Applied Optics
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Lecture: 04
Refraction from Single Interface

Hello everyone, welcome to my class. Today we will move further from our last class. In the last class, we talked about Fermat principle and then using this Fermat principle, we learned about law of refraction and law of reflection.

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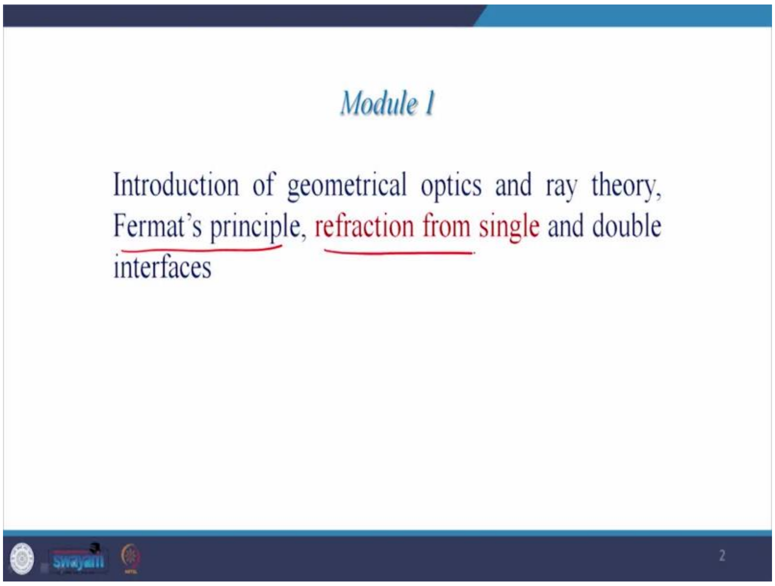

In today's class, we will talk about refraction from single interface. Now, before moving ahead, let us talk about some fundamentals of optics which helps us in deciding the ray path or which helps us in also in forming an image of a given object. Now, to form an image we will have to trace the ray path and therefore, we must understand how the ray path is traced.

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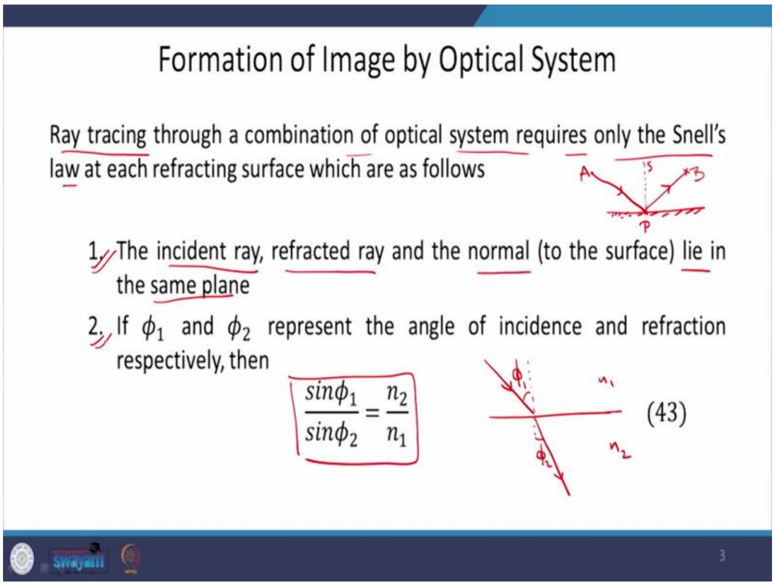

Now, the ray tracing through a combination of optical system requires only the Snell's law. If we know what Snell's law is, then we can easily trace the path of the ray in any type of medium with varying refractive index, whether the medium is homogeneous or inhomogeneous it does not matter. Not only with the in different kinds of medium, if we have some optical systems suppose, we have a lens, then too, if we know how to implement Snell's law then we can easily trace the ray through a single lens or through a combination of lenses, everything would be very easy.

But what are the Snell's law, how does it differ helps us in tracing the ray path? this is detailed here in point number 1 and point number 2 and this Snell's law says that the incident ray, the refracted ray and the normal to the surface lie in the same plane as we discussed in the refraction. Suppose, this is our mirror then suppose, the ray is starting from certain point A which is this point and then it travels to the mirror and it falls at certain point P then after a point P it reflects back to point B.

Now, we draw a perpendicular to point P at the Mirror. The first point says that incident ray means ray AP the reflected ray PB and the normal, normal means the line SP which is the dashed one which is on this surface of the mirror, they all must lie in the same plane, the AP, PS and PB they must lie in the plane of the paper. And the second point is that if φ_1 and φ_2 represents the angle of incidence and refraction respectively, then $sin\varphi_1/sin\varphi_2 = n_2/n_1$.

This we have also studied while studying the law of refraction through Fermat principle. Now, let us repeat it. Suppose this is the interface which separate two media of refractive index n_1

and n_2 and then ray which is falling on this interface which makes angle φ_1 with the normal to the interface. it will be refracted through an angle φ_2 in the second medium and then $sin\varphi_1$ and $sin\varphi_2$ the ratio between these two would be related to the refractive index through these relations. These two points constitute Snell's law. Now, we will apply these two points and see how to trace a ray through a single interface, single interface means single spherical interface.

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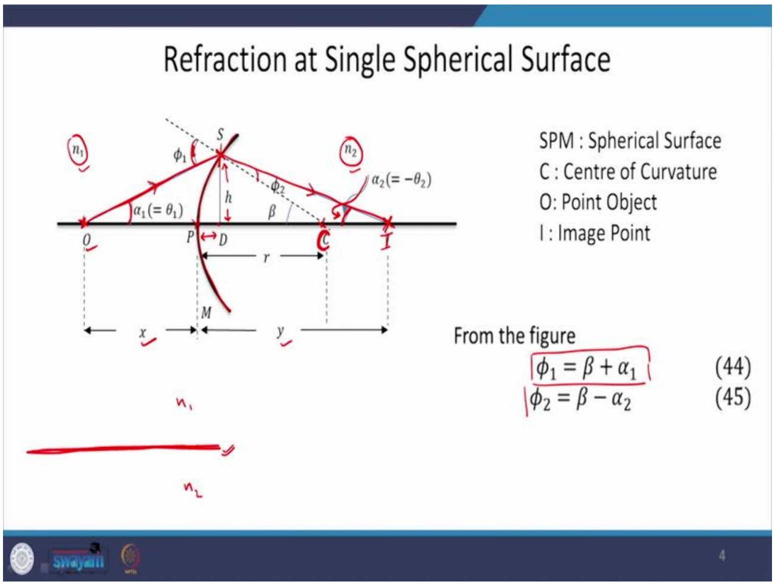

Now, what is single spherical interface. Till now, we studied that there is a medium of refractive index n_1 and there is a medium of refractive index n_2 and these mediums are joining at this by this line. This line separates the two media and this line we always drew as a straight line, but what if this line is not straight? What if this line is curved? This is the point which would be addressed in this slide. Suppose this is the curved line which separates the two media this SPM is the spherical surface which separates two media of refractive index n_1 and n_2 respectively and a source is kept at point O, an image is being formed at point I.

Now, we will analyze what is happening here in this figure. Now, since the source is point O, the ray will start from point O and it will fall at some angle at this interface, this curved interface. Now, where will the ray go now? To know the direction of the ray after refraction, we will have to first draw a perpendicular. Now, since the surface is, this SPM is curved and this is a spherical surface then it must have some center and suppose the center is situated at point C here.

Now, if we draw a perpendicular at point S. Here this perpendicular will pass through point C which is the center of this spherical surface. Now, again we will start from point O, the ray will fall and point S and then we will draw a perpendicular on this surface passing through S which will of course pass through the center of this spherical surface. Now, if the angle of incidence here is φ_1, angle of incidence is known, the refractive index of the two media is known which are n 1 and n 2 respectively.

Then by exercising Snell's law we can also calculate what is the angle of refraction which is φ_2 here. Once angle of refraction is known, we can easily draw the refracted ray which will ultimately fall at image point. Now, let us assume that incident ray which is OS ray makes an angle α_1 with the horizontal and the refracted ray makes an angle α_2 with the horizontal, α_2 is here.

Now, curvature of this curve sphere is such that length PDE is very small and this point S is at a height h from this point D. Now, since C is the center point, center of curvature then this distance, the distance from P to C would be r which is your radius of curvature. Now, we also assume that the object point O is at a distance x from the spherical surface and these distances are measured from point P. Similarly, the image point is at a distance y from point P. These are the definitions which are given.

Now, we will apply some geometry in this figure and then figure we can see that φ_1 is equal to $\beta + \alpha_1$ and similarly φ_2 is equal to $\beta - \alpha_2$ this φ_1 is here this is exterior angle and therefore, it would equal to $\beta + \alpha_1$ and similarly $\varphi_2 = \beta - \alpha_2$.

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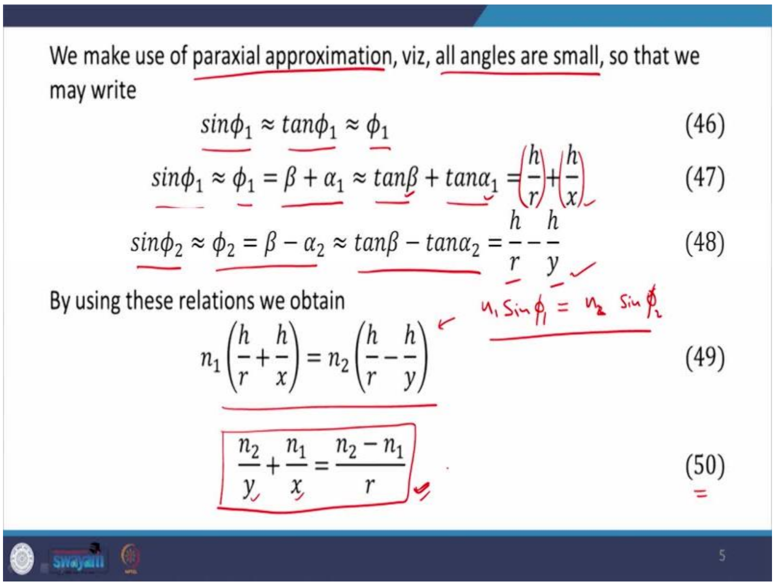

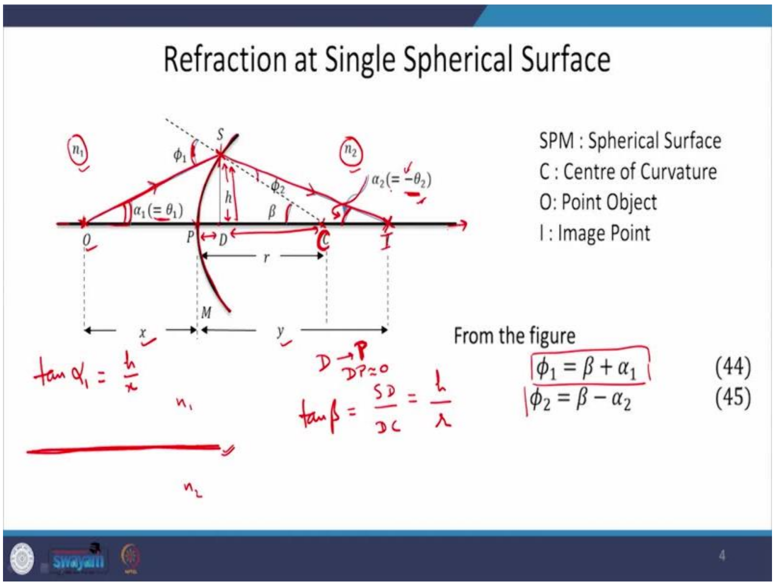

Once this quantity is known, we will know exercise paraxial approximation. what is paraxial approximation? Paraxial approximation says that we will consider only those rays which are very close to the principal axis of the system or the axis of the system and what is the axis after system? The axis of system is this axis of symmetry, the dark line which is represented by this dark line the OCI line or OI line. Now, if we consider the rays which are very close to this line only in that case α_1 would be very small.

Similarly, α_2 would also be very small. Now, what will happen if we do not assume this paraxial approximation. If we do not resolve to this paraxial approximation under that case, from point O the ray will emanate in all possible directions and it would be extremely difficult or almost impossible to converse all these rays to point I, at point I only those rays will be conversed, which are very close to OI axis or the principal axis or axis of symmetry of this optical system.

Therefore, we will resolve to, we will use this paraxial approximation, which says all angles are small or which says we consider only those rays which are parallel to this principal axis or which are very close to this principal axis. Under this approximation to φ_1 would be very small, α_1 would be very small, α_2 would be very small, and therefore, we can safely write that sinφ_1 is equal to or almost equal to tanφ_1 and which is almost equal to φ_1. If $sin\varphi_1$ is almost equal to φ_1 and which in the previous slide we saw that φ_1 is equal to $\beta + \alpha_1$.

Therefore, we can write that $\varphi_1 = tan\beta + tan\alpha_1$, let us go back to the previous slide $\varphi_1 = \beta + \alpha_1$ and if φ_1 is very small then we can safely write that this is almost equal to$tan\beta + tan\alpha_1$. What is tan β? β is this angle if you take the tan of this angle then this would be equal to perpendicular by base then beta here would be equal to SD/DC. What is SD? SD is h. What is DC? Now, if we are under paraxial approximation, the point D would be very close to point P, and this DP distance you can neglect, the DP would almost be equal to 0.

Therefore, DC would be equal to PC which is equal to r and therefore, we can write tan $\beta = h/r$. Similarly, we can write tan$\alpha_1 = h/x$ which is very much clear here in this figure. α_1 is this angle and therefore, tan$\alpha_1 = h/x$,this is what is written here in this equation number 47.

Similarly, we can calculate the expression for sinφ_2. Since $\varphi_2 = \beta + \alpha_2,$ which again would be equal to $tan\beta - tan\alpha_2$and this will ultimately give us this relation equation number 48. Now, we will apply Snell's law here what Snell's law says Snell's law says $n_1 sin\varphi_1 = n_2 sin\varphi_2$, we are in the habit of callings this angle of incidence φ_1, φ_2 and therefore, here in your view in the bracket you see that $\alpha_1 = \theta_1$ and $\alpha_2 = -\theta_2$.

This minus, I will talk later about. Now, we will apply this and once you apply this, you will see that you will land up with this relation and from this relation we can derive that $\frac{n_2}{y} + \frac{n_1}{x} = \frac{n_2 - n_1}{r}$. This equation number 50 relates the refractive index and the distance of object and image from the spherical surface. Now, this relation we are not familiar with, it seems that we are not familiar with this relation, but it would be but before moving ahead, let us talk about the sign convention which we discussed here you can see that I equated $\alpha_2 = -\theta_2$.

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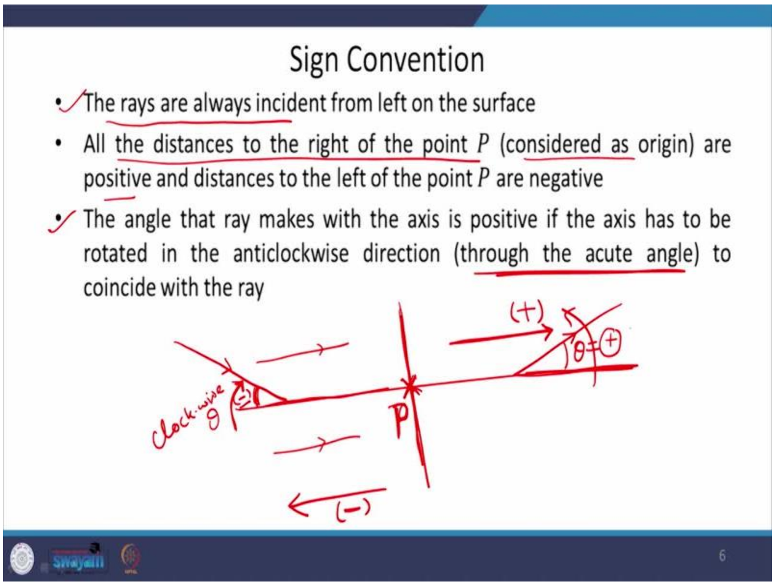

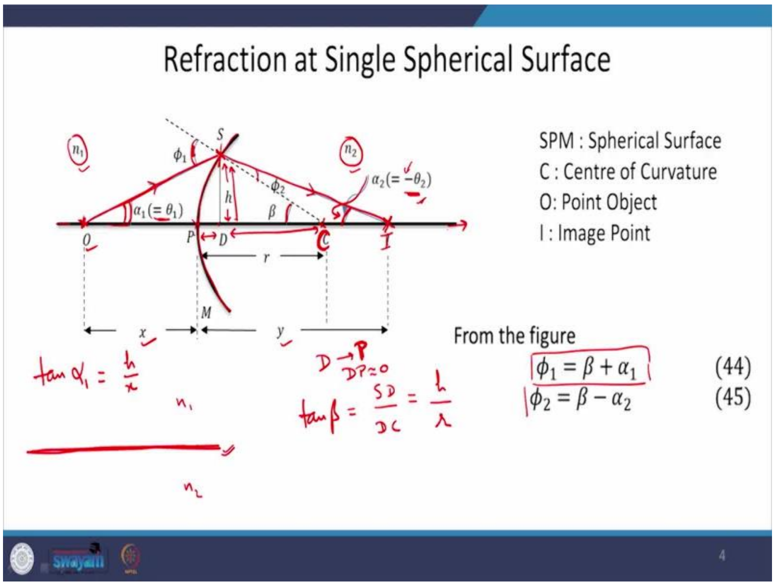

Now, what is this sign convention? Suppose you have an axis system with origin here at the center. Now, if the curvature which separates the two refractive index medium is here, which is either a straight line or a curved surface and its central point missed this point P is assumed as the origin here this point is assumed as origin O, let us not call it O let us call it P. Suppose the origin is point P then we will try to understand how the signs of distances are taken now, the convention is that the ray always incident from the left on the surface.

Now, this is a convention which is followed by the optics community and which is very much helpful in drawing the different ray path and understanding the mathematics which follows, which helps us in properly forming the image height, image distance and many other things. The first point in the ray convention is that rays are always incident from left on the surface whenever you draw a ray, you always draw from the left and make it incident on the optical element which is centered here at point P. All the distances to the right of the point P, of course, P is considered as origin then all these distances are positive, it means in this direction the distances are positive.

And the distances to the left of P, it is considered negative, that any distance if you measure in this right-hand direction to P, its a positive and any distance you measure before P which is on the left to P, it is always represented as negative. The third point the angle that ray makes with the axis is positive. If the axis has to be rotated in the anti-clockwise direction, what does it mean? Suppose on this axis a ray falls like this slanted. Now, if you want to make this horizontal axis overlap with the direction of the ray then you will have to rotate this axis in this direction. This rotation must happen always through the acute angle.

Again of course, this is our acute angle therefore, we will have to rotate this axis to clockwise direction, you can see the direction of this arrow is along the clock with this is clockwise rotation and this is of course a acute angle. Now, again I read this statement the angle that ray makes with the axis is positive if the axis has to be rotated in the anti-clockwise direction. Now, you see that here the rotation is in clockwise direction instead of being anti clockwise it is in clockwise direction therefore this angle would be represented in minus, had it been a clockwise rotation like for this ray?

This is ray which is moving in this direction then to coincide this principal axis with the direction of the ray you will have to rotate it in anti-clockwise direction through acute angle of course, if this rotation is in anti-clockwise direction then this angle would be positive this θ would be measured in positive here and here θ would be negative. Let me read, write very clearly.

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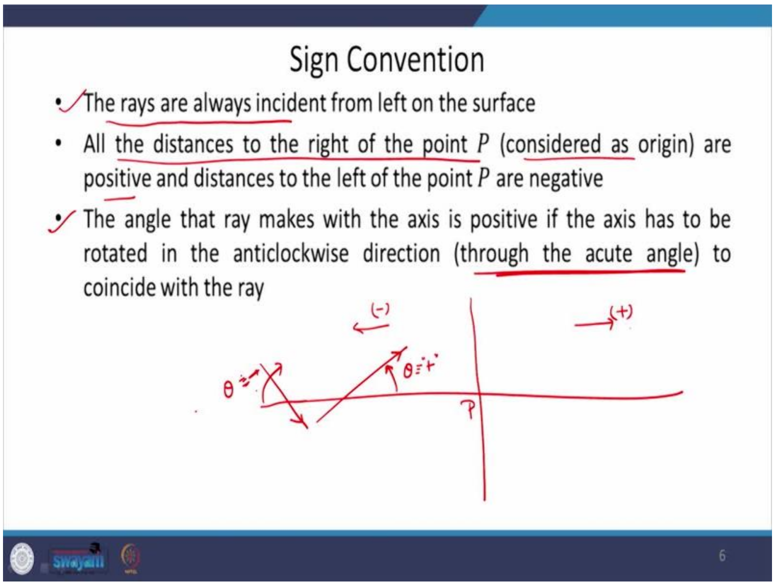

This is our axis, this is our point P. All the distances in this direction are positive. All the distances in this direction are negative. If the ray is moving in this direction then the axis has to be rotated in anti-clockwise direction therefore, θ would be positive and if the ray is moving in other directions, say this direction then the rotation would be in clockwise direction and θ would be negative here.

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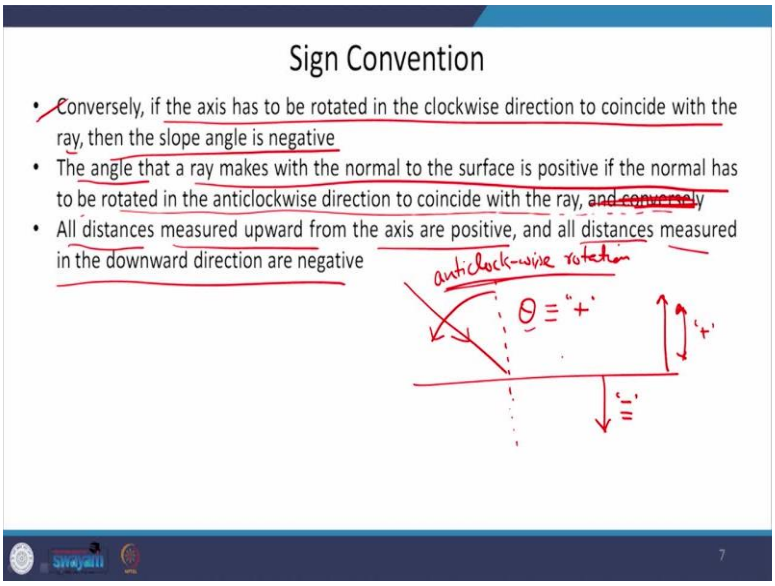

This segment negative θ is represented here if the axis has to be rotated in the clockwise direction to coincide with the ray then the slope angle is negative. Next point, the angle that a ray makes with the normal to the surface is positive if the normal has to be rotated in the anti-clockwise direction to coincide with the ray the same point which was stated for the axis of the system is now being stated for the normal. Suppose this is an interface and there is filing here and this is the normal to the interface now, if this normal has to be rotated to coincide with the direction of the ray then you see that this is the direction of rotation which is anti-clockwise.

This is anti-clockwise rotation and therefore, this angle this would be what? Since it is anti-clockwise rotation then this angle would be treated as positive angle. I repeat the angle that a ray makes with the normal to the surface is positive if the normal has to be rotated in the anti-clockwise direction to coincide with the ray instead of axis now it is the normal which we are rotating. Now, all the distances measured upward from the axis are positive and all the distances measured in the downward direction are negative this is not of any use.

Because if there is some object which is upright then this distance would be positive and if object or image is facing downward then it would be that the height or the depth would be negative quantity here. Make a point that here the rotation is in anti-clockwise direction therefore, the angle is positive if it would be in clockwise direction the angle would be negative.

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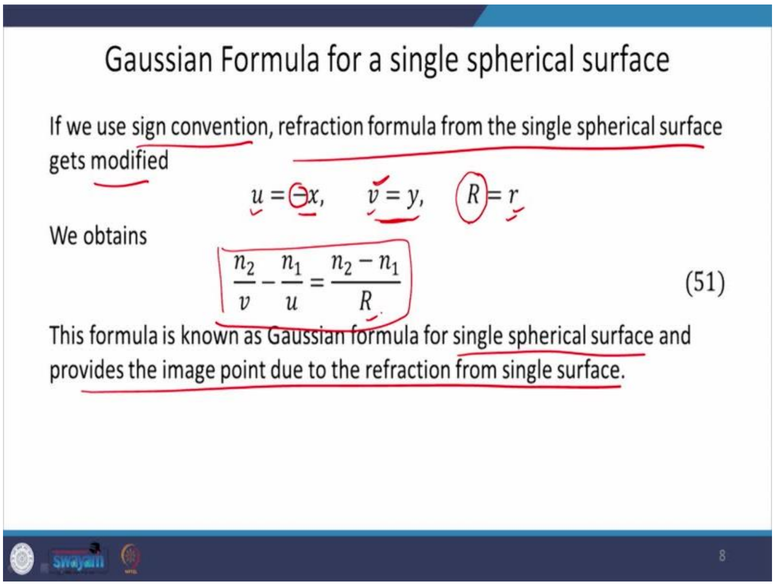

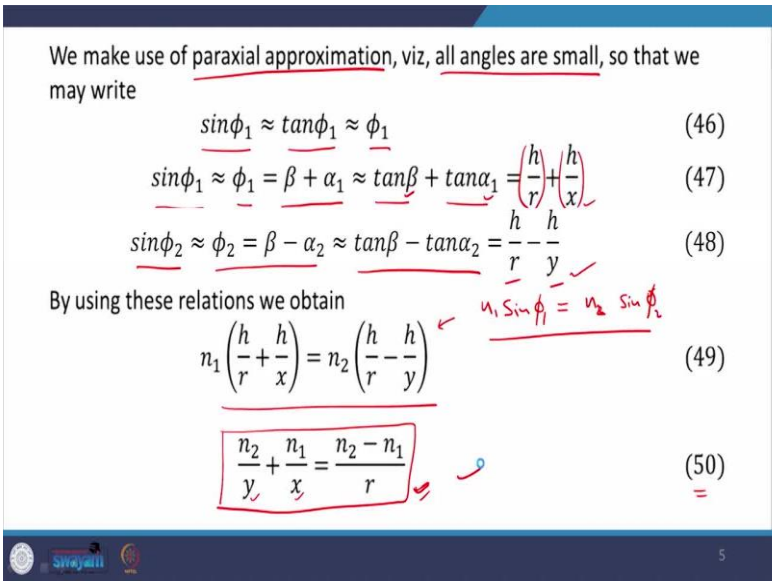

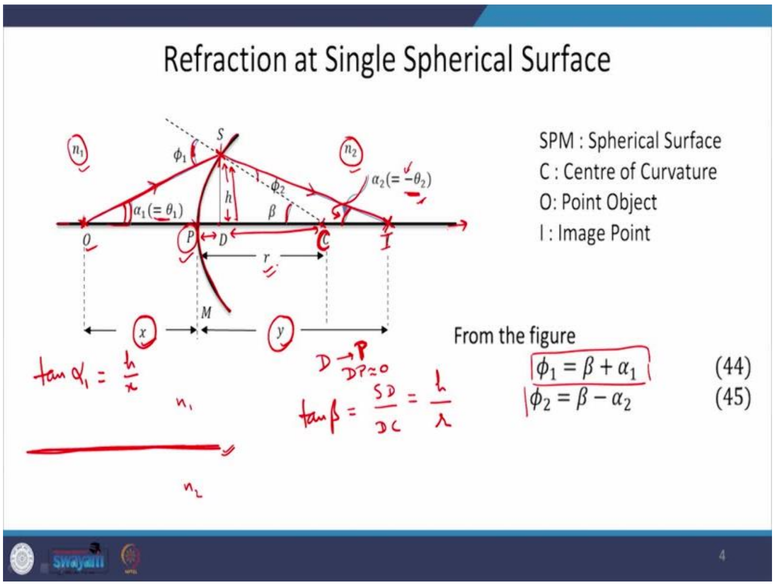

Now, if we apply this in the formula which we derived in this previous slide, equation number 50. Then let us see what do we get now, if we use the sign convention, the refraction formula from the single spherical surface gets modified. How does it modify? Now, here in this figure, you see that the object is on the left-hand side of P it means x using sign convention, x would be a negative quantity while y would be positive. Now, let us follow the sign convention and let us introduce a new variable u which represents the distance of the object from the point P.

Now, this distance would be minus x. Now, minus is coming from the sign convention. Similarly, y which is the distance of image point I from point P would be positive here and therefore, the new variable which we introduced would be equal to plus y and what is the radius of curvature the radius of curvature you can see this figure which is represented by r it is on the right hand side of point P. Point P is here and the radius of curvature is on the right hand side therefore, it would again be a positive quantity and therefore, we introduce a new parameter which is R which represents the radius of curvature and this is equal to plus r.

Now, let us replace x y and r with the newly introduced variable which takes into account the sign also which are u, v and R. Now, this equation, equation number 50 gets modified and the new form of equation 50 is this: $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$.

This is the famous Gaussian formula and you must have seen it in your in your previous classes or in your junior classes and this formula is valid for single spherical surface and this provides the image point due to refraction from single surface, single spherical surface or non-spherical even because the radius of curvature decides the surface is spherical or non-spherical. And this is all for this lecture and see you in the next lecture. Thank you.