# Applied Optics

**Professor Akhilesh Kumar Mishra**
**Department of Physics**
**Indian Institute of Technology, Roorkee**

**Lecture 47**

## Phenomenon of Double Refraction

Hello everyone, welcome back to my class. Today, we are in lecture 2 in module 10. We started module 10 in last class wherein we talked about Brewster's law and Malus's law.

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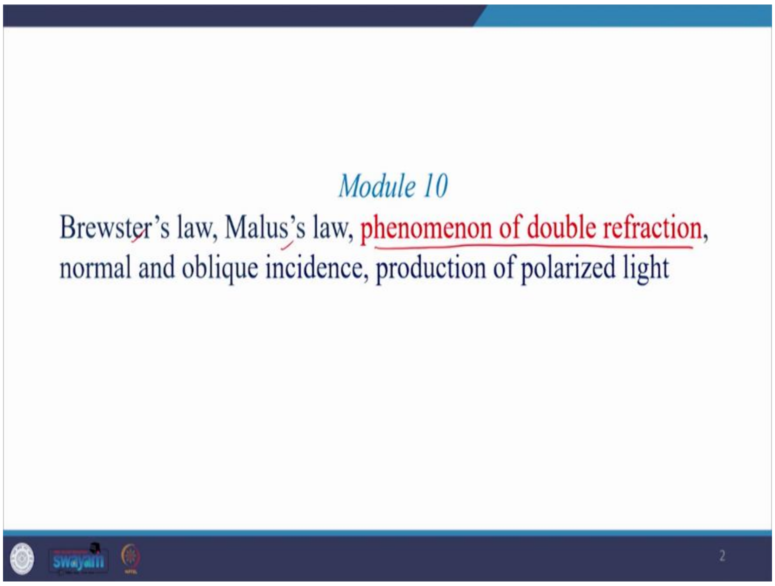


Today, we will start a very interesting topic, which is double refraction, which is quite a new phenomena for you, I guess. And in double refraction, you will see that apart from the usual refraction, there is a new type of refraction happening and this happens only in certain kinds of materials, which we call birefringent materials. And birefringent materials, they exhibits two refractive indices and due to the existence of these two refractive indices, a part of the light follows the snell's law, while the other part which is seeing the other refractive index, it does not follow the usual snell's law.

The snell's law for the second type of ray gets a bit modified. We will see all these in this double refraction topic.

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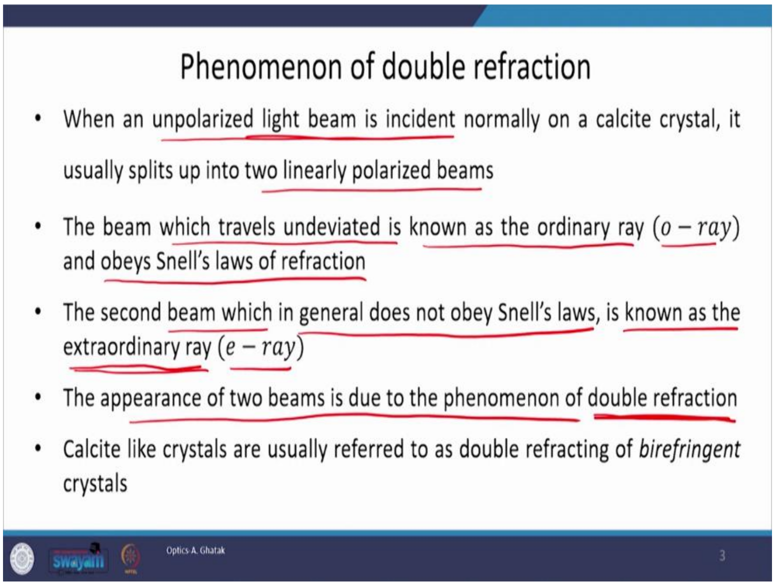


Now, when a unpolarized light beam is incident normally on a calcite crystal, it usually split up into two linearly polarized beam. Now, the property of the calcite crystal is such that it produces two linearly polarized light and do remember that unpolarized light beam is launched and it is giving two linearly polarized beam. The beam which travels undeviated on normal incidence is known as ordinary ray or o ray.

We have a crystal on which we are launching a light, unpolarized light normally, and the part of light goes undeviated, which we call o ray or ordinary ray and whatever we have studied till now, it all applies well on o ray and this ordinary ray obeys Snell's law of refraction, while the second beam which in general does not obey snell's law is known as extraordinary ray. Even on normal incidence, this extra ordinary ray, it does not go straight, it get deviated and this is why the name. This beam is called extraordinary ray or e ray.

The appearance of two beam is due to the phenomena of double refraction due to the existence of two index of refractions. The crystals like calcite are usually referred as a double refracting or birefringent crystals. Why double refracting? Because, these type of crystals are splitting the incident unpolarized ray into two parts. The one follows usual snell's law is referred to as ordinary ray, while the one which is not following the snell's law is called extraordinary ray.

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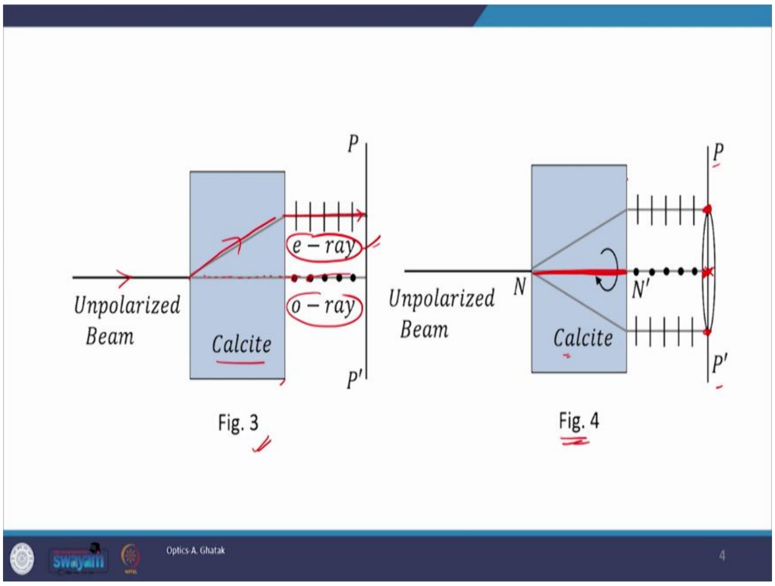


Now, this is shown schematically in Figure 3. You see that unpolarized beam is launched normally on a calcite crystal and a part of the beam goes un-deviated and this we call o ray, while other part gets deviated and it emerges out of the crystal from a different point and we named this ray as e ray. Now, you see that their polarization is also different. In o ray, the polarization is perpendicular to the plane of the paper, while in e ray, the polarization is in the plane of the paper.

It means, the double refracting crystals are birefringent crystals. They are also producing polarized light. They are converting unpolarized light into polarized one. But apart from this, another property of this crystal are that if you rotate this crystal, say in this particular case, in this figure, in figure 4, if you rotate the calcite crystal along an N’ axis, then although the spot which is observed at a screen PP’, for o ray, it remains unaffected, the spot of e ray, it rotates, it creates a circle due to the rotation.

The spot of e ray, it rotates on a circle if the calcite crystal is rotated.

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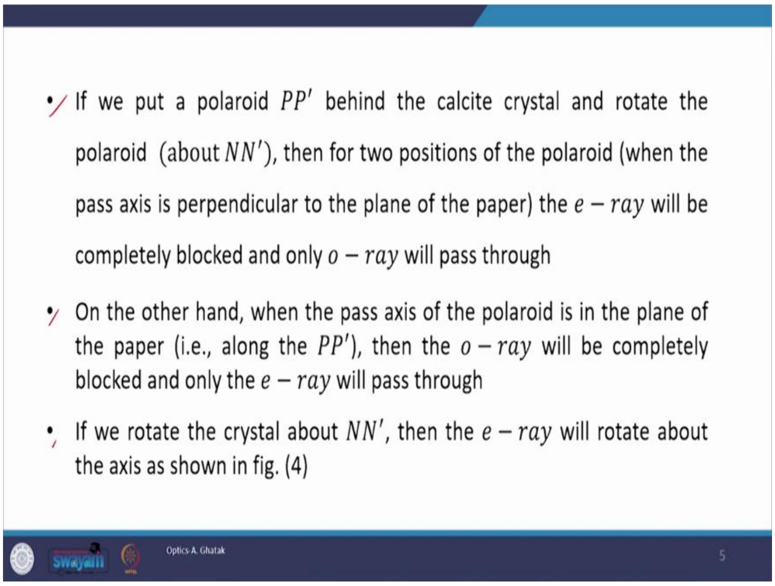


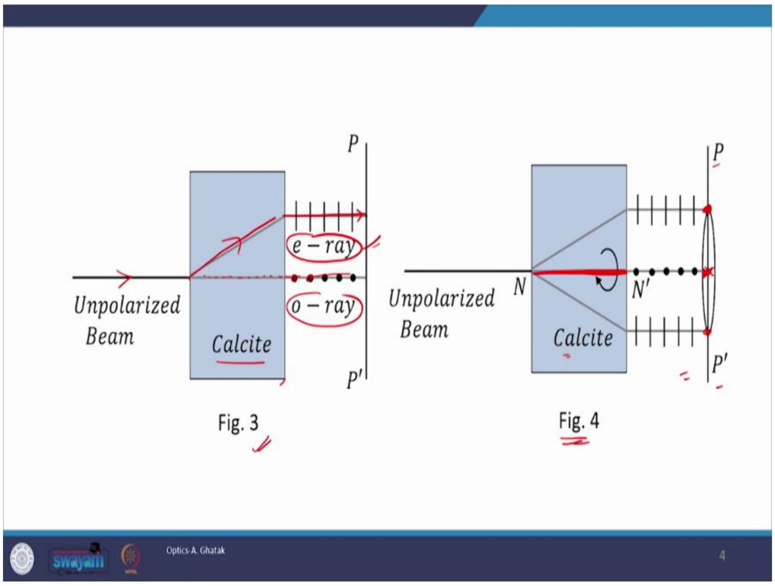


Now, this is summarized in these 3 points, which says that if we put a Polaroid PP’ behind the calcite crystal, Polaroid is a polarizer, if we put a Polaroid PP’ behind the calcite crystal and rotate the Polaroid about an NN’ axis, then for two positions of Polaroid, when the pass axis is perpendicular to the plane of the paper, the e ray will be completely blocked and only o ray will pass through.

And similarly, for two positions of the Polaroid, e ray will be passed through and o ray will be completely blocked. The second point is that when the pass axis of the polarized is in the plane of the paper, o ray will be completely blocked and e ray will be passed through. This is what I told you earlier. Now, instead of Polaroid, suppose we rotate the crystal itself about the same axis NN’.

Now, if the crystal is rotated, the e ray will rotate about the axis as shown in this figure and this rotation would be on the circumference of a circle. The spot of the e ray, it will trace a circle on the screen.

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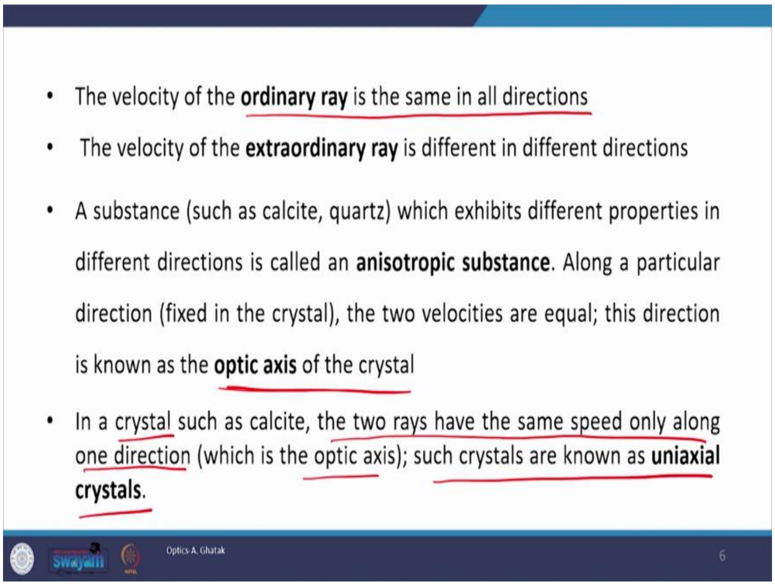


Now, with this let us move deeper. The velocity of the ordinary ray is same in all the direction. In double refracting crystals, the o ray will see same refractive index and therefore, it will travel with the same velocity. The velocity would be direction independent in case of o ray or ordinary array, while the velocity of the extraordinary ray is different in different directions. Why?

Because the extraordinary ray will see different refractive indices in different directions. Now, a substance which exhibits different properties in different direction is called anisotropic substance and which we have already discussed in one of our previous classes. I defined anisotropic material as material which exhibit different properties in different directions, while a material which exhibits same property in all the direction, it is called isotropic material.

But due to this anisotropy, the anisotropy of the crystal, the e ray will see different refractive index and which will be reflected in different velocities or direction dependent velocity of e ray. But, along a particular direction, which is fixed in the crystal, the two velocities are equal. The velocity of o ray and e ray are equal in some particular direction and this direction is known as optic axis of the crystal. I repeat.

o ray travels with the same velocity in other direction while E ray exhibit different velocity in different direction. But, there are a particular direction in double refracting crystals along which both of these ray travel with the same velocity and this particular direction is known as optic axis of the crystal. In a crystal such as calcite, the two rays have same speed only along one direction, which is of course, the optic axis as defined in the earlier point and such crystals are known as uniaxial crystal.

I repeat. A crystal which has only one optic axis is called uniaxial crystal, while a crystal which has two optic axis is called biaxial crystal. The calcite falls in first category and therefore, it is called uniaxial crystal.

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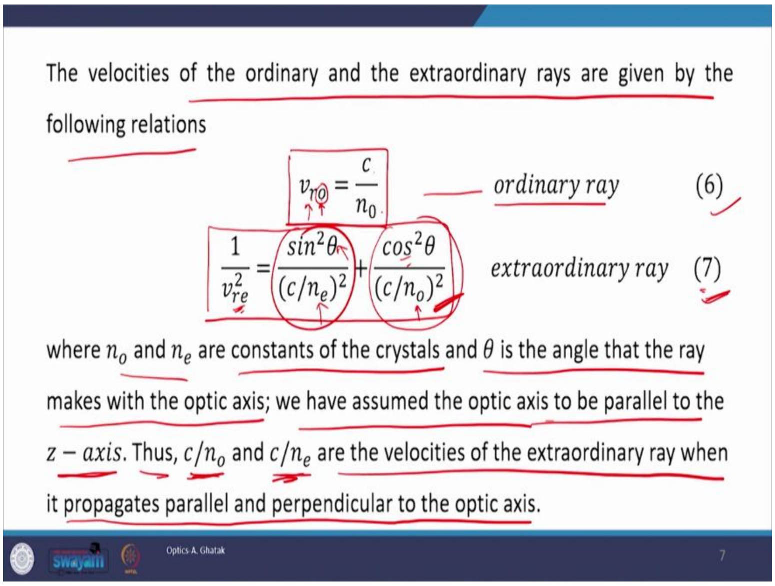


Now, the velocities of ordinary and extraordinary rays are given by following relations which are expressed by equation number 6 and equation number 7. For ordinary ray the velocity is given by this relation, which says $v_{ro} = c/n_o$. o, here in the subscript, it stands for ordinary ray, r stands for ray. Velocity of ordinary ray is equal to $c/n_0$ . This holds for ordinary ray but since the velocity of e ray is direction dependent, the corresponding expression is quite complicated and it is given by equation number 7 and this expression says that the velocity of extraordinary ray is θ dependent.

And what is θ? θ is the angle that the ray makes with the optic axis, while you also see $n_o$ and $n_e$ in the denominator which are constant of the crystal, which are some constants in the crystal. We will learn about these constants more in our coming lectures. Now, we have assumed the optics axis to be parallel to the z axis. Now, in this particular case, then $c/n_0$ and $c/n_e$ are the velocities of extraordinary ray when it propagates parallel and perpendicular to the optic axis.

I repeat. In equation 7, $\theta$ is the angle between the direction of propagation of ray and optic axis. When the ray is propagating along optic axis, then $\theta = 0$. And if $\theta = 0$, we can neglect the first term on right hand side in equation 7 because $sin\theta = 0$ when $\theta = 0$. We are left with only the second term, where the numerator would be 1 and therefore, the equation 7 would be similar to equation 6 when $\theta = 0$.

And we can see therefore, from equation 6 and 7, that along optic axis, both the ray travel with the same velocity. Now, when $\theta = 90$ degree, the second term on the righthand side of equation number 7 is 0. This term would be 0 when $\theta = 90$ degree. And in this particular case, the $v_{re} = c/n_e$. It means the velocity of extra ordinary ray would be different than the velocity of ordinary ray, when $\theta = 90$ degree or when the ray is propagating perpendicular to the optic axis.

And therefore, we can say that $c/n_0$ and $c/n_e$, they are the velocity of extraordinary ray when it propagates parallel and perpendicular to the optic axis respectively.

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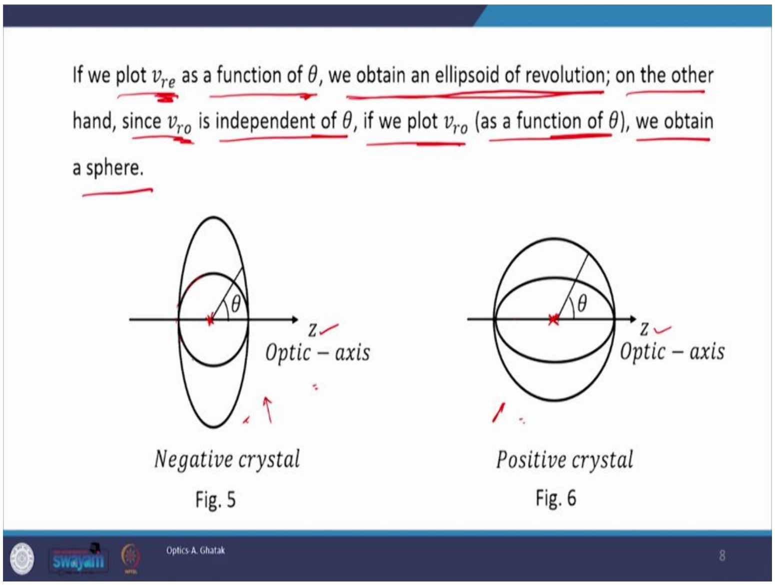


And if we plot $v_{re}$ as a function of $\theta$, we obtain an ellipsoid of evolution which is very much obvious because if you see equation number 7, then you see that equation number 7 is nothing but an equation of ellipse. Similarly, equation number 6 is nothing but an equation of a sphere in polar coordinates. Therefore, if we plot $v_{re}$ as a function of $\theta$, we obtain an ellipsoid of evolution. On the other hand, since $v_{ro}$ is independent of θ as visible in equation number 6.

If you see in equation 6, you will see that $v_{ro}$ is independent of θ. If we plot $v_{ro}$ as a function of $\theta$, we obtain a sphere and this is shown schematically here in this figure. We are having one sphere. At this point, we have a source and this source is kept in a double refracting medium or in a birefringent medium, optic axis of the crystal is along z axis in both the cases and we see that there is one circle and one ellipse.

In one case, the minor axis of the ellipse is along optic axis and the circle is inside the ellipse. And while in the second case the major axis of the ellipse is along the optic axis and the circle is outside the ellipse. This happens due to the crystal properties.

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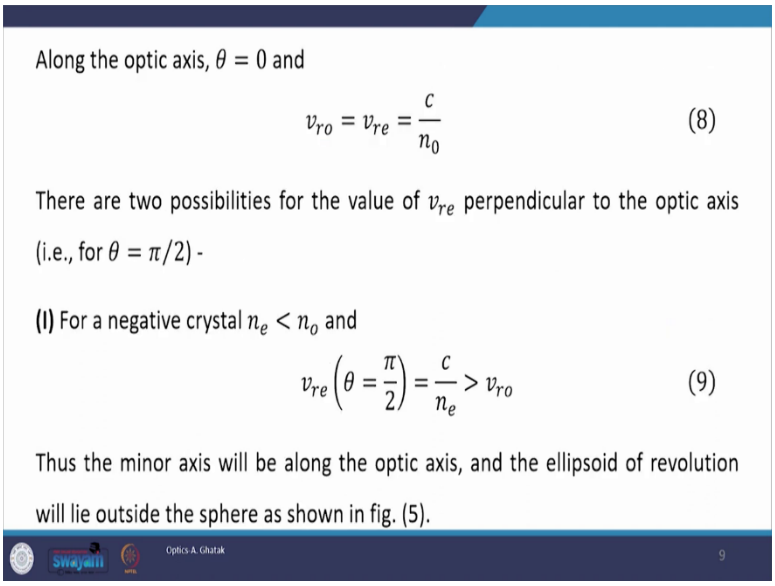


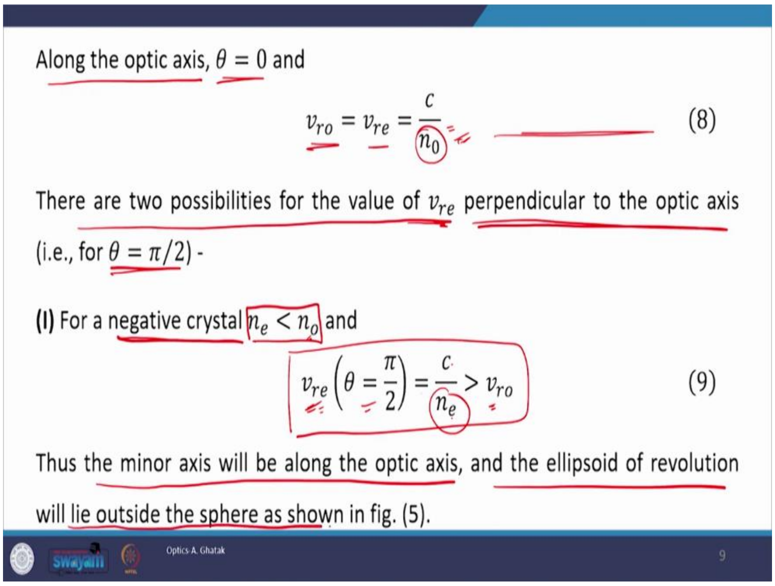


What are those crystal properties, we will see here. Now, along optic axis we know θ=0 and therefore, $v_{ro} = v_{re} = c/n_0$ and this follows from our previous equation, equation number 6 and 7. And this is why we can define optic axis. Along optic axis, the two rays travel with the same velocity and which is clear from equation number 8. Now, there are two possibilities for the value of $v_{re}$ and these two possibilities arises when we look into the direction which is perpendicular to the optic axis.

When $\theta = \pi/2$, then two possibilities arise. What are those possibilities? These possibilities arise due to the different values of $n_0$ and $n_e$, the constants of the Crystal. Now, if $n_0$ is larger than $n_e$, then the crystal is said to be negative crystal. I repeat. If $n_o$ is larger than $n_e$, then the crystal is said to be negative crystal. $n_0$ is refractive index of ordinary ray and $n_e$ is the refractive index of extraordinary ray.

And this is also clear from equation number 8. $n_0$ or $n_o$ is the refractive index of ordinary ray. If $n_o$ is greater than $n_e$, then at $\theta = \pi/2$, $v_{re}$ is given by this expression, $v_{re} = c/n_e$. Since $n_e$ is smaller, $c/n_e$ would be larger and therefore, $v_{re}$ would be larger than $v_{ro}$. It means extraordinary ray will travel with larger velocity. The velocity of extraordinary ray is larger than the velocity of ordinary ray and the crystals for which this happens, they are called negative crystals.

And since extraordinary ray is traveling faster, the minor axis will be along optic axis and the ellipsoid of revolution will lie outside the sphere, as is shown in figure 5.

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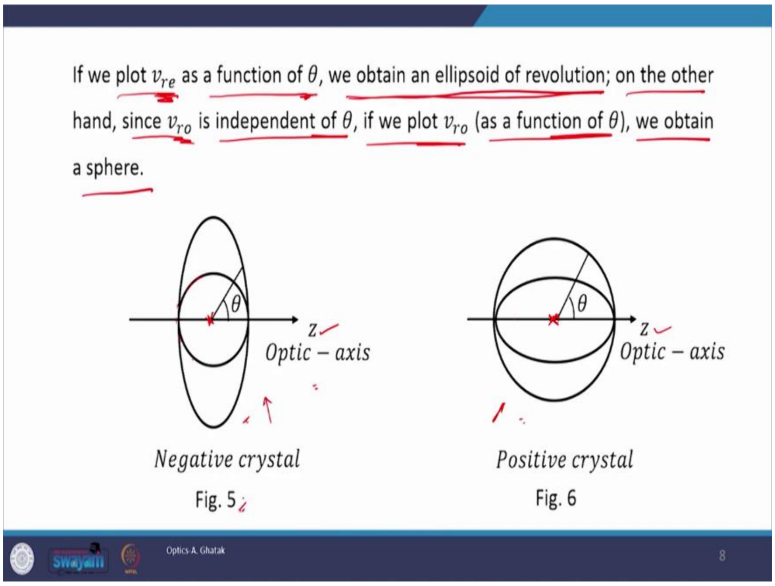


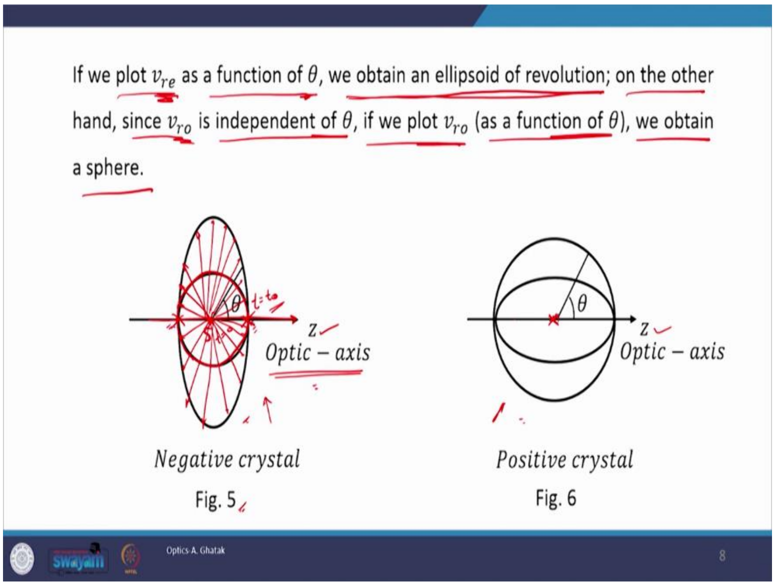


Now, this figure I will explain you in detail. The rule of thumb is that along optic axis, the two rays will travel with the same velocity. Therefore, if this center of the circle is the position of our point source S and whole of this thing, the source is kept in a birefringent material then from source S, both extraordinary ray and ordinary ray will emanate. They will start from source S.

Now, along z axis, the velocities of these two rays are same. Therefore, if you take a picture snapshot of the rays emanating from the point source S at a time which is given by $t = t_0$, then what would you see is that ordinary ray will make a sphere, while extraordinary ray whose velocity is governed by equation number 7, which is of course, an equation of an ellipse, therefore, at time $t = t_0$, at the time when we took this snapshot, the extraordinary ordinary ray would have formed an ellipse.

How it forms an ellipse? Because it travels from source S and then since its velocities are different in different direction, the rays which started at time $t = 0$ from source S, it reaches different distances in time $t_0$. Therefore, in different direction, the light has traveled different distances and it formed an ellipse. But along the optic axis, the two velocities must match. Therefore, along optic axis, the ellipse and sphere must coincide and this is what is visible from this figure, figure number 5.

Along optic axis, the two rays have reached the same position.

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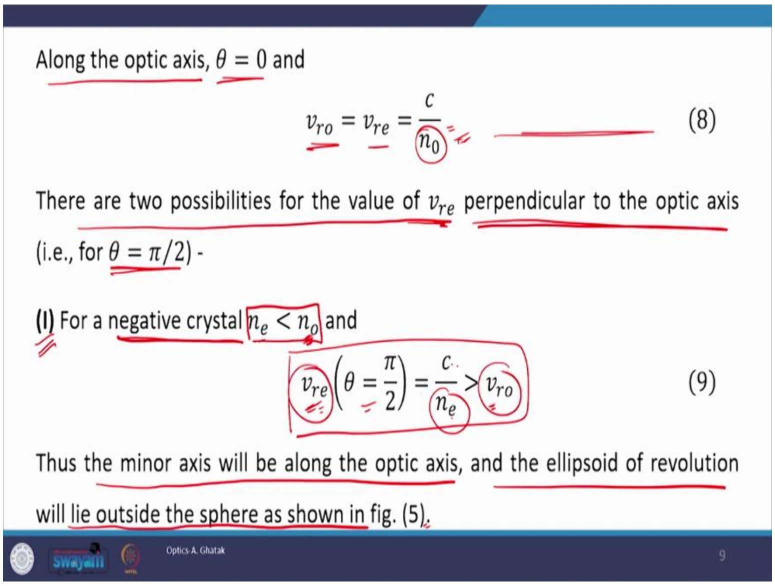


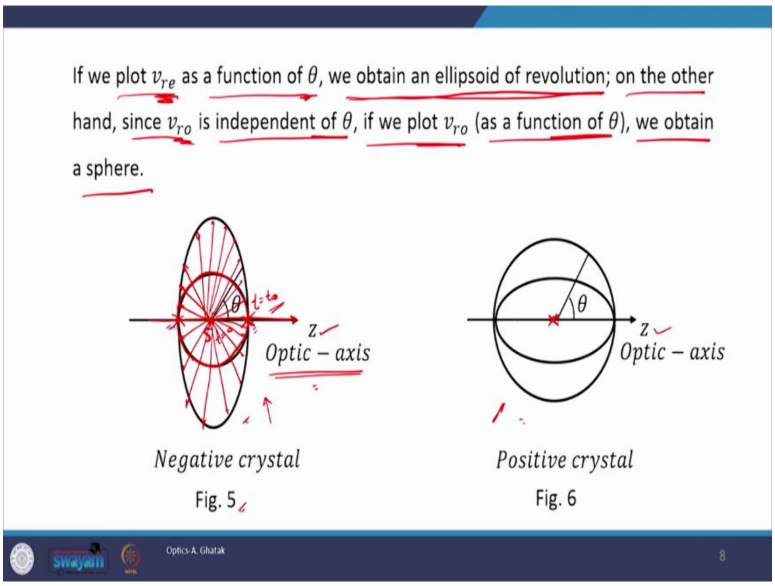

But as discussed here in case 1, when we are considering a crystal for which $n_0 > n_e$ , $v_{re} > v_{ro}$. It means extraordinary ray must travel longer distance as compared to ordinary ray in the same time interval and time interval here is $t_0$. And therefore, the sphere must be inside this ellipsoid of evolution. Ellipsoid must be outside because, the ellipsoid represents distance which the ray has traveled after starting from point source S at time $t_0$ and the starting time is t=0, while the snapshot is taken at $t = t_0$.

The light starts at t=0 and the snapshot is taken as $t = t_0$. During this time interval, the both e and o ray travel some distance and if we draw an envelope over these distances, then for o ray we get a sphere, while for e ray, we get a ellipsoid of revolution. Since e ray is traveling faster, therefore, ellipsoid of revolution would be outside of the sphere. And since along optic axis, the two velocities are the same, therefore, the sphere and the ellipsoid must touch along the optic axis.

And therefore, in negative crystal, the minor axis of the ellipsoid must be along optic axis.

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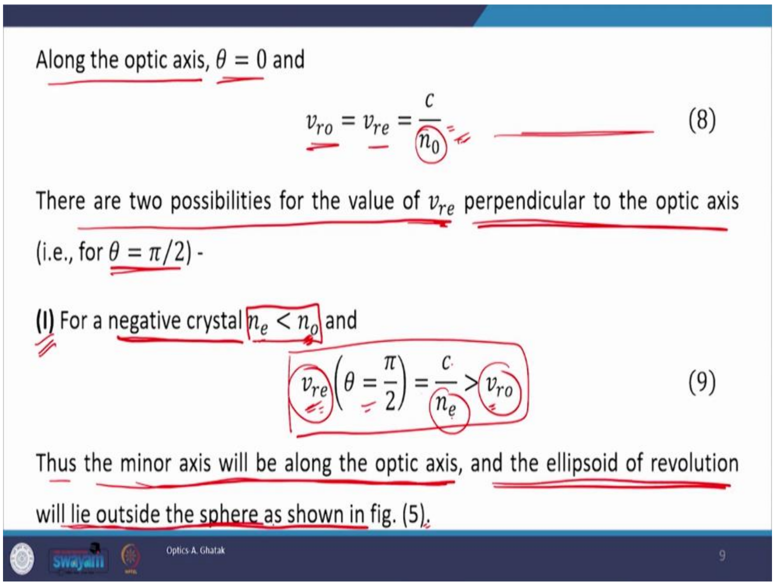


And this is what is written here, the minor axis will be along optic axis and the ellipsoid of revolution will lie outside the sphere as shown in the figure.

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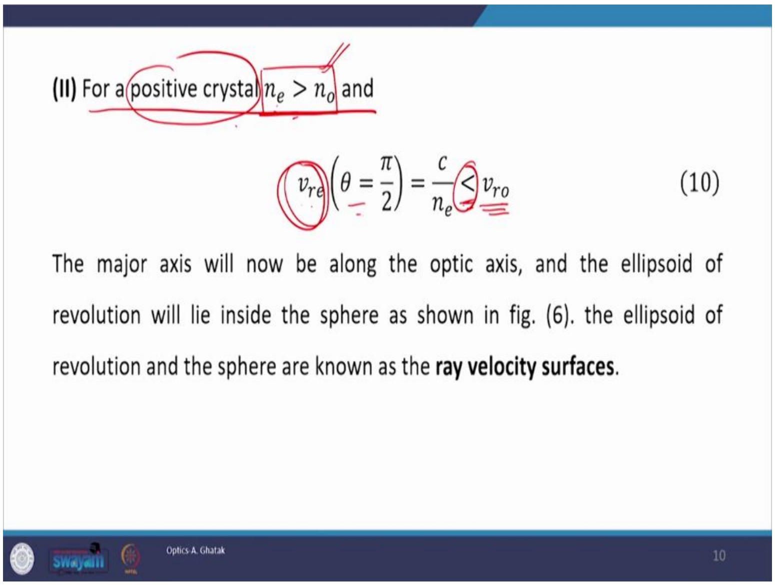


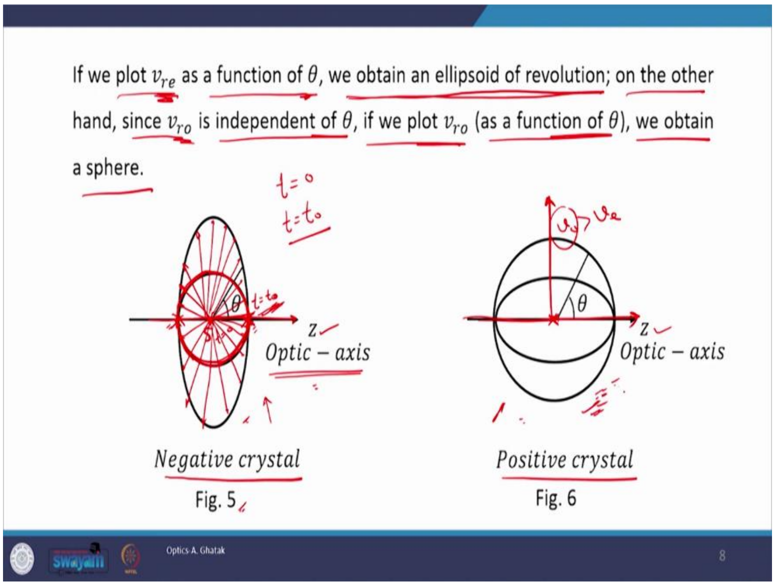


Now, let us consider the second case. The second case is when $n_e > n_o$ or $n_e > n_0$. The crystals in which this relation hold, they are called positive crystal. And in this crystal since $n_e$ is larger than $n_0$, $v_{re}$ would be smaller than $v_{ro}$. Therefore, at an angle which is $\pi/2$ degree, $\theta = \pi/2$ degree, $v_{re}$ would be smaller than $v_{ro}$. This is smaller than sign. In positive crystal $v_{re}$ would be smaller than $v_{ro}$ at $\theta = \pi/2$. And since in this case extraordinary ray is traveling slower, therefore, the ellipsoid of revolution which is made by extraordinary ray, it would be smaller than the sphere which is made by ordinary ray.

Therefore, the major axis will now be along the optic axis why, which is clear. Along the optics axis two velocities must be same. In other directions, since the velocities are θ dependent and in particular in 90 degree, the velocity of o ray is larger as compared to e ray. Therefore, the sphere would be now outside of the ellipsoid of evolution. And therefore, the ellipsoid of revolution would be oriented in such a way that its major axis would be along optic axis.

And therefore, in the positive crystal, the situation is depicted here by this figure 6.

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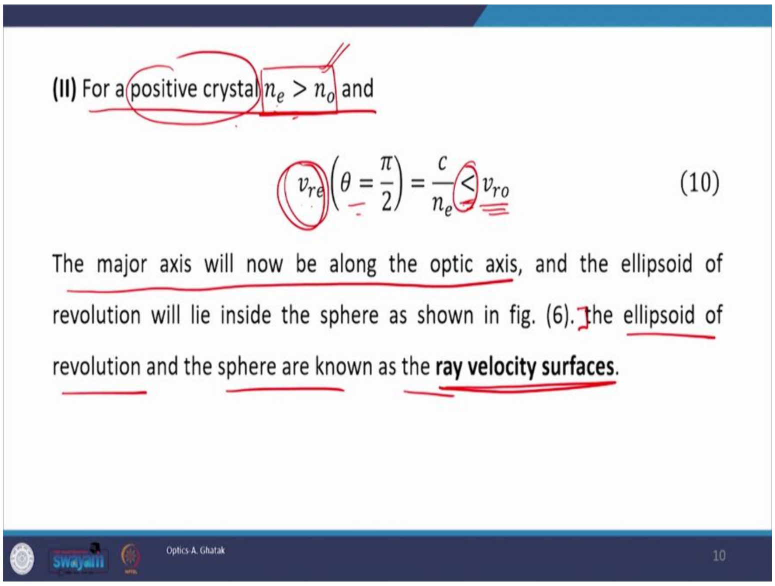


Now, the ellipsoid after evolution and the spheres are known as Ray velocity surfaces, these are the snapshot which is taken at a lapse of some time after the rays had started from point source S at time t=0. And these surfaces are called ray velocity surfaces. Now, this is all for the introduction of double refracting medium or double refraction. We will dive deeper into this topic in the next class. And thank you for joining me. See you in the next class.