Applied Optics
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Lecture 54
Antireflecting Coating

Hello everyone, welcome to my class, slowly we are approaching towards the end of this course. And today we are going to start the last module, module 12 of this course.

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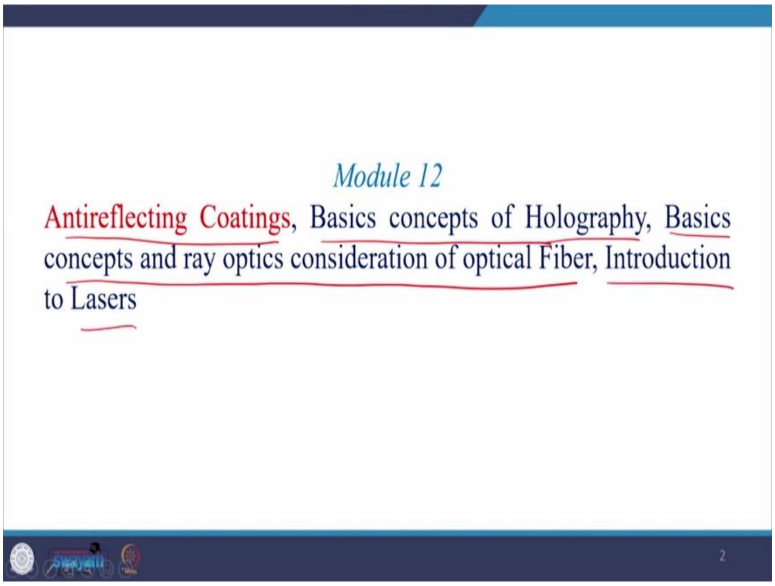

In this module, we will learn about Antireflecting coating, which would be followed by basic concepts of holography, then basic concepts and ray optics consideration of optical fiber and ultimately, we will be introduced to lasers. Today, we will start with the first topic which is Antireflecting coatings.

Now, in optical instruments we know that whenever light falls from air to a glass surface there is roughly around 4% of reflection and any optical instrument like microscope or telescope, it has a series of lenses sometimes 5, 6, 7. Now, if you consider 4% reflection from each surface of the lens, then the total light which is lost in reflection would be too much and therefore, we will have to design the lens surface in such a way this reflection loss reduces down to a minimum.

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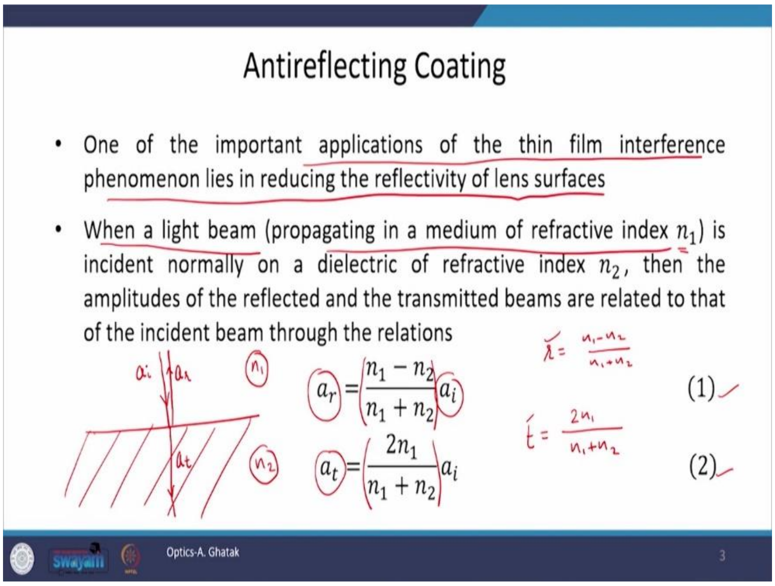

And one of the important applications of thin film interference phenomena lies in reducing the reflectivity of lens surface, we studied this thin film interference in great detail, while covering the interference part of this course. Now, using that knowledge, we will try to develop an Antireflecting coating which reduces the reflectivity of lens surfaces. Now, when a light beam which is propagating in a medium of refractive index n_1 is incident normally on a dielectric material of refractive index n_2, the amplitudes of reflected and transmitted beams are related to that of the incident beam through certain relations.

And these relations are given here by equation number 1 and 2 where a_r is the amplitude of the reflected light a_i is the amplitude of the incident light and a_t is the amplitude of the transmitted light, schematically, we can write like this, this is a refractive index of medium n_1, this is refractive index of medium n_2 and light is incident normally and amplitude of the incident light is a_i, a part of the light get reflected and its amplitude is represented by a_r while apart get transmitted and the amplitude of the transmitted light is represented by a_t.

Now, you see that here this coefficient appears, this is called coefficient of reflection and it is represented by $r, r = (n_1 - n_2)/(n_1 + n_2)$, where n_1 and n_2 are refractive indices of first and second media respectively while this term in this in the bracket is called transmission coefficient or amplitude transmission coefficient and we know we have already studied about this t and r, they are fresnal relations.

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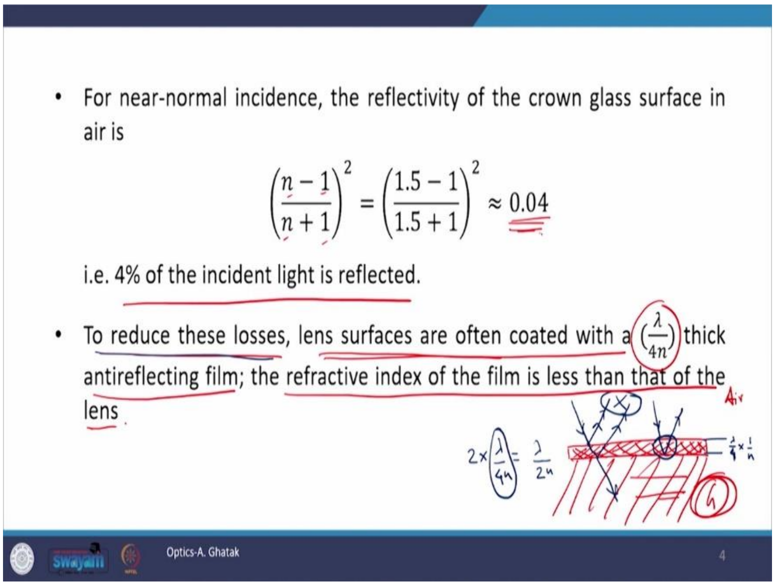

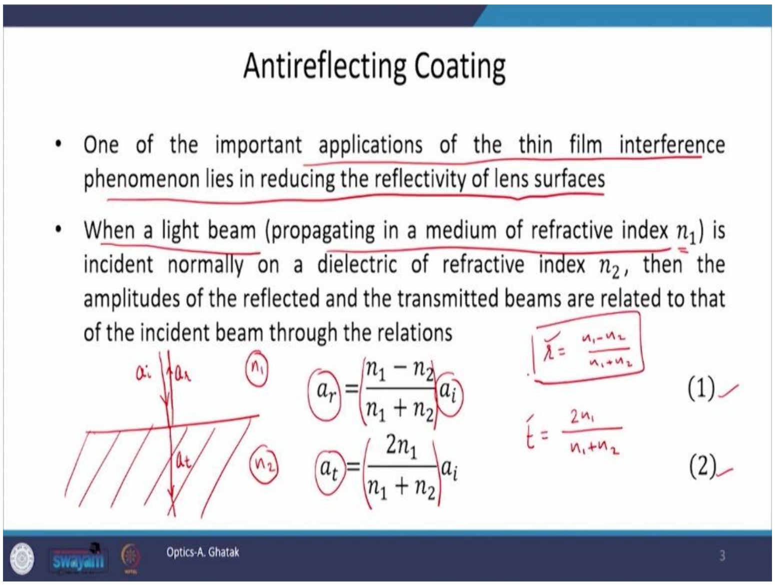

Now, for near normal incidence, the reflectivity of the crown glass surface in air is around 0.04 which can be calculated from the same relation $(n_1 - n_2)/(n_1 + n_2)$, this relation and this therefore, the reflectivity would be 0.04. This means that 4% of the incident light is reflected. Similarly, now, to reduce the losses, the lens surfaces are often coated with some thin film and this thin film is choosen in such a way that the reflection from the upper surface and reflection from the lower surface of the thin film, it destructively interfere.

What I mean by saying this is that, suppose this is the glass material, glass and this is air. Now on top of the glass we coat thin film on the top of the glass and the thickness of this thin film is chosen in such that let us say this is the incident light, then there would be reflection from the upper interface and this part of the incident light will go within this thin film and then there would be a partial reflection and partial transmission.

Now, if these two lights interfere destructively or if these 2 light beams, they are out of phase by 180 degrees, then there would not be any reflected light. There would be complete destructive interference and there would not be any light, all the light will go in the transmitted direction. Now, to have destructive interference, we require path difference of $\lambda/2$.

Now, if the thickness of the thin film is $\lambda/4$ then from this figure we are seeing that the incident light is traversing through the thickness of the thin film twice. You see that this light is falling here and then it is going inside, then getting reflected and then coming out, it means it has traversed the film thickness twice and therefore, total extra distance travel would be 2 times $\lambda/4$ which is equal to $\lambda/2$.

Now, if you take refractive index of the thin film also into the account, then the thickness of the film it would be $(\lambda/4)(1/n)$ and here too n will appear in the denominator in the denominator therefore, film thickness to have destructive interference film thickness will be equal to $\lambda/4n$. This is what is written here.

To reduce these losses, lens surfaces are often coated with a $\lambda/4n$ and thick anti-reflecting film, the refractive index of the film is less than that of the lens because here in this calculation, we did not take into account the extra phase difference due to reflection and this only happens when the refractive index of this glass are the lens which is lying below it is larger than the refractive index of the thin film. And the refractive index of the thin film is of course larger than that of the ambient material which is air.

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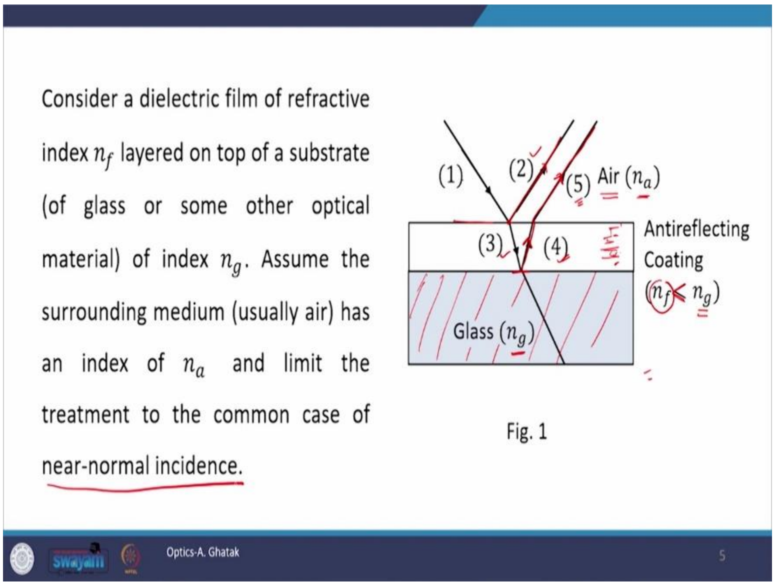

Now, the same figure is drawn here in a more clear way, you see that ray number 1 is falling under interface and then the reflected rays named as ray number 2, the transmitted rays named as ray number 3, the reflection from the lower interface generates a ray which is named as ray number 4 and then this ray number 4 transmit out into the air and this transmitted rays named as a number 5.

The material below is our lens and it is made up of glass and its refractive index is taken as n_g, the anti-reflective coating which is given by this thin film, this thin strip represents the antireflecting coating and its refractive index is considered to be n_f, f stands for film. The refractive index of film is n_f which is considered to be smaller than that of the glass. And n_f is less than n_g and the refractive index of air is taken to be n_a, a stand for air. Now, here we assume the incident beam is normal to the interface of the medium. We always consider near normal incidence.

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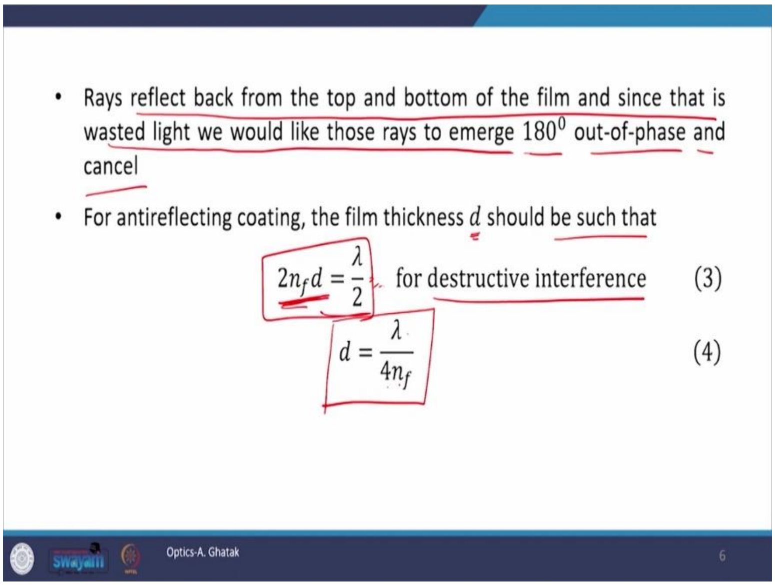

Under this condition, the rays reflect back from the top and bottom of the film and since that is wasted light, we would like those rays to emerge 180 degrees out of phase and cancel each other. And if these two reflected rays are out of phase by 180 degree they will destructively interfere and then there would be no reflected light in the air, and this is how we can cancel the reflectance, we can make reflectance 0 and save the light and then everything would be utilized better in the optical instrument.

Now for Antireflecting coating the film thickness is of prime importance and this film thickness d it should be such that the path length difference that is $2n_f d = \lambda/2$ and whenever this condition is satisfies, we get destructive interference in reflected arm. And from this equation we can easily derive the expression for d which gives destructive interference and this $d = \lambda/4n_f$.

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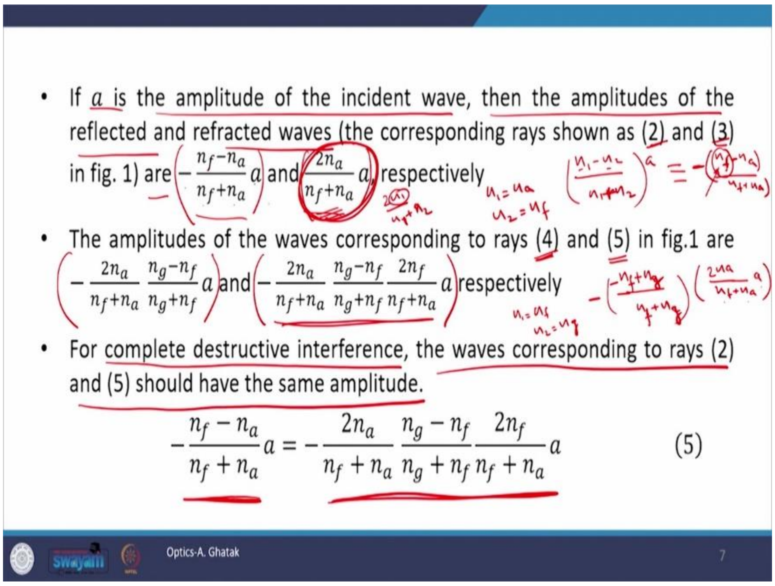

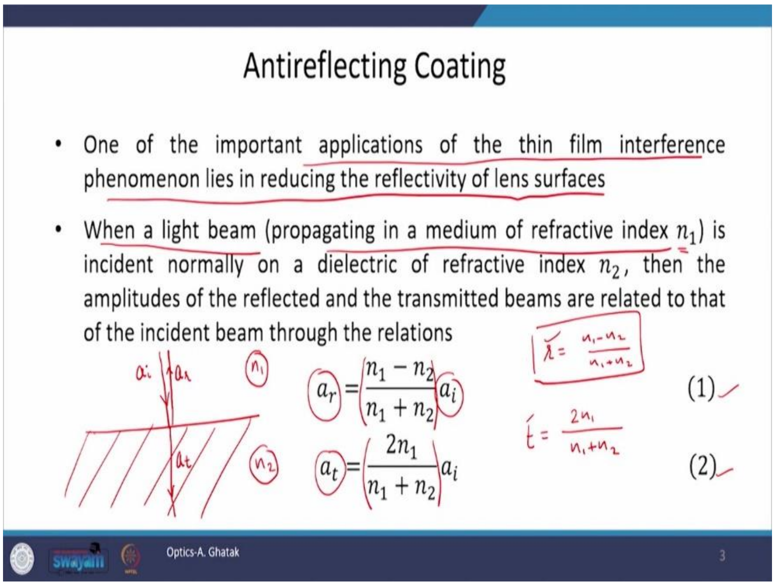

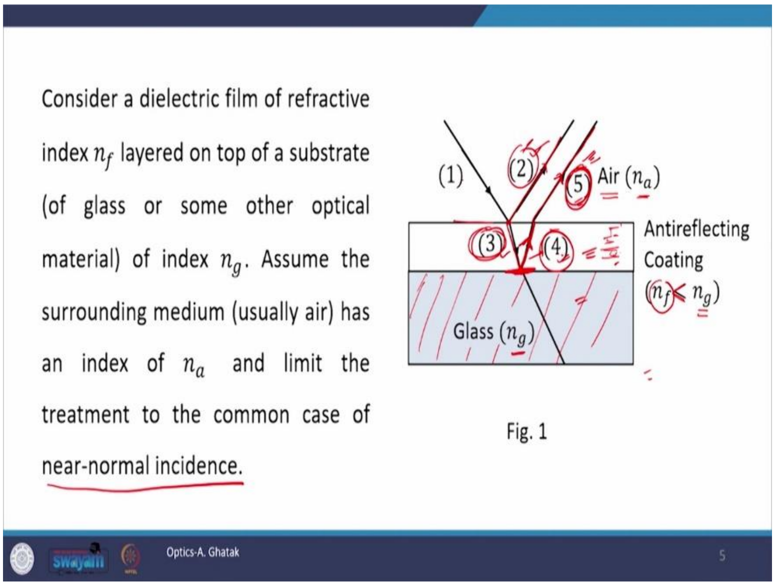

Now consider the 3-layer medium, first layer is air, second layer is thin film which is of refractive index n_f and the third medium is glass which is of refractive index n_g. Now, if a is the amplitude of the incident wave, then the amplitudes after reflected and refracted waves are $-(n_f - n_a)/(n_f + n_a) \times a$ this would be the amplitude of the reflected wave and this would be the amplitude of the transmitted wave. And this relation we got from these equations, equation number 1 and 2 respectively.

Because here while calculating the reflected amplitude we used $((n_1 - n_2)/(n_1 + n_2))a$ the incident 1, but here n_1 is air and n_2 is film and the refractive index of air is smaller than that of the film therefore air we take minus sign out and then write it as $(n_f - n_a)/(n_f + n_a)$ if we bring the minus inside the bracket then it would be $n_a - n_f$ which is $n_1 - n_2$. But since $n_f > n_a$, we want n_f to be before we want n_a to be subtracted from n_f and therefore, we are writing n_f first and then subtracting n from n_f.

And this is why n_2 is coming earlier here in this formula because minus sign is taking care of this change. Similarly, for transmitted light it is $2n_1/(n_1 + n_2)$, n_1 is n_a, n_2 is n_f here it is clear n_1 is n_a, n_2 is n_f. And this is how we can derive the reflected and transmitted amplitudes and this is far ray number 2 and ray number 3. Reflected amplitude means the amplitude of ray number 2 and transmitted amplitude means the amplitude far ray number 3.

Now, once it is done for ray 2 and ray 3, we calculated the amplitude. Now, let us calculate the amplitude for ray number 4 and ray number 5 as shown in this figure 1. Now you see that ray 3 is falling on this interface and then it is getting reflected and this reflected rays named as ray number 4. And what amplitude falls on this interface the amplitude of ray 3 and what is the amplitude of ray 3, it is given in this point, this is the amplitude ray 3. Therefore, for ray 4 we will again write this formula $(n_1 - n_2)/(n_1 + n_2)$ into amplitude and amplitude is $2n_a/(n_f + n_a)a$.

Now, here n_1 is refractive index of antireflecting coating layer, n_2 is that of glass, therefore, n_1 is n_f and $n_2 = n_g$, we will substitute this here now, you will see that in place of n_1 we will write n_f in place n_2 we will write n_g. n_f n_g but since n_g is larger than n_f we will take minus sign outside the bracket this term would be plus this term would be minus effectively we will have $(n_g - n_f)/(n_g + n_f) \times 2n_a/(n_f + n_a) \times a$ and which is what is written here.

Similarly, we calculate the amplitude for a number 5 which is the transmitted part of ray number 4, here we apply to $2n_a/(n_1+n_2)$ and then the amplitude of ray number 4 and this will give us this expression. Now, for complete destructive interference the ray number 2 and 5, they should be equal in amplitude but they should be out of phase by 180 degree therefore, the wave corresponding to ray numbers 2 and 5 should have the same amplitude. Let us compare their amplitude here on the left hand side we just put the amplitude of ray number 2 while on the right hand side the amplitude of a number 5 is written.

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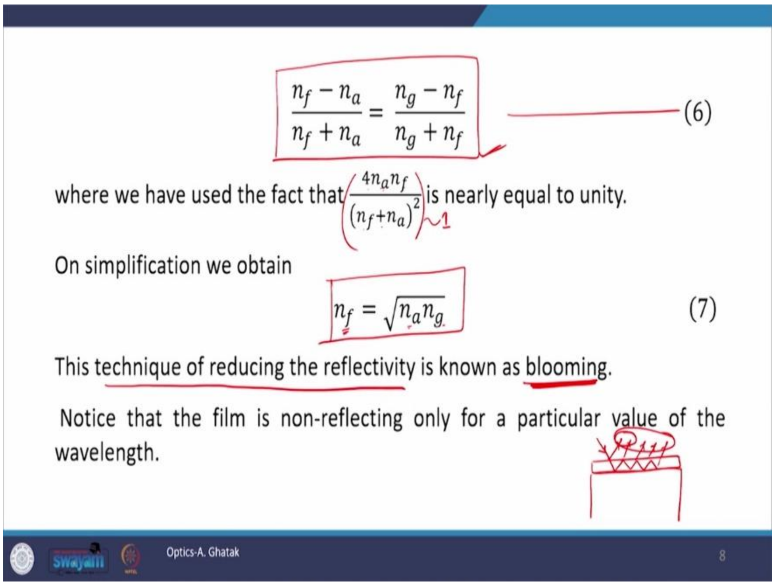

Now, we will simplify this, after simplification we get this expression and while to reach at this expression we have used one approximation and we say that $4n_f\ n_a(n_f+n_a)^2 \approx 1$, with this we equated to 1 and after this we reached equation number 6. Now, from equation 6, if you further simplify it, then you can clearly see that n_f if you calculate the expression for n_f, then $n_f = \sqrt{n_a \times n_g}$ it means, if refractive index of film is a square root of refractive index of air into refractive index of glass, then this film will work as a anti-reflecting coating.

This will work as an anti-reflecting coating and this technique of reducing the reflectivity is known as blooming. Now, notice that here we have done everything for a given refractive index and we can fix we can treat refractive index as a constant as long as the wavelength of the light is fixed, the interrogating wavelength is treated fixed here.

Now, if we change the wavelength, then this condition will not be valid anymore, this condition would be valid, but we will have to change the refractive index accordingly because refractive index is wavelength dependent quantity, but d the thickness of the film that would be fixed once designed some antireflecting coating on a lens surface. So, once the design is fixed, then the perfect condition of 0 reflectivity would be valid for a single wavelength, but if we deviate from the wavelength, there would be some non-zero reflectivity.

Now, till here what we considered is that there is air and then there is a thin film on top of a glass and this thin film is working as anti-reflective coating, but here we just considered to reflected light, but in interference part, we know that the ray suffers multiple reflection and refraction. And therefore, here there will be lot of reflected light and to have a better parameter for anti-reflective coating, we will have to treat all these reflecting rays, we will have to take into account all these reflected rays.

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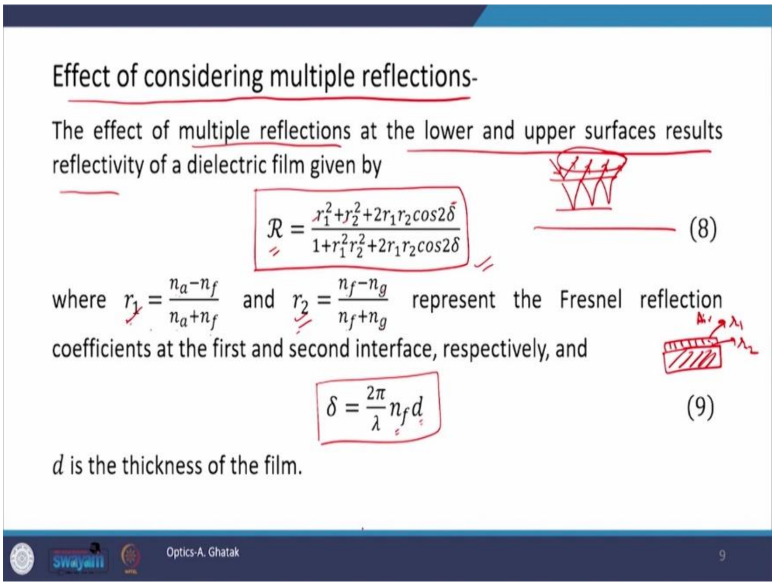

Now**,** in this part we will take into account all the reflections and the effect of multiple reflection at the lower and upper surfaces results reflectivity of a dielectric film that is given by equation number 8. If we take into account all the reflected rays, then the expression for reflectivity modifies and the ultimate reflectivity turned out to be this. Here, as I said before rays coming, it is getting reflected from here and then from here again then from here again if we take all these reflections into account, then the reflectivity of the dielectric film would be given by this expression.

Now here, we see that we have r_1 term r_2 term and there is a phase part δ the $r_1 = (n_a - n_f)/(n_a + n_f)$ while $r_2 = (n_f - n_g)/(n_f + n_g)$. From this expression, you can see that r_1 is reflection coefficients of the first interface of the thin film r_2 is the reflection coefficient of the second interface of the thin film, because this is our glass and top of this class, this is anti-reflecting coating.

And top of this anti-reflecting coating we have air, this surface reflection coefficient is denoted by r_1, while this surface reflection coefficient is given by r_2 and the expressions for r_2 and r_1 are given here, r_1 and r_2 they represent the Fresnel reflection coefficients and as I said before and these coefficients are at first and second interfaces and the phase δ is nothing but this is our conventional phase $(2\pi/\lambda)n_f d$ where $2\pi/\lambda$ is nothing but $k, kn_f d$ is the phase and if it is the refractive index of the film, d is the film thickness.

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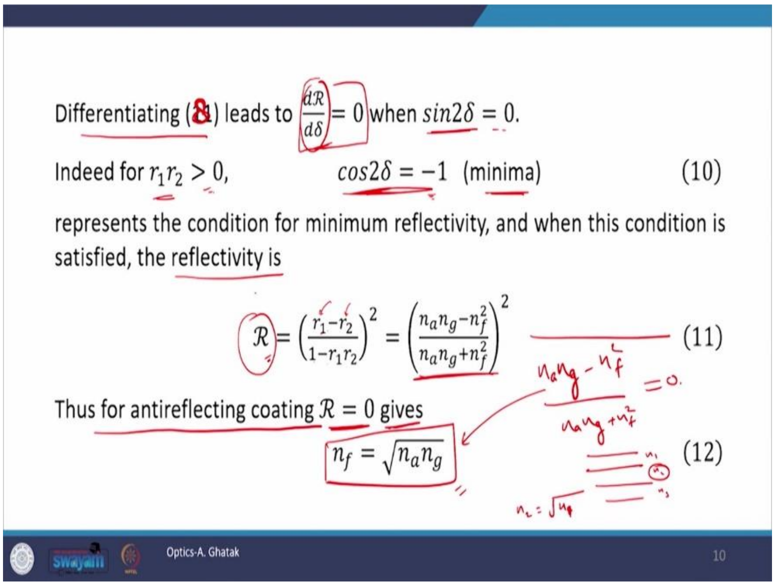

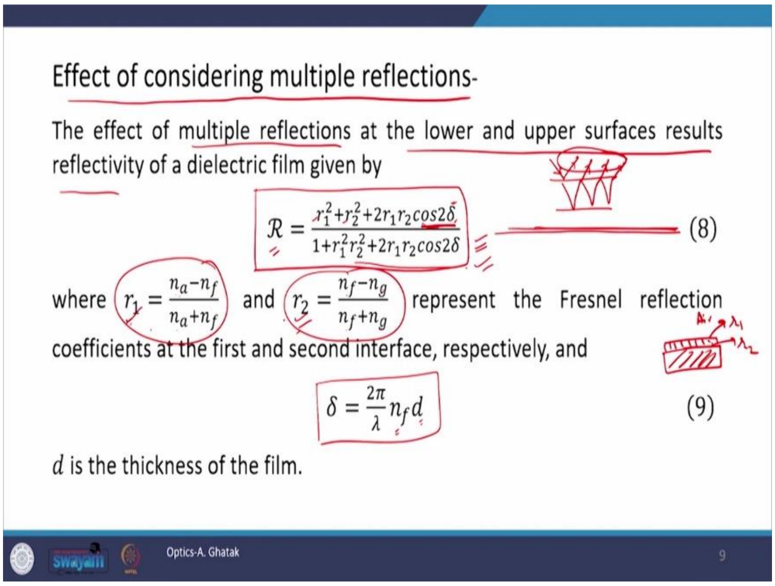

Now, if you differentiate question number 8 which is this equation, this is equation number 8 with respect to the phase δ, we want R to be 0, R to be minimum. And mathematically, if we apply the condition of maximum minima here then we can get the conditions for which R would be minimum and for that we take first derivative of R with respect to δ and equated with 0 and upon equating we will get a condition which will lead to either maxima or minima.

Now, if we equate the first derivative of R with 0, this gives $sin2\delta = 0$, the first derivative of R with respect $to\ \delta$ would be 0 only if $sin2\delta = 0$ and indeed for r_1, r_2 larger than 0, $cos2\delta = -1$ gives a minima. Now, this condition $cos2\delta = -1$, it represents the condition of minimum reflectivity. And when this condition is satisfied, the expression for reflectivity modifies and the modified expression is given by equation number 11.

The earlier expression was a bit cumbersome which is given by equation number 8. And when you modified, when you put $cos2\delta = -1$ which is the condition for minima, the new expression for reflectivity R is reduces to $((r_1 - r_2)/(1 - r_1 r_2))^2$, if you substitute the expressions of r_1 and r_2 from these expressions, these are the expressions for r_1 and r_2 if you substitute them back into equation number 11, then we get this expression for R.

Thus, for antireflecting coating, r is equal to 0 gives $n_f = \sqrt{n_a n_g}$ because, if you equate this to be 0 with 0, this is the $(n_a n_g - n_f^2)/(n_a n_g + n_f^2) = 0$, this if you equate to 0, then you get this condition directly $n_f = \sqrt{n_a n_g}$. It means even if you consider multiple reflection still we will end up on the same value of $n_f = \sqrt{n_a n_g}$ or more generally, suppose we have 3 films of refractive index n_1, n_2 and n_3 and if we want n_2 to behave as anti-reflective coating there $n_2 = \sqrt{n_1 n_3}$.

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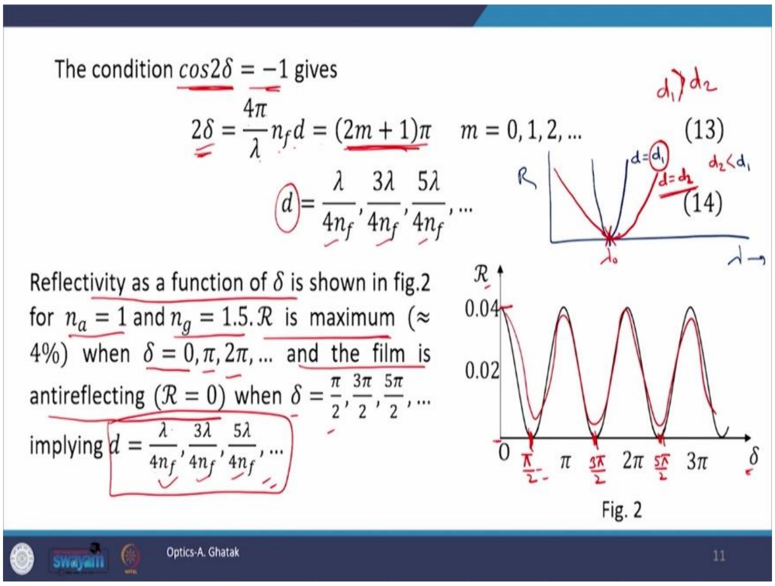

Now, the condition cos $2\delta = -1$ gives that $2\delta = (4\pi/\lambda)n_f d = (2m+1)\pi$ which is from here we can see the expression of d which is the possible values of the thickness of the thin film which will give 0 reflectivity. Now, reflectivity as a function of δ is shown here, you see that the if you vary δ then reflectivity vary periodically etc. So, periodic variation is here. The maximum value of reflectivity is 0.04 and the minimum is 0. Now, whenever they will satisfy this condition, we get a minima here.

Now, this figure is drawn for $n_a = 1$ and $n_g = 1.5$, R is maximum that is 4% when $\delta = 0, \pi, 2\pi \ldots$ and the film is antireflecting when $\delta = \pi/2$, here the value of $\delta = \pi/2$, here the value of $\delta = 3\pi/2$, here the value of $\delta = 5\pi/2$ and so on and the corresponding film thickness is given by this expression $d = \lambda/4n_f, 3\lambda/4n_f$ or $5\lambda/4n_f$ and so on.

Then can the question now, can we use any of these thickness in making the anti-reflecting coating? Now, the answer is no because, if we pick the larger values of thickness, then what will happen is that, let us plot here the wavelength and here the reflectivity. Now, for larger value of d say the minima is appearing at this point at some wavelength and this is for some d which is equal to d_1 and let us pick another color and this for this color, let us draw again minima this is for d is equal to d_2 and d_2 is smaller than d_1.

Now, in this figure what you see is that for ticker thin film the minima is very sharp, the d_1 is larger than the d_2 and therefore for thicker, thin film and thicker antireflecting coating the sharpness of the minimize very large, minimize very sharp and as soon as you deviate from this value of λ say it is λ_0 and if you deviate from λ_0, the reflectivity rises very quickly while for a smaller value of antireflecting coating, smaller thickness of antireflecting coating, a smaller value of d, you see that if you deviate from λ_0 then the reflectivity is not rising very rapidly.

The bandwidth over which reflectivity is minimum is large for smaller thickness of anti-reflecting coating and therefore, people prefer the smallest possible thickness of anti-reflecting coating while designing the optical instrument, hope things are understood. This is all for today I in my lecture here, see you all in the next class. Thank you for joining me.