Applied Optics
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Lecture: 35
Multiple Slit Diffraction

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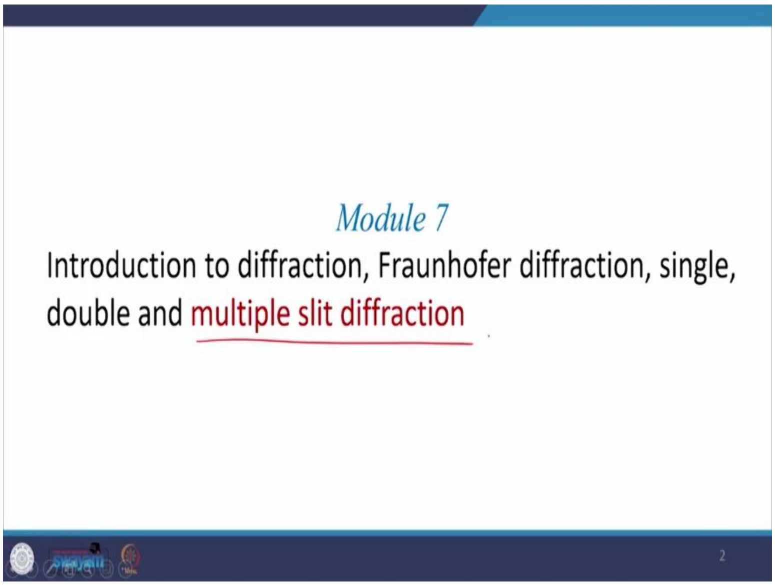

Hello everyone, welcome to my class. Today, we will hold last lecture of module 7. Now, in this lecture we will talk about Fraunhofer diffraction pattern of multiple slit. In the lecture before we talked about diffraction pattern produced by single and double slit in Fraunhofer regime, today we will generalize these two slits.

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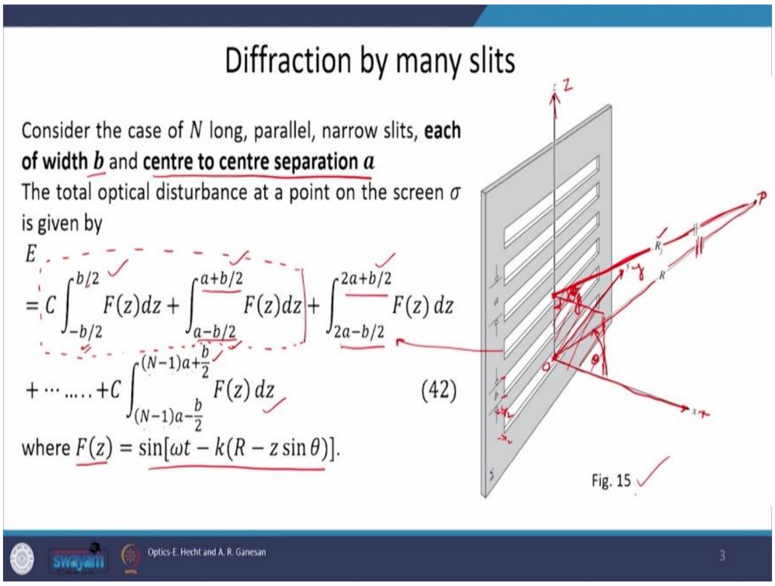

Now, this is schematic diagram of a multiple slit where you see that the slits are placed horizontally. This is contrary to the case where we discussed single and double slit, there slits were placed vertically, but the axis is unchanged. Now, you see that since we have rotated the slit therefore, the axis also got rotated y axis is in the direction of length of the slit, x is coming out of the plane of these slits or plane of the aperture and z axis is pointing upward vertical here, this is your z axis, this is your y axis and this is your x axis.

The point of observation P, similar to the previous cases, is very far from the slit and the origin is placed at the center of the lowest slit, a lower most slit, the origin O is here and the distance of the point P from the origin is R and this P is very far from the slit therefore, we have introduced these two parallel lines which represents brick. It means that this length OP is very much larger than the length which is shown here in this figure.

Now, since we are talking about many slits or multiple slits, then we assume that there are N number of such slits and for any jth slit, suppose we pick some random slit and which we assign a number j. From jth slit, the distance of point P is R_j and the line OP is at angle θ from x axis while this R_j makes an angle θ_j from a plane, this plane is xz plane, the shaded region is the xz plane and with this plane this line which is of length R_j is making an angle of θ_j .

Now, these all slits are identical and the slit width is b and center to center separation is a, this slit width and center to center separation a is same as what we studied in double slit experiment. Now, we will again apply the same mathematics which we developed for single slit and then apply it to double slit. In double slit, we solve two integrations.

Now, since here the number of slits are N, we will solve N integrations and these are the integrations, here F(z) is your this function which we already derived in previous classes, since the origin is at the center of the lowest slit, the limit of integration from the first integration will vary from -b/2 to +b/2 as shown in the figure.

Similarly, for the second slit, it would be a - b/2 to a+b/2. In case of double slit experiment, we just solve these two integrals here, we just took into account these first two terms in double slit experiment. Now in this case we have large number of slits, in this particular case we are considering N number of slits.

Therefore, N such integrals would be written with appropriate limits. Now for the third slit, the limit would be like this 2a-b/2 to 2a+ b/2 and similarly for last slit the limit would be (N-1) a - b/2 to (N-1) a +b/2. Now, we will have to solve all these integrals. Now, out of all these integrals, we will pick the integration, the integral for jth slit, we will solve this and then sum over j.

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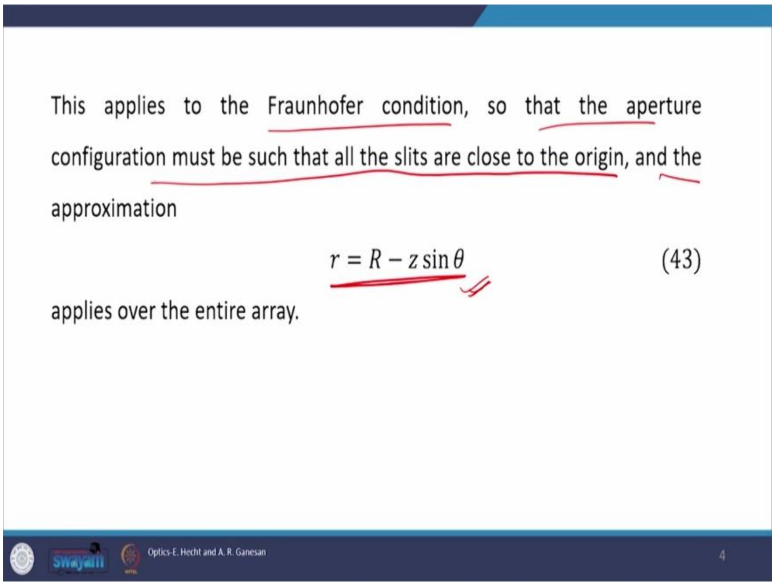

Since we are in Fraunhofer regime therefore, Fraunhofer conditions apply here. And therefore, the aperture configuration must be such that all slits are close to origin and this approximation is valid and we assume that this is applicable over the entire array of the slit, this we derived in case of single slit, this is the Fraunhofer condition.

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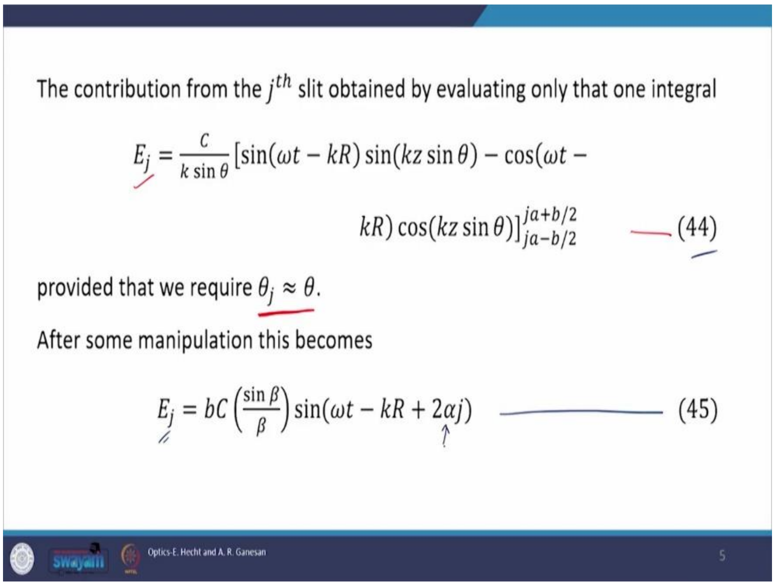

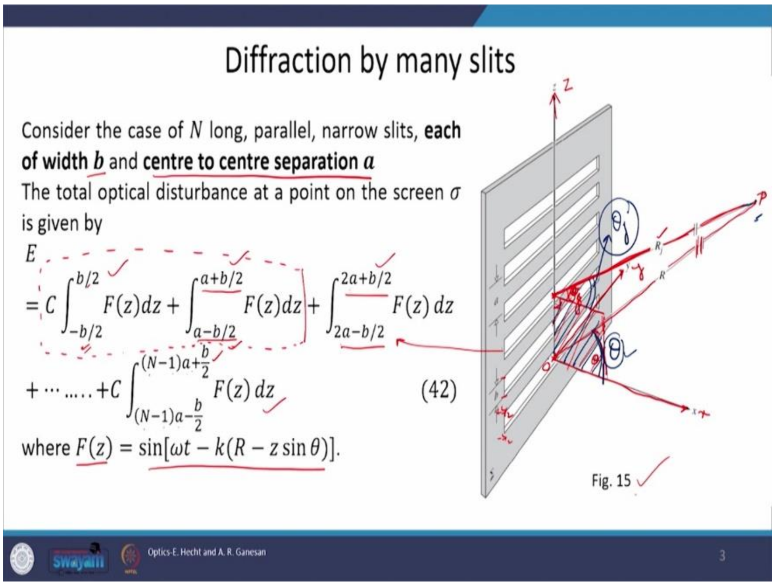

Now with this, we will solve the jth integral and after integrating, we get equation number 44 while solving this integral, we assume that $\theta_j = \theta$, let us go back to the figure, now, in this figure θ_j is this angle, this angle is θ_j while this angle is θ. Now, if you consider point P very far from the aperture and this aperture plane where N number of slits are there, this aperture or all these slits as compared to the distance between the slit and the point of observation would be very small.

Therefore, we may assume that slits are so closely spaced that angle subtended by a line starting from center of each slit to the point of observation P subtend the same angle with xz plane, this plane.

I repeat since point of observation is very far and slits are very thin, the width is very small even the center to center separation is also very small. Therefore, the line starting from center of each slit and reaching to point of observation P subtend an angle which is same irrespective of the slit numbering and this angle is being subtended from a plane which have x and z axis.

And this angle which we designate as θ_j is equal to angle θ which is the angle of point of observation P from origin with respect to x axis. With this approximation, we solve this jth integral and after some manipulation, we get equation number 45 which represents the contribution of electric field or contribution of disturbance at a point of observation P due to jth slit in N number of slits. Now, here you see a new term α.

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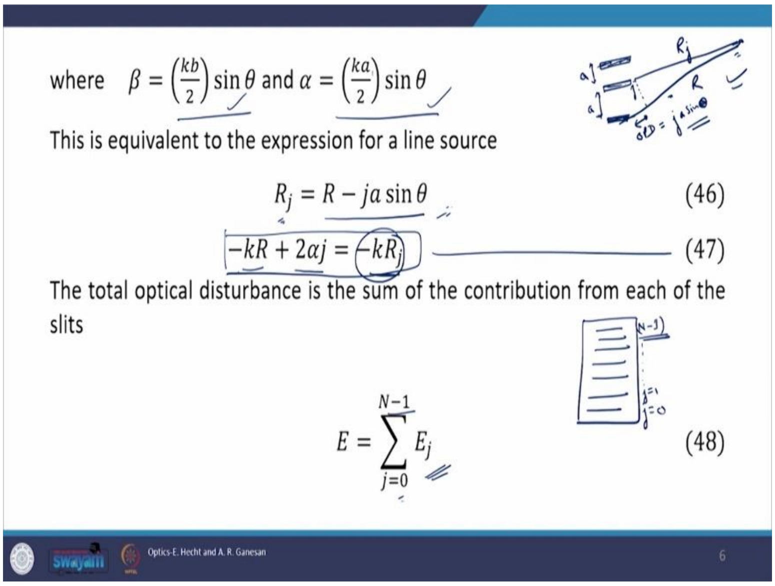

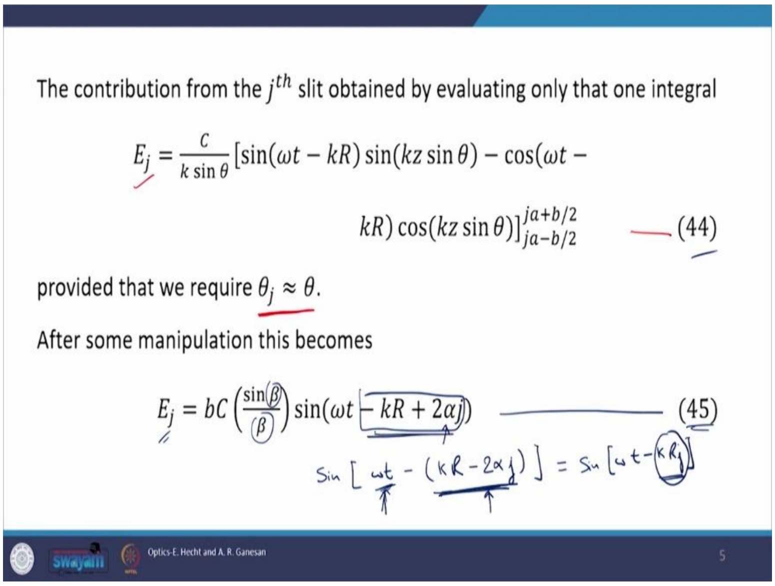

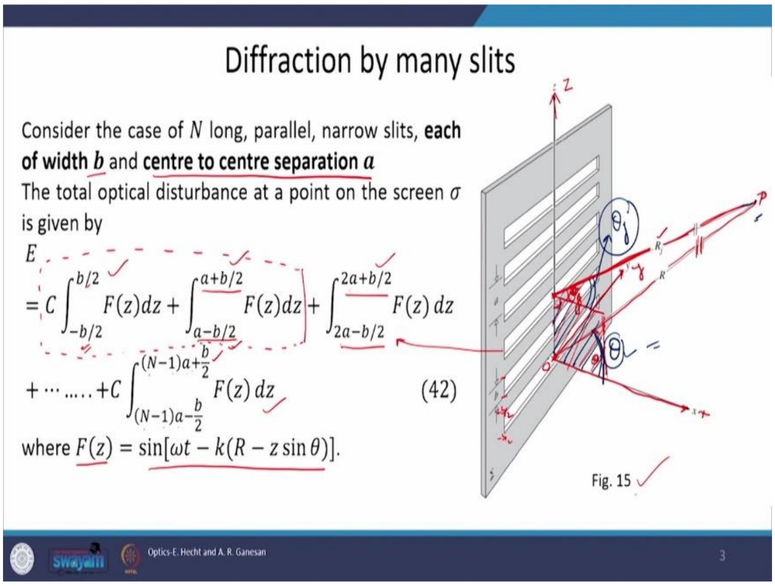

Now α=(ka/2) sinθ while β= (kb/2) sinθ, β is here in this sinc function, this we have already talked about in double slit experiment. Now, if you go to the phase part of equation number 45, then you see that we have sin function with ωt and if you take minus out then you will have this as a second term. Now (kR-2α j) is the phase part in equation number 45. This is a time varying phase part and this is the spatial phase part, space dependent phase part and it is (kR-2 αj) let us talk about this.

Now, R_j which is the distance from the center of jth slit to the point of observation P, this would be equal to (R-j$a$sinθ) and which is very much clear from this figure also because center to center separation is a and to calculate R_j, these are the slits and these slits are separated by distance a, the adjacent slits are separated by distance a and slit width is b which is very small.

Now the point of observation is P here, now, suppose this is our jth slit then this distance which designate at R_j, this would be equal to, like suppose this is your lower most slit, then this would be R and the path difference would be this much, this would be your optical path difference and this optical path difference would be equal to j which is the number of the slit into a sinθ.

Therefore, R_j would be R-ja sinθ this which is very much clear from this figure. Now, with this we can write that - kR+2 α j, this would be equal to - k R_j because α is (ka/2) sinθ, with this substitution we get this relation. Now, -kR+2 α j is equal to - kR_j, what is this, let us go back, now you see here - kR +2 α j is here. Therefore, we can equivalently write this as sinω t and - k R_j, this is what we get after the substitution.

Now, this is the phase which the light accumulates in reaching to point P, the light start from the center of jth slit and then it reaches to point P and during this propagation the light accumulates a phase which is now equivalent to k R_j and which is now very much obvious because - kR + 2 α j is equal to - kR_j which is given by equation number 47. Therefore, this equation number 45 is now well understood, and this phase part is also understood, why is it coming so.

Now, once the field due to jth slit is calculated, then we have to just sum this field up over all the slit numbers, and this summation will give us a total field, a total disturbance at the point of observation P. And therefore, once E_j is calculated with sum over E_j where j starts from 0 to N- 1, make it a point, the counting of the slits starts from 0, this is our slits and we start counting the slit from 0, j=0 here, then j=1 here and so on and at last j=N-1, it ends at N-1 since the counting starts with 0 therefore, the total number of slits in this case would be N.

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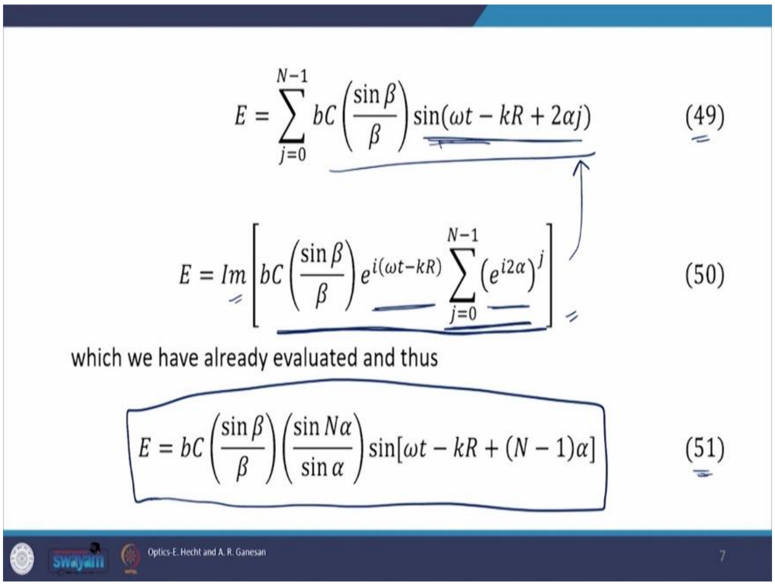

Now, let us perform the summation over the E_j and the summation gives this. Now you see that it is a sin function here we can also express it in form of complex representation. In complex representation, the imaginary part of this expression will give us sin because $e^{i\theta}=\cos\theta+i\sin\theta$ and sin is coming with Iota (i) therefore, imaginary part of this complex representation will give us back equation number 49 which have sin function.

Now, this summation which is $\sum_{j=0}^{N-1}(e^{i2\alpha})^j$ is already done, we did it when we were dealing with point oscillators in the very beginning of diffraction. We will borrow the knowledge of summation from there and directly write the results here, then after summation and doing a bit of mathematics, we land up with this value, this expression of total electric field at the point of observation P.

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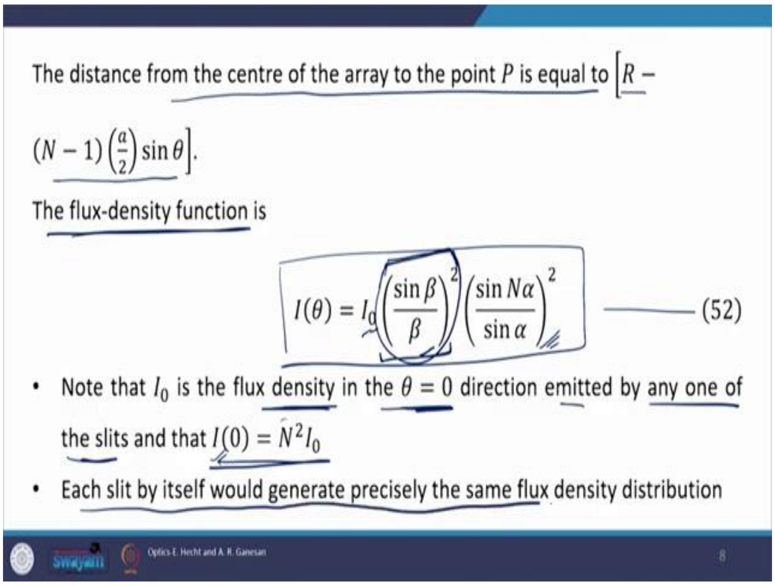

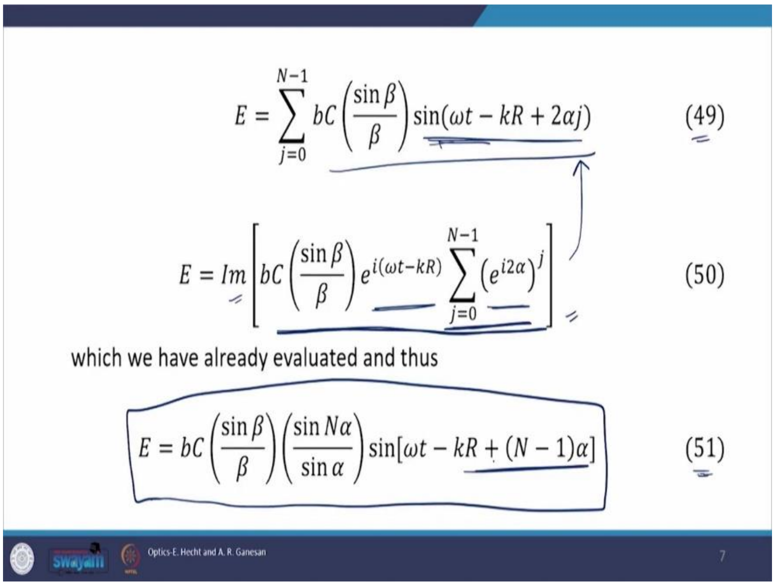

Now, the distance from the center of the array to the point P would be equal to (R-(N -1) (a/2) sinθ, this also have been done previously while dealing with the this point oscillator. Now, this distance is visible here too, in equation number 51. Therefore, ultimately, we can calculate the flux density function, which is irradiance. Once the total electric field expression is known, we can easily calculate the irradiance, the irradiance is given by equation number 52. Now, you see that we have sinc square function which is symbolizes the diffraction produced by single slit and then we have this function which is new $(sinN\alpha/sin\alpha)^2$.

This function was neither there in a single slit diffraction expression nor in double slit diffraction expression, while this function, this sinc function$(sin\beta/\beta)^2$ were both in single slit as well as in double slit expressions. Now, I_0 is the flux density or irradiance in θ=0 direction emitted by any one of the slit. And therefore, if you substitute θ=0 in equation number 52, you get $I(0) = N^2 I_0$. It means in 0 direction, in θ=0 direction, the resultant irradiance would be $N^2 I_0$, I_0 is contribution from the single slit.

If you increase the number of slits the total irradiance will increase by $N^2 I_0$. Now as the expression 52 suggest, we will have single slit diffraction pattern also in the complex pattern produced by multiple slit, we will have single slit diffraction therefore, each slit by itself would generate precisely the same flux density distribution.

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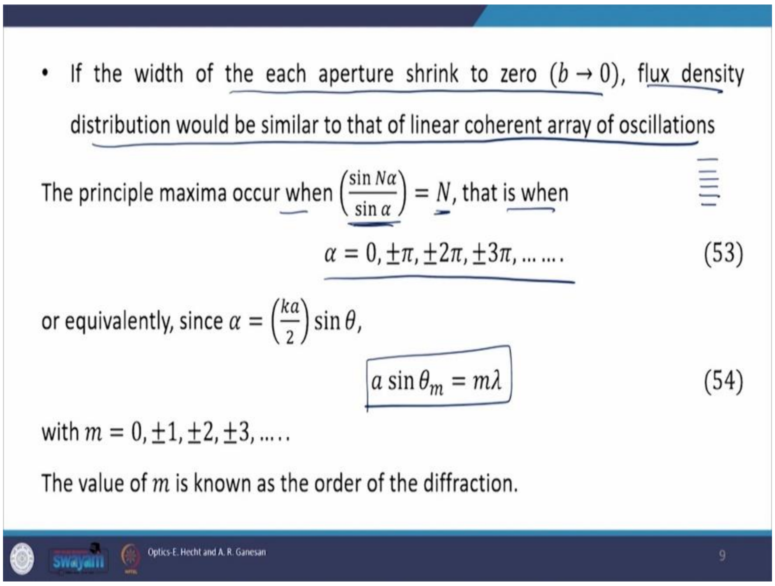

Now, if the width of each aperture shrink to 0, if we start reducing the width of each slit, then flux density distribution would be similar to that of linear coherent array of oscillator, why is it so, because suppose these are the slits which we are talking about and if you reduce the width to 0, then we will have a single line oscillator.

Now, the principle maxima occurs when this function (sin Nα/sinα) =N and when does this happen, this happen when α assumed these values which are integral multiple of π. For principle maxima α must be integral multiple of π which is 0, $\pm\pi$, $\pm 2\pi$, $\pm 3\pi$ or equivalently what we can see is that since α=(ka/2) sinθ then from this expression a sinθ_m =mλ this gives now the principle of maxima, where m is an integer.

And the value of m is known as the order of diffraction if m is equal to 0, it is a principle maxima, it will give you the intensity in this straight direction while m=1 or -1, designates first order maxima. Similarly, m=2 or -2, designate second order maxima and so on.

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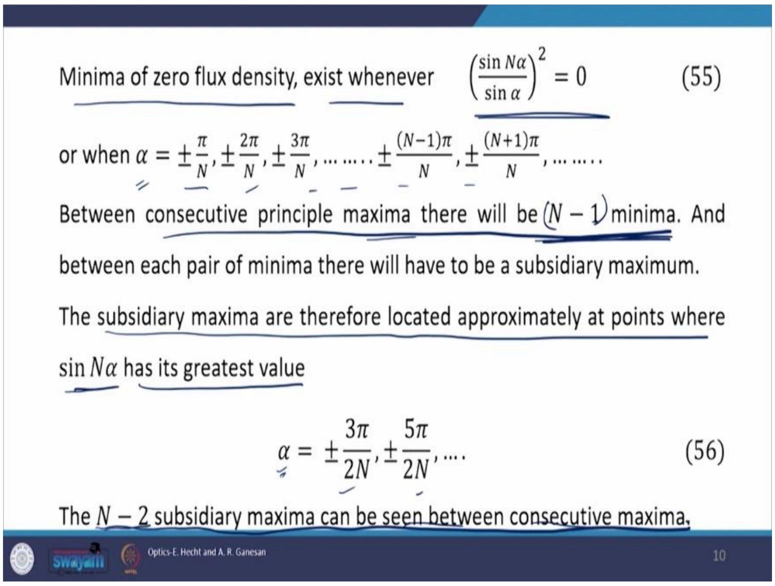

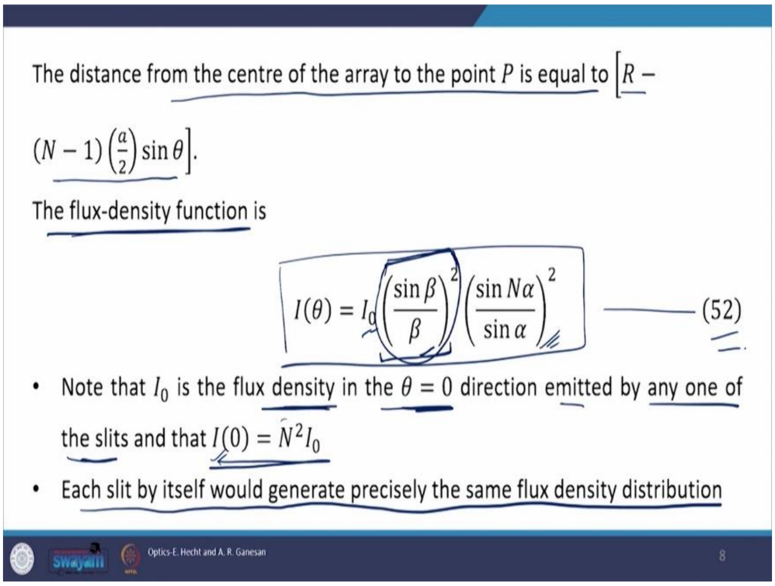

Now, for a minima, minima of zero flux density exist when this function is equal to 0, (sin Nα/sinα)=0 and this appears when $\alpha = \pm\pi/N, \pm 2\pi/N$ and so on. Now, between consecutive principle maxima, there will be (N-1) minima and between each pair of minima, there will have to be subsidiary maxima, how to decide the position of subsidiary maxima?

The subsidiary maxima are located approximately at points where sin Nα has its greatest value and what are those α, those α are given by this $\pm 3\pi/2N, \pm 5\pi/2N$ and so on. Therefore, (N-2) subsidiary maxima can be seen between constitute maxima.

I repeat between consecutive principle maxima there would be (N-1) minima and (N -2) subsidiary maxima can be seen between consecutive maxima. Now, this becomes very clear when you plot equation number 52.

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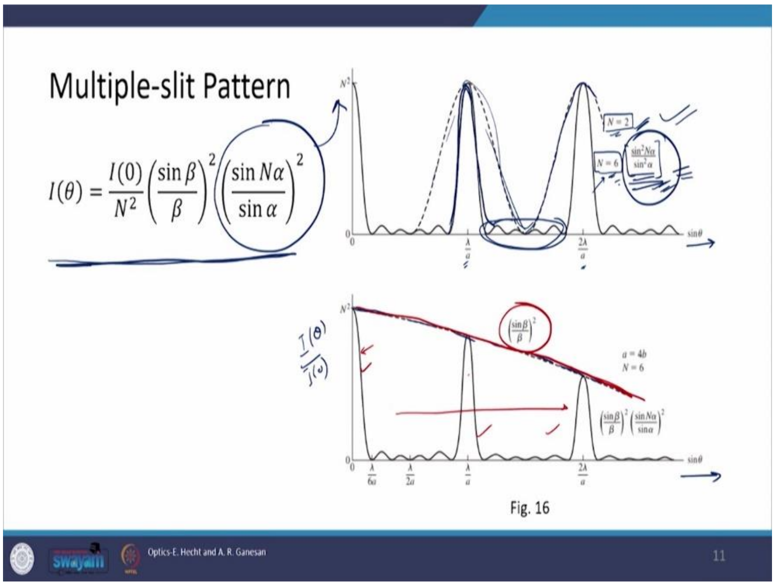

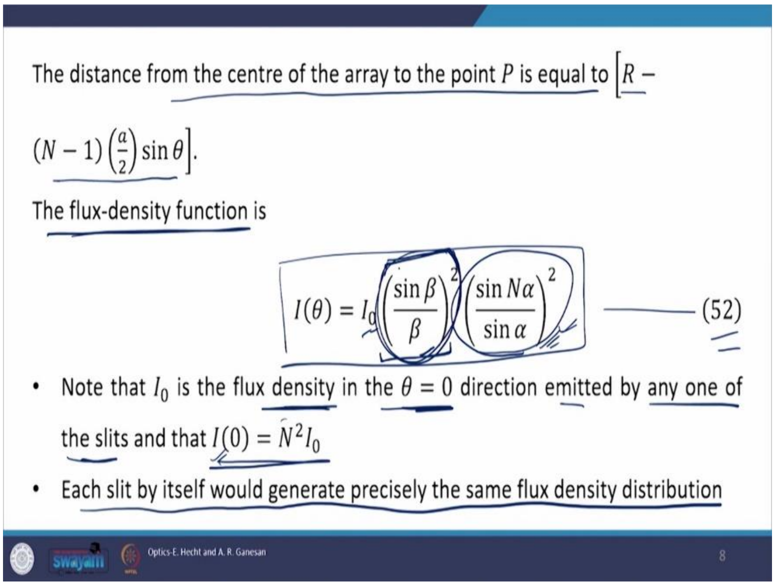

Now, here the multiple slit pattern is shown, the diffraction pattern or irradiance pattern, now, on the vertical axis I/I_0 is plotted when N or number of slit is equal to 2, then you see this dashed line pattern and when N=6, then you see this continuous pattern.

Now, in this first figure, this whole pattern is not plotted. In this first figure, we picked only this part and this part is plotted here on the vertical axis and this is you see here this function, $(sinN\alpha/sin\alpha)^2$. On the vertical axis we have plotted $(sinN\alpha/sin\alpha)^2$ while on the horizontal axis it is the value of θ which is being varied or value of sinθ which is being varied.

Now, you see that as you increase the number of slits, new features appear, for smaller number of slits, you see only big principle maxima peaks that is going down to 0 and then again the next peak appear, while for larger number of slits apart from the usual central maxima which we predicted to appear at λ/a or $2\lambda/a$ and so on. Several subsidiary maxima also appear between them and several minima also appear.

Now, if you plot the whole function which is your $I(\theta)/I_0$ then what you see is that the pattern which you see here in the first plot, it slowly decays down as you increase $\sin\theta$. Now, why is it so, this is what exactly is mentioned here. Now, you see that there is some pattern which is given by $(\sin N\alpha/\sin\alpha)^2$ and this is a rapidly varying function, this function is being modulated by a single slit diffraction pattern.

Now, this is the rapidly varying function which is $(\sin N\alpha/\sin\alpha)^2$ and if you multiply this function by $(\sin\beta/\beta)^2$ which is slowly varying function, it means that that function will now envelop it that function will now modulated and this function is plotted here. This is the diffraction, single slit diffraction, $(\sin\beta/\beta)^2$.

This single diffraction now envelop the underlying function which is$(\sin N\alpha/\sin\alpha)^2$. Therefore, the overall pattern would look like this. This is with the continuous line, the overall pattern is done where at $\sin\theta 0$, you see highest peak and as you move outward, this principle maxima intensity or irradiance, it decays down due to the single slit envelop or single slit modulation.

Now, you see that you can compare this with a diffraction grating also, we got in grating we have multiple slits, the same analysis also hold here. We will talk more about grating in next module, but you can now start having a comparison, you can see that this study is basically diffraction grating study, because there are so many opaque and transparent portion in aperture and which clearly mimics diffraction grating. Now, with this I end this lecture. See you in the next class.