Applied Optics
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Lecture 38
Diffraction Grating

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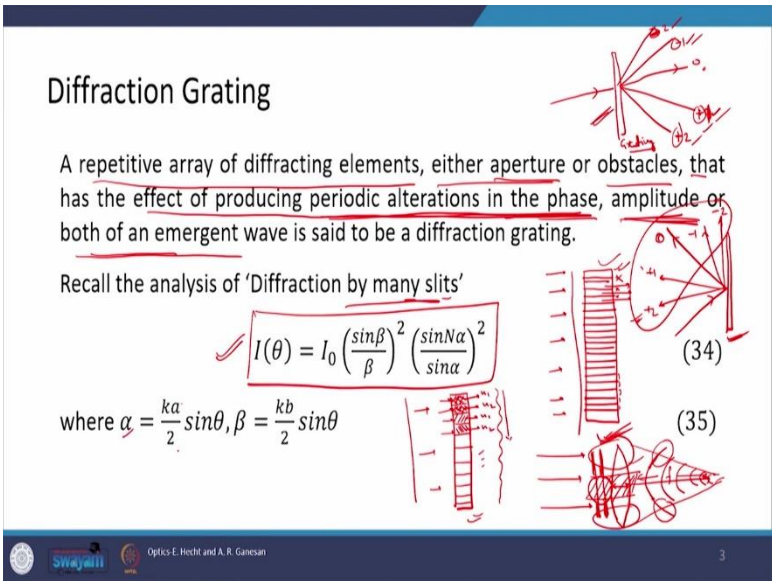

Hello everyone, welcome to my class. Today we will talk about diffraction grating. A diffraction grating, a repetitive array of diffraction element either aperture or obstacle that has the effect of producing periodic alteration in the phase, amplitude or both of an emergent wave is said to be a diffraction grating. It means that suppose we have an aperture and then if we alternatively put closed and open aperture or if we put multi slits then this arrangement is called grating, this we have also talked about while discussing multi slit arrangement. Now, here you see that these red lines, they represent closed portion while this separation between the two red line that is the white background, it represents the open portion.

Now, if you shine plane wave on this arrangement then what will happen is that the portion of the way which is falling on the opaque part or closed part of this aperture that would not pass while the portion of the light which is falling on the open part or the transparent part it will be allowed to pass such an arrangement is called grating. Now, the closed and open portion or apertures it can also be replaced with periodic alteration in the phase.

Now, here when I am saying that a part is closed and next one is open and then next part is again closed then the part which is next to this close part is again open, this alternative arrangement of open and closed or transmitting or non-transmitting apertures it leads to an amplitude modulation.

Suppose this is the plane wave which we launched and due to the amplitude modulation created by this grating, the output here you will see that a portion would be absent from here and this part will allow the transmittance of the light here again absent, present, absent, present. Then you see that some part of this diffracting element are allowing the transmission of the light while some part are stopping it, it is mimicking the multi slit arrangement, where we have open slit at few places and these open slits are separated by a closed portion, it is a periodic arrangement, the same periodic arrangement is also here in the grating too.

But, when we talk about open and closed portion, then it means that we are modulating the amplitude of the emergent wave, the same output can also be achieved if we modulate the phase, how the phase can be modulated, suppose we have a glass plate then what we can do is that we can divide this glass plate in small strips and each strip may be assigned a periodic refractive index variation. Now, here what you see is that this strip has refractive index n_1, while this n_2, this again n_1, this n_2 and so on.

Now, you see that these strips which are of refractive indices n_1 and n_2, they are periodically varying, they are alternating and due to this alternation, if we launch a plane wave then what will happen is that a part of the wave will see higher refractive index, while the other part will see lower refractive index and due to this periodic arrangement, this periodicity will also be imprinted on the wave.

And therefore, ultimately, we are getting almost same type of pattern, these two types of grating will produce the same effects ultimately. Now, you will see that just by changing the refractive index, we are playing with the phase of the wave which is passing through the strip of particular refractive index, the same phenomena was also utilised in lenses. Now, if you remember when we were talking about convex lens then what we said is that if we launch a plane wave then what happens is that due to the thickness of the lens, this part of the wave it gets slowed down while the part which is falling on the thinner ends of the lens it also gets slowed down, but the extent of slowness of these upper and lower part of the wave is smaller than that of this middle portion of the wave.

And therefore, here on the right-hand side of the lens, we again get away front but the central part of the wavefront is delayed while the upper and lower portions these are ahead in time and this is why the wavefront after refracting through to a convex lens, it gets curved and the curvature is such that it focuses the light with propagation.

And therefore, we see that the parallel beam of light which falls on the concave lens, it gets focus on the other side of the lens and this distance is known as focal distance and this is why this lens is called a convex lens. Similar analysis can also be made for concave lens. Now, here too, we are using this refractive index variation, but here in case of convex lens, the refractive index is not varying but the thickness of the medium is varying here, in the center the lens is thick while at the top and the bottom the lens is thin.

Therefore, the light which is falling on the top and the bottom, they are travelling a thinner medium of higher refractive index while the light which is travelling through the center of the lens it is travelling through a thicker portion of the glass and therefore it gets delayed and due to this relative delay, the plane wavefront gets curved and the curvature is such that the wavefront get focuses. The wavefront gets focused, the different part of the light gets modulated differently and since the refractive index is periodic due to this phase modulation, we see a type of diffraction pattern which is similar to the amplitude modulated diffraction pattern.

Therefore, to produce grating, we can either use periodic alteration in phase or in amplitude or sometimes both. Now, usually when we have a grating and we launch a light then what happens is that it produces a 0-order pattern and apart from the 0 order, it also produce higher order patterns, these are called first order diffraction, second order diffraction and these diffraction patterns are symmetric around the 0 direction or the main axis.

Therefore, first order is produced both in left and right-hand side of the central pattern these orders are differentiated by assigning plus or minus sign before the order numbers. Therefore, here we are calling +1 and while on the other side we are assigning it or we are calling it -1 order, these are first order, second order, minus first order, minus second order and so on.

And such type of grating, this is our grating and such type of gratings are called transmissive grating or transmission grating. But there are also gratings which produce diffraction pattern in reflection direction, are in the same direction from where the light is being launched that we are launching light here.

Now, in the first medium itself now you see different orders here it is 0. Now here, it is a + 1, +2, here it is -1, -2, different orders of diffraction patterns are generated in reflection mode here. Therefore, this type of grating is called reflective grating. And the first kind of grating is called transmission grating and this is called reflective grating or reflection grating. As I said before this grating is analogous to many slit or multiple slit arrangement.

Therefore, we can use the mathematical formulations which we developed in case of multiple slits. Now, recall the analysis of diffraction by many slits. Now, from there the irradiance is given by this expression where α is (ka/2) sinθ and β is (kb/2) sinθ. Do remember that b is slit width, in case of grating, it is the width of opening, open portion and a is center to center separation in between the slit. Now, the different orders of grating, it can be calculated by imposing the conditions of maxima. Now, where to find the principal maxima?

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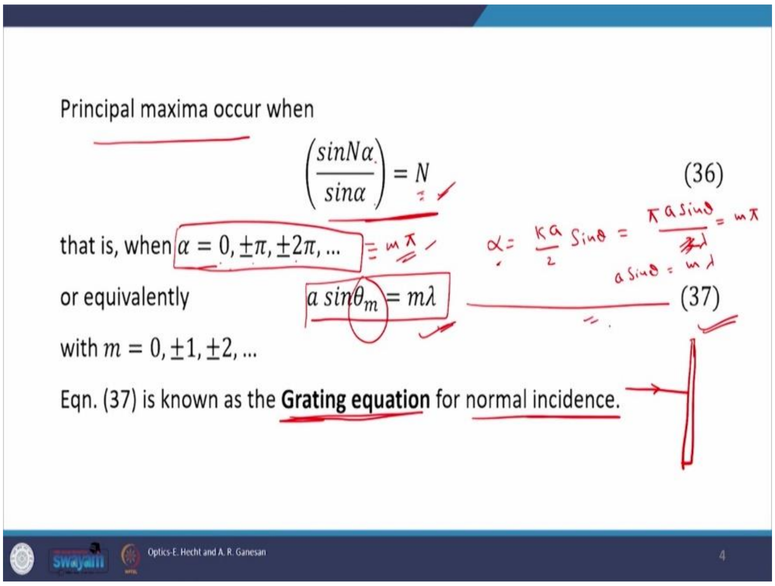

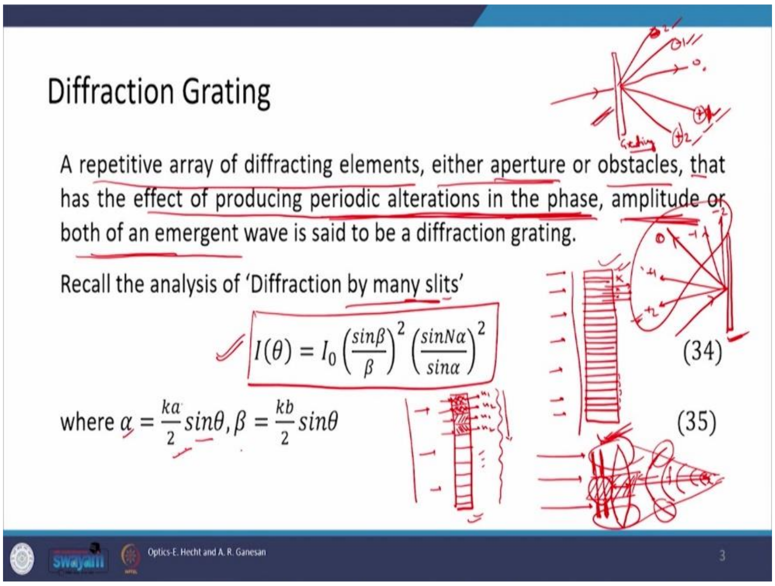

Principal maxima occur when this term is equal to N, $sinN\alpha/sin\alpha = N$, this we have already done in case of multiple slit arrangement and $sinN\alpha/sin\alpha = N$ for these values of α when $\alpha = 0, \pm\pi, \pm 2\pi$.. But we know that $\alpha = (ka/2)sin\theta$ here, $\alpha = (ka/2)sin\theta$ or $\alpha = (\pi a/\lambda sin\theta$ where k is equal to $2\pi/\lambda$.

With this substitution now, here you know that α is equal to integral multiple of π, if you equate it with integral multiple π then you get a $\sin\theta = m\lambda$ and this is the condition which you derive now, sorry 2 is gone from here, k is $2\pi/\lambda$ and 2 and 2 will go away.

Therefore, this condition where which says that α must be integral multiple of π is equivalent to a $\sin\theta_m = m\lambda$, where θ_m is angle of diffraction and m represent the order of diffraction, m can take any values varying from minus infinity to plus infinity, m is an integer it is 0, ± 1, $\pm$ 2, this equation is for finding principal maxima and diffraction grating.

And therefore, this equation is named as grating equation, $a\sin\theta_m = m\lambda$ equation is called grating equation, but this equation is valid only for normal incidence. Suppose if you have a grating here and if the incident light is falling normally on the plane of the grating, then only we can apply equation number 37. And equation number 37 is called grating equation for normal incidence.

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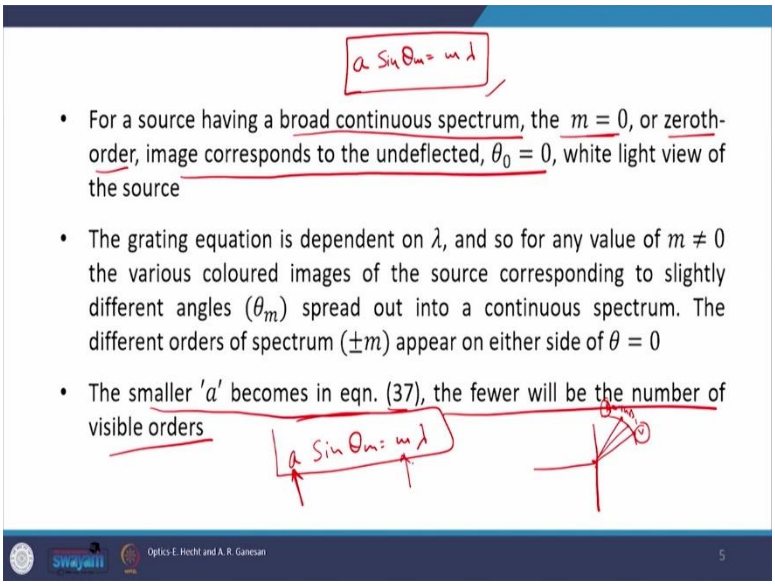

Now, there are few points which we can take away from this grating equation. I will write here the grating equation $a\sin\theta_m = m\lambda$. Now, for a source having broad continuous spectrum, the m is equal to 0 or zeroth order image correspond to the undeflected white light view of the source which is very much clear, whenever we talk about zeroth order diffraction pattern than we will have to substitute m is equal to 0 and when m is equal to 0, θ_m would be 0 or θ_0 is equal to 0 because m would be substituted by 0.

Now, irrespective of the wavelength, all the colors will go in the same direction for zeroth order diffraction pattern. And therefore, we will find white light in θ is equal to 0 direction, the second point is that the grating equation which is $a\sin\theta_m = m\lambda$ is dependent on λ, λ is there on the right-hand side of the grating equation.

Therefore, the grating equation is dependent on λ and therefore, if m is not equal to 0, the different colors will form their diffraction maximum at different angles, the various color images of the source corresponding to slightly different angle spread out into a continuous spectrum. And the different order of a spectrum appears on either side of θ is equal to 0 direction.

What this sentence says is that if the incoming light is white light and we are talking about order of diffraction, which is different from 0 order diffraction then different colors will go in different direction and they will form their respective maxima at different angle of diffraction. Now, since a $\sin\theta_m = m\lambda$, for non 0 value of m, $\sin\theta$ is proportional to λ, larger the λ is larger will be the angle of diffraction.

Now, therefore, this is how the white light pattern look like, the first comes the violet color and at last will come the red color, for the smaller λ, θ would be smaller. Now, again let us look into the grating equation. Now, suppose we are talking about a particular diffraction order with a particular wavelength, it means the right-hand side of this grating equation is fixed, m is fixed as well as λ is fixed.

In this case the smaller a becomes the fewer will be the number of visible orders, let me correct myself, in grating equation in the right-hand side there is m and wavelength. Now, suppose we have a grating and we are launching some light of particular wavelength on it. Now, for smaller a, the smaller a becomes in equation 37, the fewer will be the number of visible orders, why? Because, as the equation suggest, this is the relation, now if you reduce a then m will also be reduced.

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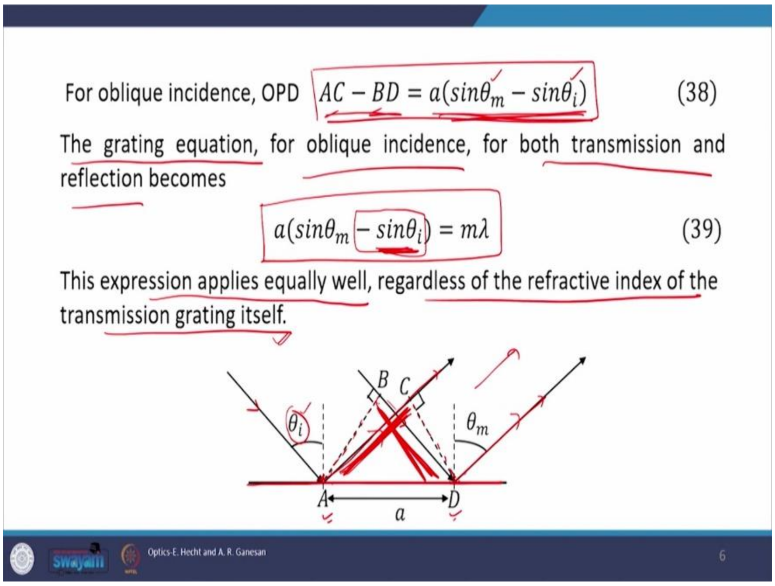

Now with this we will move ahead. Till now we were talking only about normal incidence now what will happen if light falls on the grating obliquely. Now, for oblique incidence, now suppose this is our grating and this is the ray, the parallel beam of light which is falling under grating at a certain angle of incidence here it is θ_i. Now, again suppose that this grating is reflection grating therefore different orders of diffraction would be formed in the same medium, in the first medium itself.

Now, angle of incidence here is θ_i, and these two rays are falling the beam is falling on the grating and they are reflecting and going back in this direction. Now, from this figure you can calculate the optical path length difference between the rays which is falling on the grating and then reflecting back into some direction.

The optical path length difference would be equal to AC which is this length minus BD which is this length. How did we calculate it the traditional way we dropped the perpendicular from the point of incident to the other ray. Similarly, here this is the perpendicular and on incidence the extra path is BD. On reflection the extra path is AC.

The total path length difference would be the difference between the 2 which is AC-BD. And from the figure itself, you can calculate what AC and BD is, the difference between the two point of incidence is a and the angle of incidence and reflection or diffraction is known, angle of incidence here is θ_i and θ_m is the angle of refraction for this particular ray bundle for this particular beam.

Therefore, the optical path length difference would be given by $a(sin\theta_m - sin\theta_i)$. Now, in grating equation we were having $asin\theta = m\lambda$ where $asin\theta$ if you look back then it this grating equation resembles with that of the Young's double slit experiment. In Young’s double slit experiment, we were getting some fringes, but in grating we have lot many slits therefore, the pattern would be intensified.

Now, once we know that optical path length difference then we can again write the grating equation for oblique incidence for both transmission and reflection. For both transmission grating and reflection grating, the grating equation can be written very easily once we know this optical path length difference and this would be equal to $a(sin\theta_m - sin\theta_i) = m\lambda$, this term is now extra here.

Now, you see that there is some modification in the usual grating equation for normal incidence, this term is added and it is the contribution due to the oblique incidence. Now, this expression applies equally well regardless of the refractive index of the transmission grating, the refractive index of the grating material does not play any role here. And it would be very nice exercise if you can prove why the refractive index of the material medium of the grating medium does not play any role.

With this let us move ahead now, whenever you see a grating pattern then the majority of the light or majority of the power goes to the zeroth order and then whatever is left it gets distributed in the higher order diffraction pattern, but most of the available light energy is concentrated in the zeroth order.

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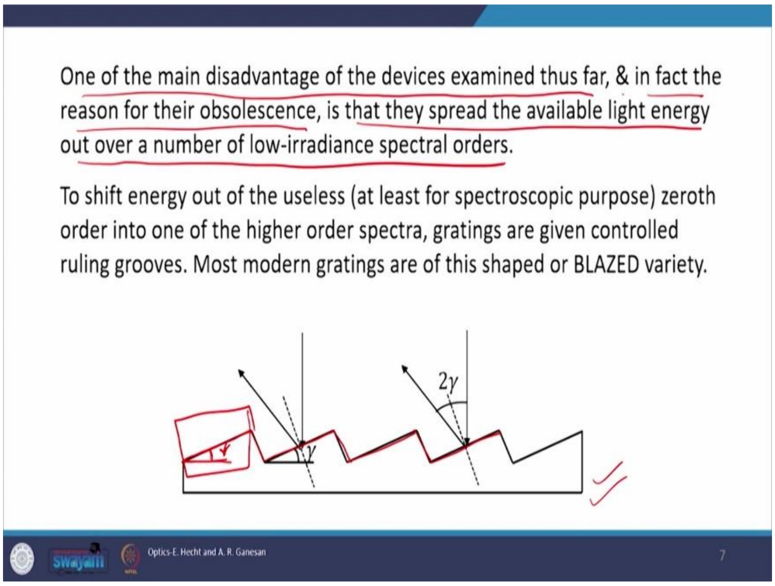

And this is what is written here, one of the main disadvantages of the devices examined thus far and in fact the reason for their obsolescence is that they spread available light energy out over a number of low-irradiance spectral order. Now, what does this sentence want to convey? This sentence says that whatever devices we have learned so far or the grating to the most of the energy is concentrated in the zeroth order and then rest of the energy are distributed in different higher order, diffraction maxima. The zeroth order has maximum energy the first a little lower, second bit more lower, third a bit more low, the energies are there in different orders, but they get reduced as you go away as you increase the angle of diffraction.

But now suppose we want to do some spectroscopy. In that case, we require sufficient intensity, sufficient irradiance in a given diffraction order, then how to play with the intensity, how to extract the intensity or the energy from the zeroth order to a given order where we want to perform some experiment or where we want to study something, this is a major concern.

Now, to shift energy out of the useless zeroth order into the higher order spectra gratings are given controlled ruling grooves. And most modern gratings are of this shaped and the grating with the ruling grooves are called Blazed grating and one such Blazed grating is shown here schematically in this figure.

Now, you see that there is some angle, the grating surface is now oriented at some angle here, it is here in this direction then it is going back then again in the same angle. Again, it is going back again the same angle going back, this type of grating is called Blazed grating. Now, while studying the point oscillator in our very first few lectures of diffraction, we first assumed that all these oscillators or this antenna array, they are oscillating in the same phase and then we did some mathematical calculations.

And thereafter, we deliberately introduced initial phase and we assumed that this initial phase is the phase difference between the adjacent oscillator. Now, if we assume say ϵ phase difference between the adjacent oscillator then depending upon this phase value, the magnitude of the phase, we saw we studied that the grating spectrum can be tuned, it can be oriented in a desired direction, the same thing is now being implemented here in case of grating.

Earlier with point oscillator or point antenna array we incorporated some initial phase and with that phase we found that the maxima, the position of maxima, can be oriented in a desired direction, the same initial phase is now here too and it is in the form of tilt of this grating element, you see this element is tilted by γ with respect to the horizontal. Now, by playing with this tilt, this γ, we can shift energy from one portion of the grating spectrum to the other. From say center portion from the zeroth order pattern to some say second order pattern.

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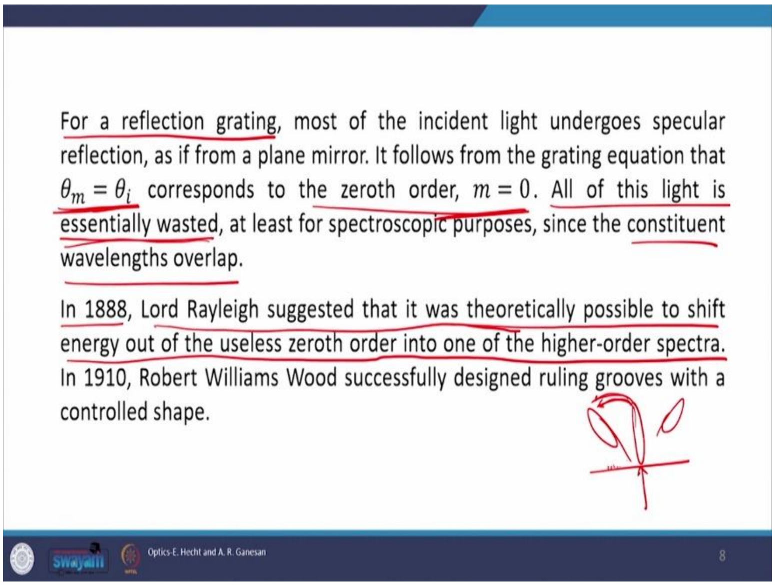

Now, say we are dealing with a reflection grating, most of the incident energy undergoes specular reflection and reflection grating. What is the specular reflection? It is the same reflection which we see in mirrors. For a reflection grating most of the incident light undergoes specular reflection as if from a planar mirror.

It follows from the grating equation that if $\theta_m = \theta_{i,}$ means angle of incidence is equal to angle of diffraction, then this corresponds to zeroth order that is m is equal to 0 and all of this light is essentially wasted, it goes in the same direction where from the light came. And what do I mean by wasted, wasted means from the sense of spectroscopy, it is not in use. Because all the constituent wavelength will now overlap.

Now, to resolve this problem in 1888, Lord Rayleigh suggested that and he also did it theoretically, Lord Rayleigh suggested that it was theoretically possible to shift energy out of useless zeroth order into one of the higher order spectra. So, this is our grating, we are launching light, this is our zeroth order and these are the higher orders, and now he is saying that just by incorporating some initial phase we can siphon out the energy from the zeroth order into some relevant order. Now, in 1910, Robert Williams Wood successfully designed ruled grooves with a controlled shape.

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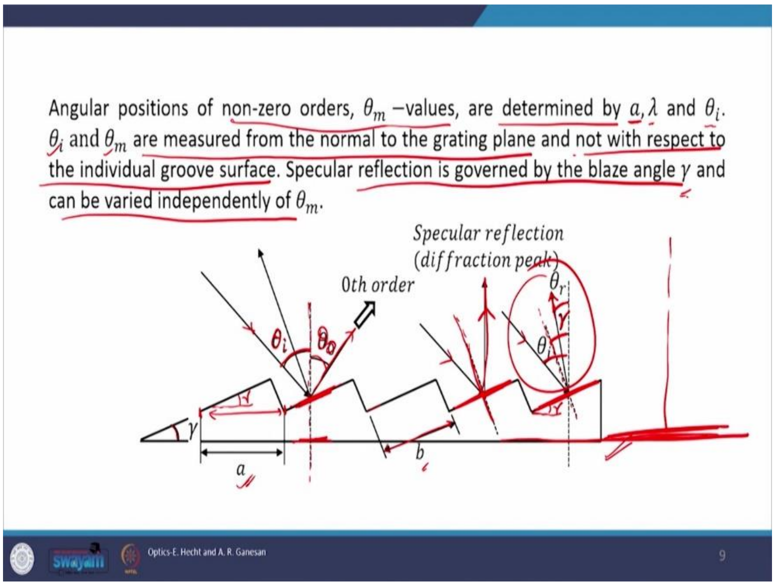

This is called the blazed grating this is what is shown here. Now, here b which is the width of the slit is shown here and slit to slit separation is again shown here represents this separation. Now, angular position of non-zero order θ_m values are determined by a which is shown here, λ the wavelength of light and angle of incidence. Do note that in case of blazed grating all the measurements of angles are performed from the normal to the grating plane, this is the grating plane the lower plane the horizontal plane.

And therefore, θ_i and θ_m are also measured from the normal to the grating plane and not with respect to the individual groove surface. The angles are not measured with respect to the normal to the groove surface, it is measured with the normal to the grating plane, the base of the grating is our grating plane, which remain the same throughout the grating.

Now, specular reflection is governed by Blaze angle γ, what is blaze angel? The blazing angle is this angle of this tilt and it can be varied independently of θ_m. Now, here you see that a light is falling on this particular groove and you see this is the normal to the grating plane which is normal to this base plane, not to this plane and all the angles are measured from this normal.

Now, angle of incidence is θ_i and say the zeroth order are appearing here then this is θ_0. Now, if you talk of usual reflection then it is shown here this is the angle incidence, this is the normal to this part of the slope and this is the reflected beam. Now, if you take all into account together then this is the ray which is falling on this section of the grating, the angle of incidence is measured from the normal to the grating plane, this is the normal to this the wedge portion of the grating.

Now, the angle of this normal with respect to the normal to the grating plane would be γ which is nothing but this angle, the wedge angle, angle of the wedge and this is our reflected beam which is at angle θ_r. Do note that that all angles are being measured from the normal to the grating plane and in this particular case θ_m and θ_r they both are on the same side of the normal. Therefore, sign kind considerations which we discussed in geometrical optics will also come into the picture here.

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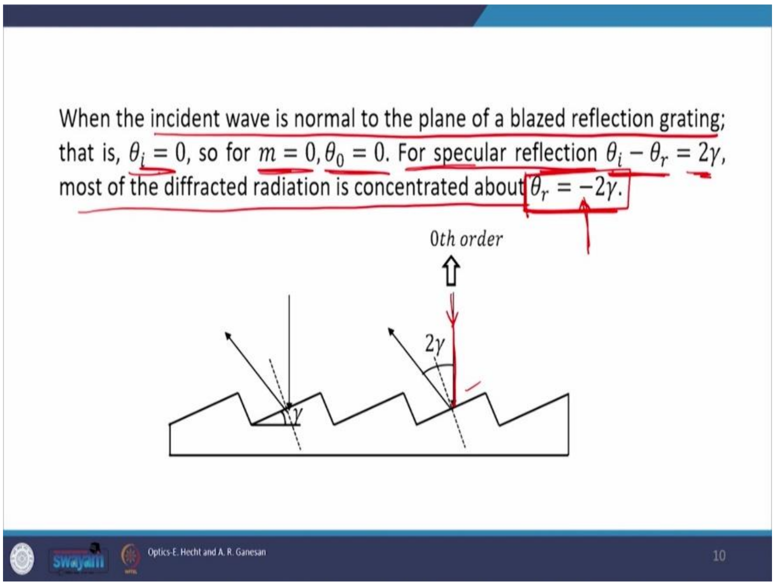

Having done this now, say that suppose that the incident wave is normal to the plane of the blazed reflection grating. In this case, since the incidence is normal to the blaze reflection grating that is this is the incidence and $\theta_i = 0$, when $\theta_i = 0$ then for m= 0, $\theta_0 = 0$, it means that 0 order, the 0 order would be in the direction of incidence itself. And in this case for specular reflection $\theta_i - \theta_r = 2\gamma$ and most of the diffracted radiation is therefore, concentrated about $\theta_r = -2\gamma$. Here minus sign is appearing because both incident and the reflected is on the same side of the normal therefore, we see this minus sign before 2γ.

I repeat for specular reflection, $\theta_i - \theta_r = 2\gamma$ which is very much clear from the figure and most of the diffracted radiation is concentrated about $\theta_r = -2\gamma$ diffraction angle, $\theta_r = -2\gamma$, it can easily be derived from $\theta_i - \theta_r = 2\gamma$ provided $\theta_i = 0$.

This is all for the introduction of the grating. In the next class we will talk about the one of the applications of the grating which is in spectroscopy. I end my lecture here, thank you for listening me, see you in the next class.