Applied Optics
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Lecture – 27
Fabry-Perot Interferometer and Etalon - I

Hello everyone, welcome back to my class and now we are in module-6. And in the last class of module-6, we discussed about multiple beam interference. And there we talked about multiple reflections and multiple transmissions and we saw that multiple refractions generate fringe pattern and similarly, multiple transmissions also generate fringe pattern. And we also learn that how an incoherent extended source generate or sustain interference fringe.

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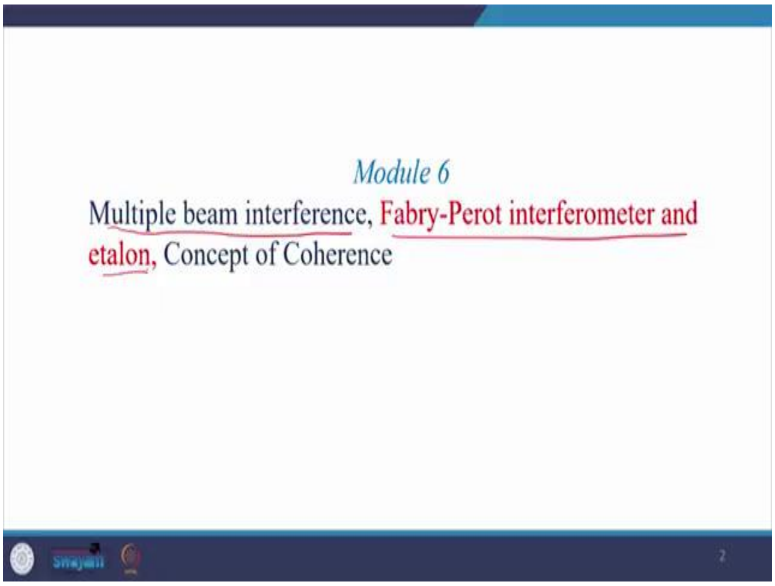

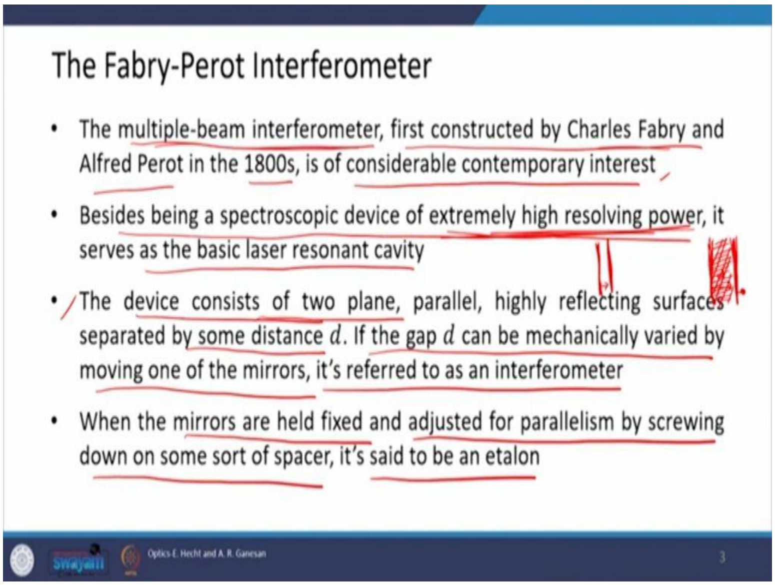

Today, we will start Fabry-Perot interferometer and etalon. Now, the multiple beam interferometer, first constructed by Charles Fabry and Alfred Perot in 1800. This is the Fabry-Perot interferometer is again interferometer, which relies on multiple beam interference which we discussed in the last class. And this type of interferometer was first constructed by Charles Fabry and Alfred Perot in 1800s.

And is of considerable contemporary interest, it has a very wide scope in spectroscopy, in microscopy and we will see here. Now, besides being spectroscopic device of extremely high resolving power, it serves as a basic laser resonant cavity.

Now, in today's or tomorrow's class, we will learn that how to calculate the resolving power of Fabry-Perot interferometer. But, how does a Fabry-Perot interferometer works as a laser resonant cavity, this we will learn while talking about lasers.

Now, the third point in the introduction of Fabry-Perot interferometer is that the device consists of two parallel planes. As we saw that in thin film interferometer, there were one film which is of course bounded by its interfaces. Here Fabry-Perot interferometer, it consists of two planes, and which are made highly reflecting and these planes are separated by a certain distance d.

Now, if the gap d can be mechanically varied by moving one of the mirrors, like suppose we have two mirrors, which are separated by certain distance d and if I fix some micrometer on the back of one of the surface, and then the d can be varied.

And if we are able to vary the separation between the two mirrors, then it is referred to as interferometer. While when the mirrors are held fixed, when these two surfaces if we fix it, and adjusted for parallelism by screwing down on some sort of spacer. Suppose we fill the space between the two mirrors or two surfaces with some fillers or some spacer and therefore, we would not be able to now change the separation between the two mirrors, here the d is fixed now.

In this case, this interferometer is called etalon. But, you can see here that you can use interferometer etalon almost synonymously, the only difference is that when d is fixed and parallelism is removed, it is etalon.

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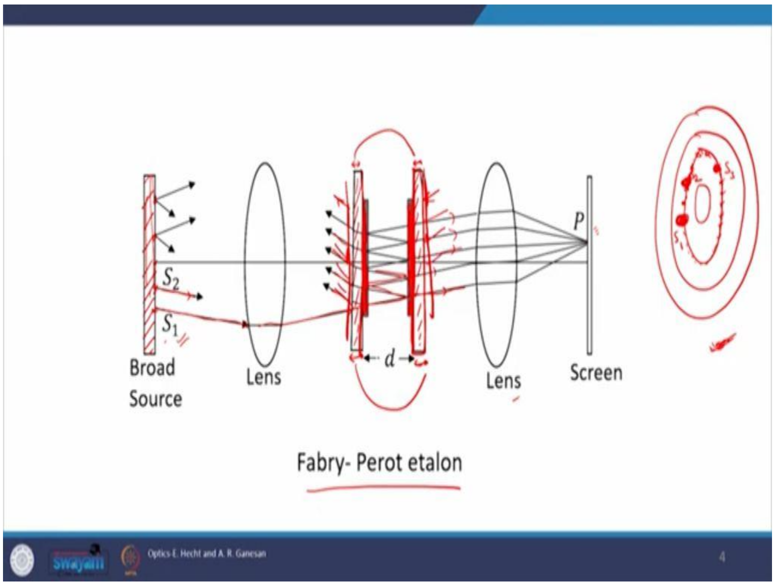

Now, this is the typical schematic diagram of Fabry-Perot etalon. Here you see that we have a glass slide and this is again a second glass slide. And on the inner surface of this glass slide, there is a coating and most of the cases, this is a metallic coating. And what does this metallic coating do? This metallic coating has very high reflection coefficients and therefore its efficiently reflects the light. Where does the light come from? The light comes from a very wide source which is very broad source and the broad sources represented here with this shaded rectangle.

And now you can see is that this is a broad source therefore here again we have multiple number of point sources. Now, say, there are two point sources which are S_1 and S_2 here; the ray from S_1 is start, and then it falls on the Fabry-Perot interferometer; and then it suffers multiple reflections and transmissions. These are the reflections and then within the film rays internally reflected and then here we are getting the transmitted light. And this transmitted light is now collected through a lens, and it falls on the screen where it overlaps and generate some interference pattern.

Now, all the rays either in reflected arm or in transmitted arm; if all these rays are emanating from source S_1. Because, there is a particular ray which is falling on the this interferometer; and then multiple reflection and refractions happen here, multiple reflection and transmissions happen. And out of this multiple reflection and transmission, out of the same source S_1, multiple reflected and transmitted rays are generated. And these rays are perfectly coherent with among each other. Why? Because they are getting generated from the same source.

Similarly, suppose there is another point source S_2, which again emitter ray which falls on this spectrometer; then that reflected and transmitted rays, which owes its origin. In S_2, they would be mutually coherent. But, the rays which are having its origin in S_1 would be incoherent with the rays which have their origins in source S_2; because source S_1 and S_2 are incoherent, they are not correlated. Therefore, the pattern which we see on the screen, this pattern, this would be the concentric ring type which we have already studied for a thin film.

And each circle, be it a dark circle or a bright one; each circle represent a set of rays, which falls on the same angle on the interferometer. And therefore, these fringes are fringes of equal inclination. The coherent rays which are parallel, they will fall on the same angle; and they will fall on a certain point of a same circle. And these circles are made by the rays which are parallel. The different points on this circle are made by rays which comes from some point source on the broad source; but, not necessarily from the same point source say S_1.

This point may have its origin in S_1, this point on this circle may have its origin in S_2. Similarly, this point on this circle may have its origin in S_3. But, still they are forming or they are falling on the same circle. They may have their origin on different points sources, which are completely on incoherent; but they will fall on the same circle, because they are parallel or they are falling on at the same angle on the interferometer. And this is what we detailed in our last class, why incoherent source is giving a sustained interference fringe pattern; and this is what exactly is happening in Fabry-Perot interferometer too.

Now, you see that these glass plates they are slightly wedge; they are not perfectly parallel. Here they are thinner and here they are thicker; here they are thicker and here they are thinner.

And this side you see the glass plates are thin; while on this side they are thicker. The inner surfaces of these glass plates are although parallel; but outer surfaces, these surfaces are not parallel; they are slightly wedge like this. This wedge shape is deliberately introduced into the system just to avoid interference of the reflections, which are happening at these surfaces. We do not want to see the reflections out of this outer surfaces in our field of view; therefore we deliberately make them wedge.

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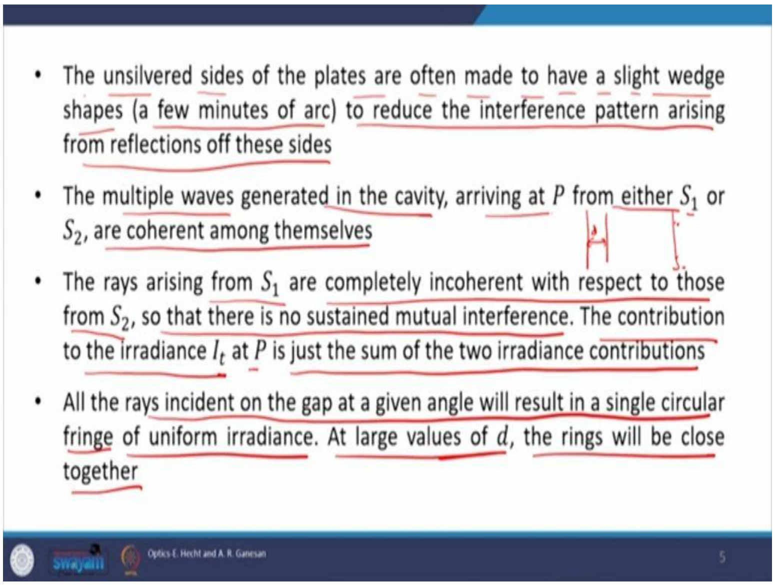

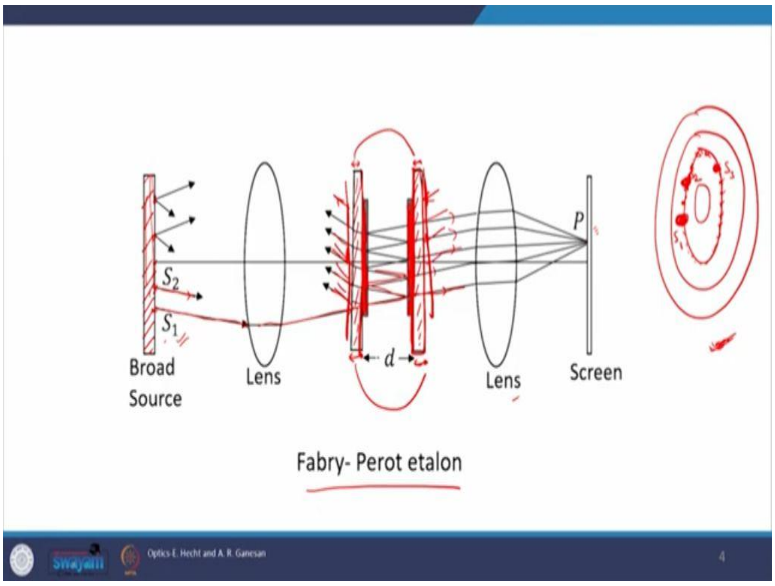

Now, the un-silvered sides of the plates are often made to have a slight wedge shape as we discussed few minutes before. And this wedge is of a few minutes of arc; the wedge angle is very small. And why we do this? We do this to reduce the interference pattern arising from the reflection of these sides. We do not want this extra reflection to come in our field of view.

The multiple waves generated in the cavity; cavity means the space between the two inner surfaces. The multiple waves generated in the cavity arriving at point P which is on the screen from either S_1 or S_2 are coherent among themselves. The light which is coming from S_1, it would be coherent; because the multiple reflections and multiple transmissions are being generated from the same source.

The rays arising from S_1 are completely incoherent with respect to those from S_2 and so that there is no sustained mutual interference. The contribution to the irradiance I_T at P is just the sum of two irradiance contribution; and this is what we talked about here. But, this distribution would be completely incoherent with this distribution, but they are independently sustained; therefore, overall pattern will look sustained.

All the rays incident on the gap at a given angle will result in a single circular fringe, this we know; and this would be a uniform irradiance, because the source is very broad. And the rays will fall from different angles, infinite many rays are there; then roughly the irradiance of the circle would be same, and it would be uniform. At large value of d, the rings will be close together. If you increase the separation between the interfaces, the air gap will increase or the cavity width will increase; and this will ultimately reduce the radius of the fringes, and fringes will sink.

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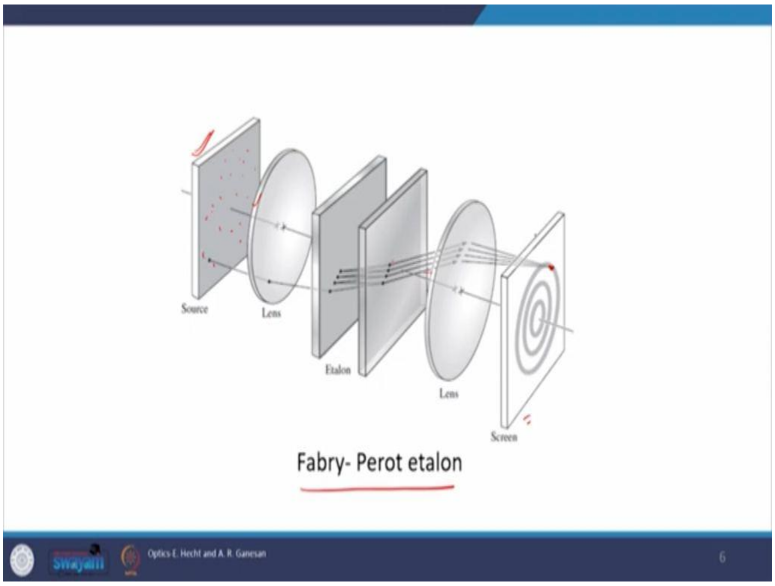

Now, this is again the schematic of Fabry-Perot etalon; this 3d schematic you see source which is wide, very broad. And then this is the lens which is falling, which is directing the light towards the etalon; and here etalon multiple reflections and transmissions are happening. All the transmitted rays are collected through this lens and they fall on certain point on the screen. And if you collect all the pattern from all the points on the source, then you will see that it is forming a concentric ring pattern, as shown here in this figure on this screen.

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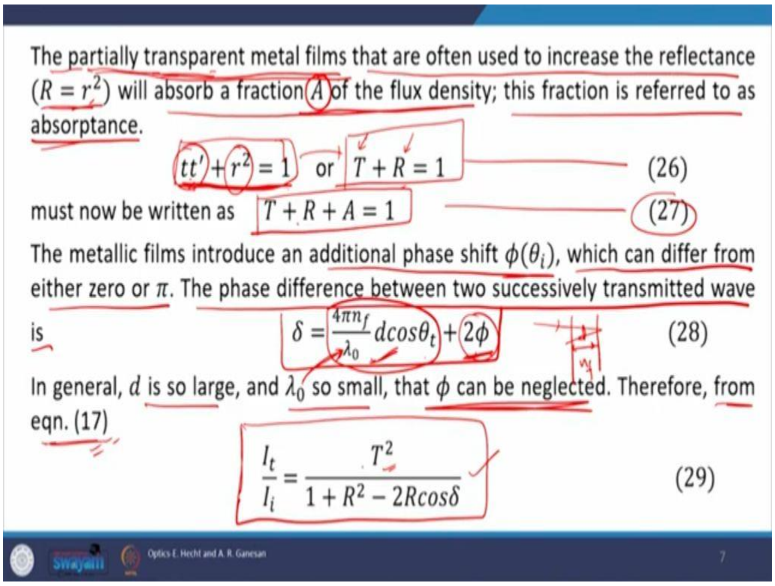

Now, to increase the reflection, as I said before, people usually use metals; but we all know metals are lossy. Therefore, apart from coefficient of reflection and coefficient of transmission, we will also have to take into account coefficient of absorption; because there is a losses, which are there due to the involvement of metal.

Therefore, partially transparent metal film, although the metal is very thin, it still would have losses. But, since the metal is thin therefore, we can say that it is a transparent metal film, partially transparent metal film.

Then, the statement says that the partially transparent metal films that are often used to increase the reflectance; r is the amplitude reflection coefficient and R is intensity reflection coefficient or irradiance reflection coefficient. And they are related through this relation $R=r^2$. And therefore, the metal will absorb a fraction A of the flux density; some part of the intensity will be absorbed into the metal, and this fraction is referred to as absorptance.

Therefore, initially, we were into the habit of seeing this relation $tt' + r^2 = 1$. This is the amplitude reflection coefficient, you can write the same equation in terms of intensity reflection coefficient or transmission coefficient, this T is the transmittance or intensity transmittance coefficient; and this is your intensity reflection coefficient which is of course in capital. And T is related to tt'. $T = tt'$ and R is r^2. Therefore, the same relation can be expressed in this form; T+ R=1.

But, since we are using metal, absorptance also comes into the picture; and therefore, this equation-26 modifies to equation to 27, which is T+R+A=1. And this shows the conservation; the total transmittance plus total reflectance plus total absorptance, they must be equal to unity.

Now, since we are using metallic film, it will introduce some additional phase shift say ϕ; and this phase shift can differ from either 0 or π. We are into the habit of considering additional phase shift of zero or π due to the internal and external reflection. But, due to the incorporation of metal, we can have either 0 or π or some other phase.

The phase difference between the two successive transmitted wave, therefore, in the case where metal is involved is equal to δ, which is given by $(4\pi n_f/\lambda_0)dcos\theta_t + 2\phi$.

This first part we are familiar with; it is k multiplied by $2n_f dcos\theta_t$. And n_f is the refractive index of the material which is between the two edges of the Fabry-Perot interferometer whose thickness is d. And n_f is the refractive index, $cos\theta_t$ is the angle of transmittance; we already know about it. And ϕ is there, it is a twice of ϕ; because the ray is falling and it is passing twice into the metal film. There are two metal films on each side of the internal interfaces; therefore, it is passing through the same thickness of the metal twice in one round trip. Therefore, two ϕ is there in the phase term.

But in general, the metal film thickness is very small, d is very large and λ is very small; λ you just notice, it is coming in the denominator. λ is small and d is large. Therefore, this first term in equation number-28 on RHS is very large as compared to the second term which is 2 ϕ. And therefore, phase term, this ϕ term can be neglected from equation number-28. And therefore, from the, this thin film relation, and we studied in our last class, the equation number-17 modifies to equation number-29.

The relative transmittance irradiance modifies to equation number-29 here. Now, here in the numerator we have T; but we know T is from equation number-27. T=1-R-A; let us substitute further.

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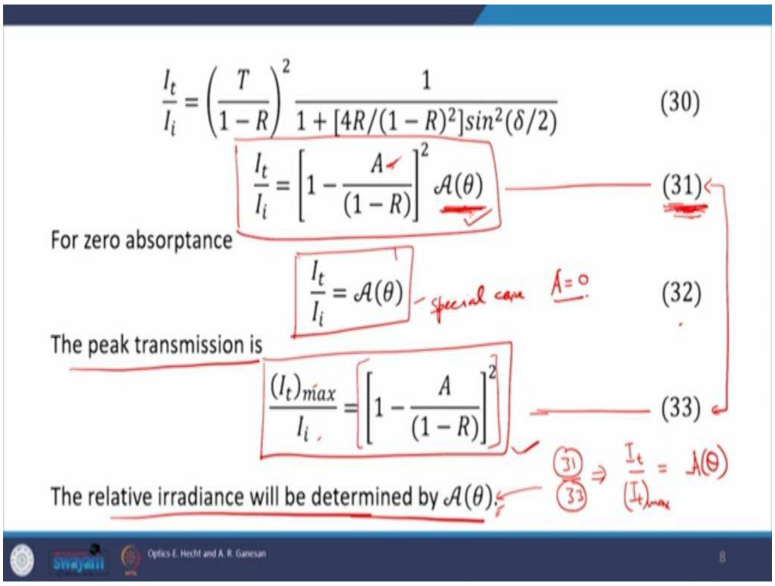

Then, if you do the substitution, then ultimately, we will have this relation for the relative transmitted irradiance; and which is given by equation-31. And this relation we have introduced again this Airy function, the typical function which we studied in our last class again.

Now, here in equation-31, we see that there is absorptance which is appearing here in this right hand side. If we neglect absorptance, then 31 reduces to 32 and which is the typical expression. The same expression which we studied in our last class, $I_t/I_i = A(\theta)$. This is a special case; it is a special case where we neglect absorptance. But this is not usually the case; and therefore we will continue with equation number-31.

Now, the peak transmittance is given by $(I_t)_{max}/I_i$; and what would be the peak transmittance? 31 maximizes, 31 peaks when $A(\theta)$ maximizes; this airy function maximizes. And what is the maximum value of $A(\theta)$? It is 1. Therefore, the maximum relative transmitted irradiance is equal to $(1\text{-}A/(1\text{-}R))^2$. This is given by equation number-33.

But, if you want to calculate the relative irradiance, then it will still be decided by $A(\theta)$, the Airy function. How? If you want to calculate the relative irradiance, then multiply the usual irradiance or divide the usual irradiance with equation number 33. Normalize all the irradiances with $(I_t)_{max}$; and if you do this, this factor will go away from the expression, and you will get $A(\theta)$. Now, if we want to do, then use the equation number 31 and 33.

If you want to write the relative irradiance, then perform this. Then, from here what you get is that $I_t/(I_t)_{max}$, which is nothing but relative transmitted irradiance. This would be equal to the $A(\theta)$; which is nothing but Airy function, which is written here in this statement. The relative irradiance will be determined by $A(\theta)$, the Airy function.

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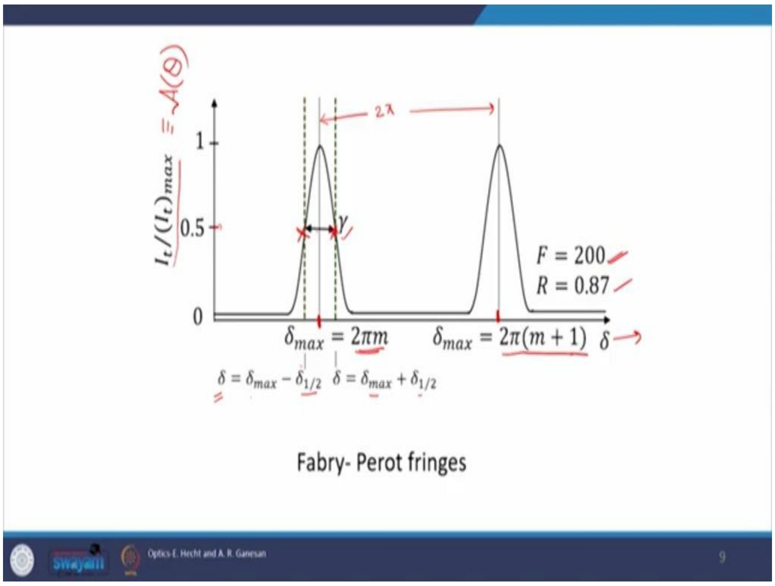

Now, let us look on the fringe pattern which we see in Fabry-Perot interferometer. Now, to see the fringes, we have to plot $I_t/(I_t)_{max}$, which is nothing but your Airy function. If you plot this Airy function with δ the phase, the things which is very much clear from our previous classes is that maxima will appear when phases integral multiple of 2π. Therefore, phase maximizes when δ =2mπ. The next peak will appear when $\delta = (2m + 1)\pi$. This is what is written here, $2m\pi$, 2π in bracket m+1 the next integer; this is how the maxima repeats itself.

And you can see that the two consecutive maxima are separated by 2 π, and all these peaks maxima peaks has certain width. And the width is represented by γ, and it is measured at value when the Airy function reduces to half; it is half width. How to calculate the half width? $\delta_{1/2}$ is the half width.

If you move from the center by $\delta_{1/2}$; then you get this edge. And if you move on the right-hand side from the center by $\delta_{1/2}$, then you get this edge. The difference between these two edges will give you the width; this would be equal to $2\delta_{1/2}$. This plot is plotted for F is equal to 200 and R is equal to 0.87. You see that F which is a coefficient of finesse is very large as compared to 1.

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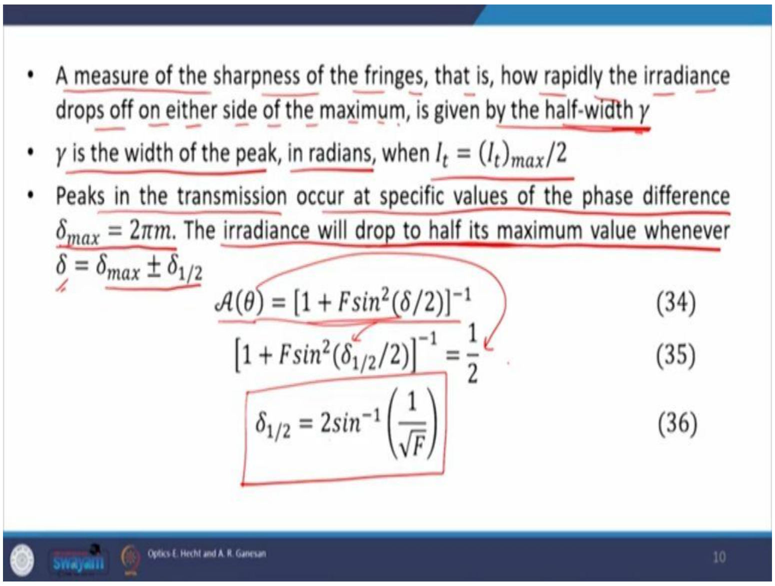

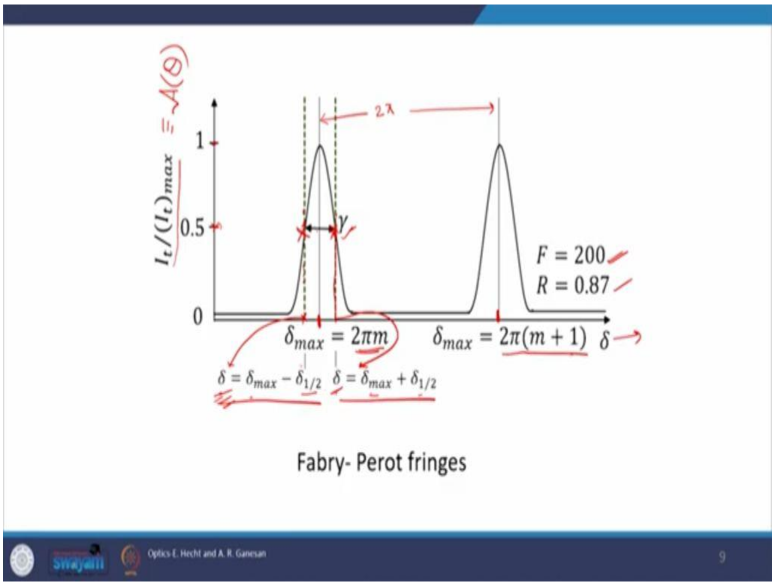

Now, you see that the fringes would be sharp if the width is smaller; and this is what is written here. A measure of the sharpness of the fringes, that is how rapidly the irradiance drops off on either side of the maximum is given by half width γ.

If the γ is small, the irradiance falls off quickly; and the width of the peak would be smaller. If γ is large, the irradiance falls off slowly and the γ would be larger. γ is the width of the peak in radian. When $I_t = (I_t)_{max}/2$,, when irradiance goes below by half then only we measure γ and there only we define γ. And this is also clear from this figure. The γ is defined where the relative irradiance becomes half its maximum value.

Now, the peak in the transmission occur at specific values of the phase difference δ. And where does this occur? Where this occurs? Where phase difference is integral multiple of 2π; that is equal to $2\pi m$, where m is an integer. So, δ_{max} when it is equal to $2\pi m$ and irradiance will drop to half of its maximum value, whenever $\delta = \delta_{max} \pm \delta_{1/2}$; and this is what is shown here. These are the edges, which defines the γ.

And here the value of $\delta = \delta_{max} - \delta_{1/2}$; and here the value of $\delta = \delta_{max} + \delta_{1/2}$. Now, if you know the value of phase difference, calculate the value of relative irradiance there; and you will see that it is half of the maxima. Now, the difference between these two phases will give you the width, the width of the peak.

Now, we know that on the vertical axis we are plotting this Airy function, $A(\theta)$. At half maximum, $A(\theta) = 1/2$; because the maximum value of a $A(\theta) = 1$. Therefore, at half maximum, the value of $A(\theta)$ would be half, therefore, this relation would be intact and here we can substitute 1/2. And from this relation, we can calculate the expression for $\delta_{1/2}$, which is the half width. And $\delta_{1/2} = 2sin^{-1}(1/\sqrt{F})$.

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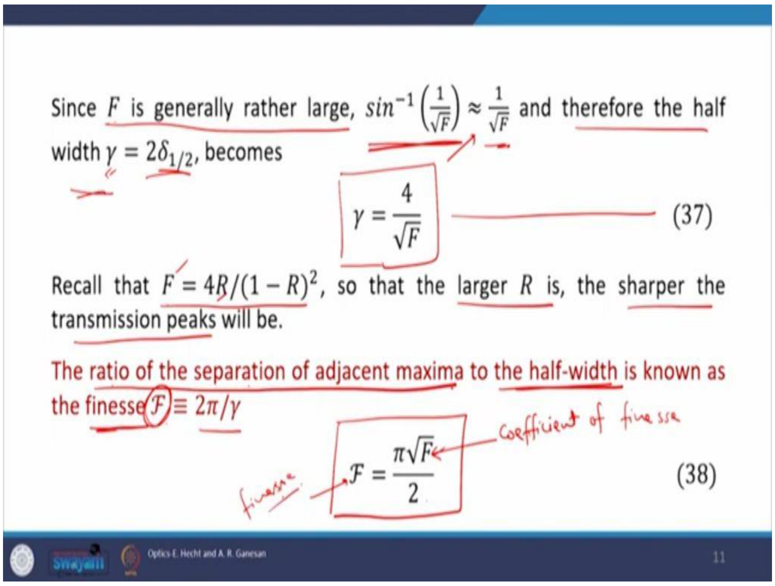

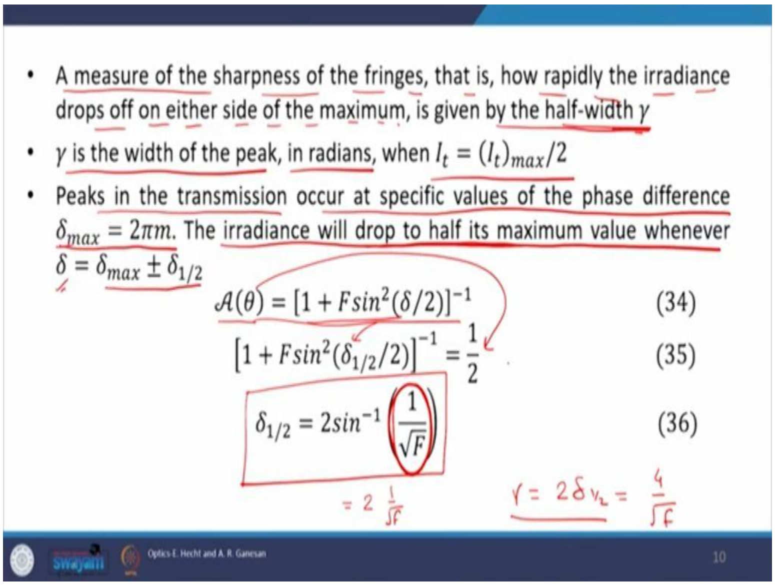

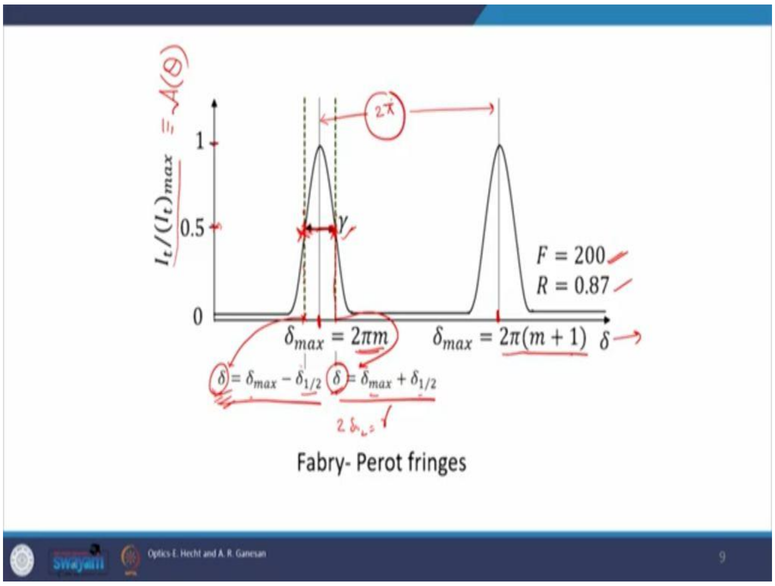

But, we also know that F is generally very large. When these resonances, these peaks are very sharp, then usually F is very high. And if F is very large, then this term would be very small. And if this is very small, then we can substitute $sin^{-1}(1/\sqrt{F}) = 1/\sqrt{F}$.

And therefore, the expression for half width would be modified; the half width as is given by twice of $\delta_{1/2}$. It will after this substitution would be equal to $4\sqrt{F}$. I repeat, here you see the $\delta_{1/2}$; but if F is very large, then you can replace it with $1/\sqrt{F}$.

And we know $\gamma = 2\delta_{1/2}$; because in this figure you see gamma is the width, F at half irradiance. And what is this width? This width is this phase minus this phase. And if you subtract these two then you will get $2\delta_{1/2}$, which would be equal to your γ. And therefore γ is $2\delta_{1/2}$, del half we know; and therefore $\gamma = 4\sqrt{F}$, which is given by equation number-37. The half width gamma is equal to $4\sqrt{F}$.

And we know F is coefficient of finesse which is given by $4R/(1-R)^2$; and $R= r^2$. Now, larger the R is, sharper will be the transmission peaks; larger the R is, sharper would be the transmission peak. Alternatively, larger the coefficient of finesse is sharper would be the peaks.

Now, there is another very important parameter which we must know; and this is defined by the ratio of separation of adjacent maxima to the half width. What is the separation between the adjacent maxima? The separation between the adjacent maxima is 2π. And what is half width? Half width is γ.

Then, we define another parameter which is known as finesse; not coefficient of finesse. Coefficient of finesse is F, and finesse is this italic *F*; notice the difference. The finesse is defined by ratio of separation of adjacent maxima to the half width; and therefore, finesse is would be equal to $2\pi/\gamma$. If you substitute for γ , then you get this expression where finesse would be equal to $\pi\sqrt{F}/2$. This is coefficient of finesse and this is only finesse, do not get confused.

This is all for today. In the next class, we will talk more about Fabry-Perot interferometer; there we will see how the resolving power is measured. What exactly is the resolving power? How it is useful for the usual optical system or spectroscopy? How can it be used as a spectroscopic tool? This all would be covered in the next class, for now it is all. Thank you for listening me.