Applied Optics
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Lecture: 29
Concept of Coherence - I

Hello everyone, welcome back to my class. Today we will talk about concept of coherence, we have already talked about interference and in the interference topic we have talked about two categories of instrument, one is called the instruments which produce interference fringes using division of amplitude and other sets of instruments produces the interference fringes using division of wave front, having talked about all these things.

Here we will define what coherence is, what are the different kinds which people talk about, this all would be addressed here in today's class. To start with, we will just repeat what we have learned till now about coherence and then we will slowly dive deep into the coherence definition and its understanding.

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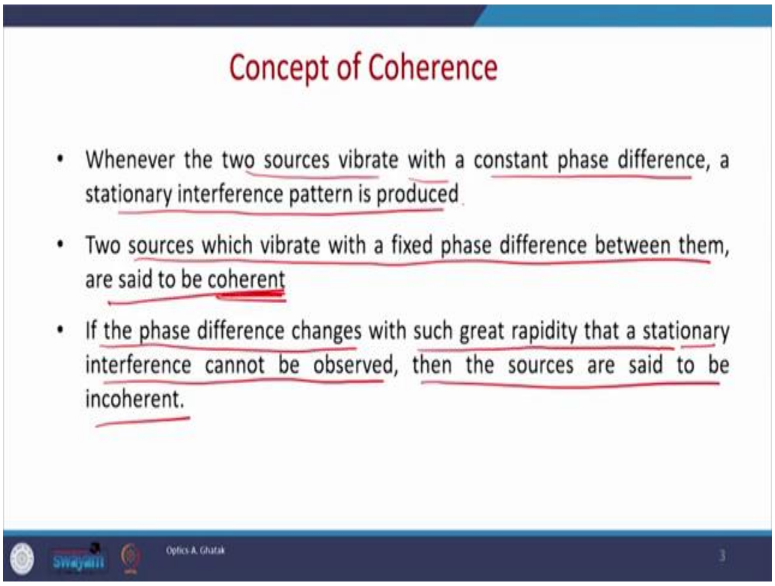

Now, whenever the two sources vibrate with a constant phase difference stationary interference pattern is produced, this we know, we have two sources and if the phase difference between the two sources is constant, then due to the overlap between the two sources or between the light emanated by these two sources, we see an interference pattern and which would be sustained. Because these sources are perfectly coherent because the phase difference between the two sources is constant in time.

Now, the two sources which vibrate with a fixed phase difference between them are set to be coherent, this reality you know till here. And if the phase difference changes with such a great rapidity, than a stationary interference cannot be observed then the sources are set to be incoherent, because while studying about interference we talked about these things, we said that if the constant phase difference is not maintained, then the fringes would not be stationary, and if this constant phase difference between the sources are is maintained. Then the fringes would be stationary.

Now, for the sources in which the constant phase difference is maintained, we named them as coherent sources or for the sources, for which we are able to get sustained interference fringes, we named them as coherent sources while for sources which does not produce sustained interference fringe or for the sources which does not maintain a constant phase difference, they are called incoherent sources, these things we know from our previous lectures.

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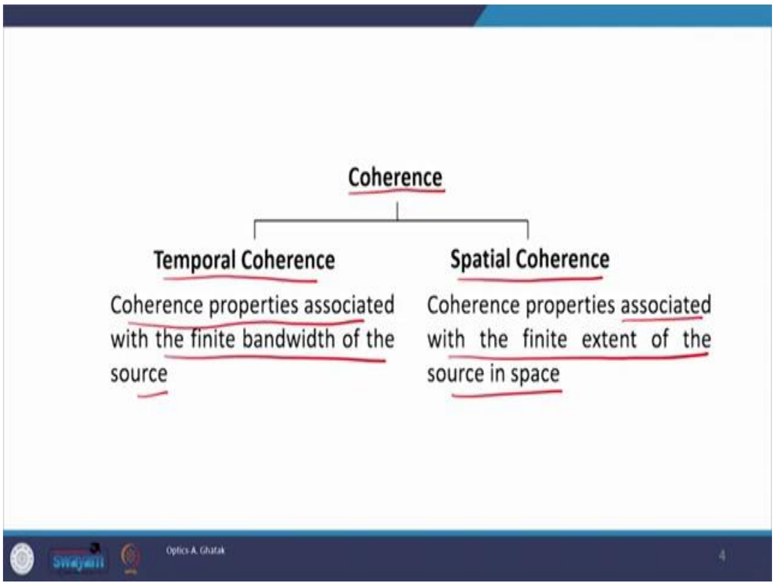

Now, the new thing here is that the coherence is categorized in two category here, the first one is called temporal coherence and the second is called spatial coherence, the coherence properties which are associated with the finite bandwidth of the source comes in the category of temporal coherence.

But if the coherence properties are associated with finite extent of the source in space, suppose we are talking about broad source then spatial coherence comes into the picture, we will talk about these two term more in the coming slides.

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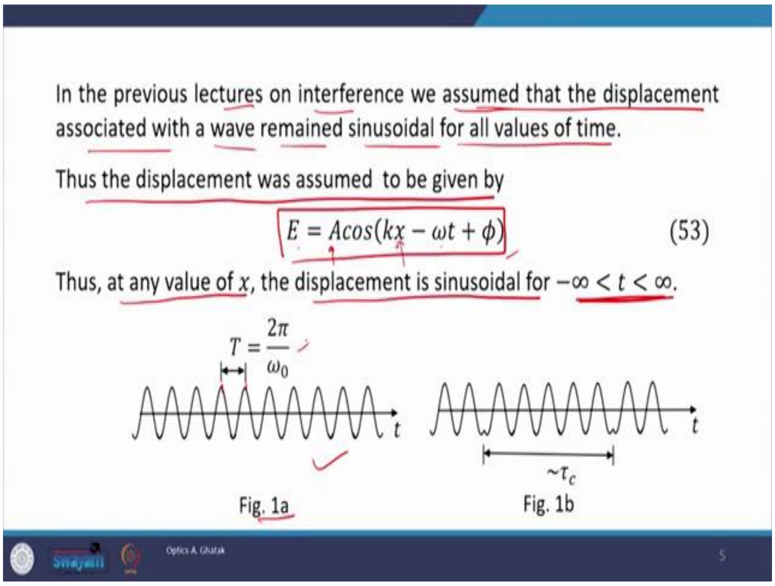

Now, in the previous lectures on interference, we assumed that the displacement associated with a wave remained sinusoidal for all values of time, because whenever we write an expression of the field we just write $E = E_0 e^{i(\omega t - kz)}$, or $E = E_0 cos(\omega t - kz)$.

And this expression is valid for a monochromatic source and this expression itself says that the wave is sinusoidal extending from minus infinity to till plus infinity. Thus the displacement was assumed to be given by this expression, this is the expression which I was starting about E=A, A is the amplitude, $cos(kx - \omega t + \varphi)$, φ some initial phase, x is the direction of propagation and this represents General Electric field displacement.

Now, at any fixed particular value of x, the displacement is sinusoidal for all values of time, if you fix particular value of x, then you will see that for all values of time the wave is sinusoidal and this is represented here in this figure, in figure 1a, we see that, with t we see that the sinusoidal nature of the wave prevails irrespective of the value of t, such a wave is called monochromatic wave and the time period is defined here this is the usual definition which we learn in our initial classes.

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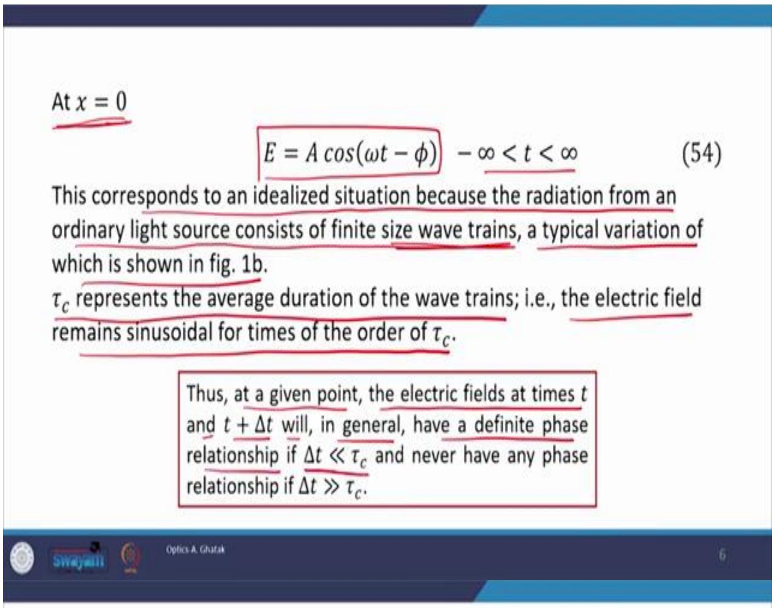

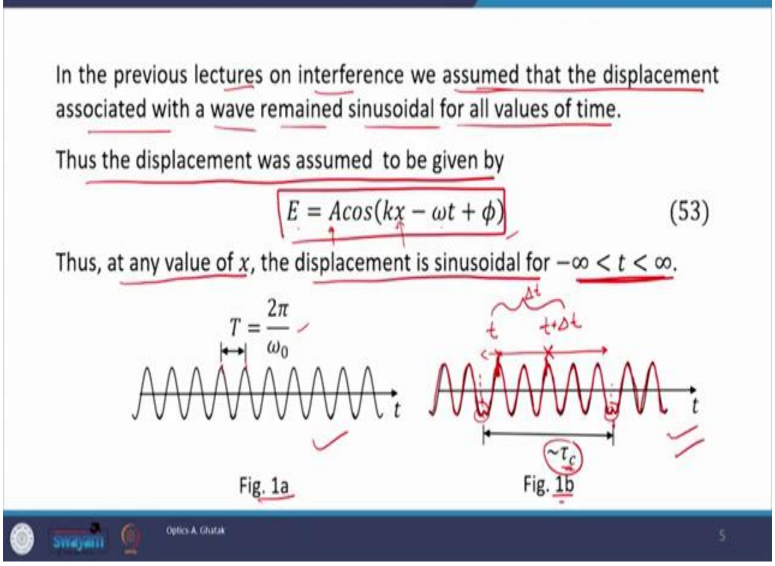

Now, as I said, if you fix x, so let us fix $x = 0$, then we see the sinusoidal nature of the wave for all values of t. And this corresponds to an idealized situation because the radiation from an ordinary light source consists of finite size wave train, the light which we receive from a source, it is not infinite wave train, it has certain length or certain time duration.

And a typical variation of wave train is shown in figure 1a, then let us go to figure 1b and in this figure what we see is that, that a wave is sinusoidal and it is oscillating like this and then all of sudden there is a jump, there you see a kink and all of sudden there is an abrupt change in phase and a new sinusoidal oscillation is start.

And this sinusoidal oscillation sustain till this point and here again we observe a jump and a new sinusoidal oscillation is start then here we see there is a phase jump, abrupt change in phase and abrupt change in amplitude. Now, the time during which the wave maintains its sinusoidal nature it is given by τ_c, and this one set of wave is called wave train.

Now, τ_c represents the average duration of the wave train that is the electric field remains sinusoidal for times of the order of τ_c. Now, we can conclude that at a given point, at a fixed point x in space, the electric fields at time t and $t + \Delta t$ will in general have a definite phase relationship if Δt is less than much-much less than τ_c.

If we pick two points in time, then those two points will be in certain definite phase relationship as long as this Δt, Δt, the time separation between the two time points is much-much less than τ_c. And these two points will never have a phase relationship if Δt is much-much longer than τ_c, and this is very much clear from this figure also.

Now, here if you choose these two points, take these two points suppose this is t and this is $t + \Delta t$ then the times separation between these two points is Δt and if this Δt is smaller than τ_c which is defined here then there is a phase relationship between these two type time point, but if Δt is much-much larger than τ_c then of course, there would be several phase jump which would be there in Δt and these two time points will not be in phase they would be totally uncorrelated.

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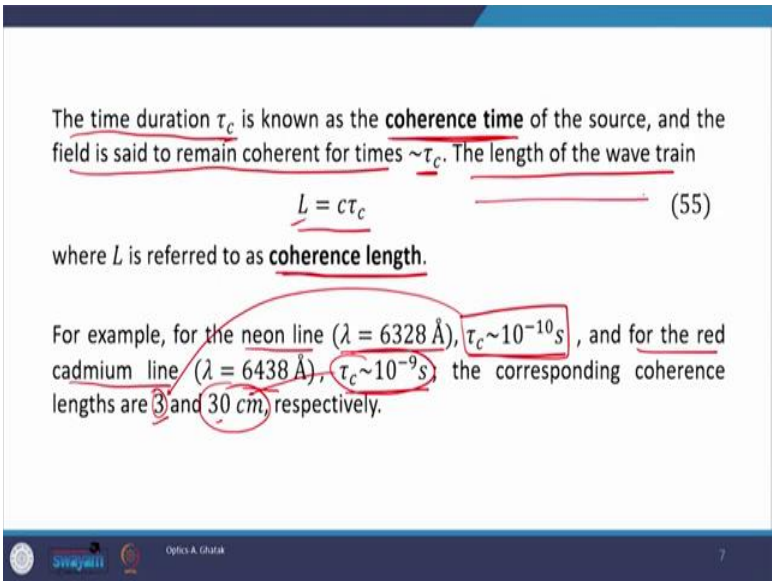

Now, this time duration τ_c is known as coherence time and the field is set to remain coherent for times which is of the order of τ_c and therefore, suppose light is emanating from some source and the wave train has a temporal length of τ_c then this τ_c defines the coherence time of the source.

Now, during this time the wave travels $c\tau_c$ distance and this is the length of a wave train and this length is referred to as coherence length, the τ_c is coherence time and the corresponding length is called coherence length, and as an example, we can take a neon line whose wavelength is given by 6328 angstrom and τ_c that is coherence time for this is 10^{-10} seconds.

And for the red light, red cadmium line, lambda is 6438 angstrom and for this τ_c 10^{-9} s while the corresponding coherence lengths are 3 and 30 centimeter respectively. Now, you see that when the coherence time is small, the corresponding coherence length is also small, when the coherence time is large, the corresponding coherence length is also large here, which is very much obvious from this relation, equation number 55.

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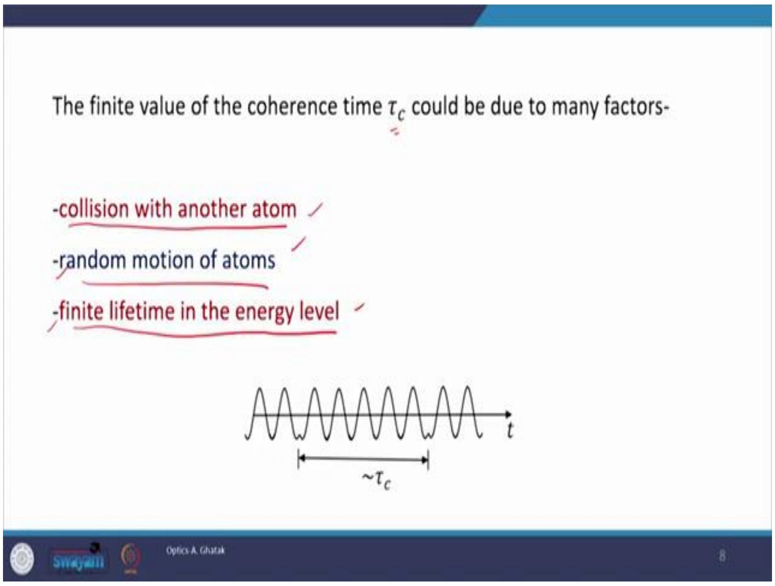

Now, in an idealized source, we will have in finite τ_c, but τ_c is finite then what is the origin of finiteness of τ_c, what affects τ_c, what limits τ_c from reaching to infinity, this limits are listed here, the first is collision with another atom, suppose an atom is there in a source and then it collides with the another atom, due to this collision, there is abrupt change in phase and that generates kink and that limits the wave train length.

The second factor which affects τ_c or which limits τ_c is random motion of atom here, the atoms moves inside the material volume and that also limits τ_c and the third one is finite lifetime of energy level, suppose an atom is an excited state and then when it goes down to its ground state, it emits radiation. But it only stays the atom within some finite time duration, it will reach down to the ground state and that also limits τ_c and therefore, the wave train which get generated will have certain time duration or certain length.

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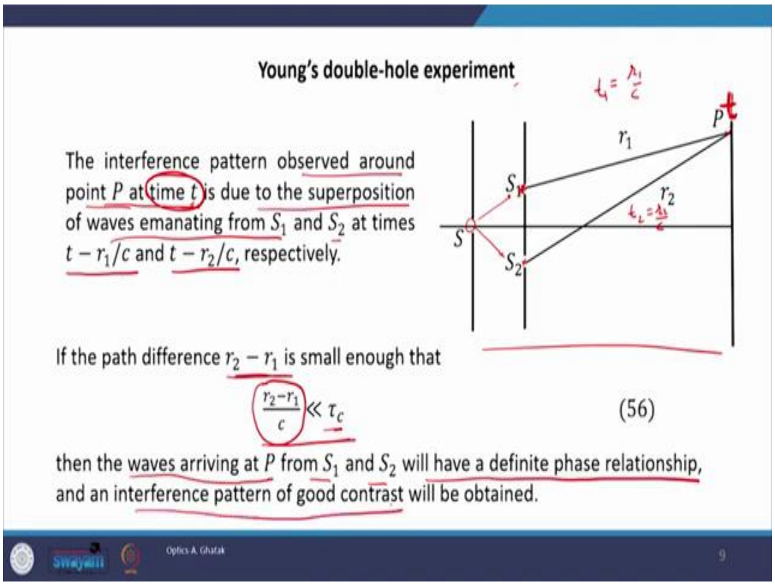

And having talked about these things, now, we will see what are the implications, how would it affect the usual interference experiments and what is the most widely studied experiments Young's double slits experiment or Young's double hole experiment and Michelson interferometer, we will implement this knowledge in Young's double hole experiment and Michelson interferometer and see what are the conclusions which we can draw.

Now, here you see usually Young's double hole experiment and there is a source S out of this source, two sources S_1 and S_2 are generated and the rays which are starting from the these two point sources are known as S_1 and S_2 the reaches to point P which is on the screen and at P due to the superposition we observed some fringe pattern, the distance between S_1 and P is r_1 while between S_2 and P is r_2.

Now, the interference pattern observed around point P at time t is due to the superposition of waves emanating from S_1 and S_2, of course. And since this distance between S_1 and P is r, the time which the light takes in reaching from S_1 to P would be r_1/c, while the time which the second ray will take from reaching from S_2 to P would be r_2/c.

Now, we are observing the interference pattern at point P at time t here, at time t the pattern is being observed but the rays which start from S_1 and S_2 it takes some time to reach at P, then what is the time when it is start from S_1, the light at S_1 is start at time $t - r_1/c$, while at S_2 it started $t - r_2/c$ which is very much obvious.

Because it is reaching at point P at time t therefore, it must have started sometime earlier and what is this earlier time these earlier times are $t - r_1/c$ and $t - r_2/c$ respectively from sources S_1 and S_2. Now, we will calculate the path difference, what is the path difference? Path difference would be $r_2 - r_1$.

Now, what is the time to cover this path difference, the time which the light will take in covering $r_2 - r_1$ is $(r_2 - r_1)$ /c, this is the time which correspond to the path difference. Now, if this time is less than τ_c then only the waves arriving at P from S_1 and S_2 will have a definite phase relationship and then only we will be able to see proper interference fringes with very good contrast.

I repeat if the time corresponding to the path difference is much-much smaller than τ_c which is the coherence time for the given sources. Then only we will be able to observe interference pattern of good contrast.

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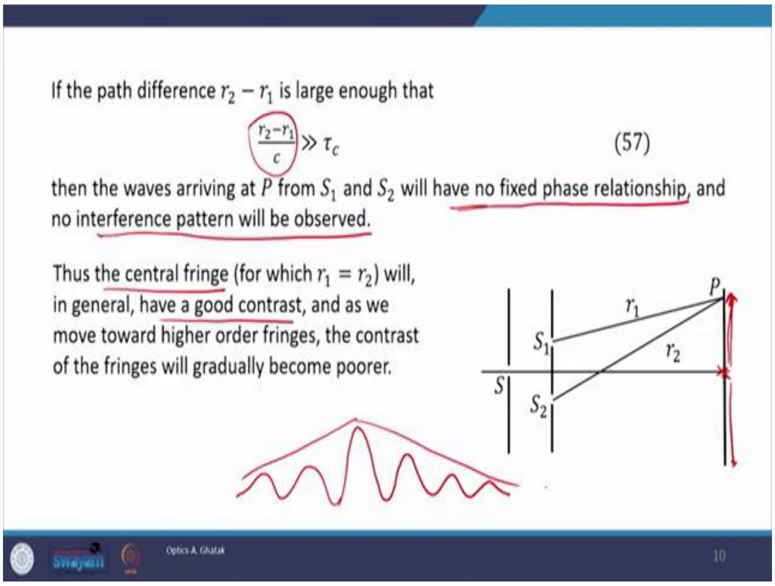

And if the time is larger than τ_c, then the waves reaching at point P will not have any fixed phase relationship and no interference pattern would be observed. Therefore, at the center means at this point we are $r_1 = r_2$, in general, we will have a good contrast fringe, because the path difference there is 0.

And which is of course smaller than τ_c, and as we move towards higher order fringes, as we move away from the center, the fringes will gradually become poorer, the contrast will decay down why the contrast will decay down because as we move away from the center, the path difference slowly increases.

And therefore, the corresponding time the light takes to cover the path difference will also increase and when it will be very close to τ_c and larger than τ_c the fringes will start to fade away and therefore, instead of getting good fringes, we will have these kinds of fringes slowly it will decay down.

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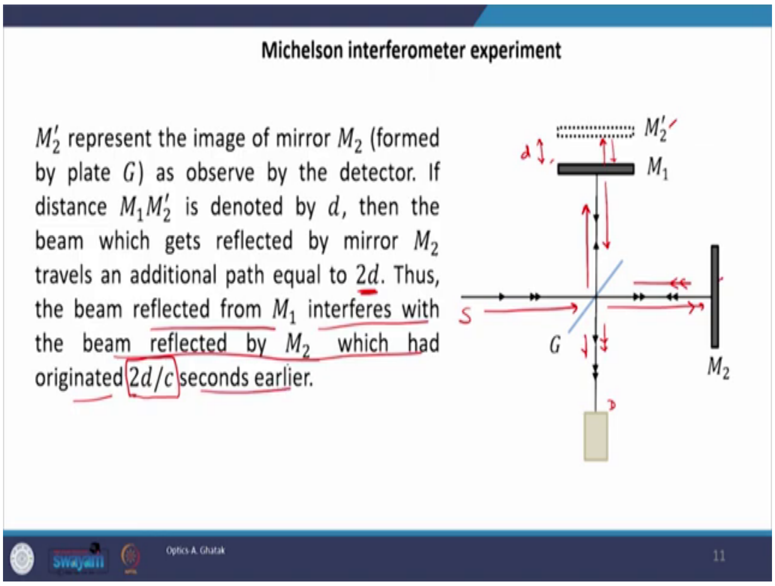

Now, we will implement this knowledge of coherence time in Michelson interferometer experiment, in Michelson interferometer, we know we have a source here and from this source the light falls on this beam splitter G and from the beam splitter, it goes in this direction as well as in this direction and then it returns back. After getting reflected and then the two lights goes into the detector, where they interfere and we observe interference fringes.

And in this figure, M'_2 is the virtual image of mirror M_2 and these two mirrors are separated by a distance d. Now the path difference between the two ray for normal incidence. Let me clearly write, these the separation between the two mirrors M'_2 and M_1, now the path difference between the rays which are interfering would ultimately be equal to 2d, twice of d.

Because the secondary which is getting reflected from M_2 mirror or its virtual image M'_2 would have to travel this thickness twice here, it will go in this direction and then return back and it is traveling d twice therefore, the part difference would be 2d. Now, how much time the light will take to cover 2d distance, it will take 2d/c second. And therefore, the beam reflected from

M_1 interferes with the beam reflected by M_2 which had originated 2d/c seconds earlier, which is quite correct.

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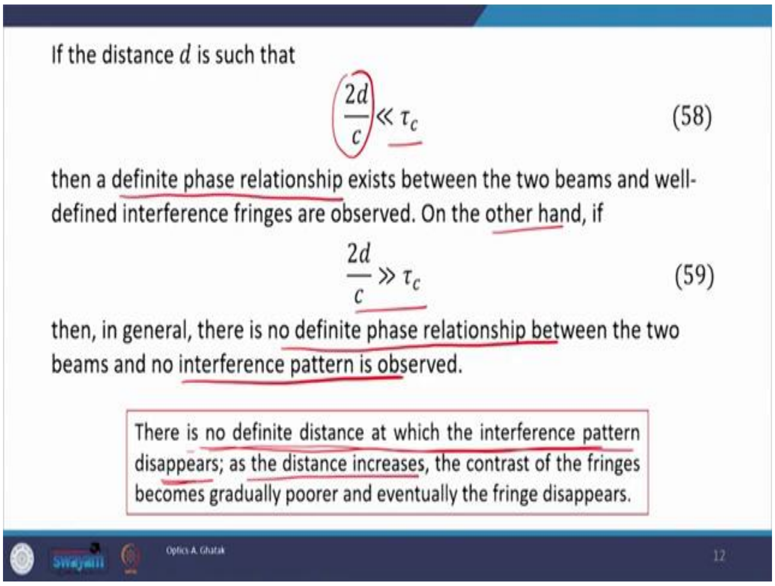

Now, again we will implement the same condition, if this time is much-much less than τ_c then only we will observe fringes because then the two interfering ray will have definite phase relationship. And on the other hand, if 2d/c is much-much larger than τ_c, two interfering beams will not have any definite phase relationship and we will not be able to observe an interference pattern.

Therefore, we conclude from this analysis is that there is no definite distance at which the interference pattern disappear, it is not so that it will there is a some critical time and critical path length difference beyond which we would not be able to observe fringes and before which are at a path length difference which is smaller than that critical value you will see interference pattern, it is not like switching, switch on and switch off like stuff.

Here as you gradually move towards τ_c as your characteristic time gradually move towards τ_c, the fringes will start to fade up and if you are far beyond τ_c if the characteristic time or our 2d/c time is much-much larger than τ_c then fringes will almost be invisible from our eyes.

And therefore, we can say that there is no definite distance at which the interference pattern disappears as the distance increases, distance means path length difference, the contrast of the fringes becomes gradually poorer and eventually the fringe disappears.

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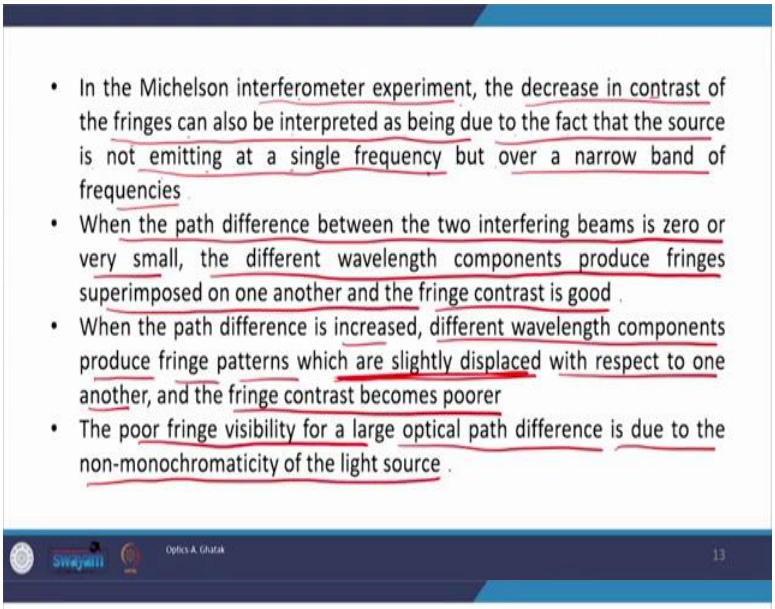

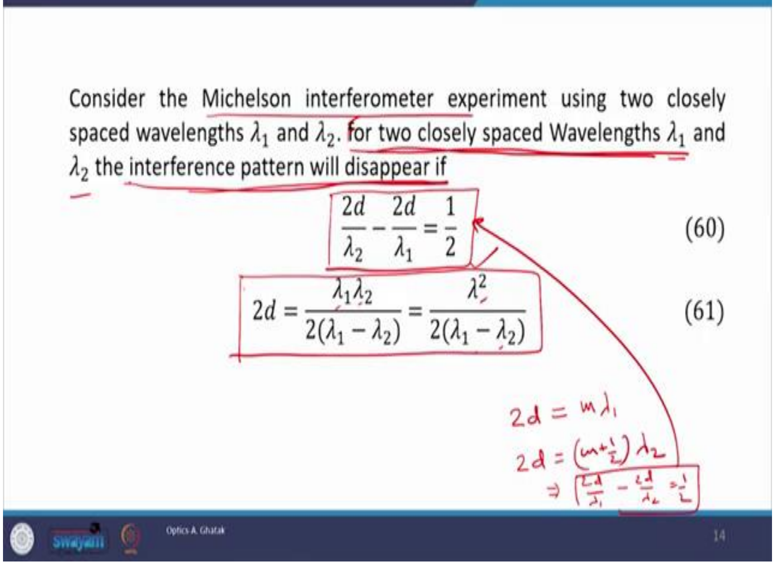

Now, the few points which we can note down is that, in the Michelson interferometer experiment, that decrease in the contrast of the fringes can also be interpreted as being due to the fact of the source is not emitting at a single frequency but over a narrow band of frequencies. I repeat in the Michelson interferometer experiment, the decrease in the contrast of the fringes can also be interpreted as being due to the fact that the source is not emitting at a single frequency, but over a narrow band of frequencies.

The same concept which we studied in the last slide, that can also be interpreted as that since, our source is a narrow band source. Our source is not ideal monochromatic source, it has several frequencies, although narrow band, but even in narrow band we have multiple frequencies and each frequency generates its own interference pattern and this blurs the overall interference pattern.

Now, how does it happen? When the two frequencies like suppose we have several frequencies, and when the path difference between the two interfering beams is zero or very small, the different wavelength component produce fringes superimposed on one another and the fringe contrast is good. Suppose we have a narrow band sources and the path length difference between the two interfering beams is very small, very close to zero.

And if this is so, then maxima and minima of different wavelengths, they fall almost on top of each other, all the maxima fall on the maxima of other frequencies. And therefore, we see a very good contrast very good fringes, but when the path length difference is huge, then what happens different wavelength components produced fringe patterns which are slightly displaced.

Therefore, the maxima does not fall on top of maxima and minima does not fall on top of minima, there would be a slight displacement which incorporate a little blurring in the fringe and which are slightly displaced, with respect to one and other and the fringe contrast therefore becomes poorer. The poor fringe visibility for large optical path difference is due to the non-monochromaticity of the light source, the same thing this is the same reasoning but it is explained in a different way.

Now, let us consider a Michelson interferometer in which the source is having two wavelengths λ_1 and λ_2 and which are very closely spaced. Now, for two closely spaced wavelengths λ_1 and λ_2, the interference pattern will disappear if maxima of one falls on the minima of other, suppose the it is normal incidence it is equal to integral multiple of wavelength here then suppose it is m λ and suppose it is for first wavelength, for second wavelength $2d = (m + 1/2)\lambda_2$.

Now, take the difference between the two and what you will get you will get the following $2d/\lambda_2 - 2d/\lambda_1 = 1/2$, this is the relation which is written here for normal incidence for two wavelength.

If maxima of one falls on the next successive minima of other, then you will have this condition and you will not see any fringe pattern, this is what it is said here for two closely spaced wavelengths λ_1 and λ_2 the interference pattern disappear if this relation holds. And from here we can get expression of the path length difference which depends upon the wavelength and on their separation, since λ_1 and λ_2 are very close, we can replace $\lambda_1\lambda_2 = \lambda^2$ where λ is average of λ_1 and λ_2.

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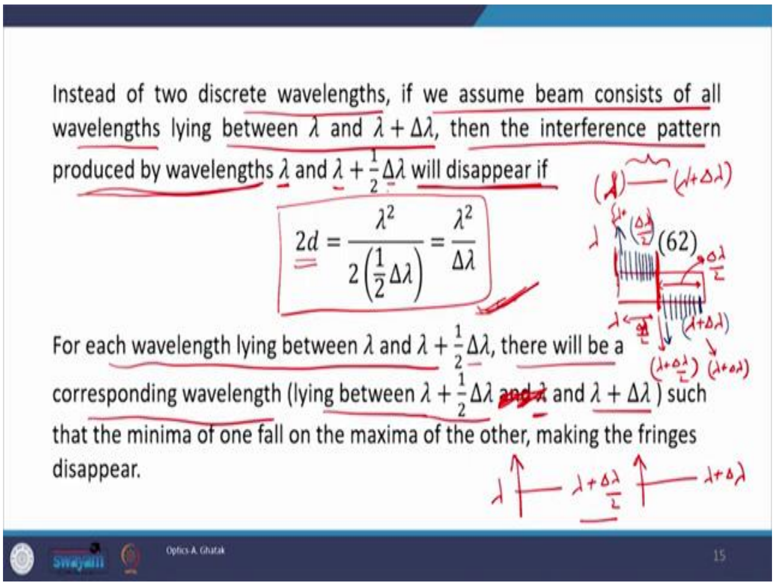

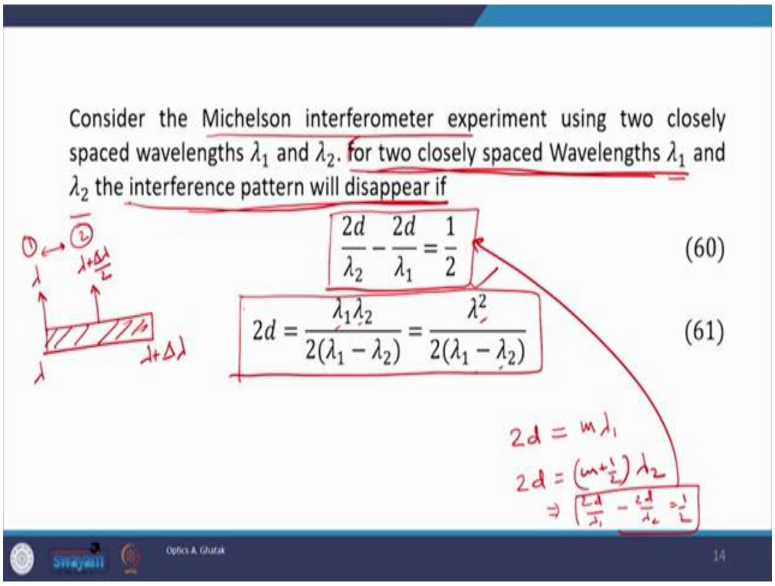

With this in hand let us do it differently. Now, instead of two discrete wavelength if we assume the beam consists of all wavelengths lying between λ and λ+$\Delta\lambda$, earlier we took only two discrete wavelengths λ_1 and λ_2 now, we are considering that we have a source which is emanating a beam.

And which have several wavelengths, a continuum of wavelength and this continuum of wavelength lie between λ and λ+$\Delta\lambda$, two limits are there, the first one is λ and the second is λ+$\Delta\lambda$ and between these two wavelengths, all the wavelength components are there in our beam.

Then, the interference pattern produced by the wavelength λ and $\lambda + \Delta\lambda/2$ will disappear if this relation holds, what exactly we are doing is here we consider two wavelengths λ_1 and λ_2 now, we are considering a band of wavelength and this band extend from $\lambda + \Delta\lambda$.

Now, what we are doing is that we are picking two values of wavelength in this band. The first one is λ and second is $\lambda + \Delta\lambda/2$. And we want these two interference pattern between these two wavelength, this is first wavelength and second wavelength between these two wavelength. We want the interference pattern to disappear and when will disappear, they will disappear when they satisfy equation number 61.

Now, let us replace λ_1 *by* λ and λ_2 *by* $\lambda + \Delta\lambda/2$ and then get this expression. Therefore, the interference pattern produced by wavelength λ and $\lambda + \Delta\lambda/2$ will disappear. If they satisfy equation number 61, we substituted the expression of λ_1 and λ_2 in expression 61 and we got this expression 62.

Now, we have a band, which start from λ ends at $\lambda + \Delta\lambda$ and out of this band we picked two wavelength λ and $\Delta\lambda/2$. And these wavelengths will the fringes for these two wavelengths will disappear if the path length difference $2d = \lambda^2/\Delta\lambda$, now for each wavelength lying between λ and $\lambda + \Delta\lambda$. There will be a corresponding wavelength lying between $\lambda + \Delta\lambda/2$ this is extra written and $\lambda + \Delta\lambda$.

Now, what I am saying is that we pick two wavelengths which is λ and $\Delta\lambda/2$. Now, for each wavelength lying in this band, there will be a wavelength lying in this band for which equation number 62 will be satisfied or alternatively for each wavelength in this band.

There would be a corresponding wavelength in the other half of the band which extend from $\Delta\lambda/2$ to $\lambda + \Delta\lambda$ sorry this is $\lambda + \Delta\lambda$, this width is $\Delta\lambda/2$, but the value of this wavelength is $\lambda + \Delta\lambda/2$ and similarly, this width is again $\Delta\lambda/2$ but the value of wavelength is, here it is $\lambda + \Delta\lambda/2$ and here it is $\lambda + \Delta\lambda$, these are the two bands.

The first band extend from λ to $\lambda + \Delta\lambda/2$ and the second extend from $\lambda + \Delta\lambda/2$ to $\lambda + \Delta\lambda$ only. So, for each wavelength, in this band there exist a wavelength in the second band such that minima of one falls on the maxima of other and this produces no fringe, this disappears the fringe.

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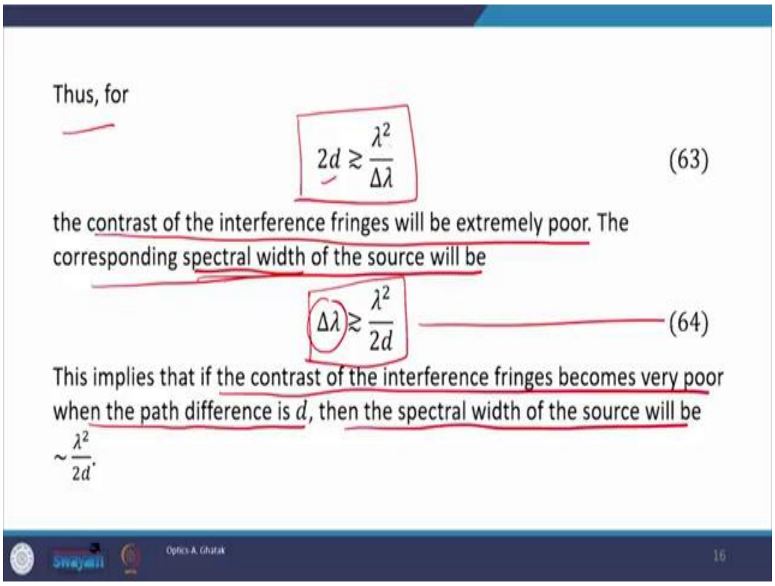

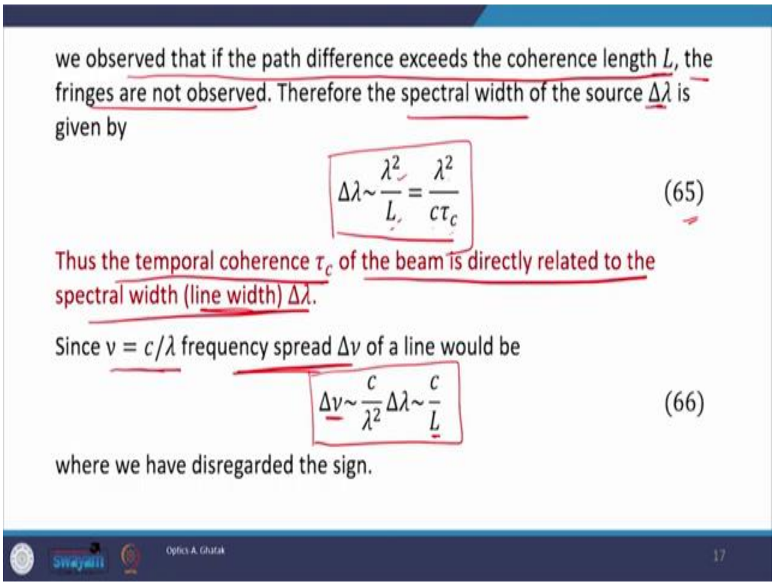

Now, therefore, as long as 2d is much larger than are equivalent $\lambda^2/\Delta\lambda$, the contrast of the interference fringes will be extremely poor. I repeat as long $2d \geq \lambda^2/\Delta\lambda$, the contrast of the fringes would be poor. The corresponding spectral width of the source will be $\Delta\lambda = \lambda^2/2d$.

Till now, we were only talking in terms of path length difference or the time which the light take in covering the path length difference and the coherence time, we were only relating these two terms. But now, the similar condition can also be imposed on the spectral width of the source and what is this condition, this condition is given by 64, and it says that if the contrast of the interference fringe becomes very poor, when the path difference is d then the spectral width of the source will be $\lambda^2/2d$.

The spectral width must not be larger than or equal to $\lambda^2/2d$, if the spectral width is satisfying equation 64, the fringe pattern would be very poor, the $\Delta\lambda$ must be smaller than the right hand side quantity in equation number 64 to have very good interference fringes.

And we observed that if the path difference exceeds the coherence length, the fringes are not observed, this is same statement. Either you say that the path difference must not exceed coherence length or the time taken to cover path length difference must not exceed the coherence time, the same thing therefore, the spectral width which is $\Delta\lambda$ can also be written as λ^2/L.

You see here $\Delta\lambda = \lambda^2/2d$, what is 2d, 2d is optical path difference and optical path length difference is a length which must be smaller than coherence length, the maximum value is L which is coherence length, and if it goes beyond the coherence length fringes will start to decay down.

Therefore, we can write $\Delta\lambda = \lambda^2/L$ which is equal to $\lambda^2/c\tau_c$, where L= $c\tau_c$. Does the temporal coherence time τ_c of the beam is directly related to the spectral width which we also call as line width. Once a source of given line width is given then we can quickly predict its coherence time using this relation using equation number 65.

Now, once we know the relation between wavelength width and coherence time, then we can quickly calculate the relation between $\Delta\upsilon$, the frequency width are frequency spread with a coherence length or coherence time.

How to do this we know that frequency $\upsilon = c/\lambda$ then $\Delta\upsilon$ which is frequency spread, it can be calculated by just taking the derivative of this relation and from here we get this relation between frequency spread and coherence length, the sign we have neglected because it does not play any role here we are just interested in the numbers.

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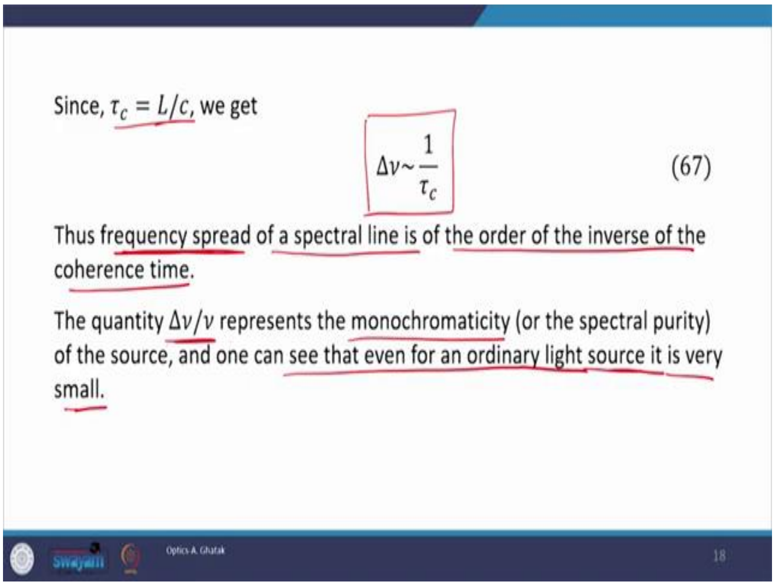

Now, since $\tau_c = L/c$, we substitute the expression here and $\Delta\upsilon$ is inversely proportional to τ_c the coherence time therefore, the frequencies spread of a spectral line is of the order of inverse of the coherence time. It means that once we are given a source we know its central frequency or central wavelength and its bandwidth, be it a $\Delta\lambda$ or $\Delta\upsilon$ then we can quickly calculate the coherence time.

The quantity $\Delta\upsilon/\upsilon$ is called monochromaticity of the source or spectral purity of the source. And one can see that even for ordinary light source it is very small, $\Delta\upsilon/\upsilon$is very small for ordinary light source but if we talk about lasers where $\Delta\upsilon$ is very small then this spectral purity increases. This is all for today. Thank you for listening me.