Applied Optics
Professor Akhilesh Kumar Mishra
Department of Physics
Indian Institute of Technology Roorkee
Lecture: 28
Fabry-Perot Interferometer And Etalon - II

Hello everyone, welcome back to my class. Today we will learn about uses the applications of Fabry Perot Interferometer.

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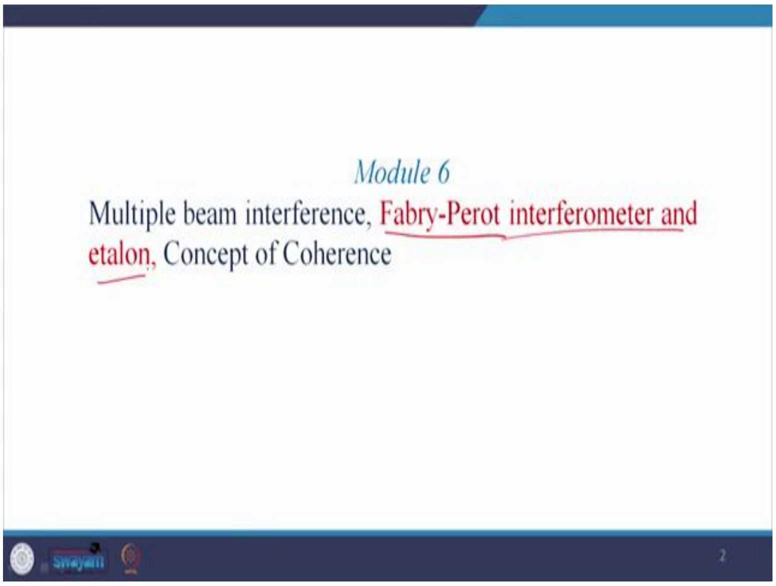

In the last class, we were introduced with Fabry Perot Interferometer and etalon, we also understood the difference between the interferometer and the etalon. Today we will learn about its applications.

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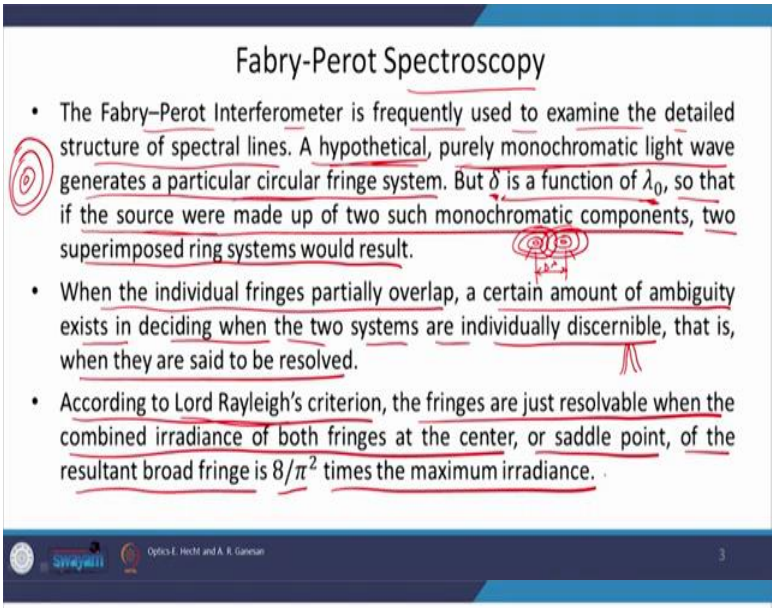

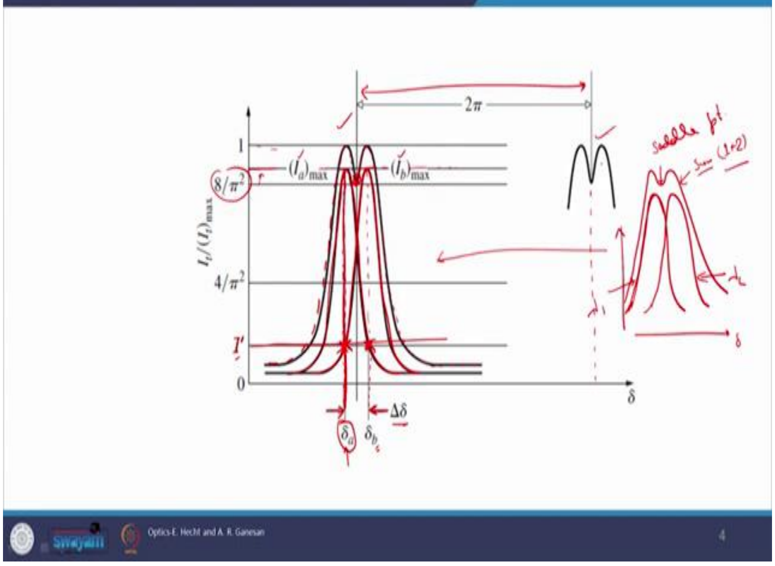

Application of Fabry-Perot interferometer is predominantly in spectroscopy. The Fabry Perot interferometer is frequently used to examine the detailed structure of spectral lines, a hypothetical purely monochromatic light wave, just observed this word hypothetical because it is impossible to have a purely monochromatic light source.

The hypothetical purely monochromatic light wave generates a particular circular fringe system, we already studied in the last class that this is the type of the fringe pattern we observe in Fabry Perot interferometer provided we have only one wavelength, but if we have more than one, then for each wavelength. We will observe such type of fringes a pattern and the screen it would be quite complicated looking and we know that the phase difference is a function of wavelength, the wavelength appear in the expression of phase difference.

So that if the source were made up of two such monochromatic components, two superimposed ring system would result. Because we have two wavelength one wavelength will make this type of ring pattern, the other wavelength which is adjacent to the first, it will also make its own ring pattern and they were would be very closely sitting if the wavelengths are very close.

Now, using this spectroscopic tool, this Fabry Perot Spectrometer or Fabry Perot Interferometer, we can investigate the separation between the two wavelength, the values of these wavelength and the resolving power of the spectrometer.

Now, when the individual fringes partially overlap, a certain amount of ambiguity exist in deciding when the two systems are individually discernible, that is when they are set to be resolved. The statement says that suppose we have two ring patterns, which are say this far. Now, if the wavelength which generated these two fringe patterns, if it is very closely spaced, then the fringes would also be very close, these rings would also be very close, the center of the rings will come very close to the other.

And in this situation probably we will not be observed or we will not be able to trace the width of the fringes or the peak of the fringes, because in this situation, the peaks which we studied in our last class, they would be sitting very close to each other and it would be so close that probably we mistakenly say that they are one peak. But they are two in reality, therefore, a proper definition is required when the two peaks would be called resolvable.

And the definition came from Lord Rayleigh, he gave a criteria which is called Lord Rayleigh criteria. According to Lord Rayleigh's criterion, the fringes are just resolvable when the combined irradiance of both fringes at the center or saddle point of the resultant broad fringe is $8/\pi^2$ times the maximum irradiance.

Then the Rayleigh says that the fringes are just resolvable, just resolvable means, if they are a bit more close, then we would not be able to resolve or the interferometer would not be able to resolve them and it may read them as 1. But if they are placed in such a way that the irradiance at the center of the two peaks is equal to the maximum intensity by $8/\pi^2$, then they would be set to be just resolved.

Now, this can easily be understood through this figure. In this figure, we have two peaks, which are from two wavelengths. Here suppose this is from wavelength λ_1 and this is from wavelength λ_2 and these two peaks are very close, because the corresponding wavelengths are very close. Now, if you plot δ and the relative irradiance, then you see that two peaks are sitting very close to each other.

And we know that if we have two sources, which are totally incoherent then the resultant irradiance distribution at the screen would be sum of the two intensities or sum of the two irradiance therefore, the total irradiance in this case would be the sum of the two irradiances and therefore, roughly we will see something like this.

This overlap is sum of the two irradiance, the first irradiance plus second irradiance and here we see there is a saddle point, at the center we have a saddle point. And this is what exactly is being shown here also and this is your first peak, first maxima which correspond to the first wavelength.

And this is second maxima which correspond to the other wavelength, the second one and if you sum these two irradiance then you get this, the dashed line, and if you sum these two irradiance then you see that at the center you get a saddle point and Lord Rayleigh said that if the saddle point is $8/\pi^2$ times the maximum.

Suppose here the maximum is 1 and this saddle point is $8/\pi^2$, then the two peaks would be resolvable, we can say that they are resolvable and if they are closer then saddle point would be a bit up and then we cannot resolve them, if the saddle point is lower than they of course, are resolvable.

Now, suppose that δ_a represents the phase value for first wavelength for which we get maximum at this phase value, the first wavelength gives us a maxima and at δ_b value of the phase the second wavelength gives us maxima, there are two maxima and also assume that the separation between these two are the difference between these two phases is $\Delta\delta$. The two maximas which owe their origin in two wavelengths, they are also supposed to have same maximum irradiances say there irradiances are $I_a$and I_b and they are here supposed to be equal, $(I_a)_{max} = (I_b)_{max}$ and the maxima of the two peaks fall on this line.

Now, here what do you see that there is another intensity irradiance point which is represented by I'. I' represents the irradiance of second peak or the irradiance value of the second wavelength, this is the irradiance peak of the first wavelength and at this phase value, the irradiance of the second is given by I'.

Now, here since the peaks are symmetric, they have same widths and the same height are same irradiance the, I' value for both the wavelength for both the irradiances is same. Therefore, we can see if we draw a horizontal line and these are the two values of the intensities of the other wave wavelength.

Now, one more thing you must keep into the mind is that the second peak will appear at a phase separation of 2π, because on each 2π we will get a maximum because δ is equal to integral multiple of 2π this is the one maximum this is the second successive maximum.

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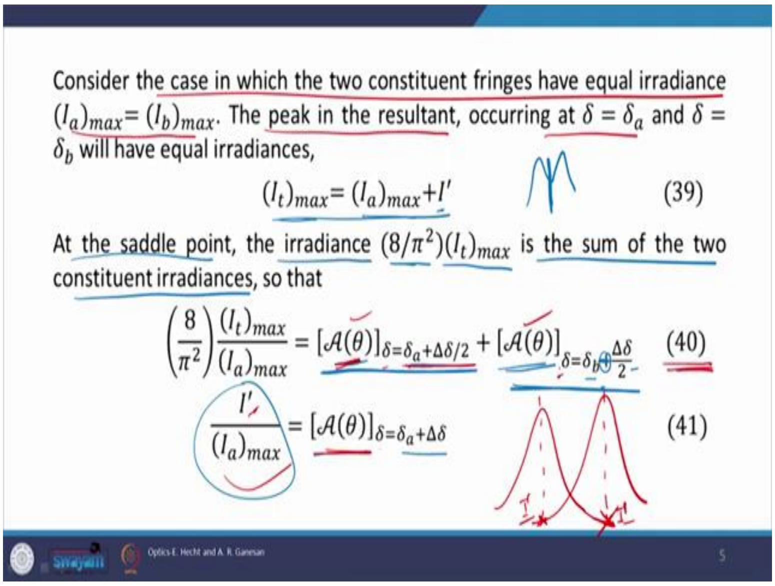

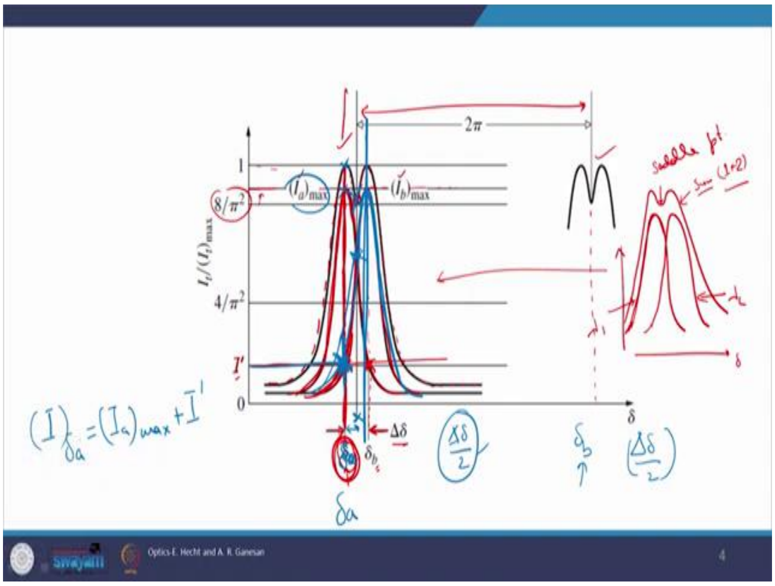

Now, consider the case in which the two constituent fringes have equal irradiance, this we have already talked about, the peak of the resultant occurring at $\delta = \delta_a$, now we are talking about this value of phase. Now the question is what would be the value of total irradiance and this δ value.

The total irradiance at $\delta = \delta_a$ would be the irradiance due to the first peak and irradiance due to the second one, but second one is this one, let us choose a different color, probably this would increase the visibility, this is the second peak. And we see that the value of irradiance of the second peak at $\delta = \delta_a$, is I', this we have already talked about.

Therefore, the total irradiance would be $(I_a)_{max}$, which is this value, and then plus I', and this would be equal to 1 here. The total irradiance I at δ_a would be equal to $(I_a)_{max} + I'$. Now, we can write it safely, $(I_t)_{max}$ the total irradiance maximizes because at $\delta = \delta_a$ we have maxima the peak of irradiance.

Therefore, we write $(I_t)_{max} = (I_a)_{max} + I'$. Similarly, at $\delta = \delta_b$, $(I_t)_{max} = (I_b)_{max} + I'$, but since $(I_a)_{max} = (I_b)_{max}$, we can use them alternatively. At the saddle point means at this dip, at this dip the irradiance $8\pi^2 (I_t)_{max}$ is the sum of two constituent irradiances. Now, here you in the figure, you see, this is the saddle point, and here we will have to add up the intensity of the red and the intensity of the blue or irradiance of the red and irradiance of the blue.

And therefore, this is the irradiance of the first peak, and this is the irradiance of the second peak. Now, in this figure irradiance maximizes at $\delta = \delta_a$, this is δ_a, where first irradiances maximizing and here is the center, center is this one which is $\Delta\delta/2$ unit away from δ_a. Similarly, center from the second peak would be $\Delta\delta/2$ unit away from δ_b.

Now, if you calculate the irradiances at the center for two peaks and add them up, then we will get the irradiance at the saddle point and this is what is being done here, we calculate irradiance after first at a phase point of $\delta_a + \Delta\delta/2$ and we calculate irradiance of the second peak at a phase point of $\delta_b + \Delta\delta/2$.

Now, I should have used here minus sign, $\delta_b - \Delta\delta/2$, this plus sign is also correct, but because the irradiance value irrespective whether we are going in plus direction or minus direction it is same because the, this peak is symmetric. Now, the $I'/(I_a)_{max}$ would be equal to the value of irradiance at $\delta_a + \Delta\delta$.

Because this I' is being calculated at the center of one of the peaks, not center of the two peaks, at the center of the one of the peaks this is the second peak and its center is here I' is calculated here. Similarly, for red color or the similarly for red peak the I_{max} is calculated here, I' is calculated here and this is the, I' value.

And if we want to calculate I', the relative irradiance I' then you calculate irradiance at $\delta_a + \Delta\delta$ and this will give you the relative irradiance value at the second peak, we have two peaks and we want to calculate this irradiance here at this peak. Similarly, this irradiance here at this peak.

At the peak of other wavelength what would be the value of irradiance of the first one, this is given by I' and its relative value would be irradiance at $\delta = \delta_a + \Delta\delta$, these are I', which are same if the peaks are symmetric, identical, then both I' would be the same. Now, if we know this, then we can solve now equation number 40, substitute for these two Airy functions and then substitute the value of the phases.

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We can solve equation (40) for $\Delta\delta$. For large values of F,

$$\Delta\delta \approx \frac{4.2}{\sqrt{F}} \tag{42}$$

This represents the smallest phase increment, $(\Delta\delta)_{min}$, separating two resolvable fringes. From eqn. (28)

$$m\lambda_0 = 2n_f d \cos\theta_t + \frac{\phi\lambda_0}{\pi} \tag{43}$$

Dropping the term $\phi\lambda_0/\pi$, which is negligible, and then differentiating, yielding

$$m(\Delta\lambda_0) + \lambda_0 \Delta m = 0 \tag{44}$$

$$\frac{\lambda_0}{\Delta\lambda_0} = -\frac{m}{\Delta m} \tag{45}$$

Fabry-Perot Spectroscopy

- The Fabry–Perot Interferometer is frequently used to examine the detailed structure of spectral lines. A hypothetical, purely monochromatic light wave generates a particular circular fringe system. But δ is a function of λ_0, so that if the source were made up of two such monochromatic components, two superimposed ring systems would result.
- When the individual fringes partially overlap, a certain amount of ambiguity exists in deciding when the two systems are individually discernible, that is, when they are said to be resolved.
- According to Lord Rayleigh's criterion, the fringes are just resolvable when the combined irradiance of both fringes at the center, or saddle point, of the resultant broad fringe is $8/\pi^2$ times the maximum irradiance.

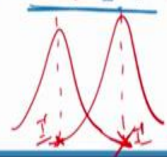
Consider the case in which the two constituent fringes have equal irradiance $(I_a)_{max} = (I_b)_{max}$. The peak in the resultant, occurring at $\delta = \delta_a$ and $\delta = \delta_b$ will have equal irradiances,

$$(I_t)_{max} = (I_a)_{max} + I' \tag{39}$$

At the saddle point, the irradiance $(8/\pi^2)(I_t)_{max}$ is the sum of the two constituent irradiances, so that

$$\left(\frac{8}{\pi^2}\right)\frac{(I_t)_{max}}{(I_a)_{max}} = [\mathcal{A}(\theta)]_{\delta=\delta_a+\Delta\delta/2} + [\mathcal{A}(\theta)]_{\delta=\delta_b-\frac{\Delta\delta}{2}} \tag{40}$$

$$\frac{I'}{(I_a)_{max}} = [\mathcal{A}(\theta)]_{\delta=\delta_a+\Delta\delta} \tag{41}$$



And after a bit of calculation, we can get the expression of $\Delta\delta$ and which for the large value of F, which is coefficient of fineness would be equal to $4.2/\sqrt{F}$, what does this represent? This represents the smallest phase increment, $\Delta\delta_{min}$ separating the two resolvable fringes.

Why does this represent the smallest phase increment because of equation 40, you see on the left hand side this is $(I_t)_{max}$ is the maximum intensity and then we are multiplying with $8/\pi^2$ which is the Rayleigh criteria, Rayleigh criteria for fringes that are just resolvable, for just resolvable fringes, we have equation number 40 and it is giving the minimum $\Delta\delta$, if we solve equation number 40 we will get the value of $\Delta\delta$ which is the least for the observable, for the resolvable fringes and therefore, equation number 42 gives the minimum value of $\Delta\delta$.

Now, we know from our last lecture, that the path difference is equal to $2n_f dcos\theta$ and plus there was a term 2φ which was equation number 28 then we rewrite that equation and in this form and m Γ is the integral multiple of wavelength which is again path difference, φ is, it is $2n_f dcos\theta$ is intact and this is θ, it is just rewritten equation number 22 is rewritten and now it is named as equation number 43.

And now, after writing equation number 28 we said that, that there was a in the denominator there was wavelength and we said that the wavelength is very small and d is large. Therefore, the second term is relatively very small and therefore, neglected and this we do here too, we will drop the second term on right hand side of equation number 43 and then differentiate the rest of the term.

Now, if you differentiate equation number 43 after dropping this term, then you will get $m\lambda = 2n_f dcos\theta_t$, here n_f is the refractive index of the film which is in the cavity, d is the thickness of the cavity, θ is the angle, they all are constant quantity, they are fixed they will not vary and therefore, they are differentiation would be 0, m is the order and λ is the wavelength and since we are in using our Fabry Perot as a spectroscopic tool.

The wavelength may vary, we may have several wavelengths and we may have several orders too, one order may coincide with like m^{th} order of one wavelength may coincide with m^{th} order of second wavelength and there are several other possibilities therefore, both λ and m the order of the fringe are variable here.

And therefore, we can after differentiation, we can write this $m\Delta\lambda_0 + \lambda_0\Delta m = 0$, Δm is differentiation of m,$\Delta\lambda$ is the differentiation of λ_0 and from equation 44 we can write this

$\lambda/\Delta\lambda = -m/\Delta m$. Now, minus sign does not hold any significance it just says that if one quantity is increasing, the other is decreasing.

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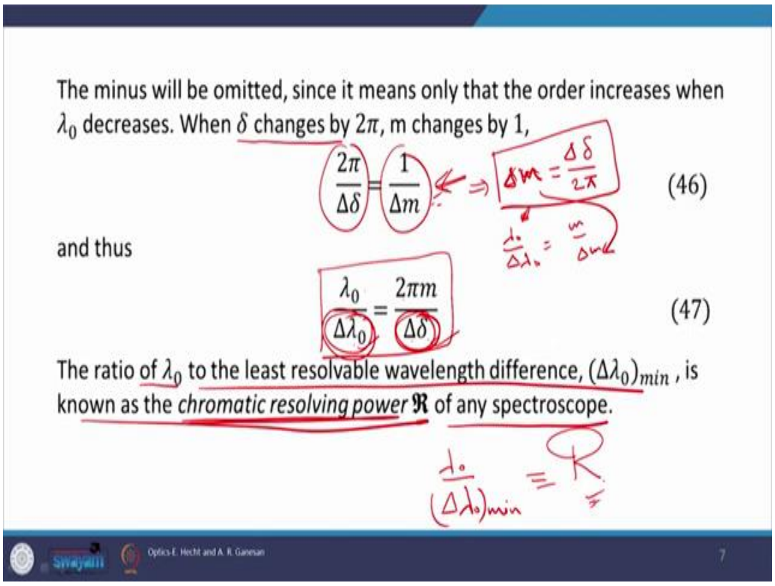

Now, we also know then when phase changes by 2π, the m which is an integer it changes by 1, 1 unit. Therefore, by $2\pi/\Delta\delta = 1/\Delta m$, this relation we can easily guess. Now, once we do know this relation then from here we can get the expression of Δm and from here $\Delta m = \Delta\delta/2\pi$.

And if we know this relation then we will substitute this in this relation $\lambda_0/\Delta\lambda_0 = m/\Delta m$, we will substitute here for Δm and this substitution will give us equation which is $\lambda_0/\Delta\lambda_0 = 2\pi m/\Delta\delta$.

The ratio of λ_0 to the least resolvable wavelength differences known as chromatic resolving power italic R of any spectroscope. Now, here while doing so, we picked here $\Delta\delta$ it is least smallest for least resolvable separation, the smallest resolvable separation, if $\Delta\delta$ is small, then we will have $\Delta\lambda_0$ minimum, once $\Delta\lambda_0$ is minimum, then this relation $\lambda_0/\Delta\lambda_0$ minimum.

This is defined as italic R, which is called chromatic resolving power. And chromatic resolving power defines the power of resolving the different wavelength of any spectroscopic tool, here in our case it is Fabry Perot Interferometer. Therefore, if we know the smallest value of $\Delta\delta$ we can calculate the smallest value of $\Delta\lambda_0$ and from there the resolving power, the chromatic resolving power of the Fabry Perot Interferometer.

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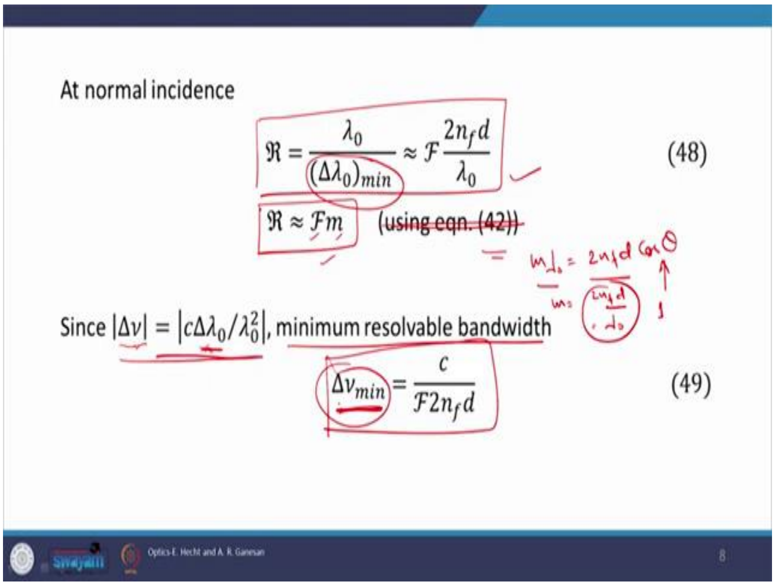

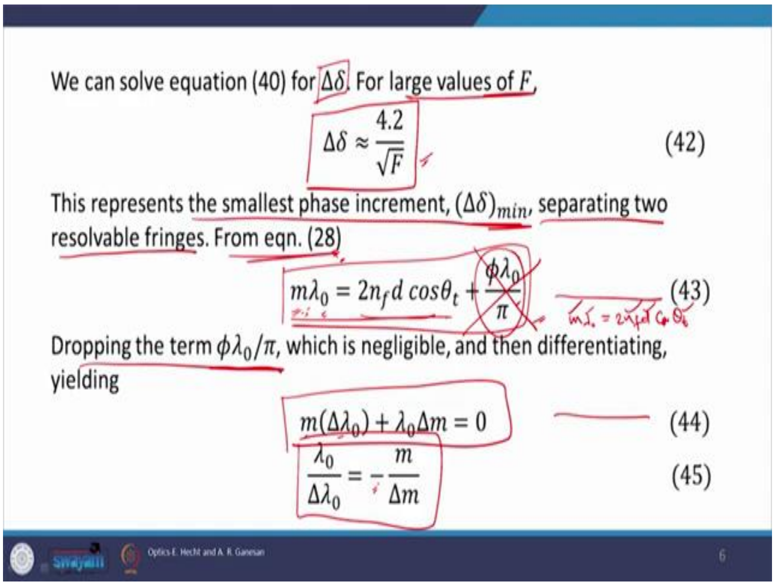

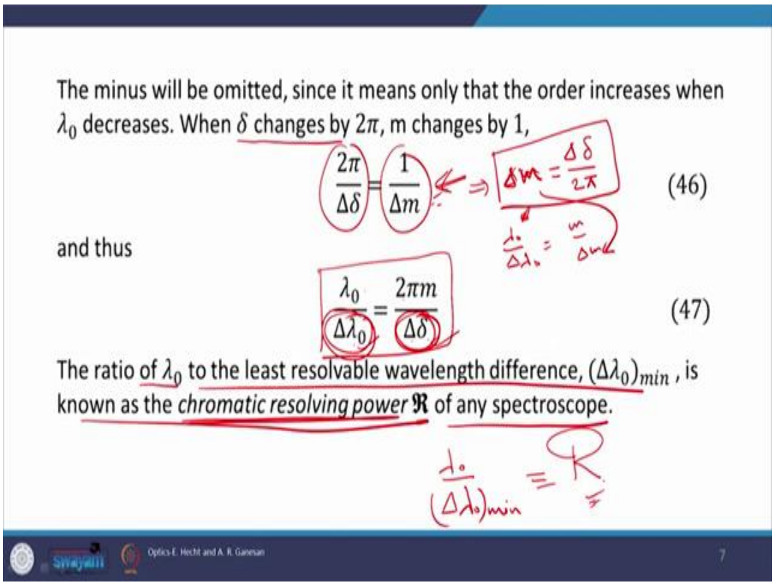

Now, for normal incidence we can use equation number 42 which is nothing but this relation, this is $4.2/\sqrt{F}$ and the relation should be because here we are using $\lambda_0/\Delta\lambda_0$ which is $2\pi m/\delta$ and we know that mλ this is the relation.

Equation number 43, $m\lambda_0 = 2n_f dcos\theta$a and $m\lambda_0 = 2n_f dcos\theta$ for normal incidence, this term would be 1 and $m\lambda_0 = 2n_f d$ or $m = 2n_f d/\lambda_0$, we can replace for m and once you replace for m in this relation, in relation 47.

And then substitute for $\Delta\delta$ when you get the solution equation number 48 and from here you can also get that resolving power is equal to fineness into m, m is the integer order of the fringe once you know this and are once you know what is the least dissolvable wavelengths separation then you can also calculate the corresponding least resolvable frequency separation because $\Delta\upsilon$ is related to $\Delta\lambda_0$ through this relation.

Therefore, minimum resolvable bandwidth would be given by equation number 49 here, we just substituted for $\Delta\lambda$, you can express $\Delta\lambda_0$ in terms of $\Delta\upsilon$ and once you do this, you can get the minimum value of $\Delta\upsilon$, the minimum dissolvable bandwidth.

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As the two components present in the source become increasingly different in wavelength, the overlapping peaks separate. As the wavelength difference increases, the mth order fringe for one wavelength λ_0 will approach the $(m+1)th$ order for the other wavelength $(\lambda_0 - \Delta\lambda_0)$. The particular wavelength difference at which overlapping takes place, $(\Delta\lambda_0)_{fsr}$ is known as the **free spectral range**.

From eqn. (47), a change in δ of 2π corresponds to $(\Delta\lambda_0)_{fsr} = \lambda_0/m$, at normal incidence

$$(\Delta\lambda_0)_{fsr} \approx \lambda_0^2/2n_f d \qquad (50)$$

or

$$(\Delta\nu)_{fsr} \approx c/2n_f d \qquad (51)$$

The minus will be omitted, since it means only that the order increases when λ_0 decreases. When δ changes by 2π, m changes by 1,

$$\frac{2\pi}{\Delta\delta} = \frac{1}{\Delta m} \qquad (46)$$

and thus

$$\frac{\lambda_0}{\Delta\lambda_0} = \frac{2\pi m}{\Delta\delta} \qquad (47)$$

The ratio of λ_0 to the least resolvable wavelength difference, $(\Delta\lambda_0)_{min}$, is known as the *chromatic resolving power* $\mathfrak{R}$ of any spectroscope.

At normal incidence

$$\mathfrak{R} = \frac{\lambda_0}{(\Delta\lambda_0)_{min}} \approx \mathcal{F}\frac{2n_f d}{\lambda_0} \qquad (48)$$

$\mathfrak{R} \approx \mathcal{F}m$ (using eqn. (42))

Since $|\Delta\nu| = |c\Delta\lambda_0/\lambda_0^2|$, minimum resolvable bandwidth

$$\Delta\nu_{min} = \frac{c}{\mathcal{F}2n_f d} \qquad (49)$$

Now, as the two components present in the source become increasingly different in wavelength or alternatively if you keep varying the wavelength in such a way that $\Delta\lambda_0$ increases slowly then the overlapping peaks separate, initially the wavelengths were close then the peaks were like this and if you increase that $\Delta\lambda$ then what will happen, this peak will be separated.

As the wavelength difference increases, the m^{th} order fringe from one wavelength will approach to the m plus 1th order film for the other wavelength, what it says is that suppose for one wavelength you were having such type of fringe pattern and if you increase the wavelength or if you change the wavelength then what ultimately will happen is that, that suppose this is the fringe pattern for the second wavelength.

Now, if you keep increasing the wavelength separation then the two fringes which were initially very close, they will open up, they will start going away from each other and a situation may come when this fringe may start overlapping with the red one, the next one. The situation may come that the blue one falls on the red one if you keep varying the wavelength.

And in this situation the m^{th} order fringe for one wavelength that is λ_0 will approach to the m plus 1th order of the other wavelength $\lambda_0 - \Delta\lambda_0$, the smaller wavelength. The particular wavelength difference at which overlapping takes place that is known as free spectral range and is designated by $(\Delta\lambda_0)_{fsr}$. It means if this is your fringe pattern of a particular wavelength.

And if you keep wearing the $\Delta\lambda_0$ then it may so happen that one wavelength may start overlapping with the second one and this overlapping happens for different orders, the mth order fringe of one wavelength may start to overlap with the m plus 1th order fringe of the other wavelength.

Now if you keep varying then what will happen that it will slowly again separate and then it will start to overlap with the next one, then the separation at which this overlapping takes place is called $\Delta\lambda_0$ and this is known as free spectral wavelength and designated as $(\Delta\lambda_0)_{fsr}$, this is wavelength to wavelength separation this is $(\Delta\lambda_0)_{fsr}$.

Now, from equation number 47, this is our equation a change in δ of 2π correspond to $(\Delta)_{fsr}$ is equal to $\Delta\lambda_0$/m, here the δ is change by 2π that $(\Delta\lambda_0)_{fsr}$ you say if you substitute Δ this del by 2π then $(\Delta\lambda_0)_{fsr}$ would be λ_0/m, and this is the expression for $(\Delta\lambda_0)_{fsr}$ and this is true only for normal incidence.

And therefore, $(\Delta\lambda_0)_{fsr}$ can be expressed byλ ${\lambda_0}^2/2n_f d$ because we know that $(\Delta\lambda_0)_{fsr}$ as given above is this and $m\lambda = 2n_f d$, in case of normal incidence therefore, m would be $2n_f d/\lambda$ and if you replace a substitute for this m then you will get this relation and once $\Delta\lambda_0$ is known the corresponding spectral width can also be calculated.

And which is given by here $(\Delta v)_{fsr}$ would be c/$2n_f d$ and here what we see is that $(\Delta\lambda)_{fsr}$, it is inversely proportional to the thickness d, the width of the Fabry Perot cavity. But, if you go back and check for the expression of $(\Delta\lambda)_{min}$ then $(\Delta\lambda)_{min}$ is again inversely proportional to d they both are inversely proportional to d.

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We need $(\Delta\lambda_0)_{min}$ as small as possible and $(\Delta\lambda_0)_{fsr}$ as large as possible, but

$$\frac{(\Delta\lambda_0)_{fsr}}{(\Delta\lambda_0)_{min}} = \frac{\pi\sqrt{F}}{2} = \mathcal{F} \qquad (52)$$

At normal incidence

$$\Re = \frac{\lambda_0}{(\Delta\lambda_0)_{min}} \approx \mathcal{F}\frac{2n_f d}{\lambda_0} \qquad (48)$$

$\Re \approx \mathcal{F}m$ (using eqn. (42))

Since $|\Delta\nu| = |c\Delta\lambda_0/\lambda_0^2|$, minimum resolvable bandwidth

$$\Delta\nu_{min} = \frac{c}{\mathcal{F}2n_f d} \qquad (49)$$

As the two components present in the source become increasingly different in wavelength, the overlapping peaks separate. As the wavelength difference increases, the mth order fringe for one wavelength λ_0 will approach the $(m+1)th$ order for the other wavelength $(\lambda_0 - \Delta\lambda_0)$. The particular wavelength difference at which overlapping takes place, $(\Delta\lambda_0)_{fsr}$ is known as the **free spectral range**.

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or

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But what do we want, we want an instrument, which can resolve the two peaks which are very closely spaced. Therefore, we want $(\Delta\lambda)_{min}$ to be as small as possible we want $(\Delta\lambda)_{min}$ very small, we want our instrument to have very high resolving power which means it can separate even very closely spaced wavelengths.

And alongside we also want that $(\Delta\lambda)_{fsr}$ be very large because if we vary the wavelength the one of the fringes must not start overlapping with the another order of the other wavelength. Therefore, this peak to peak separation this $(\Delta\lambda)_{fsr}$, we want this to be very huge.

But how can we achieve it, from this relation what we know is that if you want to decrease $(\Delta\lambda)_{min}$ then we need to increase d here and this can be written like this if we increase d, then what this will lead to $(\Delta\lambda)_{min}$ decrease and this is favorable for us we want this to happen.

If we want smaller value of $\Delta\lambda_0$t we will have to increase d but let us see what would be the effect of increasing d in the $(\Delta\lambda)_{fsr}$ and $(\Delta\lambda)_{fsr}$ is given here and here too if you increase d then $(\Delta\lambda)_{fsr}$ will reduce down, it means the fringes in a spectral domain will come closer to each other. But we do not want this, if they will be very close. Then there would be a possibility of overlap. Therefore, we cannot randomly play with the d, the width of Fabry Perot Interferometer.

Now if you take the ratio of $(\Delta\lambda)_{fsr}/(\Delta\lambda)_{min}$ then this is found to be equal to italic F which is fineness, this is a constant, then there is a trade-off between the 2 and depending upon the requirement of a particular application, we can play with the values so as coefficient of fineness remains constant. And this is all for this lecture, and thank you for listening me.