Applied Optics
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Lecture 45
Diffraction of a Plane Wave by a Long Narrow Slit and Transition o Fraunhofer Region, Diffraction at a Straight Edge, Diffraction by a Narrow by a Narrow Obstacle, Babinet's Principle

Hello everyone, welcome to my class. Today we will hold the last lecture in module 9.

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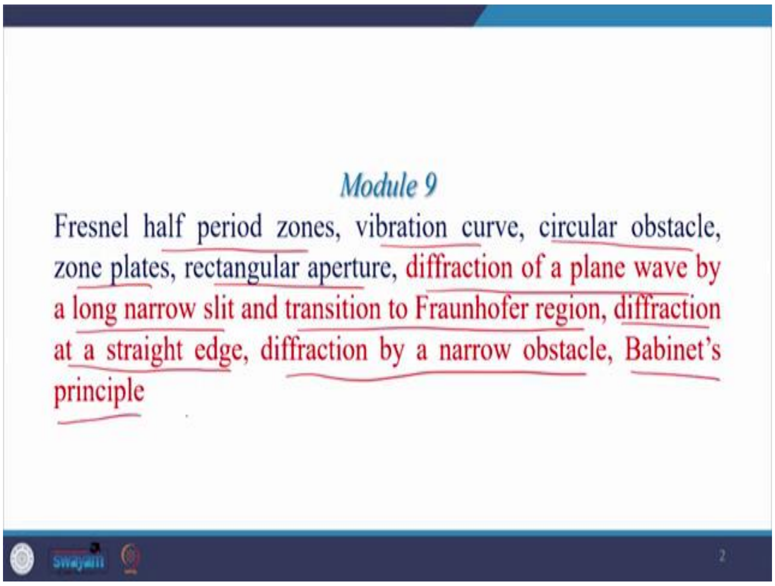

Now in module 9 till now we have covered Fresnel half-period zones, vibration curve, then we studied Fresnel diffraction due to circular obstacle. Then we were introduced zonal plates and in the last lecture we talked about rectangular aperture. Today you will learn diffraction of a plane wave by a long narrow slit and transition to the Fraunhofer region.

Thereafter we will talk about diffraction at a straight edge, then diffraction by a narrow obstacle which would be followed by Babinet's principle. And this will conclude today's lecture and module 9 as well.

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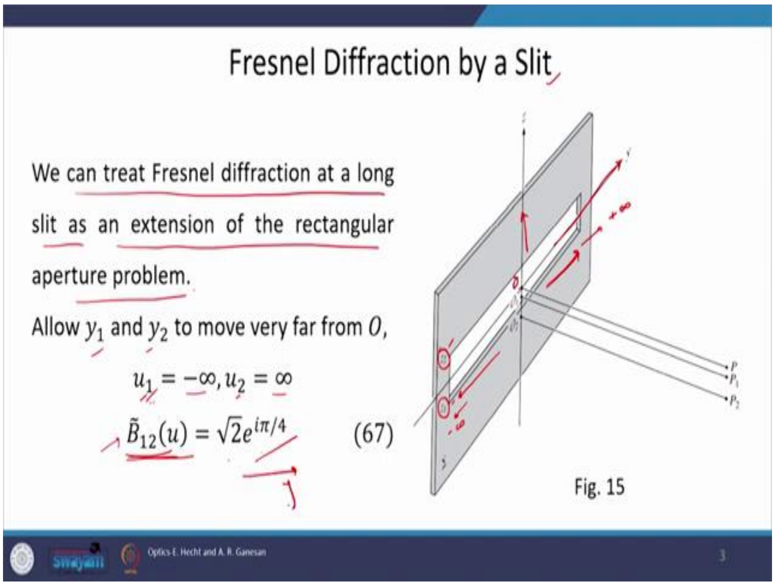

Now we will start with Fresnel diffraction by a slit. Now slit is nothing but it is a particular case of rectangular aperture where in the one side of the rectangular aperture is extended till infinity. Now this is what is written here also. We can treat Fresnel diffraction at a long slit as an extension of the rectangular aperture problem. Now in this case, the slit length or the length of one arm of the rectangular aperture is assumed to be extended till infinity.

Now in this figure the slit is extended till infinity in π direction. Now this edge goes to $-\infty$ while this edge goes to $+\infty$, while in the vertical direction along z axis the slit coordinates are z_1 and z_2 as depicted in this figure. Now therefore, here the extremities of the slit in y direction which are y_1 and y_2, they would be moved very far from the origin O, O is the origin of the coordinate system placed in this aperture plane.

Now since y_1 and y_2 are extending till infinity, therefore u_1 which is directly related to y_1 and u_2 which is directly related to y_2, it will also extend to $-\infty$ and $+\infty$ respectively. And we know phasor therefore would be equal to $\sqrt{2}e^{i\pi/4}$. Till here we have already calculated. We know how once the u_1 and u_2 are $-\infty$ and $+\infty$, then what would be the values of respective integral and we know it would be equal to $-1/2$ and $+1/2$.

Using that we calculate the phasor $\tilde{B}_{12}$ would be equal to $\sqrt{2}e^{i\pi/4}$. But this is only for horizontal direction which is along y axis. What about the integral which is in the vertical direction, the v dependent integration?

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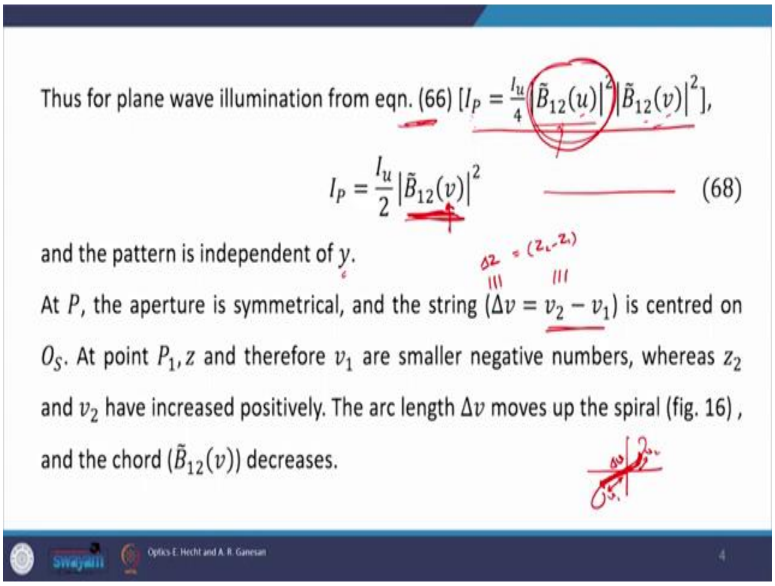

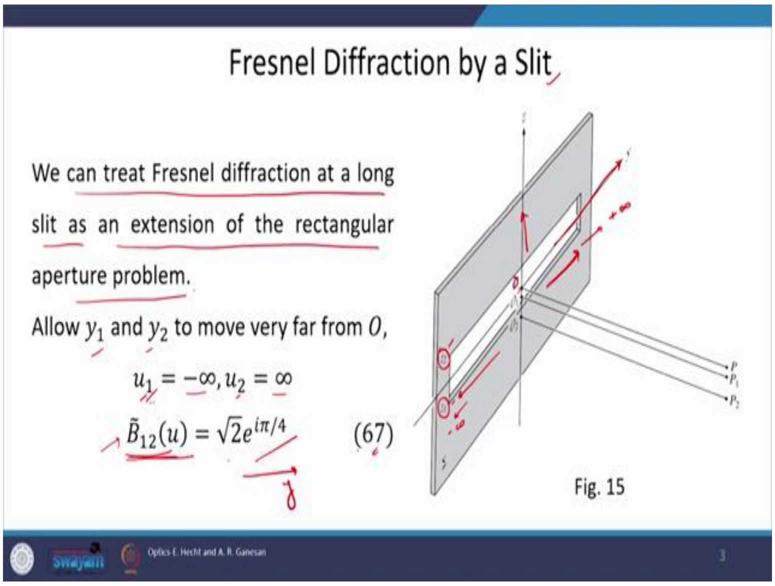

Now the integration which we are talking about is equation number 66 and which is given by this expression I_p that is irradiance at point of P is equal to $I_u/4$ and $|\tilde{B}_{12}|^2$ which is a function of u and $|\tilde{B}_{12}|^2$ which is function of v. This we have already calculated in the equation 67. Now this we have still to evaluate. The value of $\tilde{B}_{12}$ which is function of u is given by equation number 67, this is still to be evaluated.

Now we can see that equation number 68 is independent of u or equation number 68 is independent of y, because we have assumed the extremities of the slit to be at infinity and therefore we calculated the corresponding integration and using that we calculated the $\tilde{B}_{12}$. Now the value of $\tilde{B}_{12}$ which is function of v it depends upon the width of the aperture in vertical direction and that width would be $v_2 - v_1$ or $z_2 - z_1$. This is equivalent to $z_2 - z_1$.

That is Δz, which is directly related to Δv. Now you see that in this figure O is the origin and P is the point of observation. At this point P, the z_1 and z_2 extremities, these lower and upper extremities of the aperture along z direction they are symmetrically placed. This angle is equal to this angle. This θ is the same if the slit is viewed from point of observation P.

Therefore, if we draw these things on cornu spiral, then we will see that we will get a spiral like this and then the Δv would be represented by this curve. And you can see that the length of the curve here and the length of the curve here in the third quadrant, they would be same due to the symmetry. v_1 is in the negative direction, v_2 is in the positive direction and therefore v_1 would be here, v_2 would be here and the difference is $v_2 - v_1$.

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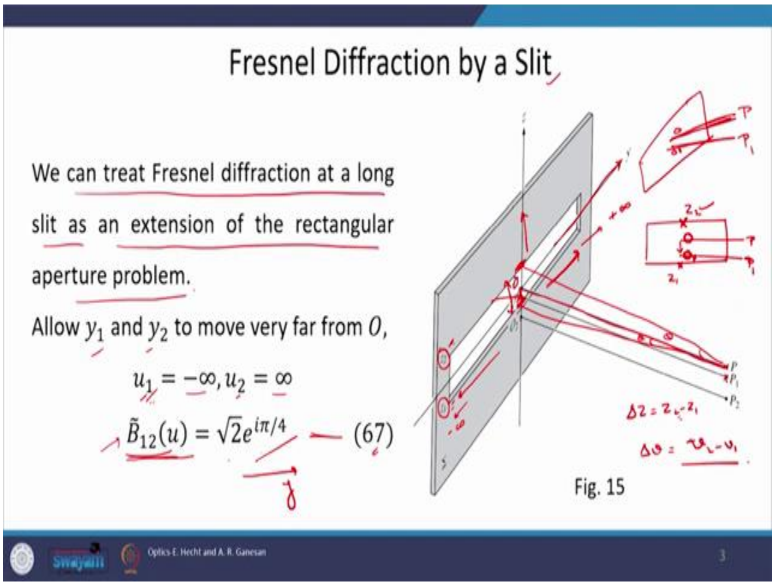

Now if we want to observe the irradiance at point P_1 which is slightly downward with respect to point P then what will happen, O would be shifted to point O_1 because we know instead of calculating irradiances at different point on the screen we move the aperture plane. Now if you move the aperture plane the point O will shift to new origin which is point O_1. Initially this was the slit and then O was here now O_1 is here.

For O the point of observation was P, for O_1 the point of observation is P_1. These are the parallel lines here, so sorry for my drawing. Let me try it again. This is our slit, these are, this is point O; this is point O_1. For point O we were having point of observation P, for O_1 we will have point of observation P_1. Now from P_1 if you treat O_1 as new origin, then this edge would be more positive and this edge would be now less negative.

For O this is a positive edge, this is z_2 and this was z_1, but since the origin is shifted downward, z_2 would be more positive while z_1 would be less negative because whole reference frame got shifted down. Therefore, the coordinates of the edges would also move, the coordinates of the edges will shift upward. Initially the origin was here and then we were seeing that it is say +2 and the lower edge was at -2.

But if we shift the origin by one unit down, then the upper extremity, which initially was at +2, it would be now +3 and the lower extremity, which initially was at -2, it would be -1, because origin got shifted by 1 unit. And this also happens here in this case too. But this length $z_2 - z_1$, this would not change. And similarly the Δv the arc length, which is equal to $v_2 - v_1$, this will also remain fixed.

Therefore, on cornu spiral the Δv which was initially symmetrically placed, it would be shifted upward in the first quadrant. Why? Because v_1 is more positive and v_2 is less negative, therefore things are shifting up in the positive direction.

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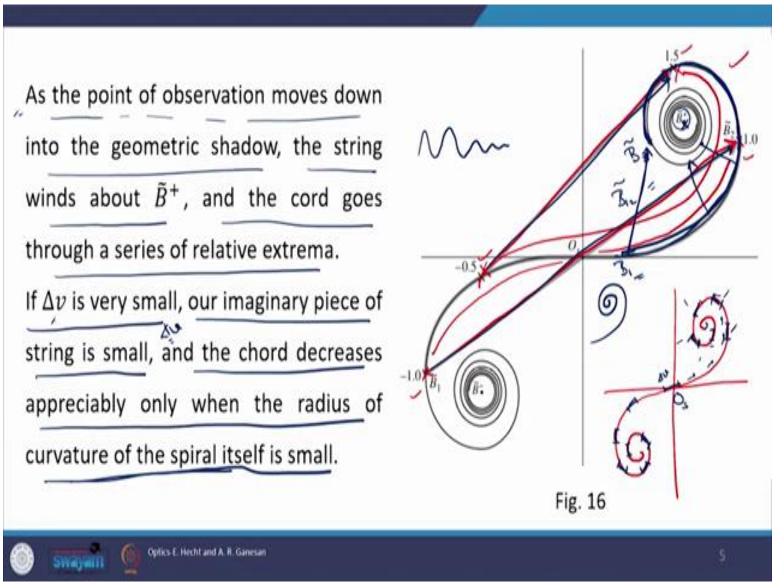

And this is what is shown here. Initially due to the symmetry, the point B_1 and B_2, they were symmetrically placed. This arc was equal to this arc in length, when the point of observation was P, when origin was exactly at the center of the slit. But now if we want to calculate the irradiance at a point P_1 and for P_1 origin has to be shifted at point O_1. In that particular case the arc is shifted, the new arc is this.

This is now new arc. You can see that this point B_1 is shifted to this place and B_2 is shifted to this place. B_1 initially was at -1 and B_2 was at +1, the new position of B_1 is -0.5 which is less negative and new position of B_2 is +1.5 which is more positive. And therefore, as we move down, let me show here these things in this figure. Now initially, the point of observation of P, if we move a bit down, the new point of observation is P_1.

And at P_1 the arc, the curve is spiraling up, it is going up in the first quadrant. If we go more down at point P_2, then at point P_2 the new origin would be O_2. O_2 would be here in the plane of aperture, but it would be not in the open portion of the aperture. O_2 is still in the aperture plane, but it is on the closed portion now and the point of observation for O_2 is P_2, which is a bit more down as compared to P_1.

And therefore, the curve will move up because from O_2 this edge is far, it is positive but more positive, while the lower edge, it is also positive because origin has shifted even more down, here the origin is here, and the slit is like this. Therefore both edges of the slit are positive now. It means our curve in cornu spiral picture it is shifted in the first quadrant. The whole curve, whole Δv now got shifted in into the first quadrant.

Now let us see what happens exactly? When the origin was at the center of the slit, this line represents the irradiance $\tilde{B}_{12}$. Now when we are at a point P_1 this line represents the new irradiance, which is $\tilde{B}_{12}$, but at point P_1. It means at point P_1 the irradiance is decreasing, it is going down. Now if suppose we are at a point P_2 in that case when both the edges are in the positive, the whole curve is in the positive quadrant.

And this would be the new position of the curve where B_1 is represented here and B_2 is here. Now in this case the resultant irradiance would be given by this phasor $\tilde{B}_{12}$. Now you can see that this phasor is still smaller. Now as we move below then what will happen? The Δv or this curve, it will spiral around. Spiral around which point? Spiral around this point $\tilde{B}^+$. And what we will see is that the irradiance it will go through several extremum, it will go through several maxima and minima and it will slowly decay down.

Now this is what is also written, as the point of observation moves down into the geometric shadow, the string winds about $\tilde{B}^+$ and the chord goes through a series of relative extrema. And this is what I said sometimes it will see maximum, sometimes minimum, sometimes maximum, sometimes minimum, but it will decay down. And this you can see this is the maximum length and then the length got reduced here even smaller.

And if you keep going then you will see that sometimes you are seeing some maximum and sometimes minimum as the chord which is of length Δv, it winds around $\tilde{B}^{+}$, which is this point. Now if Δv is very small or if the slit is very thin, our imaginary piece of string is small. String means this Δv, a chord, which is being plotted on the cornu spiral.

And the chord decreases appreciably only when the radius of curvature of the spiral itself is small. Now if Δv is very small, in this same picture, suppose this is the cornu spiral and let us represent the chord with blue color. Now this is our Δv. Since slit is very thin, therefore the length of Δv is very small as you can see in the figure and OS is the origin. Now what will happen if we go down then this chord will move up slowly.

And what we will you see is that this length is not decreasing appreciably, this is the position of the chord at different point of observation, if the point of observation is going down slowly. Then you will see that if the chord length is very small, then the resultant phasor or resultant irradiance is not decaying as the point of observation is moved down, the length almost remains constant therefore irradiance would also be almost constant.

And if we want to observe any reduction in the irradiance, then this cornu spiral it should be very small, if it is quickly winding up with a very small radius of curvature, then only we will be able to see some appreciable change in the irradiance. Similarly, if we move point of observation in the upward direction, then the chord will shift like this.

With the moving positions of the point of observation the chord will wind up around this spiral, around this point $\tilde{B}^{+}$ or $\tilde{B}^{-}$. And this is what exactly happened with the thinner slits. You see that light diverges and a lot of light goes into the shadow region and this is what exactly we are seeing it decays very slowly.

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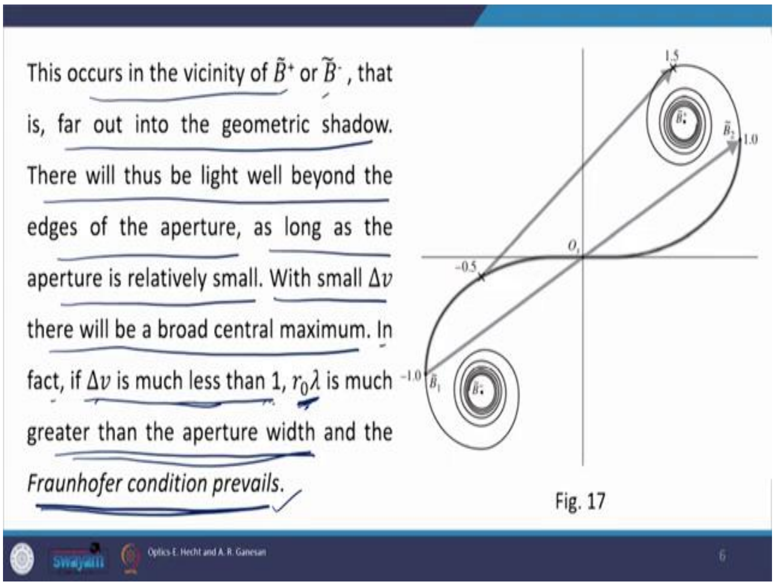

Now having done this, now let us see, this phenomena this occurs in the vicinity of $\tilde{B}^+$ and $\tilde{B}^-$. What occurs around $\tilde{B}^+$ and $\tilde{B}^-$? The decrement of the irradiance for thin slit. Because the arc length or the chord length is very small and therefore, any reduction in the irradiance would be seen if only we are in the vicinity of $\tilde{B}^+$ or $\tilde{B}^-$ or if the spirals are very tight and very small.

If the radius of curvature is very small then only we see a slight reduction in the irradiance. And this will happen far out into the geometric shadow, which is very much clear. Now there will thus be light well beyond the edges of the aperture, as long as the aperture is relatively small. With small Δv, there will be a broad central maximum.

In fact, if Δv is much less than 1, $r_0\lambda$ is much greater than the aperture width. Why? Because if you remember in the definition of u and v, $r_0\lambda$ comes in the denominator. And if $r_0\lambda$ is much greater than the aperture width, then we are effectively in the Fraunhofer regime.

Therefore, we see that as we shrink the aperture width, then slowly we move from the Fresnel regime to the Fraunhofer regime because there the Fraunhofer condition prevails. Why? Because $r_0\lambda$ is much-much larger and Δv is less than 1. In this particular case we slowly move from the Fresnel region to the Fraunhofer region.

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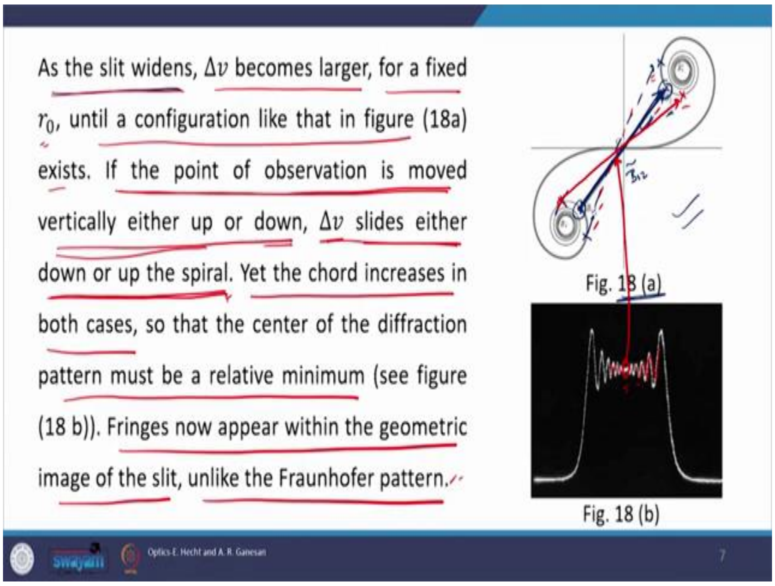

Now let us see what happens if we opens up the slit, if we widen the slit. Now as the slit widens what will happen, Δv or the chord length it will increase, it will become large. And while doing this broadening what we do is that we fix r_0. Now if we fix r_0 and keep opening the slit, then a situation comes when the cornu spiral looks like this figure 18 (a). The lower edge of the slit is shown by point $\tilde{B}_1$ while the upper edge is shown by $\tilde{B}_2$, and the resultant phasor is given by this straight line which we call $\tilde{B}_{12}$.

Now this happens only for a particular width of the aperture. Now in this particular case if you widen up the aperture or shrink the aperture in both the cases B_1 will move from this position as well as B_2 will move from this position. And if there is a shift in the position of $\tilde{B}_1$ and $\tilde{B}_2$, then what will happen this length of the phasor will increase now. This is the minimum possible length of the phasor in this particular case.

This is the minimum possible length of the phasor. Now if suppose the B_1 is here and then B_2 would be here, the new position would be this one, which is larger than this earlier position. Similarly, if you shift B_1 in this point, let us pick different color. Now if you shift B_1 here and B_2 here, the new position would be given by this red line, which is again larger than this continuous red phasor.

Therefore, if the point of observation is moved vertically, either up or down, Δv slides either down or up the spiral, which is shown by this point. Now yet the chord increases in both cases, which we understood, the new position of the chord would be given by either this dashed line or this continuous red line.

And therefore, the center of diffraction pattern must be a relative minimum. In this particular width of single slit, the center would be minimum, which is given by this figure here and this represents this, blue continuous blue line. And as we move away, play with the slit width, either it is decrease or increase, then what will happen, the phasor will increase or what we can say is that as we move the point of observation up or down, then what will happen?

Then we will see a increase in the intensity. Then let me reread it. As the slit widens, Δv becomes larger, for a fixed r_0, until a configuration like that in figure 18 (a). Now in this configuration if the point of observation is moved vertically either up or down, Δv slides either down or up the spiral. Yet the chord increases in both cases, so that the center of diffraction pattern must be a relative minimum, which is shown here.

Now in this position as I said before as soon as you move the chord up or down, the intensity should go up and this you see here, the intensity is going up. Now while going up you see that it is again going through several maxima and minima if the chord winds up around $\tilde{B}^+$ or $\tilde{B}^-$. Now fringes now appear within the geometric shadow of the slit, unlike the Fraunhofer pattern, in this particular case only.

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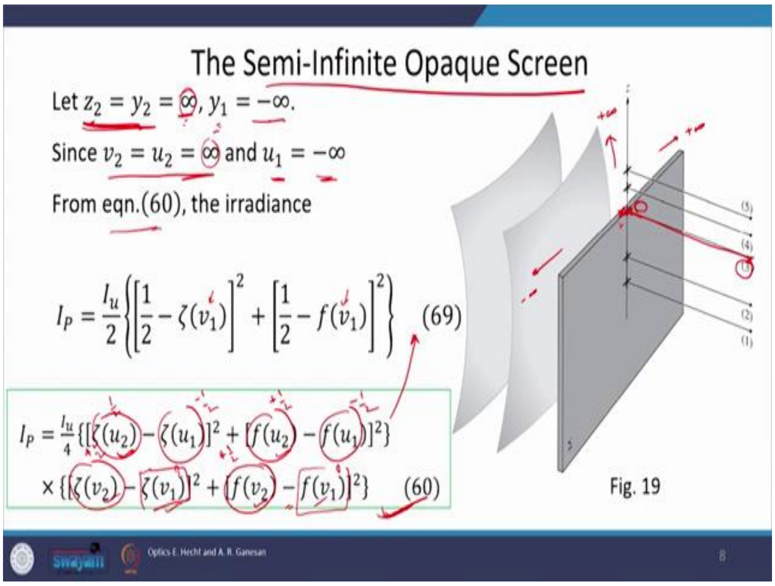

Now this is all about single slit or thin slit. Now we will start a new topic, wherein we will see the fringe pattern due to semi-infinite opaque screen. In this particular case the edges z_2 and

y_2 is extended till infinity and y_1 is extended till $-\infty$. In this direction the edge is extended till $-\infty$, here till $+\infty$, in this direction it is $+\infty$ and here it is kept at 0.

But let us not say it is 0, let us say that it is not extended till $-\infty$, it has some finite value. And therefore, z_2 and y_2 would be $+\infty$, y_1 at $-\infty$. Therefore, v_2 and u_2 would be equal to $+\infty$ and u_1 would be $-\infty$. And if this is the case, then from equation 60, which is given here in this green bracket, we will substitute for u_1, substitute for u_2 and v_1, v_2 and we know that these values are $+\infty$ and $-\infty$.

If you substitute here, then you will get that value of this function is +1/2, value of this function is again +1/2, the value of this function is -1/2, this is again -1/2, and this function is equal to +1/2, and this is again +1/2 and $f(v_1)$ is not known. This is something which is left, this is also something which is left. These two functions would be left and after all this substitution and bit simplification we get equation number 69.

Now in this equation, suppose that point of observation is here at point number 3, now if observer is at a point number 3 in Fresnel domain, then it will see that the edge is directly in the field of view, here it is, and we will see that origin is here at the edge. Therefore, the point v_1 for point 3 would be equal to 0.

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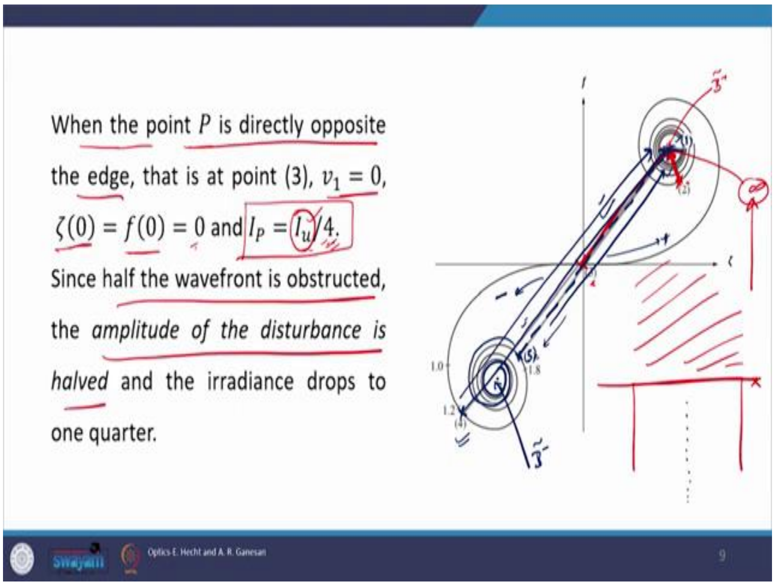

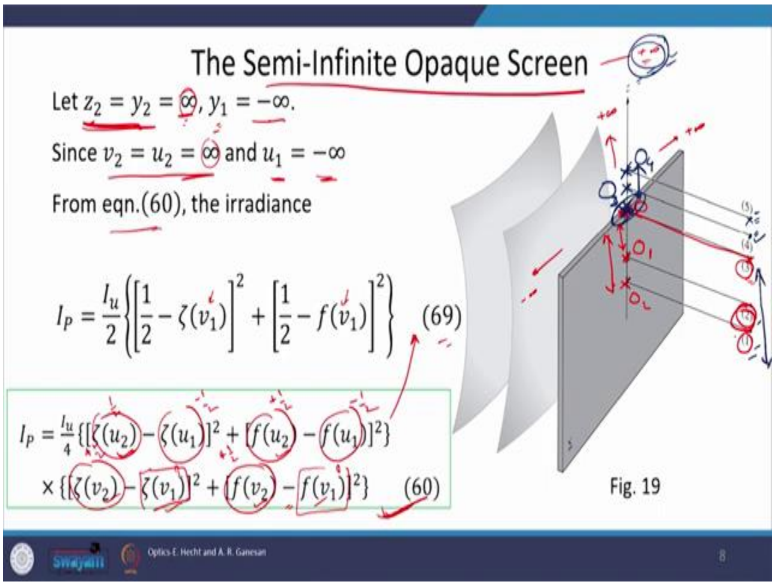

Now, therefore in this particular case point P is directly opposite to the edge and v_1 is 0. If $v_1 = 0$, $\zeta(0) = f(0) = 0$. And the value of the function or the value of equation 69 would be equal to $I_u/4$. Now you see that had there been no aperture we would have received a irradiance which is equal to $I_u/2$ but, now here we are getting $I_u/4$ and this is quite intuitive also.

Because we are covering half of the space with a screen. Now since half of the wavefront is obstructed, the amplitude of the disturbance is halved and the irradiance drops to one quarter. Had there be no obstruction we would have observed irradiance which is equal to I_u, but since half of the wavefront is stopped or half of the wavefront is obstructed, the electric field reduced down by half, therefore irradiance is reduced by one fourth.

And which is, we derived from our mathematical calculation, which says $I_p = I_u/4$. Now on cornu spiral if you want to represent this then the point 3 is given here, which represents the case when the origin is rightly at the edge. Now from point 3 if you want to calculate the irradiance, the upper edge is at infinity because this is the edge and the opening is extended till infinity.

And infinity is represented by this point here, which is our $\tilde{B}^+$, therefore the resultant phasor would be this, the phasor which start from point 3 and extending till point $\tilde{B}^+$. Now let us again go back into the figure. Now if we go to a point 2 which is a bit down as compared to point 3, then if you place your new origin here, then in this particular case the edge, the coordinate of edge would be some non-zero positive number.

Initially when the origin was at edge, the coordinate of this edge was 0, but from point O_1 the coordinate of this edge would be some positive number. And while the upper edge, which is at $+\infty$, it will remain as it is. Therefore, the spiral or the chord which initially was starting from origin from point 3, it will now be shifted upward.

The new position of the chord starts now from point P and then the new vector will be from point 2 to point $\tilde{B}^+$, because the other edge, the higher upper edge is till ∞, therefore we will draw a vector which will start from point 2 and it will extend till point $\tilde{B}^+$, because our aperture is this. This is the open portion, which is our aperture, it start from this line and extend till ∞.

The ∞ is always represented by $\tilde{B}^+$, $+\infty$. And the position of this aperture is changing as we are moving in the screen. Now you see that for point 2 the new position of the phasor is this. Now for point 1, which is even more down, more in the shadow region, we will again see that from new origin the position of this edge is now again a positive number, which is greater than its previous value.

It means the chord will now move a bit more up, it would be now closer to $\tilde{B}^+$ point. Therefore, this new point, let us pick a different color, new point is represented by this phasor here, the new point is point 1 and the corresponding phasor is given by this blue line, which is even smaller. This line is largest, smaller, smallest. It means as we move into the shadow region, as move down into the shadow region the irradiance will drop off.

It will drop slowly. Now what would be the situation if we move upward? Say we are observing now the pattern from point 4 as shown here in this figure. For point 4 the new position of the origin would be here, say it is point O_3. For point O_3, this edge it would be negative number. v_1 for O_3 is negative, while the position of upper edge is still $+\infty$, because ∞ is ∞ anyway.

Therefore, for point 4, the chord will start from negative value and it will go till $\tilde{B}^+$. And this is shown by point 4 in this cornu spiral plot. Now from point 4 we draw this vector which extend till point $\tilde{B}^+$ and this would be the phasor which will represent the resultant irradiance at point of observation 4. Now if we move a bit more up at point 5, then this would be the position of new origin, say it is O_4.

For point O_4, the coordinate of edge would be more negative number, while the coordinate of this upper edge again its $+\infty$, while this lower edge is more negative, this distance is larger. In this case the point 5 would be represented by this point on the cornu spiral and the resultant vector would be given by this dashed blue line. Therefore, what we saw is that in the positive direction or in the shadow region if we move down the phasor length is reducing.

And while in the negative direction, this is our negative direction. Why we do we call it negative direction? Because we are moving our point of observation upward, therefore the edge is going into the negative direction, the coordinate of the lower edge, here in this case it is only 1 because the other edge anyway at ∞, therefore the coordinate of edge is going more into the negative direction as we move the point of observation upward.

Now you see that at point 4 the phasor is very long. This was the length of the phasor, while for point 5 the phasor is this long, which is smaller than the phasor for point 4. And therefore as we move more up we will wind around the point $\tilde{B}^{-}$, this is our point $\tilde{B}^{-}$. We will wind around this point. And while winding around this point the phasor will go through several maximum or minimum.

It will go through several extrema. Therefore, the fringes in upward direction, it will see several maxima and minima, while in the lower direction it will decay smoothly.

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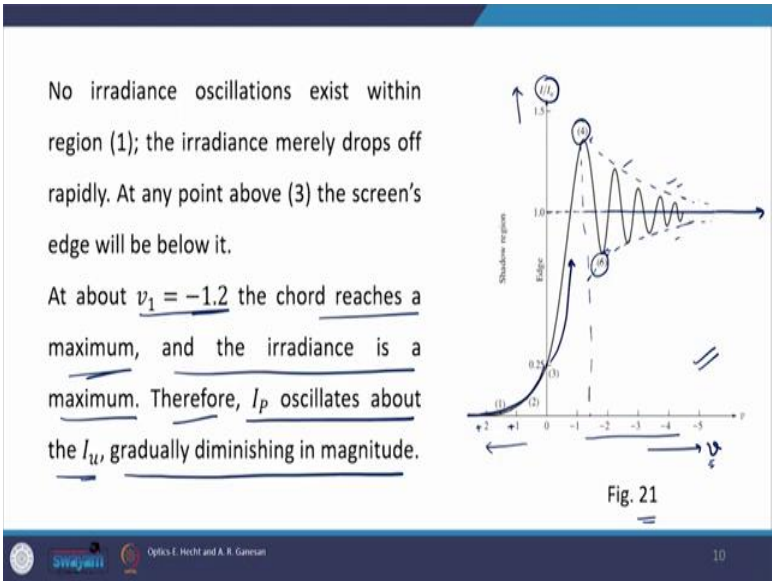

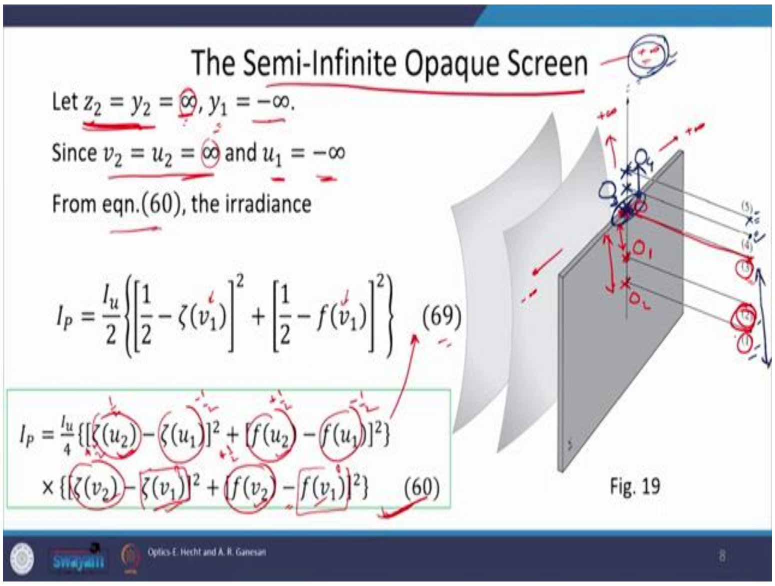

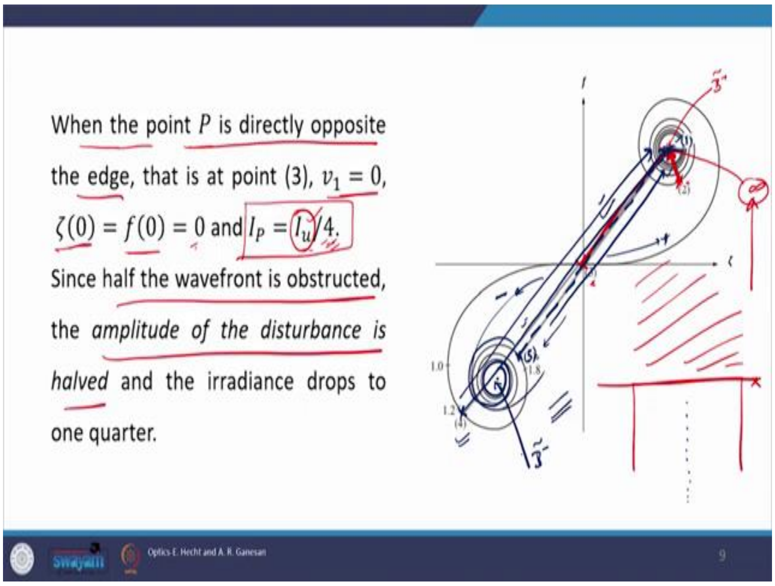

And which is being plotted, which is plotted here in this figure number 21. You see this is the position, this is the axis we are plotting v, while in the vertical direction we are plotting relative irradiance I/I_u. As long as v is positive, these are the positive number, v positive means, the point of observation is in the shadow region, in this region. As long as we are in the shadow region, then only the coordinate of edge is positive.

Then now as long as we are in the shadow region we see that the irradiance is dropping down smoothly. Now if we are in the other side, if we are in the illuminated region then v is negative, because if we are in here, if we are up, if we are above this edge, then the coordinate of edge will always be negative. And if the coordinate of edge is negative in that case we saw that the length of phasor which is plotted here, it goes through several maxima and minima, therefore we will see oscillations in irradiance.

And these oscillations are shown here. And this is the position of point 4, the irradiance at point 4, this point represents the irradiance at point 5. Several maxima and minima would be observed here if v is negative. Now at about $v_1 = -1.5$, which is this value, the chord reaches a maximum, and the irradiance is a maximum, which is shown here at point 4, which is shown here by point 4 here in this cornu spiral picture.

Therefore, I_p oscillates about I_u gradually diminishing in magnitude. This is I_u, this horizontal line is I_u value and this oscillation is about this I_u and slowly the amplitude is decaying down. If you plot the envelope, you see the amplitude is slowly decaying down as you move more into up. And if we are in some up point, which is very far from the edge, then it will ultimately lead to I_u, which is the irradiance due to an obstructed source.

Which is very much obvious also, if we are very far from the edge it is almost unobstructed and we will see irradiance which is equal to I_u. Now let us consider another case of diffraction, which is by a narrow obstacle. We have a studied diffraction pattern due to slit, narrow slit, but what will happen if we remove the slit with a wire. What I mean to say is that a slit is a opening in an aperture.

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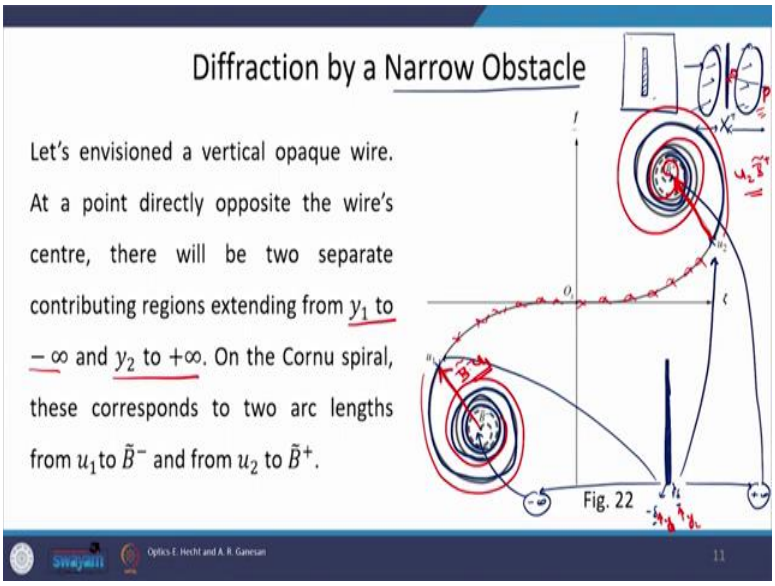

But what will happen if we move to a case which is almost, opposite means, the open portion in the slit is now replaced with a wire and the closed portion is replaced by opening. Here in the case of slit, the light is only allowed passed through this slit, but here in this case of wire, the light is allowed to pass through all spaces, except for the places where wire is there. Now this is the wire, this is the point of observation P, O is here.

Now you see that light is coming from this region and this region, but light is not coming from the places where wire exist. Now the 2 regions, we have 2 regions where from we are receiving light. And they will give some diffraction pattern. And therefore, the corresponding chord, which we can draw in cornu spiral would be given by these lines. This is the first spiral, and this spiral is extending very close to $\tilde{B}^{-}$.

And the second spiral start from u_2 and it is going to almost to $\tilde{B}^{+}$, why, because this is the wire which is inverse of singles slit. Now you see that the wire is obstructing the light which is coming from the source and you can assume that the light which is received at the screen it extend from this point, say this point is named as $+\delta$, and then it is extending till $+\infty$ and this edge say coordinates $-\delta$ and it is extending till $-\infty$.

Now this plus infinity point would be expressed by $\tilde{B}^{+}$, $-\infty$ point would be expressed by $\tilde{B}^{-}$ and this $-\delta$ and $+\delta$, say these are represented by these 2 points u_1 and u_2. It means we have 2 chords now due to wire, narrow obstacle, and in between these 2 chords there are a chord or there is a region wherein there is no chord, the chord is nonexistent between point u_1 and u_2 on Cornu spiral.

Here in I repeat, here in what we considered is that we envision a vertical wire and the point of observation is a point which is directly opposite to the center of the wire, which is shown here in this figure. The origin is at the center of the wire and P is directly opposite to O. It means from point of observation P, we are seeing or the system is very much symmetric. Now if you plot this case in Cornu spiral, there we get 2 chords.

Why? Because there are 2 contributing regions extending from y_1 till $-\infty$ and y_2 till $+\infty$. These values are nothing, but these are y_1 and y_2, $-\delta$ and $+\delta$, since the wire is very thin, therefore I prefer to write δ in their width, but these are the coordinates, y_1 and y_2. They are extending from y_1 to $-\infty$ and y_2 to $+\infty$, and therefore on the Cornu spiral these corresponds to 2 arc length from u_1 to $\tilde{B}^{-}$ and from u_2 to $\tilde{B}^{+}$.

These would be the 2 chords or 2 arc lengths, which would represent the irradiance at point of observation P due to the narrow obstacle or narrow wire. Now if we move our self laterally, then these spirals will slide up or down as per the rules which we discussed in last class as well as today. Now since earlier we were having only one arc in the Cornu spiral and then we know how to calculate the irradiance.

There we have to just draw a phasor and the length of that phasor depicts the irradiance at the point of observation P, but here we have 2 chords. Now in this particular case the situation would be quite different. For the lower chord the resultant would be given by this phasor, which is $\overrightarrow{B^- u_1}$. For upper ah chord the resultant would be given by this phasor, which is$\overrightarrow{u_2 B^+}$. And this is $\overrightarrow{B^- u_1}$.

Now we have 2 phasors. What would be the resultant of the 2? We will have to add them up vectorially. Now as you can see from the figure, from the geometry that they will never add up to 0, you will always see some finite value of irradiance at the point of observation P as long as it is symmetric.

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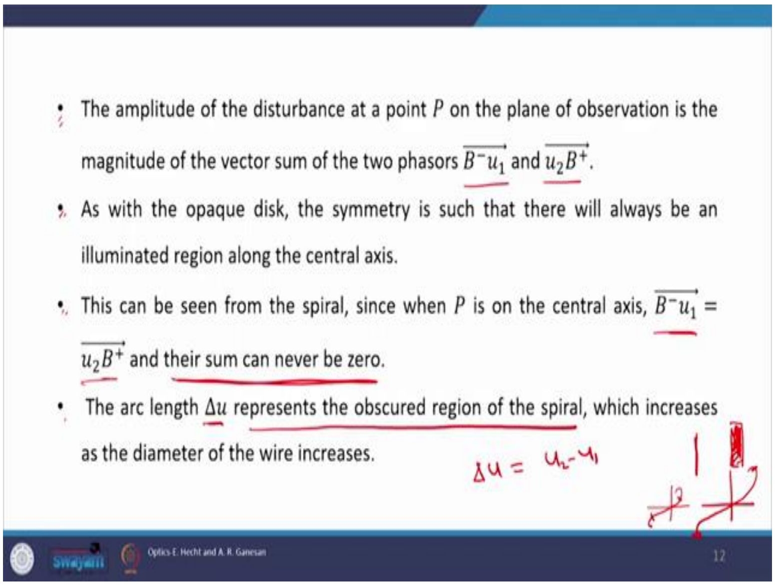

There are few points which we should note the point number one says, 'The amplitude of the disturbance at a point P on the plane of observation is the magnitude of vector sum of the 2 phasors $\overrightarrow{B^- u_1}$ and $\overrightarrow{u_2 B^+}$. If you add up these 2 phasors vectorially this will give us the resultant irradiance at a point of observation P. As with the opaque disk, the symmetry is such that there will always be an illuminated region along the central axis.

Here we will always get some light, some irradiance at the central axis and this is also seen in the case where we covered the source with some opaque disk and observed irradiance along the axis and there too, the irradiance was not 0. And this can be seen from the spiral, since when P is on the central axis $\overrightarrow{B^{-}u_1}$ arc is equal to $\overrightarrow{u_2B^{+}}$ and their sum can never be 0 and therefore we will always see some non-zero irradiance along an axis which is passing through point of observation P and starting at origin O.

The arc length Δu represents the obscured region of the spiral and what is Δu, Δu is nothing but $u_2 - u_1$. It represents the obscure region of the spiral and which increases as the diameter of wire increases, which is very much obvious, if the wire is thin then in the Cornu spiral these points are very close, but if wire is thick, for thick wire on the Cornu spiral these points would be very far. And therefore, it directly corresponds to the obscured region of the spiral.

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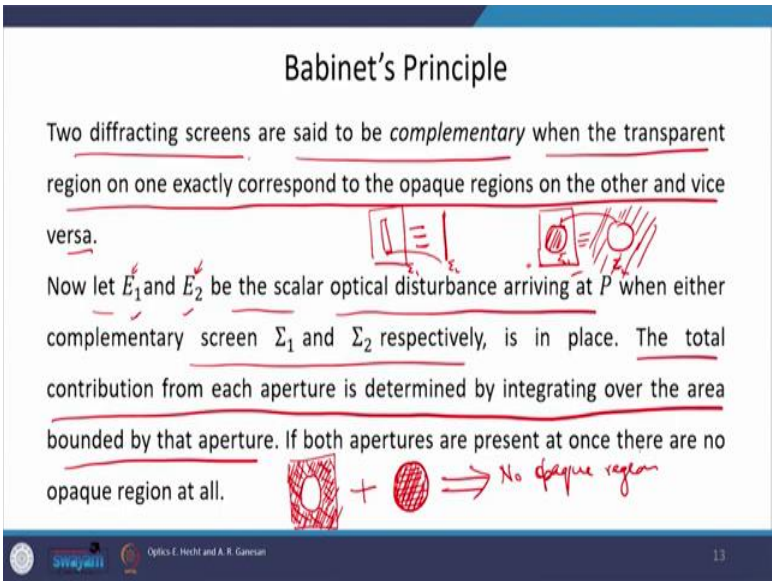

With this let us move to a new topic which is Babinet's principle. Now the Babinet's principle says that 2 diffracting screens are said to be complementary when the transparent region on one exactly correspond to the opaque region on the other and vice versa. Now in this Babinet's principle let us define what a complimentory screens are. Now 2 diffracting screens are said to be complementary when the transparent region on one exactly corresponds to opaque region on the other and vice versa.

And the best example would be the example of seeing this thin slit and thin wire. Now these 2 screens are said to be complementary because if you place this thin wire on top of this single slit then this wire will cover up the slit and we will have a completely opaque screen. Now therefore, these 2 screens are said to be complementary. These 2 screen which are Σ_1 and Σ_2. Similarly say we have a circular aperture and circular disk.

Now the circular aperture has this opening and circular disk has this opening, it is extending till ∞. These 2 screens will also be said to be complementary. Why? Because if you take this disk and put it on this circular aperture then it will cover up the aperture and you will have a opaque screen till ∞. Then any such screens are said to be complementary.

Now let us assume that E_1 and E_2 be the scalar optical disturbances which are received at point of observation P, when either complementary screen, Σ_1 and Σ_2, respectively is placed. Now this statement says that if we put a, say for example, circular aperture and then observe it diffraction pattern, then say it is E_1 and if we put a circular disk and then observe the pattern on the point of observation P then the corresponding field is E_2.

And if these E_1 and E_2 are the field due to this complementary screen then the total contribution from each aperture is determined by integrating over the area bounded by the aperture. And if both aperture are present simultaneously then there are no opaque region at all. This can be understood as follows. Now suppose this is a circular aperture wherein this portion is opaque. Now suppose this is circular disk where this portion is opaque.

Now if both the apertures simultaneously, then it will lead to a case where everything is open, then we will get no opaque region. Because this disk stops the light which falls on the area of the disk, while this circular aperture stops the light which falls anywhere except for this opening and the shaded region represents the opaque portion.

Now if we overlap, if we put these 2 apertures simultaneously, then there would not be any opaque region, it would be completely free. The light will come unobstructed.

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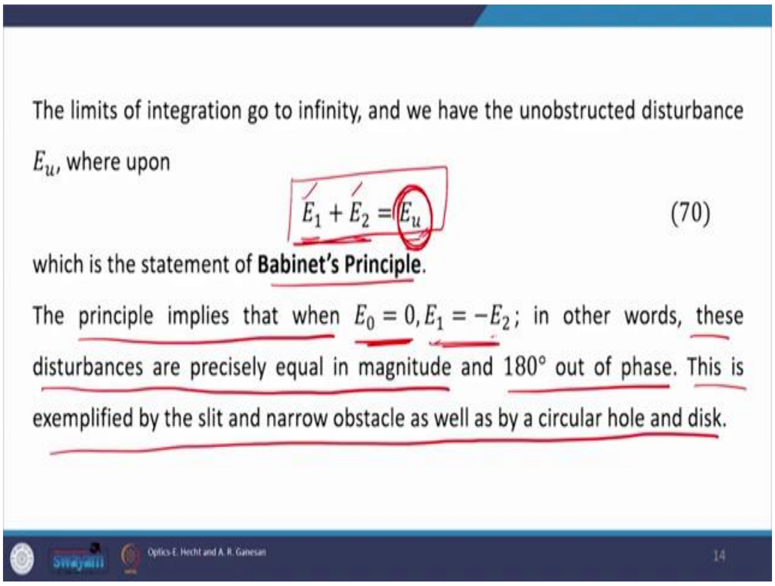

Now with this we can write that E_1, which is contribution from 1 aperture and E_2, which is contribution from the complementary aperture, then these 2 fields would be equal to E_u, which is the field due to unobstructed source. I repeat, suppose we have a circular aperture, in circular aperture, light can pass through this aperture while if light falls on the other portion of the screen it will be stopped.

And similarly suppose we have a circular disc and on this light falls, then it will not pass through while if it falls at anywhere else, then it will be allowed to pass. Now if we overlap these 2 screens then the portions which initially was obscured it would be open. Now say we have a disk screen if it is overlapped with the circular aperture screen, then what will happen? The whole aperture would be open now.

There would be no opaque regions because at the aperture by its definition, whenever we say circular aperture, then circular apertures means we have a big screen and there is a circular hole there in the screen. And this hole means there is a circular aperture. Now we have a disk, now the disk is a aperture wherein apart from the region which is covered by the disk everything is open.

Therefore, why these 2 screens are called complementary. Now if you put these 2 screen all together at a single place, same place, then what will happen everything would be open because the aperture for the disk would be region, which would be beyond the disk, apart from the disk everything is defined as aperture, everything which is open which is which is allowing the light to pass through.

While in the circular aperture the aperture is the circle, now if the 2 screens are placed together the circular aperture and the aperture in the case of disk would come together and it will form an unobstructed aperture, which is extending from minus infinity to plus infinity. In this case the light will go through without facing any obstruction.

And therefore, if we put the 2 complementary screen very close to each other or on top of each other, then we will see a field which would be equal to the field from unobstructed source. And this is the statement of Babinet's principle, this is what Babinet said. The principle implies that when $E_0 = 0$, when there is an obstructed source, then $E_1 = -E_2$.

Or when $E_u = 0$ then $E_1 = -E_2$. It means these disturbances are precisely equal in magnitude and 180 out of phase. The disturbance due to one aperture would be equal in magnitude, but would be out of phase by 180 degree, if they are produced by complementary screens. Now this is exemplified by the slit and narrow obstacle as well as by circular hole and the disk.

This is what we studied earlier in our last classes, where we saw that the pattern produced by single slit is almost equal to the pattern produced by narrow wire. Similarly, the pattern produced by circular aperture is almost equal to the pattern produced by disk, but this statement is only true if we are in the Fraunhofer region.

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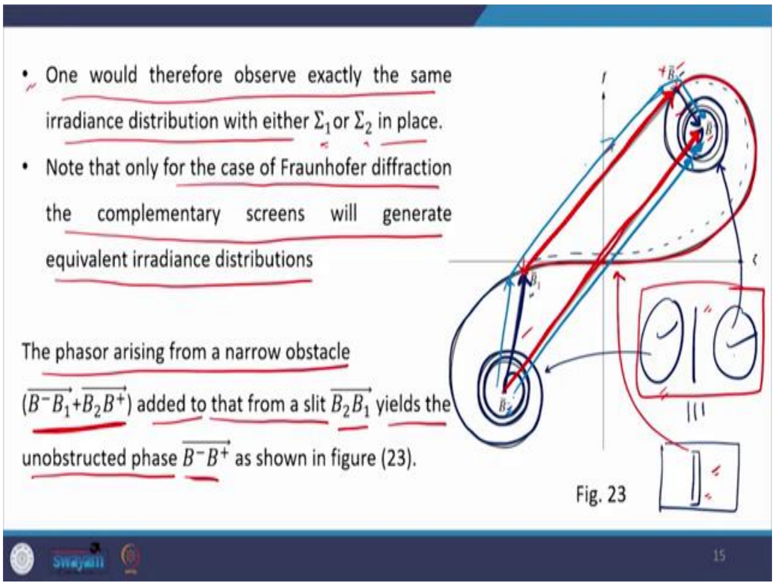

Now let us see what does these mean on Cornu spiral plot? In Cornu spiral we see that the point, if you join point $\tilde{B}^-$ and $\tilde{B}^+$ you get this phasor and this phasor represents irradiance due to unobstructed source. and this points which are $\tilde{B}_1$ and $\tilde{B}_2$, they are the points due to some aperture.

Now say one aperture are the points which it tells that that one would therefore observe exactly the same irradiance distribution with either Σ^+ or Σ^- in place where Σ^+ and Σ^- are complementary screens. Note that only for the case of Fraunhofer diffraction, the complementary screens will generate equivalent irradiance distribution.

While in the Fresnel region the irradiance would not be exactly equal or the Babinet's principle would not be exactly true. It would be more accurate if we are in the Fraunhofer domain. Now let us consider an example of single slit in narrow obstacle. We know that we have 2 phasors. Now let us represent these 2 phasors in blue color.

The first phasor it here are represented by this blue color, this starts from point $\tilde{B}_1$ and reaches to point $\tilde{B}^-$, while the second phasor starts from $\tilde{B}_2$ and it then goes till $\tilde{B}^+$. Now these 2 chords represent contribution from narrow obstacle and the corresponding phasor would be represented by these 2 vectors, which starts from $\tilde{B}^-$ to $\tilde{B}_1$ and $\tilde{B}^+$ to $\tilde{B}_2$.

These 2 vectors represents the corresponding phasors because in the narrow wire we see that there is 2 contribution, one from this left part of narrow obstacle and one from the right part of the narrow obstacle. From this part we get this chord and from this part we get this arc or this chord. And from just by measuring this vector phasors, which are extending from $\tilde{B}^-$ to $\tilde{B}_1$ and $\tilde{B}_2$ to $\tilde{B}^+$ we can have an estimate of the resultant irradiance at the point of observation P.

And to have the exact value of this irradiance at point P, we will have vectorially add up these 2 phasors and this region which is given by red arc, this represents a contribution from a thin slit which is a screen complementary to narrow obstacle. And the contribution from thin slit is plotted here by the red arc. Therefore, the phasor arising from a narrow obstacle which is given by this expression $B^-B_1 + B_2B^+$ added to that from a slit $B_2\ B_1$.

$B_2,\ B_1$ represents a phasor due to narrow slit now when it is added to B_2B_1 this yields an unobstructed phasor, which is equal to B^-B^+. When you add these to 3 phasors you get this line, which start from $\tilde{B}^-$ and its ends at $\tilde{B}^+$. Let me repeat from narrow obstacle we get 2 phasors which starts from $\tilde{B}^-$ and ends at $\tilde{B}_1$ and second starts from $\tilde{B}_2$ ends at $\tilde{B}^+$.

These 2 phasors are coming from narrow obstacle or narrow wire, when you add them up you get the resultant phasor. The complementary screen which is of narrow slit, it gives a phasor which starts from $\tilde{B}_1$ and its ends at $\tilde{B}_2$ which is given by this line. Now if you add all 3 phasors, then you can say that this phasor, let us pick a different color for addition. Now let us add this vector with this vector and then finally add this vector.

If you add all these 3 vectors then what you get is this line, which starts from at $\tilde{B}^-$ and it ends at $\tilde{B}^+$, which is a phasor due to unobstructed source, which is what Babinet's principle said. If you add the contributions from 2 complementary screen you will get a disturbance, which is of unabstracted source. Now this is all for today. Thank you for joining me. See you all in the next class.