Applied Optics
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Lecture: 34
Double Slit Diffraction

Hello everyone, welcome back to the class in the last lecture, we talked about single slit diffraction, we will extend that knowledge and now, today we will discuss Double Slit Diffraction.

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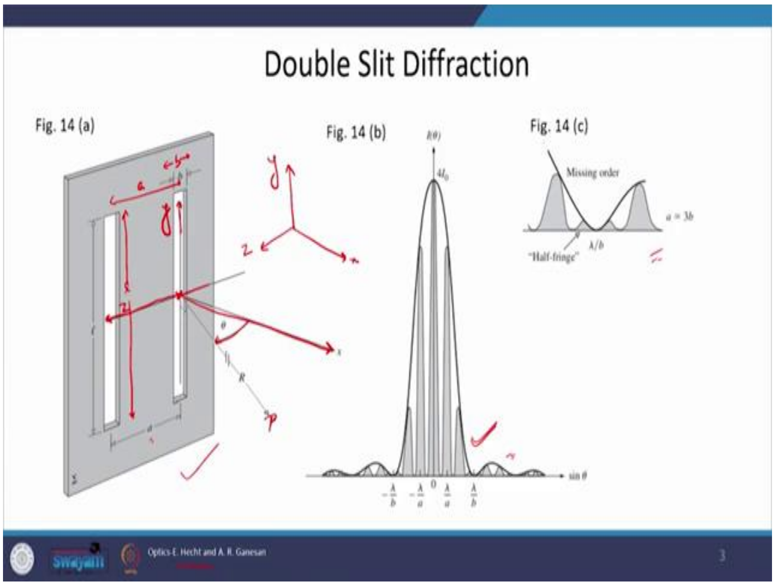

Now, this is the schematic of the double slit, each slit is of length l, this length is l and each slit is width b, but slit to slit separation is a here, the slit to slit separation is being measured from the center of one slit to the center of the other slit, this is also shown here, this figure I took from the book Hecht here, optics by Hecht.

Now also pay your attention to the orientation of axes, the x axis is in this direction and the z axis is along in the plane of the aperture which is your double slit and this is your z axis and y is pointing upward which is shown here. Therefore, I will just draw it for the clarity y is in this direction, x is here and z is in this direction.

Now, the origin is placed at the center of right slit, it can be placed at the center of any of the slit but I chose to place it on the right mode slit. Now, the angle in this case is being measured from yx plane which is shown here and P is our point of observation. Now, this is how the fringe pattern of the slit look like, but we will discuss it in detail after the mathematical analysis is complete and we will also discuss this figure 14 (c) after the mathematical analysis is done.

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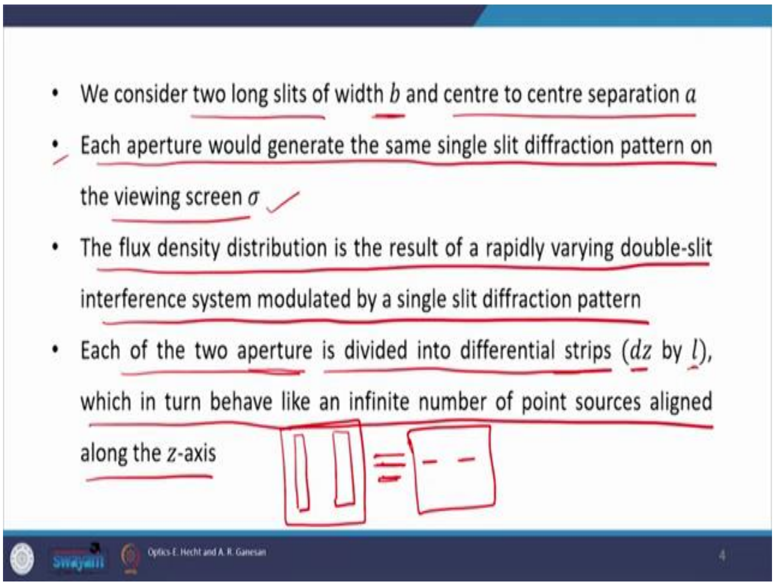

Now, here we consider two long slit each of width b and center to center separation a. Now, once we were having single slit then we divided the width of each slit into finitesimally small width which was of Δz length long and then using the formal analysis analysis which we perform in the last class, we reduced the each line source with a point source which was emitting circular ray. Now, we will do the same here to, now each slit will generate a single slit diffraction pattern on the screen and this is what is written here, each aperture would generate a same single slit diffraction pattern on the viewing screen σ.

The flux density distribution is the resultant of rapidly varying double slit interference system modulated by single slit diffraction pattern, says that since we have two slits, each slit will generate its own diffraction pattern and these diffraction patterns are slowly varying the maxima and minima the separations are slow.

Now, since we are having two slits and the diffraction pattern between these two slits, the light which is coming out of these two slits, they will interfere among themselves and this will create a rapidly varying maxima, minima, a rapidly varying fringe pattern, this rapid variation of fringes would be enveloped by a diffraction pattern, this rapid variation would be modulated by a slowly varying diffraction pattern, we will see in the forthcoming slide we will see how does this happen.

Now, we will follow the same analysis as we did for the single slit each of the two aperture or each of the two slit is divided into differential strips of width dz and length l which in turn behave like an infinite number of point sources aligned along the z-axis. What I mean to say is that suppose this is our two slit arrangement then this would be replaced by two horizontal line sources. This is exactly what we also did in case of single slit, this is the aperture and this is the modified line sources, because we know that each vertical line source behave like a point source which emit a circular wave.

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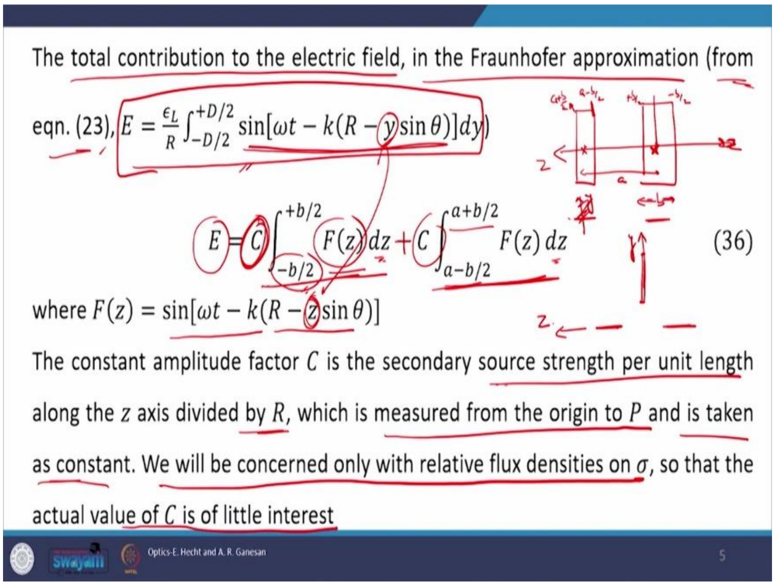

Now, the total contribution to the electric field in the Fraunhofer approximation will be given by equation number 23, which we derived in the last lecture and what is equation number 23, this is our equation number 23 and this is derived for a slit whose length is d and the origin is at the center of this line source.

Now, since we are two line sources into slit case therefore, we will have two such integrals, one is extending from -b/2 to +b/2, why, because these are the two slits which we have right now and origin is here and the slit width is b and the center to center separation is a therefore, for first slit where in our origin is placed the z which is this axis, this varies from -b/2 to +b/2 which is very much clear, the b is here sorry the z axis is in opposite direction, it is pointing in this direction.

Therefore, it would be +b/2 here and -b/2 here, but, if we move to this edge, this edge a distance away from the origin is the center of this and then you will have to subtract -b/2, a-b/2 is this edge a+b/2 is this edge and this is how the limits are chosen in this particular case. And if you sum these two integrals up, this will give you the resultant field on the screen.

Now, what is F? F is nothing but this integrand $sin[\omega t - k(R - zsin\theta)]$. We are having z here in this state of y because initially the line source were along the y axis, but now, the new line sources which we found by reducing the two slits, they are along the z axis.

Therefore, we replaced y with z here, there is a constant C is appearing here, this represents the source of strength per unit length along z axis and which is divided by R of course and what is R? R is measured from the origin to P point where the observation is being done. We will be concerned only with the relative flux density and σ, we will not pay our attention on C, why? because the actual value of C is of little interest, once the relative value is considered, then we will have C both in the numerator and denominator and it will go away.

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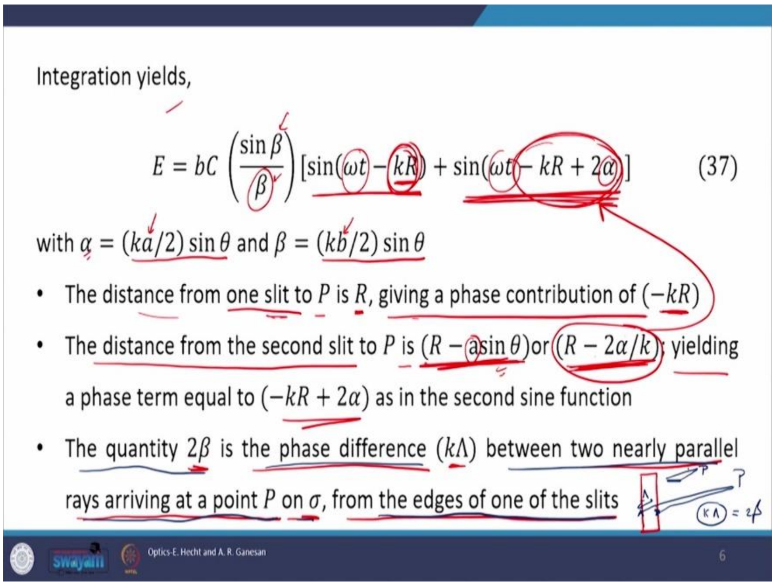

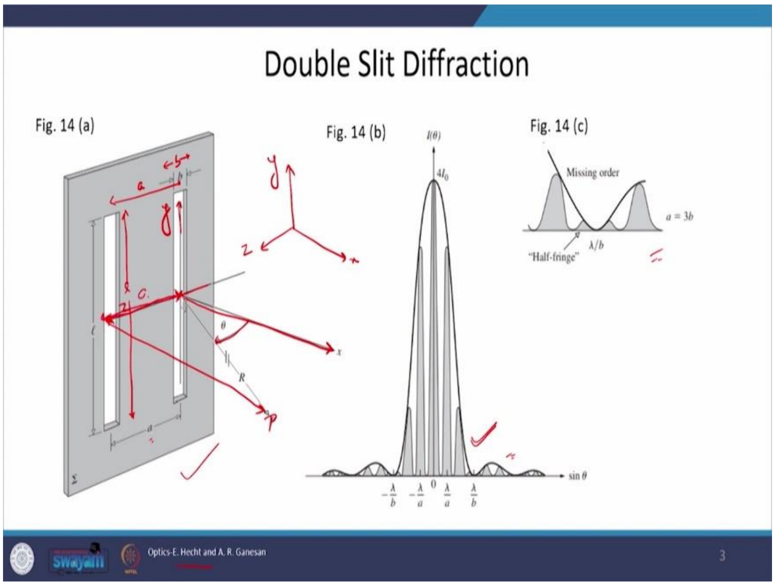

Now, we will perform the integration and integration yields two terms, this is the first term and this is the second term. First term is from first integral, second term is from the second integral. Here in you see there are two terms which are appearing α and β, α is defined by (ka/2) sinθ and β is defined by (kb/2) sinθ. Just remember b is the width of the slit in double slit, width of the each slit and center to center separation is a. Now, the distance from one slit to P is R giving a phase contribution of -kR which is appearing here in the first term.

It is the first slit at which our origin is situated and since we know that the slit is now reduced down to a line source which is now horizontally placed and R is being measured from this horizontal line source to the point of observation P and therefore, this phase part is coming due to the origin to the observation point distance. The distance from the second slit to the P is R-a sinθ, which is very much clear from this figure, this is the second slit and the point of observation P is here and this would be R-a sinθ, a is this distance, center to center distance is a.

Therefore, R-a sinθ where a is also appearing in α we can write it as R- 2aαk. Now, if you see that this R-2α/k is associated with the phase term which is appearing in the second term of equation number 37. Therefore, this term which is responsible for this phase is again due to the distance from the center of the second slit to the point of observation P. Therefore, these expressions are very much clear, this is a time dependent phase and the second term which is appearing in the phase part is due to the separation of the center of slit from the point of observation P.

Now, we also have the quantity β in the expression. The quantity 2β is the phase difference kΛ between the two nearly parallel rays arriving at point P on screen from the edges of one of the slit, this point says that suppose this is one of the slit. Now, suppose let us pick different color and this is our point of observation P then one ray is coming from this edge to the point P, other ray is coming from this edge to the point P.

Now, there is a path difference between these two rays, say this path differences Λ now, then you can calculate the phase difference between these two rays and the phase difference would be kΛ and this is equal to twice of β and this is what is being said in this line.

The quantity 2β is the phase difference between two nearly parallel rays arriving at point P on σ from the edges of one of the slit, these two points are on the edges of the slit and the rays originating from these edges are coming to point of observation P but do remember these slits has now reduced down to this horizontal line source and these lines these rays are coming from these edges.

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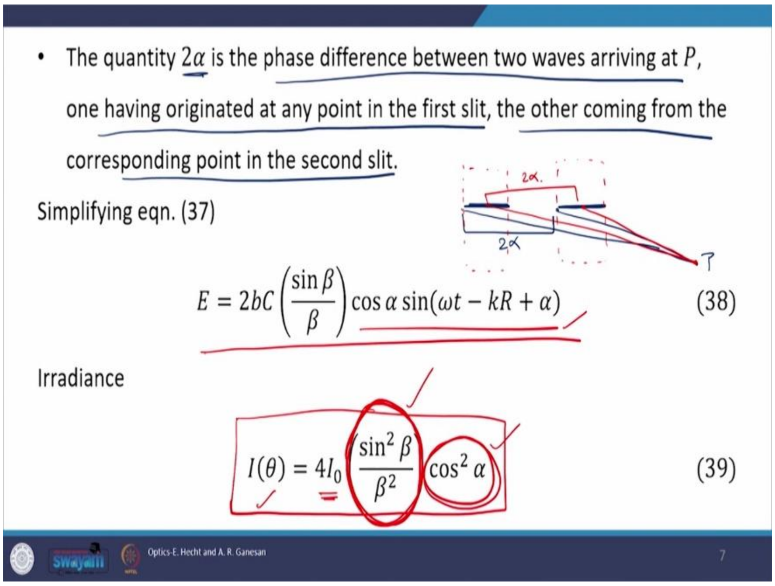

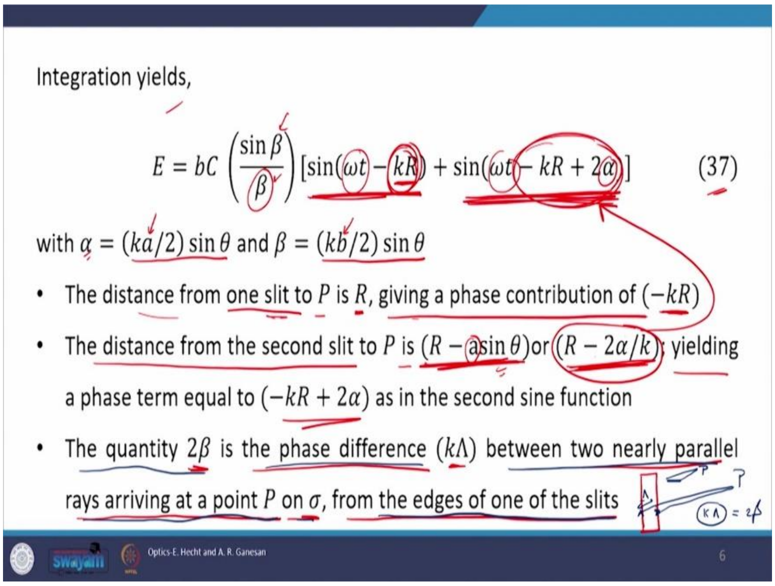

Now, the quantity 2α, what is the 2α here, the 2α is the phase difference between two waves arriving at point P, one having originated at any point in first slit, other coming from the corresponding point in second slit. Now suppose the line sources which we generated from the two slits are represented by these two horizontal lines then suppose the point of observation P is here then if this is the ray which is coming from one of the slit then the corresponding ray means this ray will also start from the left edge of the second slit.

If we pick different point in the first slit then if this is the ray which is coming to P then the corresponding ray would be here at the center of the second slit and the phase difference between these two or between these two now give 2α, hope this is understood and if it is unclear to you then you can draw the whole slit like this, same thing, same explanation is still valid here too.

Now, from this expression 37, now we can write this expression, what is the difference, we just use the sin C+sin D formula and then we have a bit simplified expression here. Once the field is known, we can calculate irradiance easily which is given by equation number 39.

Now, here what we see is that we have a constant term which is $4I_0$ and then we have sinc function here, which is contribution from the single slit, each slit have their own diffraction pattern and this resembles the single slit diffraction pattern because in the single slit diffraction pattern too, we saw this $(sin\beta/\beta)^2$ term, but this term is something new which is coming due to the presence of the other slit, had there been only a single slit this term would have been there only, but $cos^2\alpha$ term is appearing due to the presence of the other slit the second slit.

It means, had there been only single slit we would have called it diffraction pattern but now due to the presence of the other slit, the beams coming from the two slits is now interfering somehow and they are giving some interference along with a diffraction, mix up. And this cos term is the interference, it represents the interference. If you remember in the Young's double slit experiment also we were having this cos term, there we were having $I = 4I_0 cos^2\delta/2$, the same term is appearing here we will clarify it right now.

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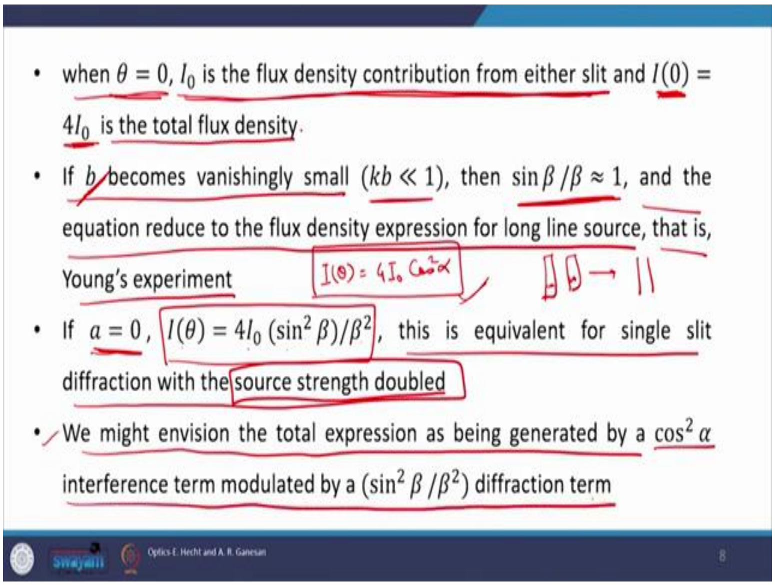

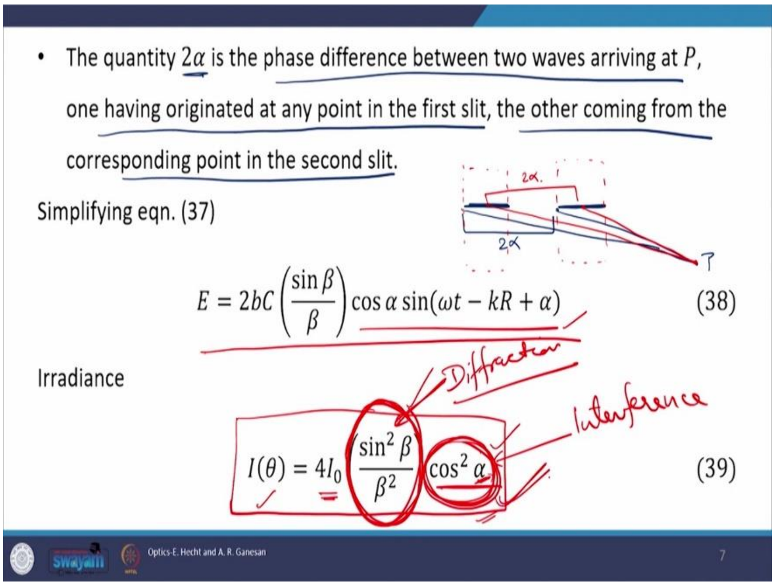

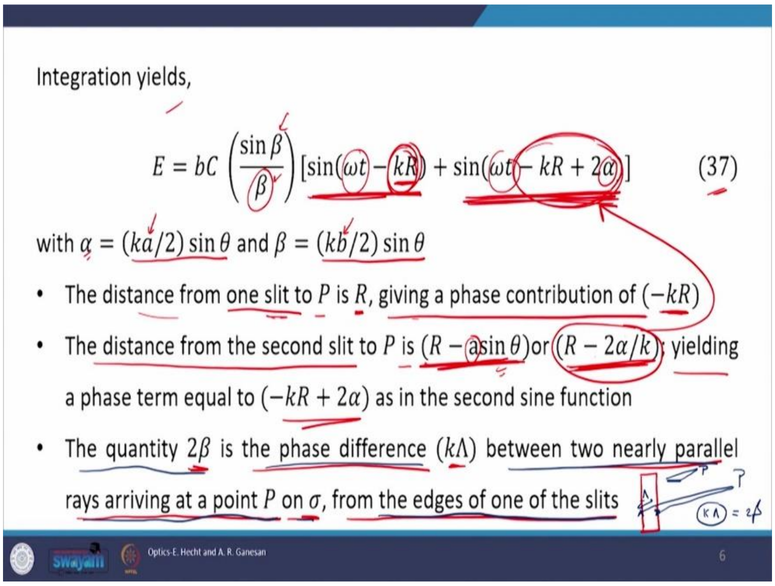

Now, suppose $\theta = 0$, what would be the irradiance in $\theta = 0$ direction, when $\theta = 0$,, I_0 is the flux density contribution from either slit and therefore, $I(0) = 4I_0$ and which is the total flux density in 0 direction, $\theta = 0$ direction. I repeat when $\theta = 0$, I_0 is the flux density contribution from either slit and $I(\theta = 0) = 4I_0$ is the total flux density, total irradiance in $\theta = 0$ direction would be equal to $4I_0$ which is also clear from equation number 39.

The second point if b becomes vanishingly small and what is b, it is the width of the slit then kb would be very-very small and in this particular case $sin\beta/\beta = 1$. If $sin\beta/\beta = 1$ then we are only left with this term $cos^2\alpha$ term and the equation reduces to the flux density expression for a long line source that is Young's experiment.

Because now, we will have $I(\theta) = 4I_0cos^2\alpha$ which is nothing but irradiance expression of Young's double slit, this expression we derived in Young's experiment and this is very much obvious because we were having two slits and then we started reducing the width of the slit then this will reduce to two line sources and therefore, we will get Young's type fringes which is also obvious with this expression.

If $a = 0$, what is a, it is center to center separation it is separation between the two slits if a is equal to 0 then two slits will merge. And if two slits will merge then we will have single slit diffraction pattern and if you substitute a=0 then α=0 and therefore $cos^2\alpha = 1$ and what we are left with is this, $I(\theta) = 4I_0(sin\beta/\beta)^2$ and this is equivalent for single slit diffraction with source strength doubled, the sources strength here is now doubled because two slits are being merged.

Now, last point here is that we might envision the total expression as being generated by $cos^2\alpha$ interference term modulated by $(sin\beta/\beta)^2$ diffraction fraction term which is what I said earlier, this is your interference contribution and this is very obvious diffraction, single slit diffraction.

This is the interference term, $cos^2\alpha$ term is rapidly varying function, α is here ka/2 sinθ, a is large as compared to b therefore, $cos^2\alpha$ will vary rapidly. β is a smaller, kb/2 sinθ. Therefore, here diffraction is a slowly varying function and interference is rapidly varying function therefore, diffraction will modulate the interference term.

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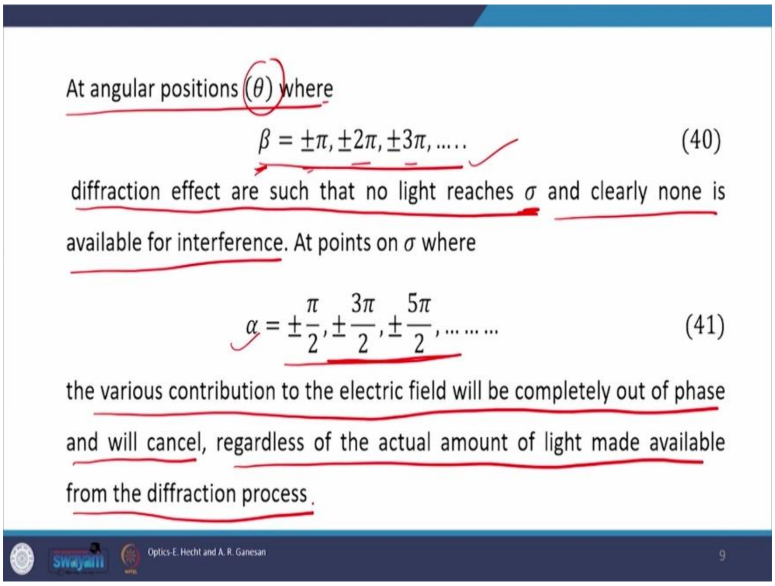

Now, let us talk about maxima minima, at angular positions where $\beta = \pm\pi, \pm 2\pi, \pm 3\pi$, that is integral multiple of π then what will happen, diffraction effects are such that no light reaches to the screen and clearly none is available for interference, at these values of β the diffraction minima appear, this is the condition for diffraction minima.

Now, at these values of α, the interference minima appear and the various contribution to the electric field will be completely out of phase and will cancel regardless of the actual amount of the light made available from the diffraction process. It means at few certain angles, diffraction minima will appear while at certain angle interference minima will appear. Similarly, at certain angles interference maxima will appear, at certain angle diffraction maxima will appear.

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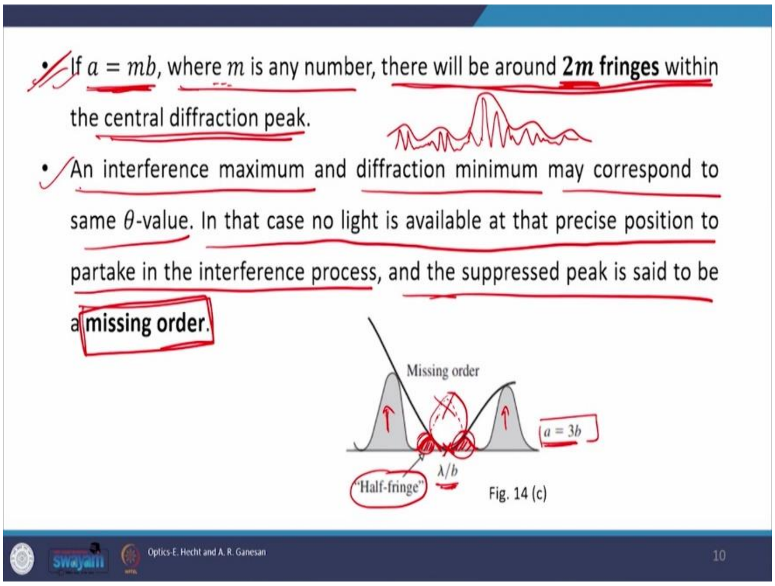

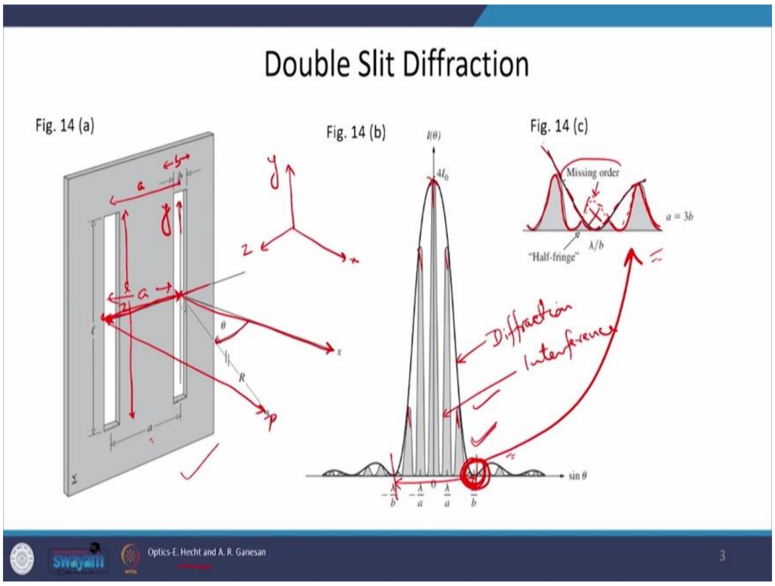

Now, if a=mb then you can see that here m is a number integer then there will be around 2m fringes within the central diffraction peak. Suppose this is our diffraction peak then in this diffraction peak we have multiple number of interference peaks, as shown here in the first slide, you can easily calculate the maxima and minima of diffraction and interference from the conditions and you see that this envelop is diffraction envelope while these the shaded peaks are interference fringes. Due to diffraction, these interference fringes gets modulated you see that you get several interference maxima.

But only one diffraction maxima within certain angle range, interference fringes are sharper, they are thinner while diffraction fringes are broader therefore, diffraction vary slowly while interference vary very fast rapidly, in terms of θ of course. Now, you see that it may so, happen that interference is having its maxima here coincide with the diffraction minima then what will happen, this is entertained here in this point. At interference maximum and diffraction minimum may correspond to same θ value then what will happen, interference says that there should be irradiance maxima while diffraction says that there should be irradiance minima then what will happen.

In that case no light is available at that precise position to partake in the interference process, since there is a diffraction minima, there will not be any light to interfere and the interference peak would be suppressed, it would be absent from there and the suppressed peak is said to be missing order, the interference order would be missing, this suppressed peak. Now, you see here the same thing in the first slide. In this slide, this is the zoomed in figure, this area is zoomed in here you see that these are the interference fringes, now, interference says that there should be a maxima here, this is how it should go.

But what happens exactly is that the diffraction minima falls on the interference maxima therefore, there is no light to partake in the interference process. Therefore, we get a missed interference order and therefore, this is called missing order and this figure is also shown here in this slide, sinθ value of λ/ b, we see a missing order when a=3b, or when fringe to fringe separation is 3 times the fringe width, in this particular case at sinθ value of λ/b, we see a missing order, had there not been a diffraction minima, we would have seen some peak, interference peak here, but it is not there due to this phenomena, we see only a partial peak here, partial interference peak here and a partial interference peak here, these are called half fringes and also make it a point.

Now, you see that the fringes are here it is a complete fringe here, this is a complete fringe. This is a complete fringe, but this is a half fringe here. Therefore, while counting the number of fringes within central diffraction maxima using this condition if a=mb where m is any number, it says that there would be around 2m fringes within the central diffraction peak, this 2m number includes these half fractional fringes, they treat the fractional fringes as one. Now this is all for today. And this completes double slit diffraction pattern or two slit diffraction pattern. Thank you for listening me.