theta$. Now, as long as $sin\theta$ is less than this right hand term, we will get total internal reflection.

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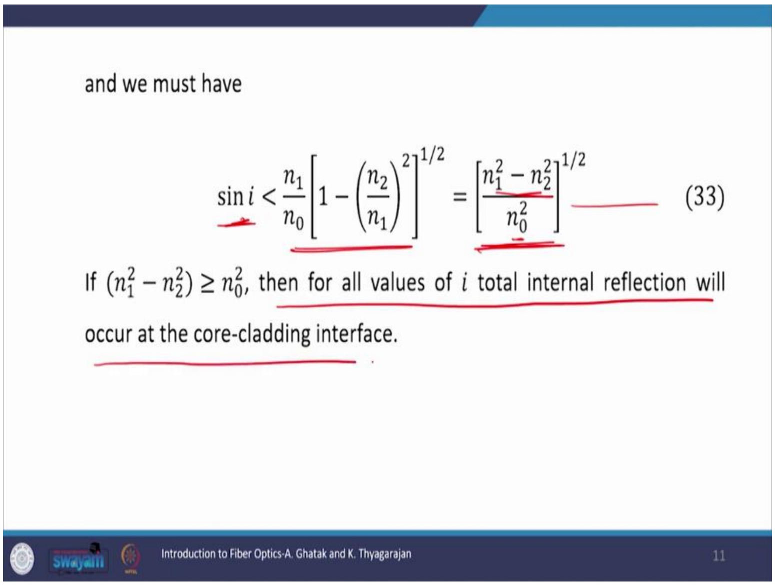

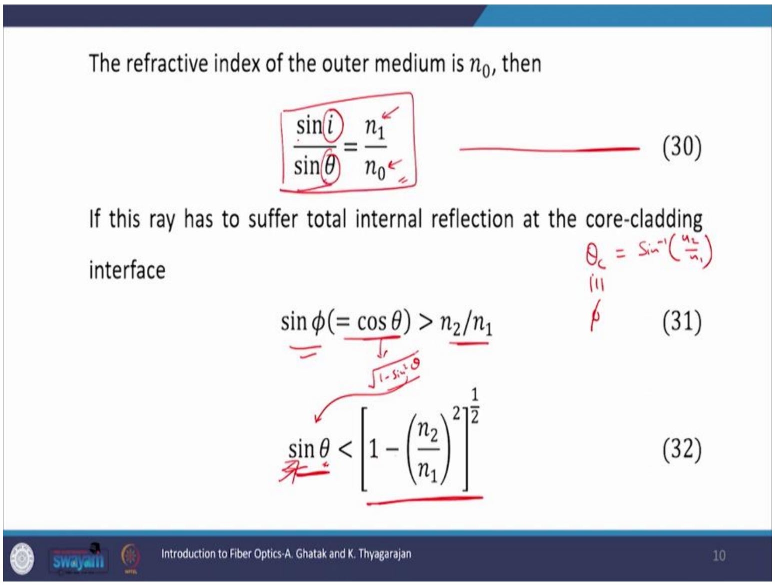

Now, if we have conditioned on θ, and we know how θ is related with i from equation number 30, this is the relation between θ and i, therefore, we can replace $sin\theta$ with $sini$ using equation 30 and therefore, we get $sini$ for TIR to happen $sini$ must be less than this term, bit simplification leads to equation number 33, we want $sini$ to be less than this term. Now, if the numerator is larger than the denominator, then for all values of i total internal reflection will occur at the core cladding interface.

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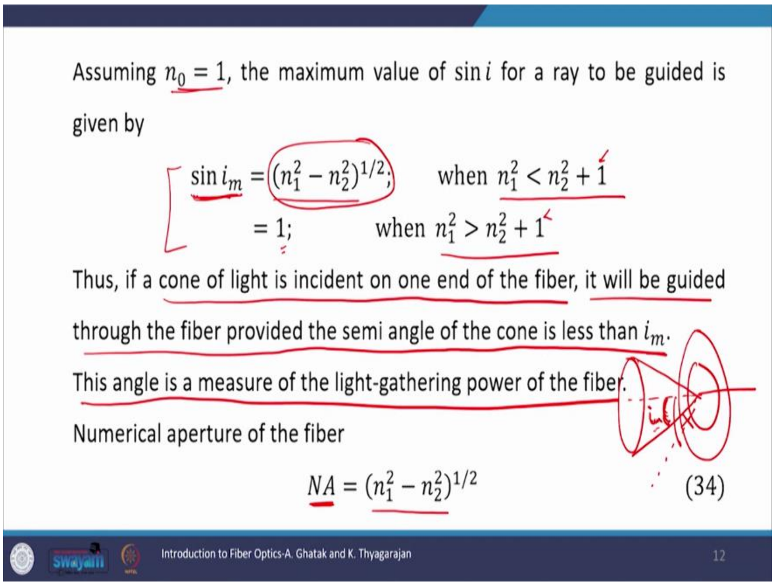

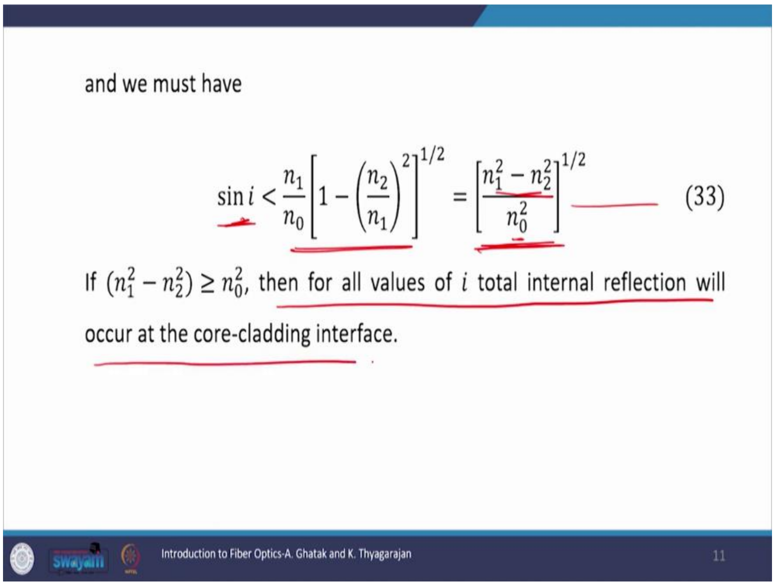

Now assume that n_0 the outer medium is air. There in this case the maximum value of $sini$ for a ray to be guided would be given by this expressions, $n_0 = 1$ we are only left with the numerator. Now, the maximum value of $sin\theta = 1$ therefore, in this condition you are seeing 1 here, then as long as $n_1^2 < n_2^2 + 1$, $sini_m$, where m stands for maximum value of i, then $sini_m = \sqrt{n_1^2 - n_2^2}$.

While where $n_1^2 > n_2^2 + 1$, we will get 1, the maximum value of $sini_m = 1$, which is very much obvious. Thus, if a cone of light it is incident on one end of the fiber, it will be guided through the fiber provided the semi angle of the cone is less than i_m. What does this sentence says is the following, this is the core this is the axis of the core and this is our cladding, and say a light is coming at this cone, and if this angle i_m if this is the maximum angle of incidence, m stands for the maximum.

Now, if the light would be guided if this half of the cone angle is less than i_m and if angle of incidence is larger than this value, then this would not get guided. This will not lead to total internal reflection within the fiber at core cladding interface. Therefore, this angle is a measure of the light gathering power of the fiber.

Now, we defined since we see that this term on the right hand side plays a decisive rule and therefore, we name it as numerical aperture of the fiber and it defines the our measures the light gathering power of the fiber. And NA which is acronyms for numerical aperture it is therefore, defined as the $\sqrt{n_1^2 - n_2^2}$ where n_1 and n_2 are refractive indices of core and cladding respectively.

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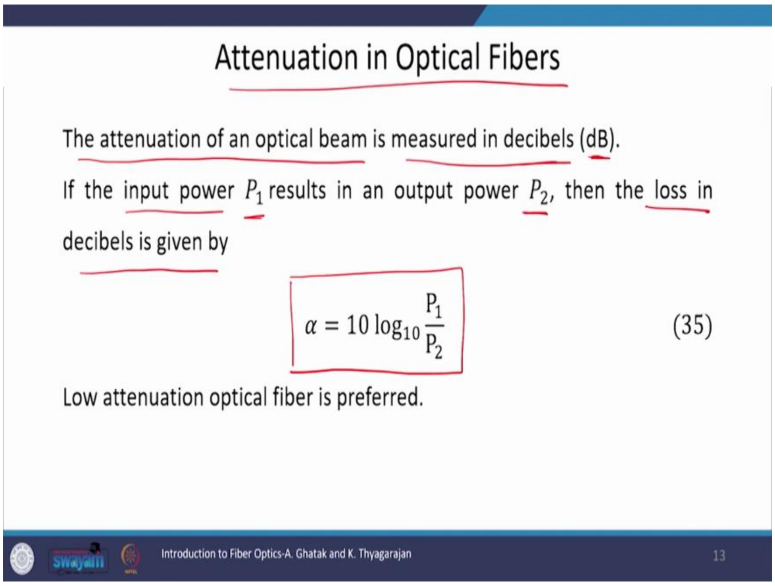

Now, let us talk about attenuation in the optical fiber. We know that optical fibers are made up of silica glass, because we want the light to propagate inside it and it must be transparent otherwise that light would be lost. But irrespective of the transparency of the glass, there are few defects, there are few impurities which are always there in the fiber. And due to these impurities, a part of the light get lost, it never reaches to the other end of the fiber. And therefore, we define attenuation or loss inside the fiber.

The attenuation of an optical fiber is usually measured in decibels. The acronym for decibel is dB. Now, if the input power which is getting coupled into the fiber is P_1 and if at the output we receive power P_2, then the loss in the fiber in decibel is given by this formula, here loss $\alpha = 10log_{10}(P_1/P_2)$, P_1 is in coupling power and P_2 is out coupling power. The fiber with a small attenuation or a small loss is preferred and therefore, people use different techniques to avoid these losses.

And with the advent of new techniques, the glass is cleaned in such a way that all the losses the defects are removed, but still the losses are unavoidable, they would be lying within the score of the optical fiber and lead to some losses although the fiber losses are negligible, but if we use 1000s of kilometer of the fiber, then they become appreciable and we cannot neglect.

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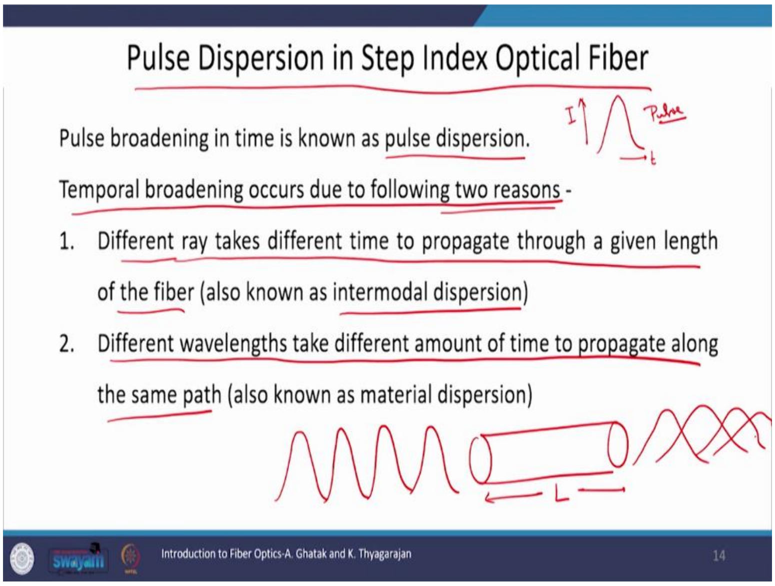

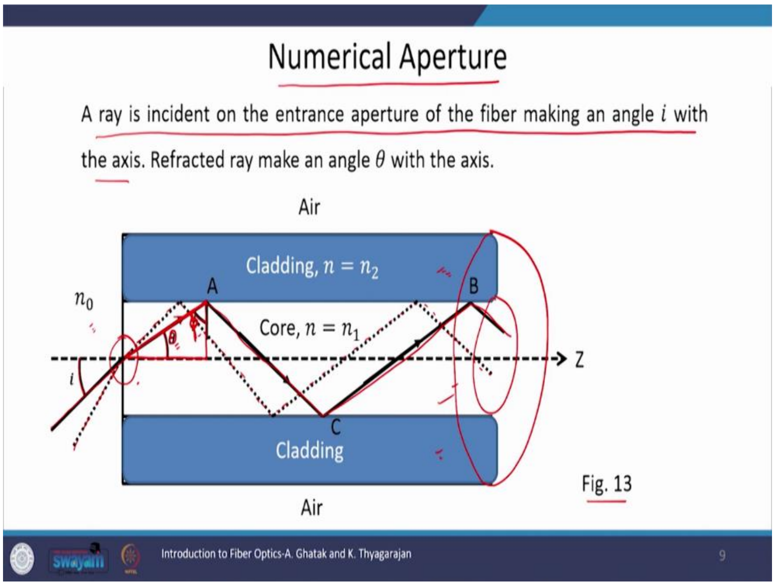

Now, let us move to the next sub topic which is pulse dispersion in step index optical fiber. As I told before step index means core has one constant value of refractive index and cladding has another constant value of refractive index which is a bit smaller than that of the core. Now, what is dispersion? The different colors of light take different amount of time in traversing the same distance, this is called dispersion.

Now, whenever we communicate then all the information's are sent in form of optical pulses within the fiber, and they are represented by this, this is the optical pulse here on the x axis we plot time while in the vertical axis we plot intensity. You must be knowing that all the information goes in the form of bits 01, 01.

And whenever there is a 1, we may assume that there is a pulse, when there is a 0, we may assume that there is no pulse, and this is called pulse. Before coupling the light into the fiber, it is encoded and then after receiving the light at the other end of the fiber, it is decoded. Now, this pulse is while traveling inside the fiber they get broadened, why? Because different colors travels with different velocity, the pulse broadens in time with propagation and therefore, we must talk about pulse dispersion.

Now, the temporal broadening occurs due to following two reasons. The first is that different rays takes different time to propagate through the given length of the fiber. Why is it so? It is so because if you see in this figure, this dark ray it is taking less time because the path length here is smaller while this dash line ray.

It will take longer time, because the path length for this dashed ray is longer. Therefore, different ray takes different amount of time in traversing the same length of fiber and why is it so, because different rays are inclined at different angle with respect to the fiber axis and this is why at the output we get broadening.

Now, this is called inter modal dispersion. Now, alongside different wavelengths also take different amount of time to propagate along the same path, this is called material dispersion and this also leads to pulse broadening, but why it is of our concern to take into account the pulse broadening. This is of our concern, because say, add the input field launch this train of pulse inside the fiber, this is our fiber and this pulse is launch here and at the output we are measuring it.

Now, due to the pulse dispersion the pulse may broadened and merged together at the output what we may see is this. Now, you see that all the pulses are merged together. And sometimes what happened is that they are merged so badly, then it is very difficult to distinguish them it is very difficult to separate them out and therefore, information is lost. Therefore, it is very important to know about pulse dispersion and also know about the ways to correct them.

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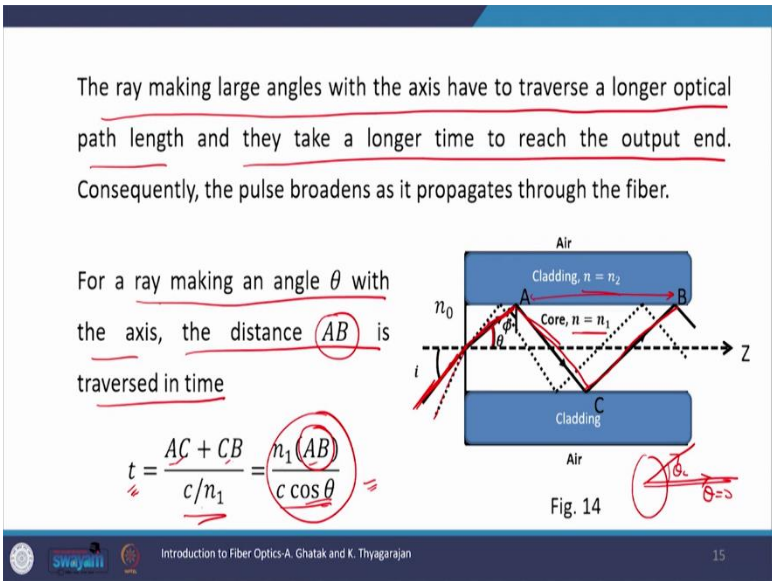

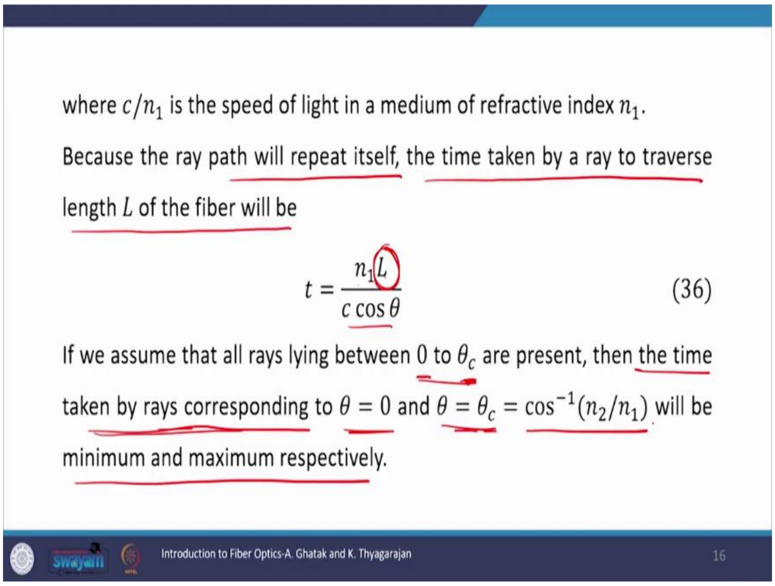

Now, the ray making larger angles with the axis have to travel longer optical path length and they take longer time to reach at the output end, this we already discussed the dash ray it is making larger angle with respect to the optical axis of the fiber while the continuous black line black ray, it is making smaller angle therefore, the dash ray will take longer time to reach the output. And consequently, the pulse broadens as it propagates through the fiber.

Now say for a ray that makes an angle θ this continuous dark line now, let us calculate the time which the ray take from traveling from point A to point B. Now for ray making an angle θ with the axis the distance A B is traversed in time which is given by this expression $(AC + CB)/c/n_1$, where c/n_1 is speed of light in core where n_1 is the refractive index of the core while n_2 is refractive index of the cladding.

Now, AC + BC is the total path it travels between point A and B but $AC + BC = AB/cos\theta$ from geometry we can see this. Therefore, total time the ray takes in covering distance AB would be $n_1 AB/ccos\theta$. Now, because the ray path will repeat itself, it will go through several such TIR. So, total internal reflection the time taken by a ray to traverse length L of the fiber would be this time multiplied by the length.

Now, again this t is the time which a ray takes in travelling distance AB therefore, the time the ray will take in traveling distance L would be $n_1 L/ccos\theta$. Now, if we assume all the rays lying between θ to θ_c here we only consider rays which are lying between θ to θ_c because only these rays will suffer TIR and will get guided. Now, if only these rays are present, then the time taken by the rays corresponding to $\theta = 0$, and $\theta = \theta_c$ will be minimum and maximum respectively see, the angle is θ is varying between θ to θ_c, this θ as you can see in the figure is measured with respect to the axis.

Now, we have one fiber for which the fastest ray would be in this direction for which $\theta = 0$, and the slowest ray would travel in this direction for which $\theta = \theta_c$, the ray which is going straight for this $\theta = 0$, it will take least time, therefore, the corresponding time would be minimum while for the other ray for which $\theta = \theta_c$ the corresponding time would be maximum and θ_c we know it is equal to $cos^{-1}(n_2/n_1)$.

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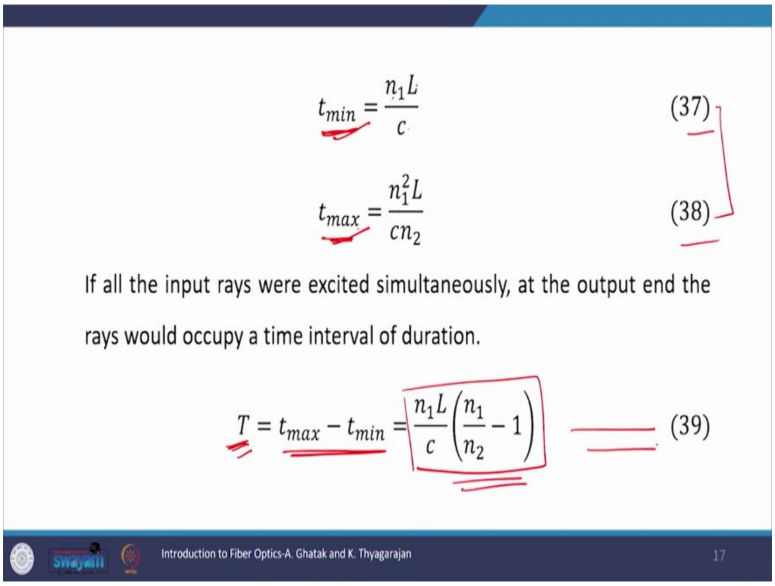

Now, we can quickly write the expression for time minimum and time maximum as $n_1 L/c$ and $n_1^2 L/cn_2$. Where we use this formula, equation number 36 here. In 36, in first case we replace θ by 0, and in the second case we replace θ by θ_c which is equal to $cos^{-1}(n_2/n_1)$. This gave us θ_{min} and θ_{max}.

Now, if all the input rays were excited simultaneously at the output end the rays would occupy a time interval of duration this, we launched cone of the ray and they coupled rays inside the fiber which is varying from $\theta = 0$ to $\theta = \theta_c$, and if all the possible paths are covered or filled with the ray, then at the output we will receive ray in varying time and they would be received between t_{max} and t_{min}.

The time interval between the earliest and the latest ray would be T which is given by $t_{max} - t_{min}$, and that is after substituting the values of t_{max} and t_{min} from these two equations to 37 and 38 we get equation number 39, and here thus we see is that if the rays are inclined a different angle they appear at different time and they lead to pulse broadening and broadening here is equal to given is equal to this value, this is given by equation number 39, this is how we calculate the pulse broadening.

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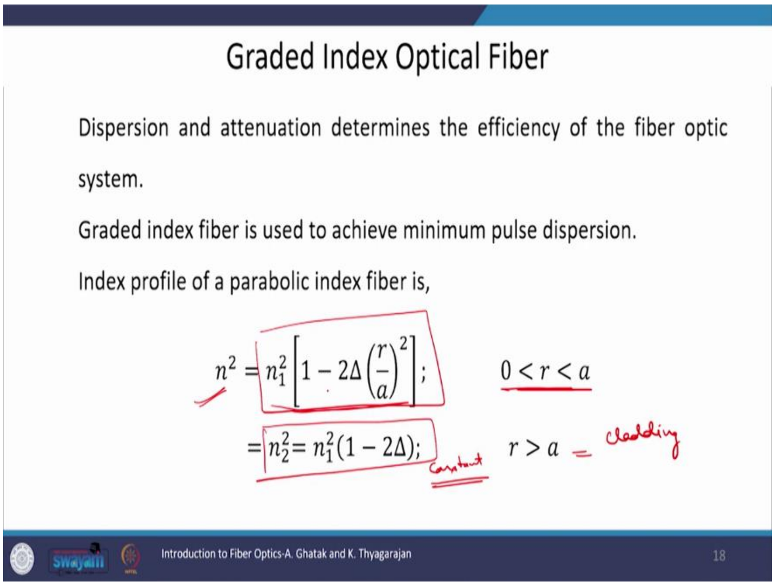

Now, the pulse broadening can be avoided if we use graded index fiber instead of step index fiber. Now, what is graded index fiber? In step index fiber what we saw is that the core has a constant refractive index and the cladding has another constant refractive index. While in graded index fiber, the refractive index of the core it varies gradually and this variation is given by this expression here inside the core, this is the variation of refractive index, while right outside the core in the cladding, this is the variation of refractive index.

Now, you see that outside the cladding, the refractive index is again constant, outside the core that is in cladding region, the refractive index is again constant even in graded index fiber, but in the core the refractive index is varying slowly as you move away from the axis of the fiber.

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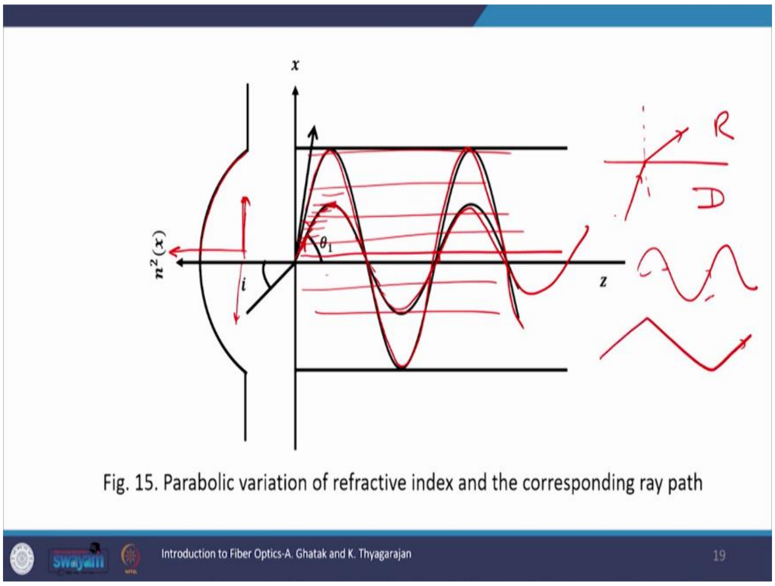

Now, the refractive index profile of graded index fiber is shown here. Now, this fiber is, this profile in particular is called parabolic index profile. Now, you see that as you move away from the center, the refractive index goes down it is decreasing, because here in this axis refractive index square is being plotted, while here in the vertical direction the radius of the fiber is plotted. As you move away from the center both, in upward direction and downward direction the refractive index goes down and at the center the refractive index is highest.

Now, if in this condition if you launch a light, then you will see here is that since the refractive index is larger here around the center and as you move away, the medium becomes rarer and rarer. This is the same situation which we see in deserts in summer season and there we see Mirage, why? because the light get reflected.

Due to this graded index, it suffers refraction in such a way that it feels that light is getting reflected and this is what exactly happens here to, say we launch a light at this angle then what will happen due to continuous variation of refractive index and as the light is moving up it is seeing rarer medium.

Therefore, every time it is moving away from the normal, because if the light moves from the denser medium to rarer medium, we know that it moves away from the normal. And therefore, slowly it will move away from the normal and then here it will turn back and comes down and same thing happens here to in the other side. And this is why it will propagate like this and the ray path depending upon its angle with the axis, it would be sinusoidal as shown here in this figure, the ray would move like this. Now, instead of having this type of ray path, we are having this continuous type variation.

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Ray Path in Parabolic Index Media

Ray equation

$$\frac{d^2x}{dz^2} = \frac{1}{2\tilde{\beta}^2}\frac{dn^2}{dx} \quad (40)$$

$$n^2(x) = n_1^2\left[1 - 2\Delta\left(\frac{x}{a}\right)^2\right] \quad (41)$$

$$\frac{d^2x}{dz^2} + \Gamma^2 x(z) = 0 \quad (42)$$

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where $\Gamma = \frac{n_1\sqrt{2\Delta}}{\tilde{\beta}a}$, and the general solution of eqn. (42) is given by

$$x(z) = A\sin\Gamma z + B\cos\Gamma z \quad (43)$$

In a parabolic index fiber the meridional ray paths are given by this eqn. (43), thus we may assume

$$x(0) = 0 \quad (44)$$

which implies $B = 0$. Thus

$$x(z) = A\sin\Gamma z \quad (45)$$

If the ray makes an angle θ_1 with the $z-axis$ at $z = 0$, then

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$$\tan\theta_1 = \left.\frac{dx}{dz}\right|_{z=0} = A\Gamma \quad (46)$$

$$A = \frac{\tilde{\beta}a\tan\theta_1}{n_1\sqrt{2\Delta}} = \frac{a\sin\theta_1}{\sqrt{2\Delta}} \quad (47)$$

$$A = \frac{a}{\sqrt{2\Delta}}\left[1 - \left(\frac{\tilde{\beta}}{n_1}\right)^2\right]^{1/2} \quad (48)$$

where, we have used the following

$$\tilde{\beta} = n_1\cos\theta_1 \quad (49)$$

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Now we can derive the equation for a path from the ray equation which we studied in geometrical optics section of the course. In initial few lectures, we derived the equation and we will now use that ray equation to derive the equation for ray path in this graded index optical fiber. Now, this is the ray equation which we studied in our initial classes.

Now here, on the right hand side, you see that n^2 is being differentiated with respect to x and what is n^2? It is the refractive index of the medium. And we know that its fiber core refractive index is given by this expression. Therefore, we will substitute here after a bit of mathematics we get equation number 42 which is familiar kind of equation, this equation appears in simple harmonic motion also and we know the solution.

Now here Γ which you see here in this expression it has this form which you can easily derived and the general solution of the previous equation, equation number 42 is given by $Asin\Gamma z + Bcos\Gamma z$. Now in parabolic index fiber or in graded index fiber, the meridional ray paths are given by this equation 43 the meridional rays means there rays which are crossing the axis of the fiber.

Now when they cross the axis of the fiber at that position, here let us go into the figure let us say the rays start from here, in this situation at x=0 at the meridional position x would be equal to 0 and of course, at input where z=0, x is 0 for meridional rays now in this situation when z=0 and x also leads to 0 if you substitute it in equation number 43, then from there you get that this equation number 44 gets satisfied if B= 0.

Substituting again back B=0 in equation number 43, we get $x(z) = Asin\Gamma z$. Now if the ray makes an angle θ_1 with the z axis at z=0, then $tan\theta_1 = \frac{dx}{dz}_{z=0}$. Because this is the ray which is going, this is our z direction, this is our x direction and if this angle is θ, then $tan\theta$ are if this is equal to θ_1 and $tan\theta_1 = \frac{dx}{dz}$, this is would be you dx this would be a dz and this is at z is equal to 0.

Now $\frac{dx}{dz}_{z=0}$ from equation 45 gives us. Because if you differentiate equation 45 to calculate $\frac{dx}{dz}$ let us differentiate it here. Now $\frac{dx}{dz} = A\Gamma cos\Gamma z$, now this at z=0 is equal to $A\Gamma$ because cos 0=1 therefore this value would be equal to $A\Gamma$.

And from here we can calculate the expression for unknown, the value of unknown A and $A = \tilde{\beta} tan\theta_1 / n_1\sqrt{2}\Delta$ where we just substituted the expression of Γ here, because we know Γ is given by this expression, this is Γ a bit of simplification lead to this expression.

And from here if you substitute also the value of $sin\theta_1$ then we get this expression for unknown A, once the unknown A is known. Then we know everything in equation number 45 and therefore the equation for a ray propagating in a gradient index fiber is known now here which is given by equation number 45 where A is given by equation number 48. While deriving equation 48, we have used this $\tilde{\beta} = n_1 cos\theta_1$ which is constant for ray this we have already discussed in geometrical optics.

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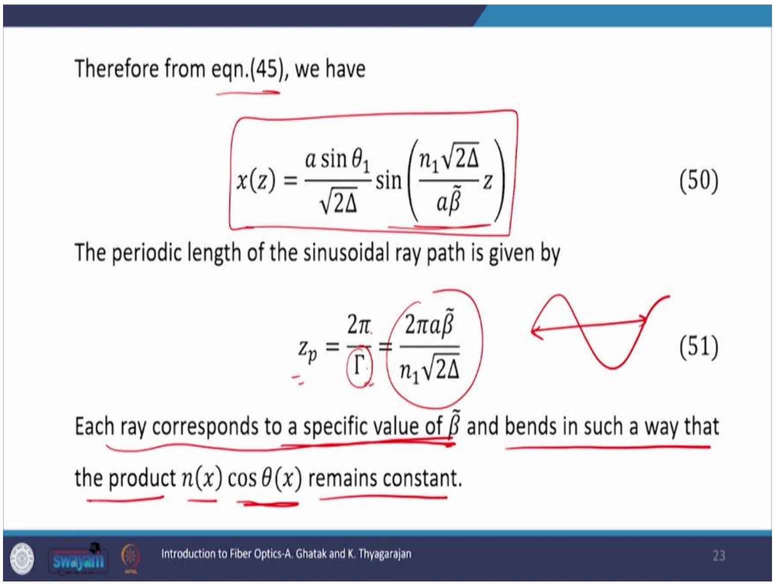

Now substituting the expression of Γ back into equation number 45 get this ray equation. Now you see that it is a sinusoidal function, then therefore we can easily calculate the periodicity which is given by $2\pi/\Gamma$ and from here, after substituting the value of Γ we get this value of z_p, the periodic length of the sinusoidal ray this made this means this length.

Now each ray correspond to specific value of $\tilde{\beta}$. As you see, $\tilde{\beta} = n_1 cos\theta_1$, θ_1 is the angle which the ray makes with the axis as you see here in this figure therefore for each ray θ_1 is different, but each ray corresponds to a specific value of $\tilde{\beta}$ and bends in such a way that the product $n(x)cos\theta(x)$ remains constant. If we pick a ray, then the $\tilde{\beta}$ value which is equal to $n(x)cos\theta(x)$ it would be constant for a given ray therefore it is called ray constant.

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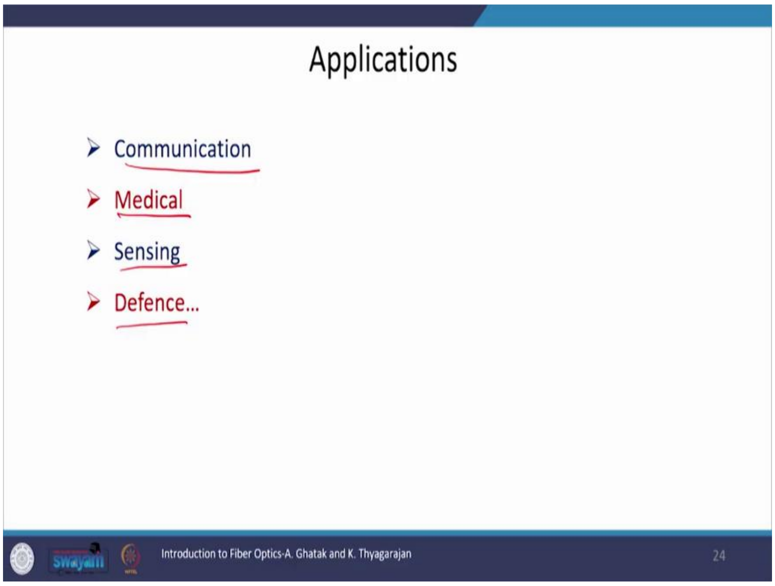

Now the fibers have very wide applications in almost all fields of science and engineering as I discussed before they are used in communications, they are used in imaging now fibers are also used in sensing, in monitoring the health of a building people use fiber brag grating or long period grating which is these gratings are inscribed in fibers, and therefore whenever there is a variation in the pressure.

Then the transmittance in the fiber changes and that can be detected at the output at the other end of the fiber and this all can be done by a person who is sitting very far from the building or very far from a bridge therefore, health monitoring or structural monitoring of a building can be done using this fiber.

Now of course they have a very important applications, they have very important application in defense. So, there are lasers which are made up of fibers, there are fiber based lasers and the beauty of this fiber based laser is that you can guide the power of the laser to a very long distance and these fiber lasers they are so strong that they can be used in cutting or drilling metals. And therefore, they have a wide a very prominent and very good application in defense.

And apart from this there are so many applications, they are used in secure communications they are used in making different type of sensors there are lot many applications. And with this I end my lecture, thank you for being with me.